$u^* = \sqrt{uv}$

Pascal Michaillat, Emmanuel Saez

December 2023

HOW TO INTERPRET LEGAL CONCEPT OF FULL EMPLOYMENT?

- Employment Act of 1946
  - "policy and responsibility of the federal government…to promote maximum employment, production"
- Federal Reserve Reform Act of 1977
  - responsibility of the Federal Reserve "to promote effectively the goals of maximum employment, stable prices"
- Full Employment and Balanced Growth Act of 1978
  - "responsibility of the federal government…to foster and promote…full employment and production"

EXISTING INTERPRETATIONS OF FULL EMPLOYMENT

- Boston Fed's Rosengren (2014): u^* = CBO's NRU
  - but a slow-moving average is not socially desirable
- Joint Economic Committee (2019); Fed's Powell (2022): u^* = NAIRU
  - "full employment is…synonymous with the non-accelerating inflationary rate of unemployment (NAIRU)—the rate of unemployment that neither stokes nor slows inflation"
  - "maximum employment in the sense of the highest level of employment that is consistent with price stability"
  - but inconsistent with dual mandate: subsumes employment mandate into price mandate

THIS PAPER: FULL EMPLOYMENT = EFFICIENT UNEMPLOYMENT

- maximizes productive use of labor
  - consistent with standard economic theory (Hosios 1990)
  - consistent with spirit of law ("promote maximum production")
- given voluntary labor-force participation
  - consistent with standard economic interpretation (Rees 1957)
  - consistent with spirit of law ("promote employment opportunities for those able, willing, and seeking to work")
- formula for u^* is easily applicable
  - simplification of Michaillat-Saez (2021) formula for US economy
  - can be applied to historical data
  - can be applied in real time

THEORY OF FULL EMPLOYMENT

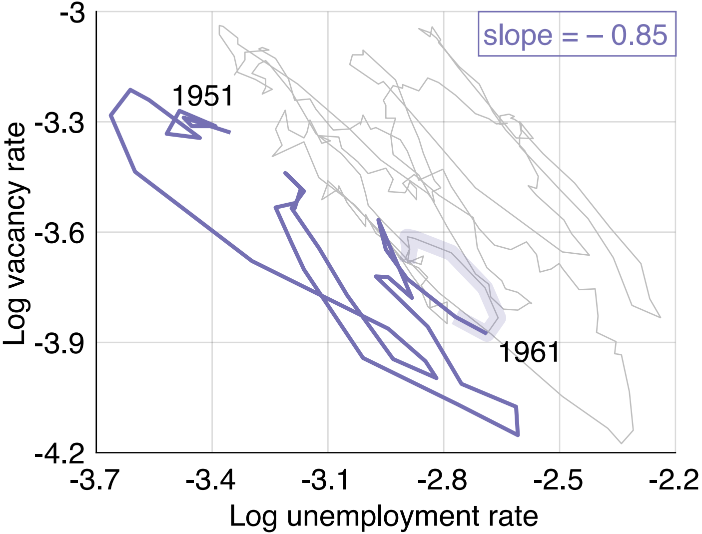
COMPOSITION OF LABOR FORCE

1. share u of labor force is unemployed
   - no home production (Borgschulte, Martorell 2018)
2. share v of labor force is employed and recruiting
   - one worker per vacancy (National Employer Survey 1997)
3. share $1 - (u + v)$ of labor force is employed and producing

‣ labor force participation rate

‣ marginal attachment rate

US BEVERIDGE CURVE ≈ HYPERBOLA



‣ Time series on log scale

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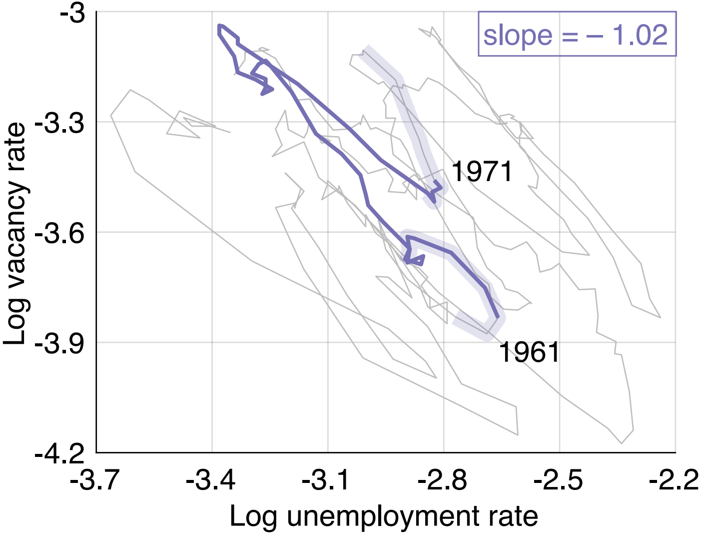

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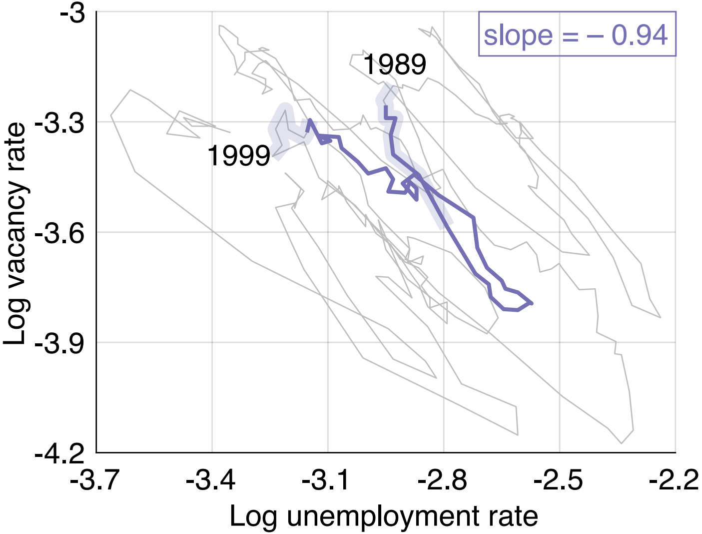

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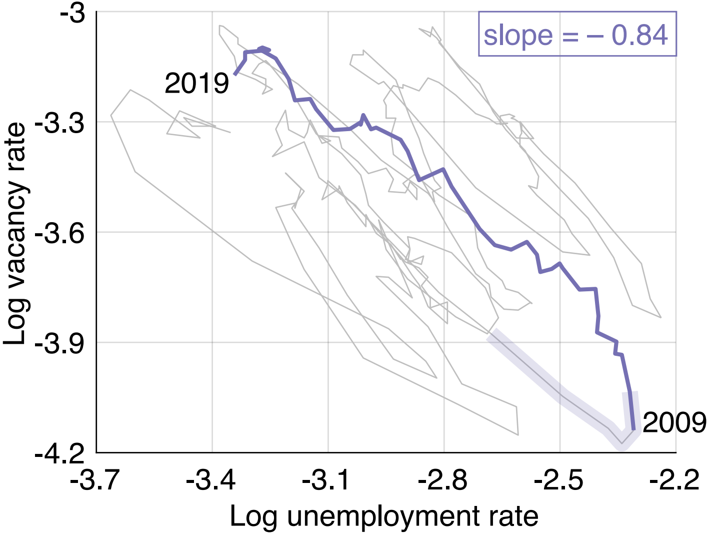


‣ Time series on log scale

COMPUTING FULL-EMPLOYMENT ALLOCATION

- minimize nonproductive use of labor $u + v$
- subject to hyperbolic Beveridge curve $uv = A$
- unconstrained minimization with convex objective: $u + A/u$
- first-order condition gives solution:

$$\frac{d[u + A/u]}{du} = 0 \;\Rightarrow\; 1 - A/u^2 = 0 \;\Rightarrow\; u = \sqrt{A}$$

- solution is full-employment, efficient unemployment rate:

$$u^* = \sqrt{uv}$$

CRITERION FOR FULL EMPLOYMENT, EFFICIENCY

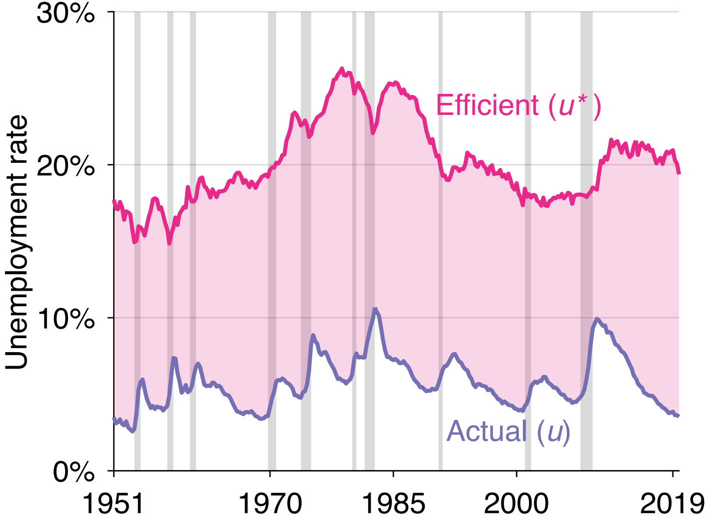
- recall: $u^* = \sqrt{uv}$ is geometric average of u and v
- economy is at full employment, efficient when $u = u^*$
  - $\rightsquigarrow$ efficient when $u = v$
- economy is above full employment, inefficiently tight when $u < u^*$
  - $\rightsquigarrow$ inefficiently tight when $u < v$
- economy is below full employment, inefficiently slack when $u > u^*$
  - $\rightsquigarrow$ inefficiently slack when $u > v$

MORE GENERAL FORMULA (MICHAILLAT, SAEZ 2021)

- home production per unemployed worker: $0 \rightarrow \zeta$
- recruiters per vacancy: $1 \rightarrow \kappa$
- elasticity of Beveridge curve: $v = A/u \rightarrow v = A/u^{\epsilon}$
- efficient unemployment rate:

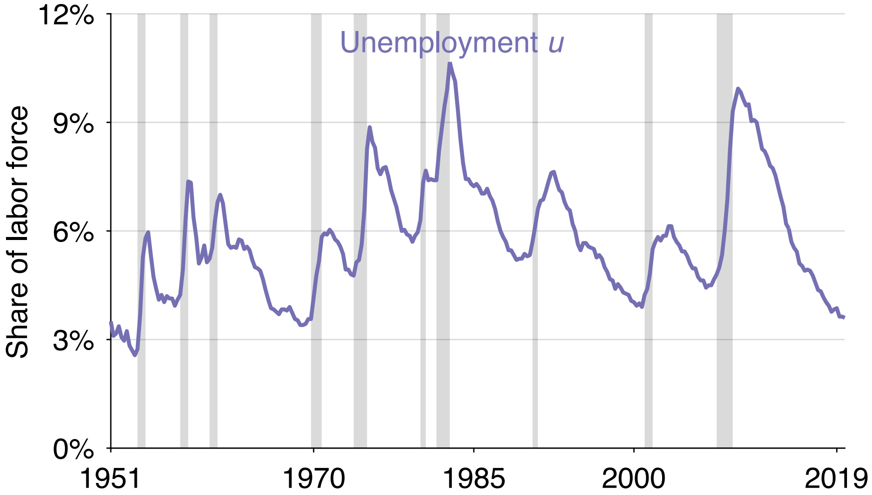
$$u^{*} = \sqrt{uv} \quad \rightarrow \quad u^{*} = \left(\frac{\kappa \cdot \epsilon}{1 - \zeta} \cdot v \cdot u^{\epsilon}\right)^{1/(1+\epsilon)}$$

u^* WITH $\zeta = 0.96$ (HAGEDORN, MANOVSKII 2008)

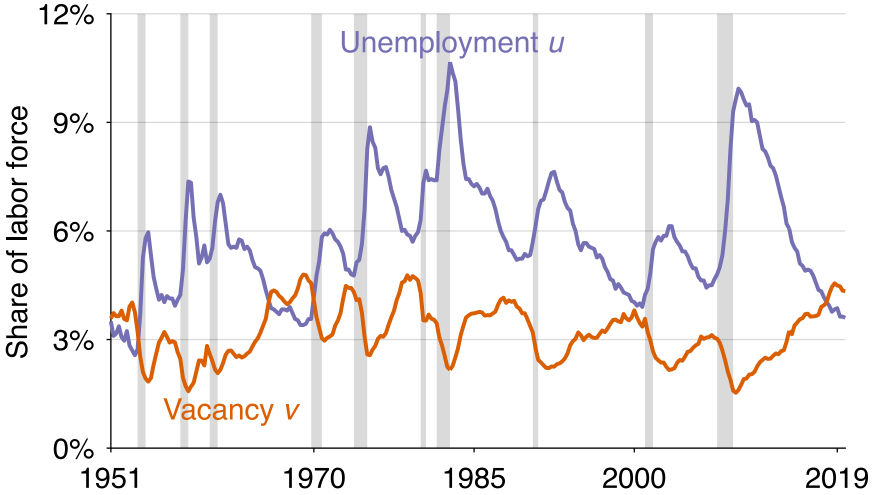


POSTWAR IN THE UNITED STATES

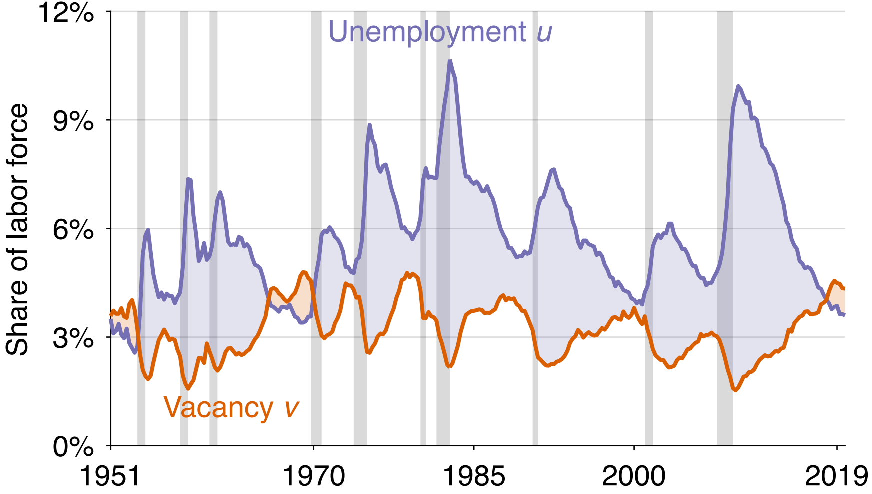
UNEMPLOYMENT RATE (CPS)



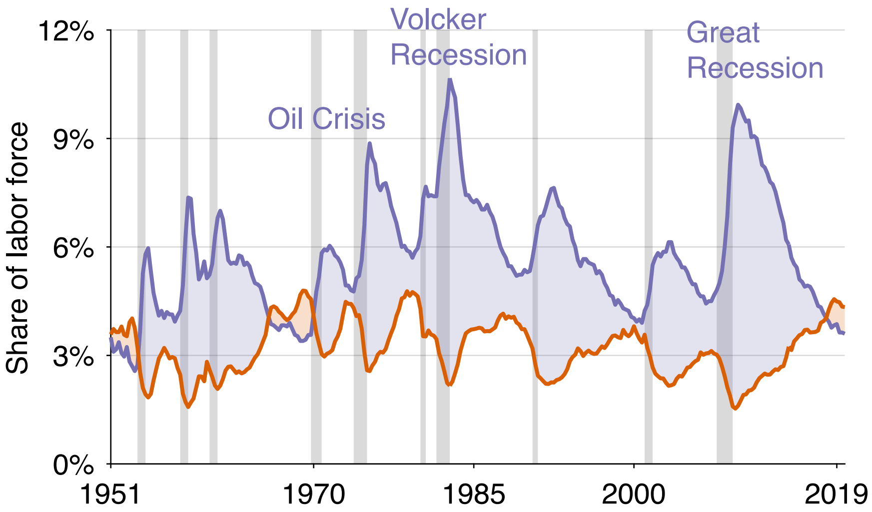
VACANCY RATE (BARNICHON 2010, JOLTS)



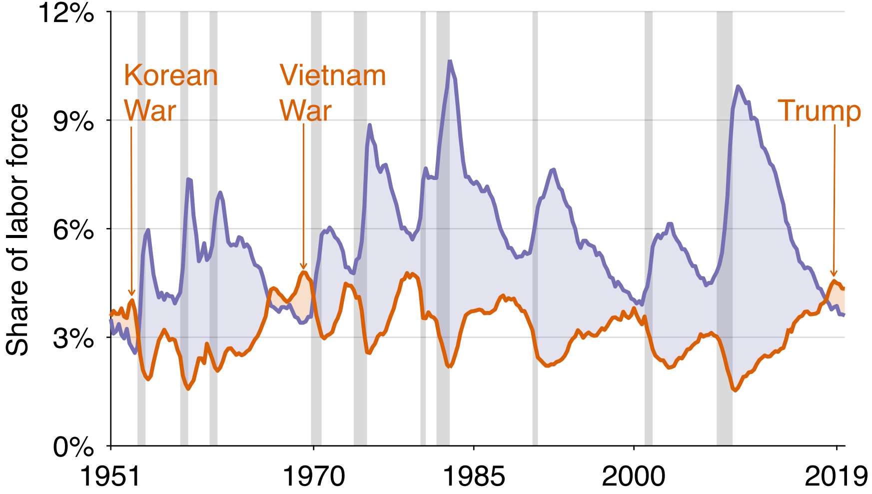
LABOR MARKET IS GENERALLY TOO SLACK...



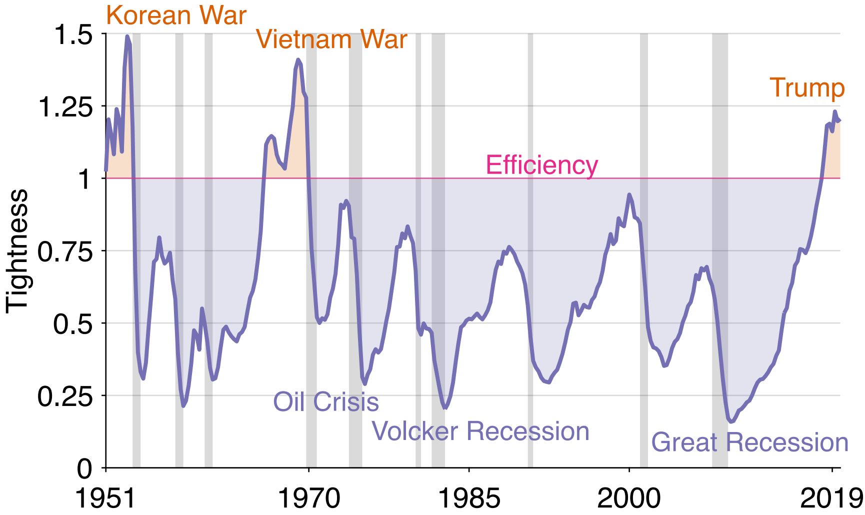
...AND IS ESPECIALLY SLACK IN SLUMPS



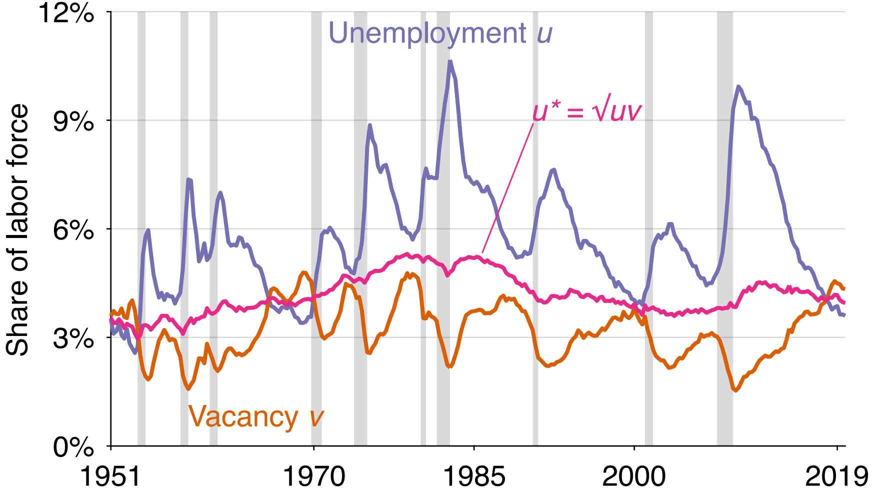
LABOR MARKET IS TOO TIGHT DURING WARS



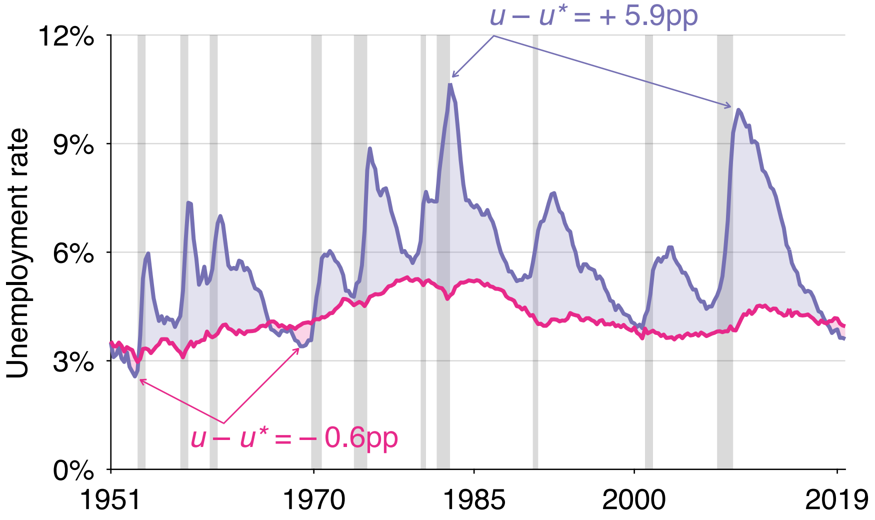
TIGHTNESS v/u SUMMARIZES STATE OF LABOR MARKET



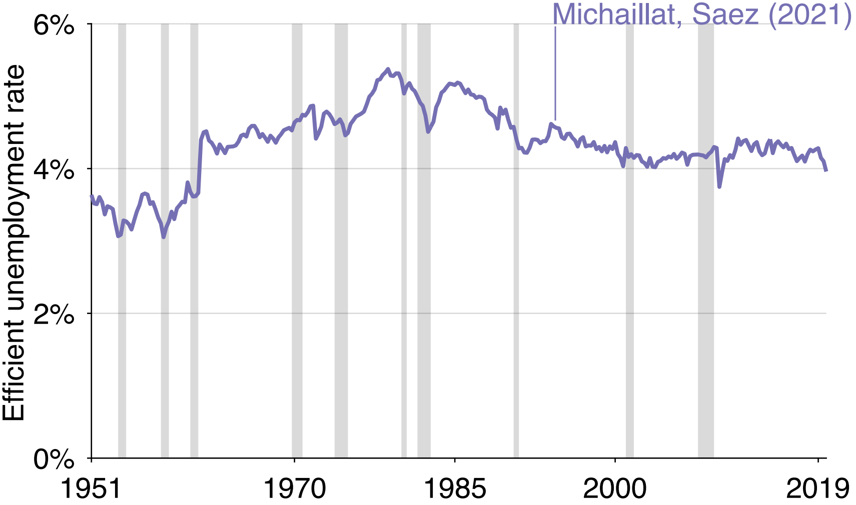
u^* REMAINS IN 3.0%–5.3%, AVERAGES 4.2%



UNEMPLOYMENT GAP IS COUNTERCYCLICAL



COMPARISON WITH MICHAILLAT, SAEZ (2021)



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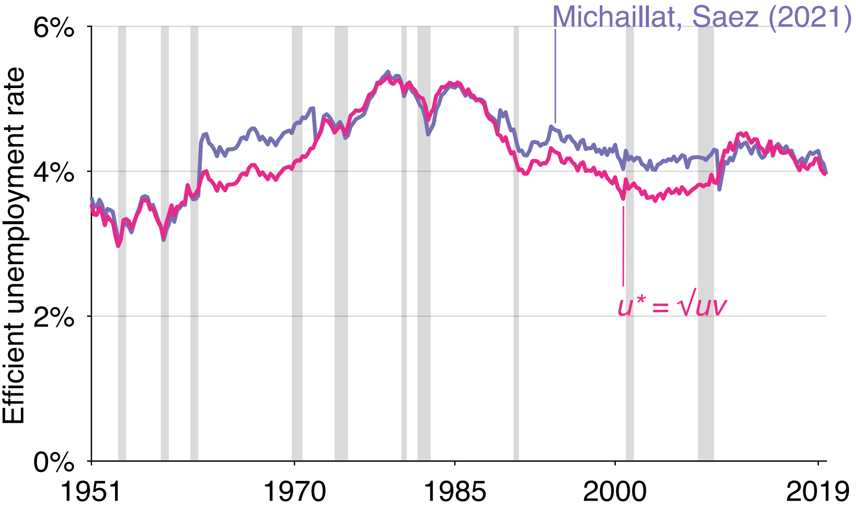
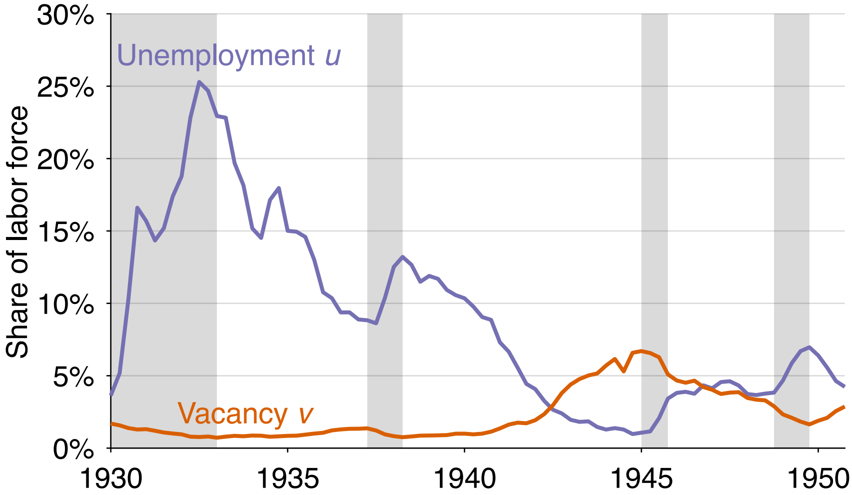


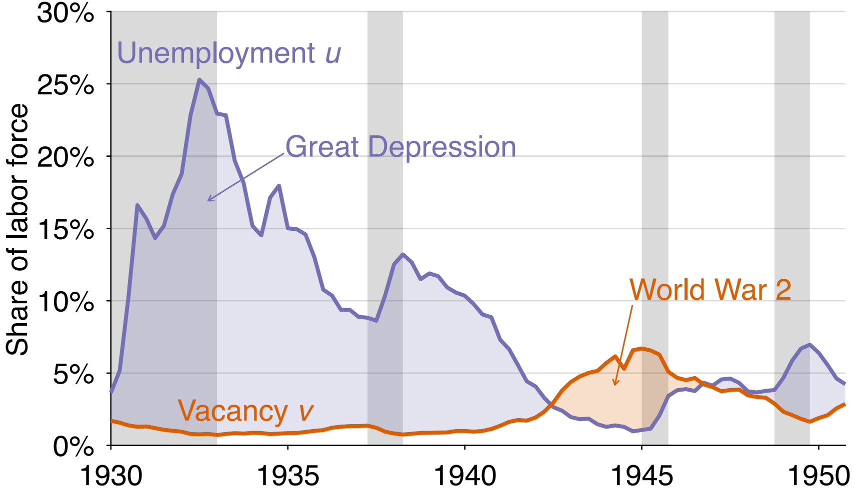
GREAT DEPRESSION IN THE UNITED STATES

NBER DATA (PETROSKY-NADEAU, ZHANG 2021)

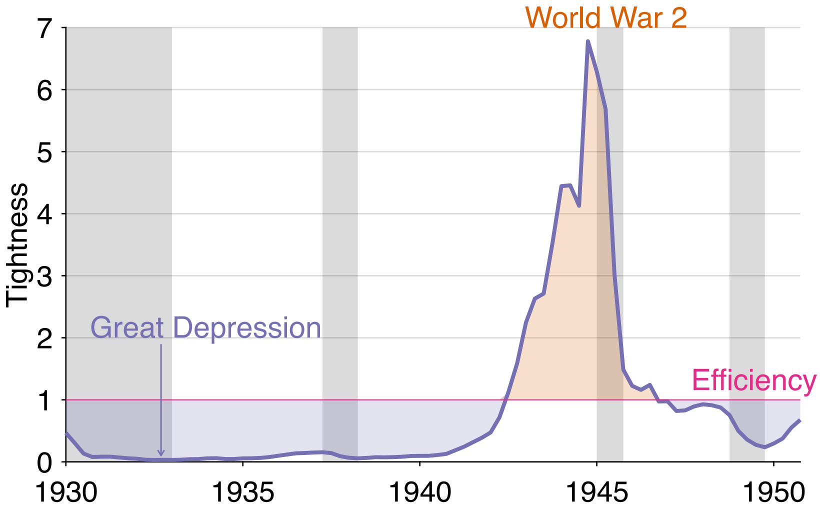


BEVERIDGE CURVE ≈ HYPERBOLA

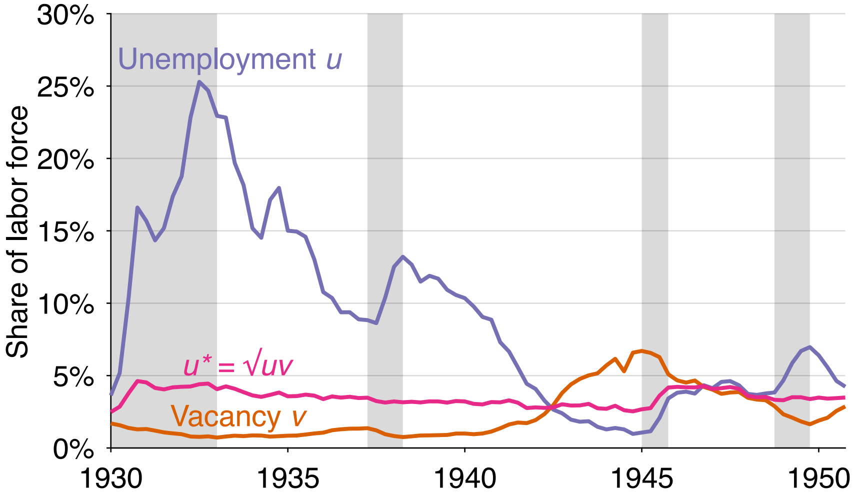
LABOR MARKET WAS TOO SLACK UNTIL WW2



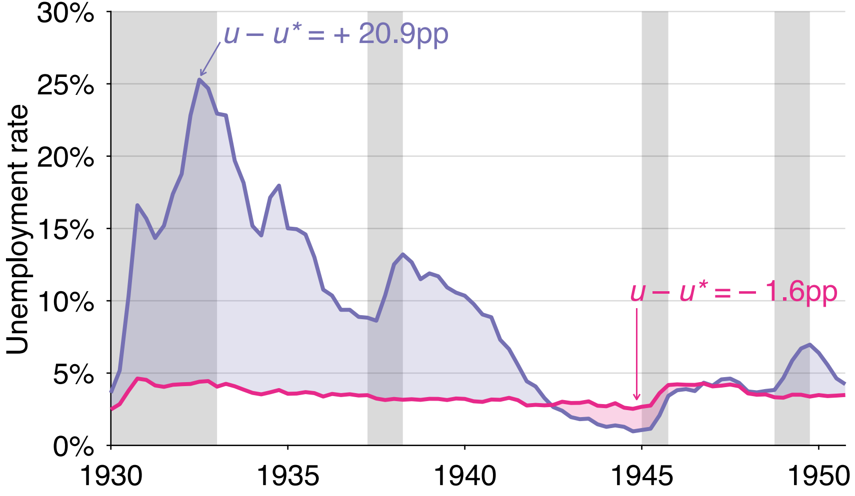
LOWEST AND HIGHEST TIGHTNESS ON RECORD



u^* REMAINS IN 2.5%–4.6%, AVERAGES 3.5%

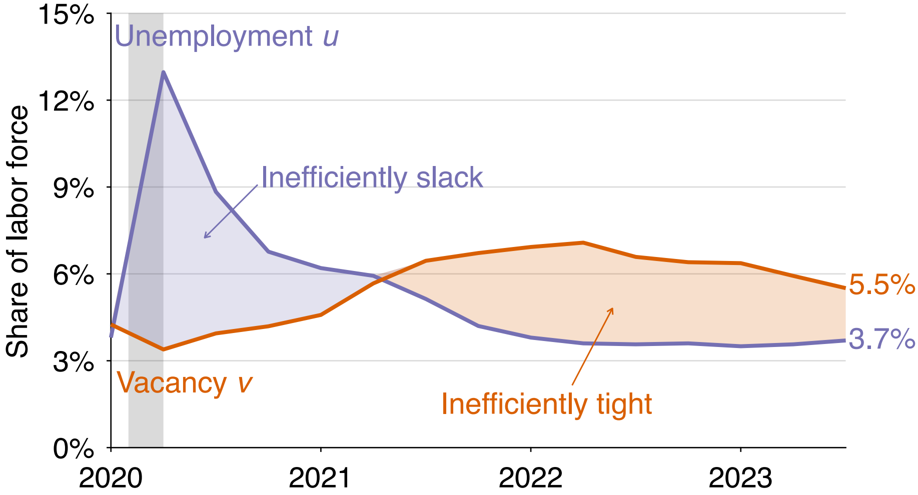


MOST EXTREME UNEMPLOYMENT GAPS ON RECORD

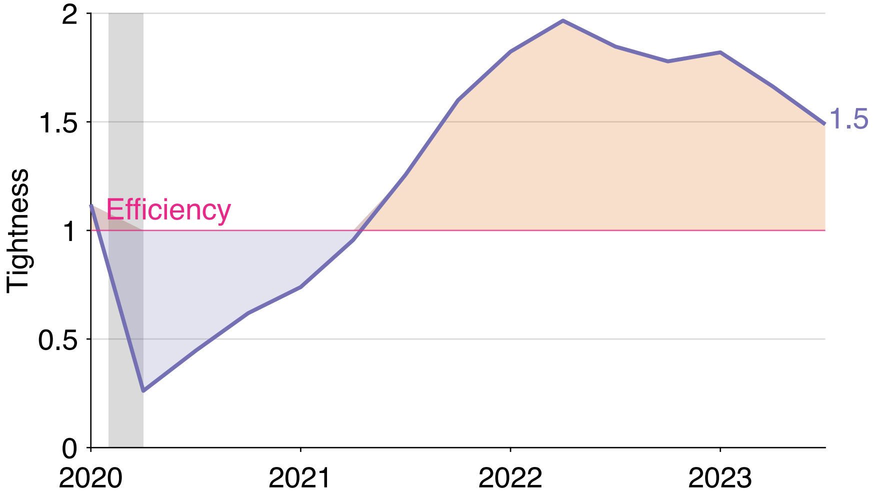


PANDEMIC IN THE UNITED STATES

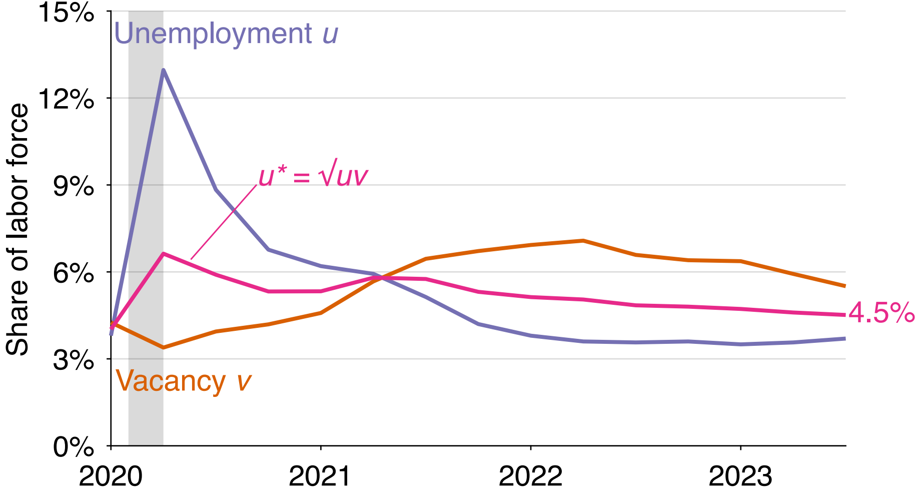
LABOR MARKET HAS BEEN TOO TIGHT SINCE 2021Q3...



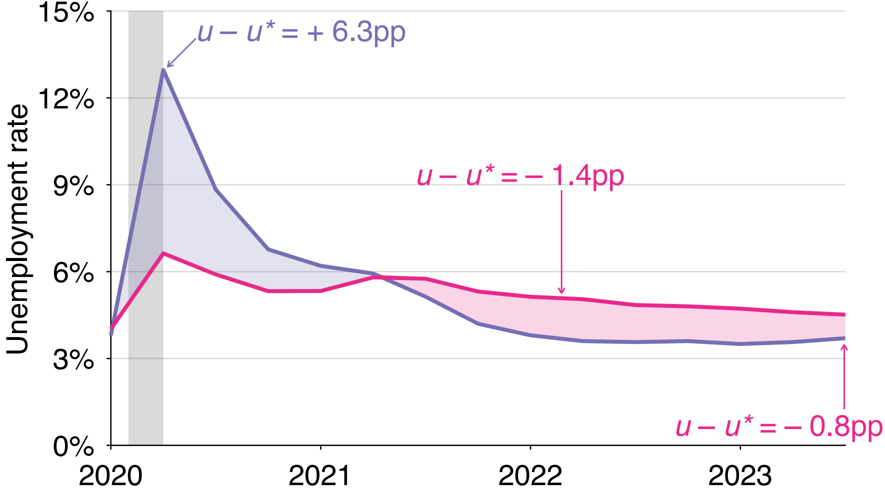
...BUT IT HAS BEEN COOLING SINCE 2022Q2



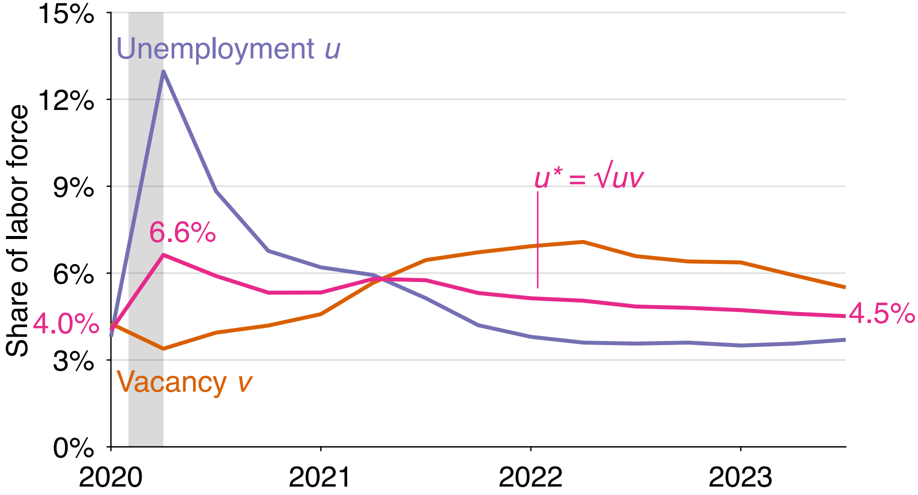
CURRENT TARGET FOR MONETARY POLICY: $u^* = 4.5\%$



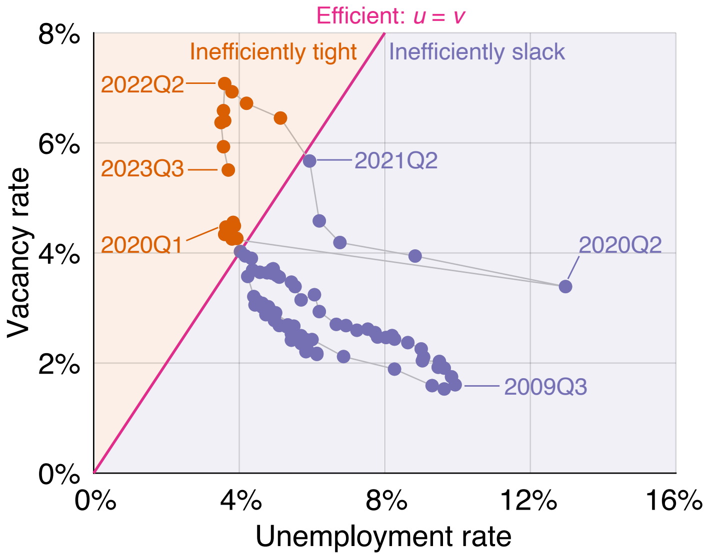
MOST EXTREME UNEMPLOYMENT GAPS SINCE WW2



WHY DID u^* INCREASE SO MUCH IN 2020?

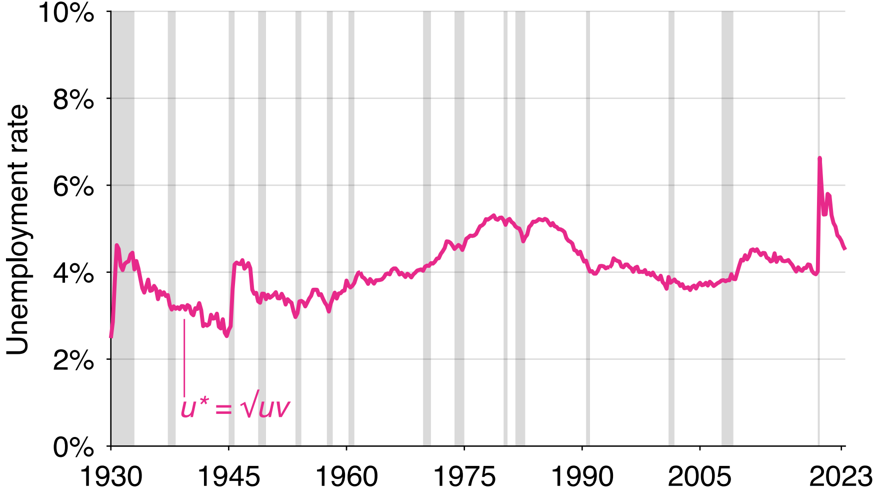


BECAUSE OF LARGE SHIFT OF BEVERIDGE CURVE IN 2020Q2

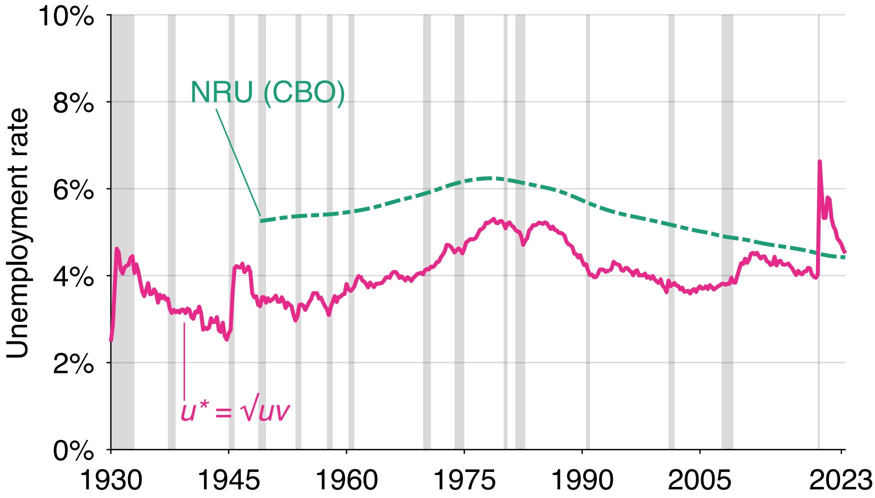


SUMMARY

$u^* = \sqrt{uv}$ AVERAGES 4.1% OVER 1930–2023



$u^* = \sqrt{uv}$ IS LOWER THAN EXISTING TARGETS



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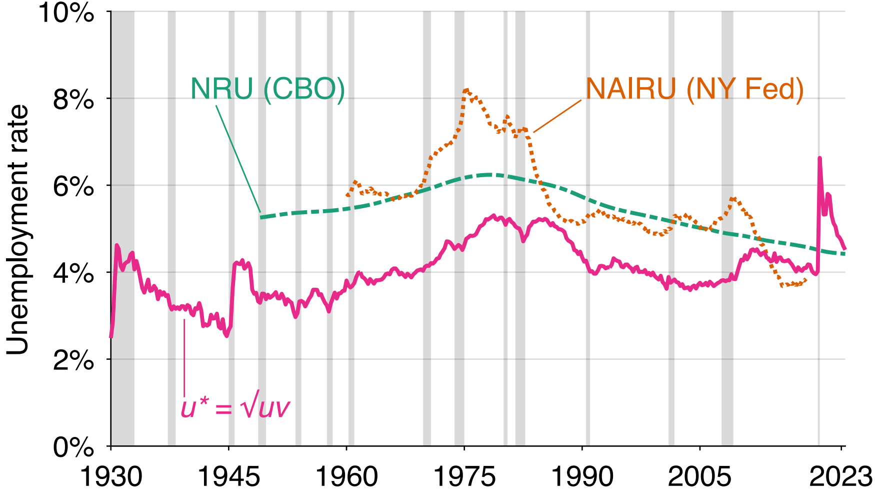
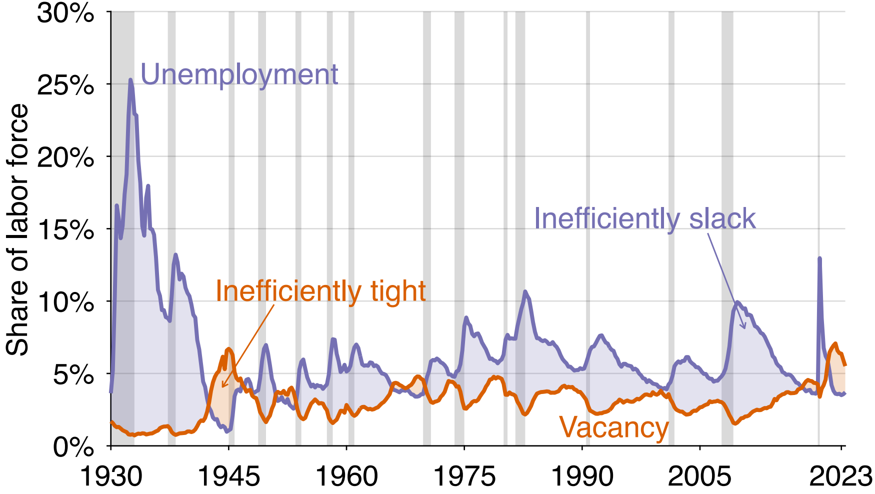
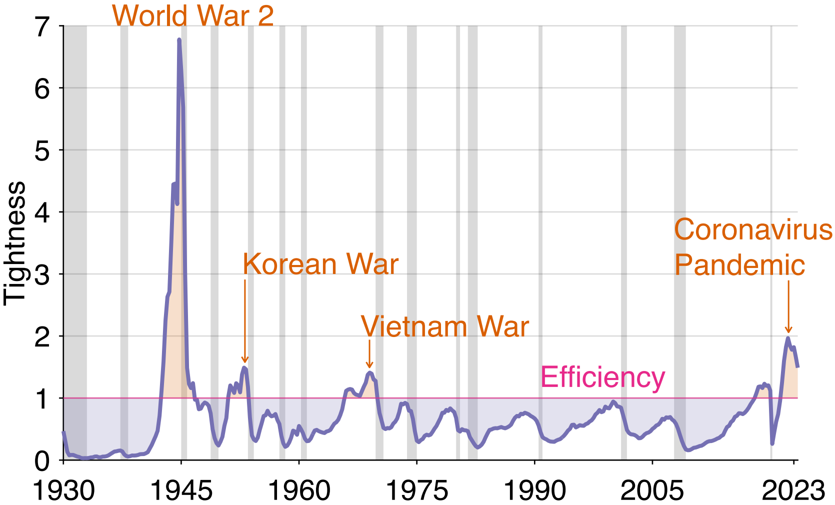


EFFICIENCY CRITERION FOR US LABOR MARKET

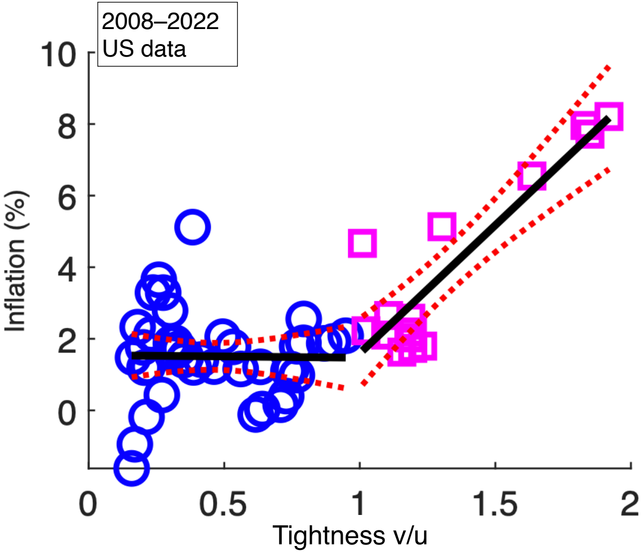


AN EQUIVALENT EFFICIENCY CRITERION

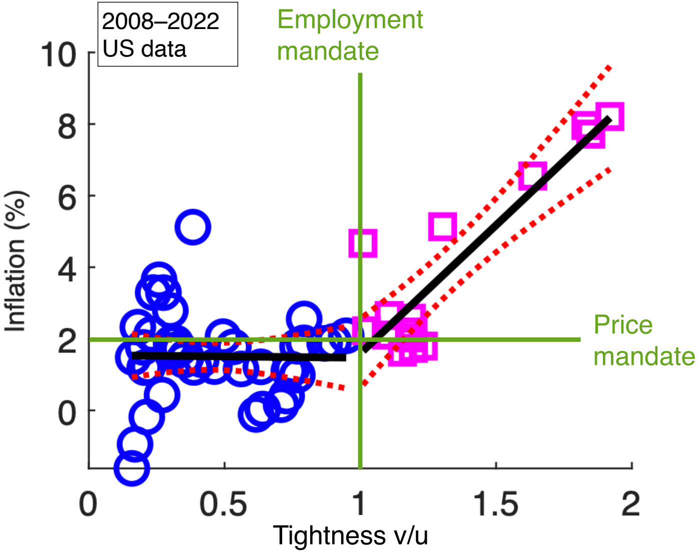


WHAT ABOUT THE PRICE MANDATE?

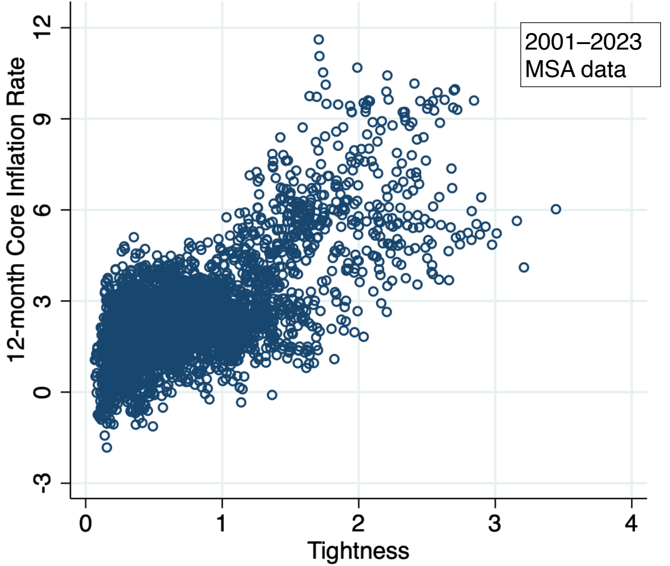
BENIGNO, EGGERTSSON (2023): DIVINE COINCIDENCE?



BENIGNO, EGGERTSSON (2023): DIVINE COINCIDENCE?



GITTI (2023): DIVINE COINCIDENCE?



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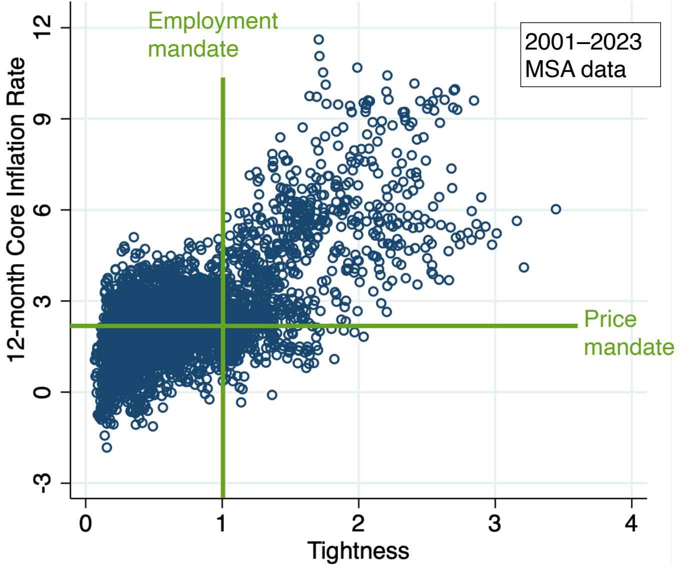
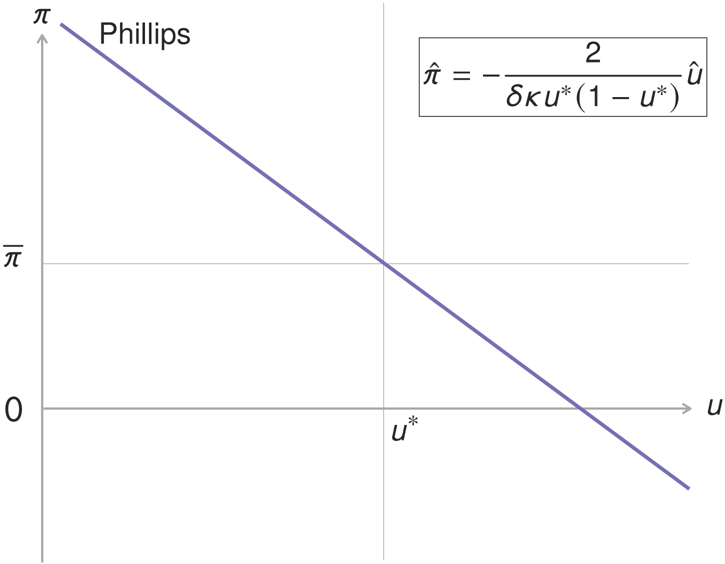


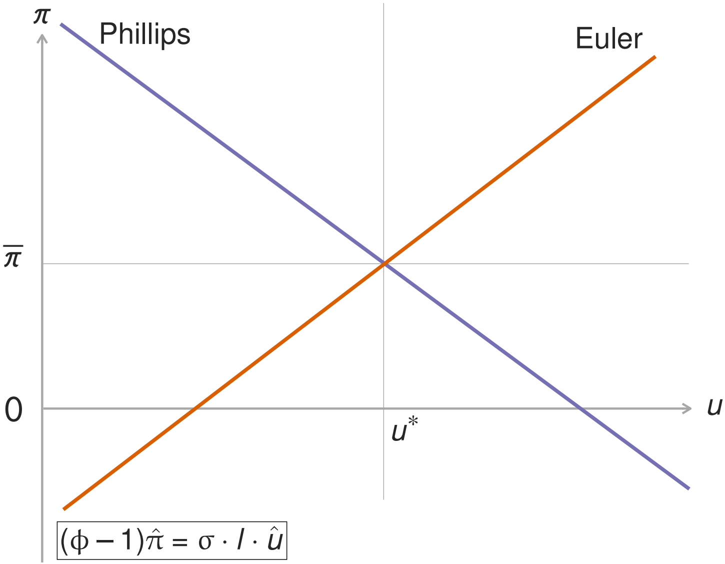
AN EARTHLY MODEL OF DIVINE COINCIDENCE

- economical business-cycle model structure (Michaillat, Saez 2022)
  - identical households sell and buy chauffeur services
  - drivers find customers through matching $\Rightarrow$ unemployment
  - utility from being chauffeured and wealth $\Rightarrow$ AD curve
- price competition through directed search (Moen 1997)
  - chauffeurs with higher prices are hired more slowly
  - chauffeurs with lower prices are hired more quickly
- price rigidity from quadratic price-adjustment costs (Rotemberg 1982)
- divine coincidence appears: $\pi = \bar{\pi} \iff u = u^*$

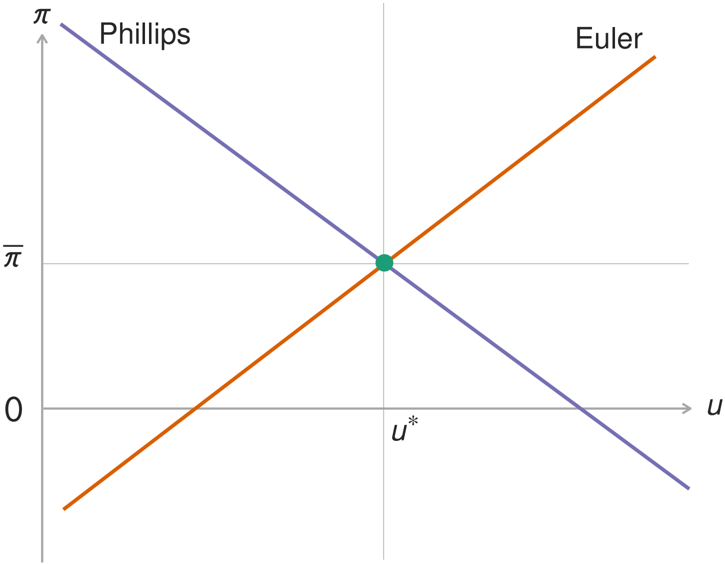
DIVINE COINCIDENCE IN THE EARTHLY MODEL



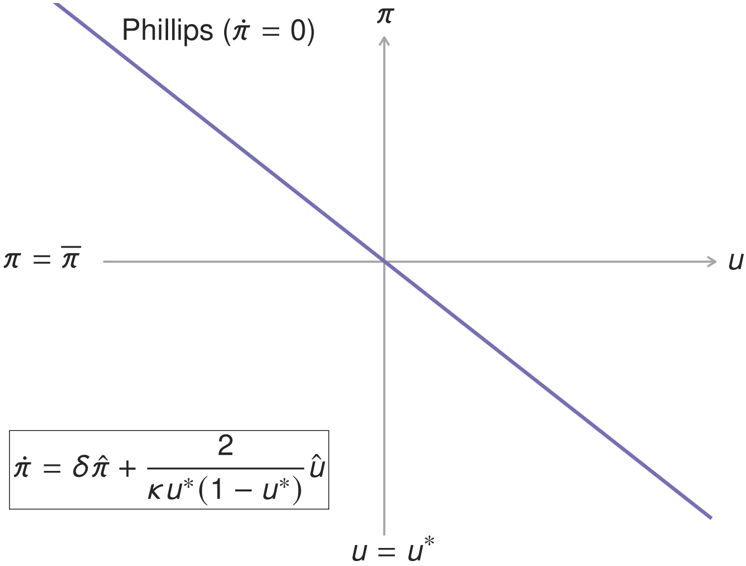
DIVINE COINCIDENCE IN THE EARTHLY MODEL



DIVINE COINCIDENCE IN THE EARTHLY MODEL

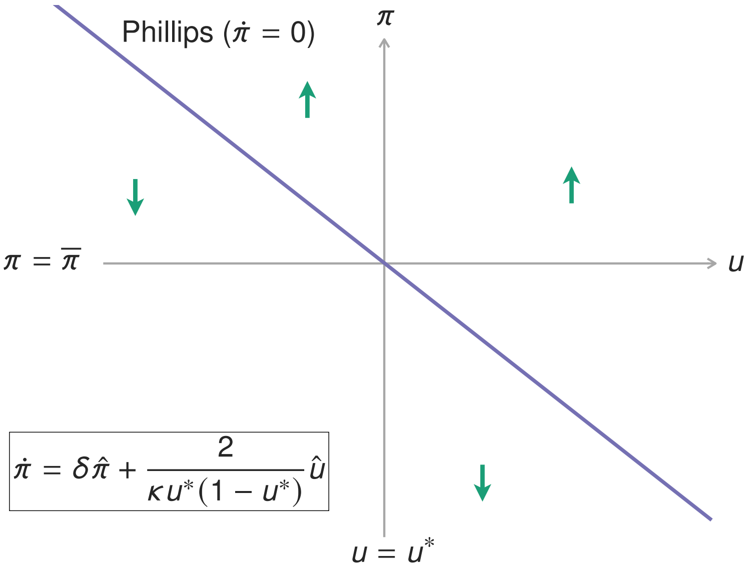


PHASE DIAGRAM OF THE EARTHLY MODEL

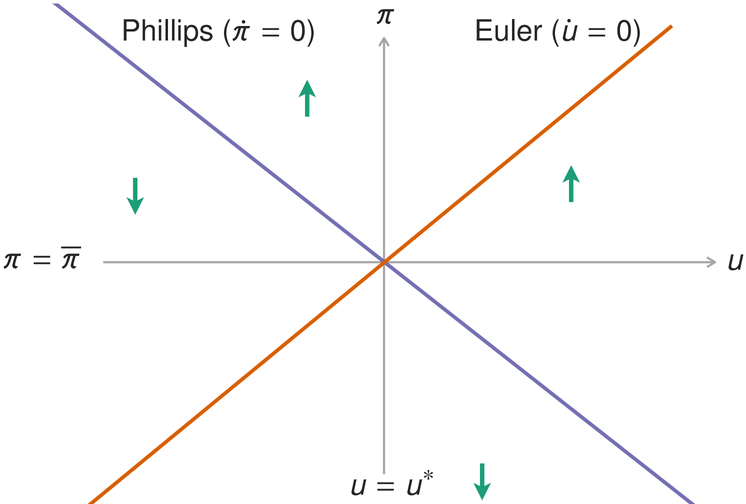


$$\boxed{\dot{\pi} = \delta \hat{\pi} + \frac{2}{\kappa u^* (1 - u^*)} \hat{u}}$$

PHASE DIAGRAM OF THE EARTHLY MODEL

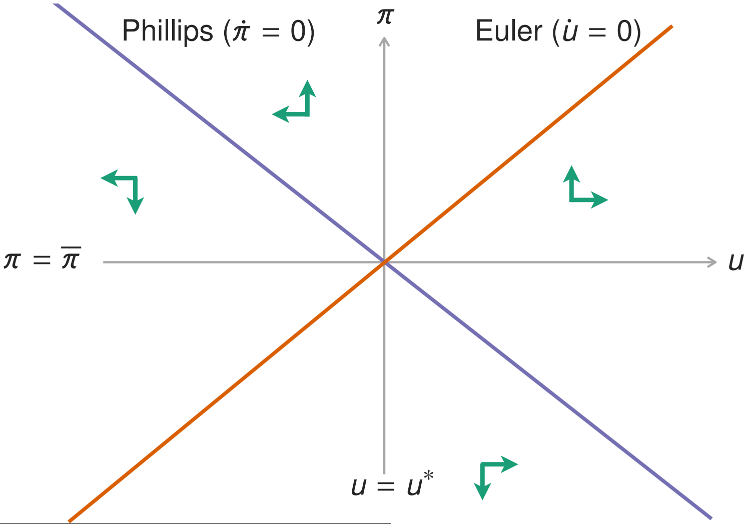


PHASE DIAGRAM OF THE EARTHLY MODEL (TAYLOR)



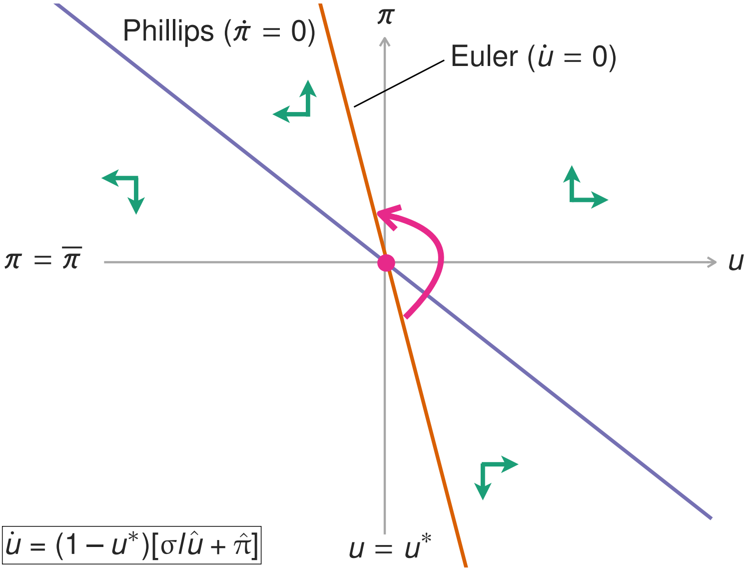
$$\boxed{\dot{u} = (1 - u^*)[\sigma / \hat{u} + (1 - \phi)\hat{\pi}]}$$

PHASE DIAGRAM OF THE EARTHLY MODEL (TAYLOR)

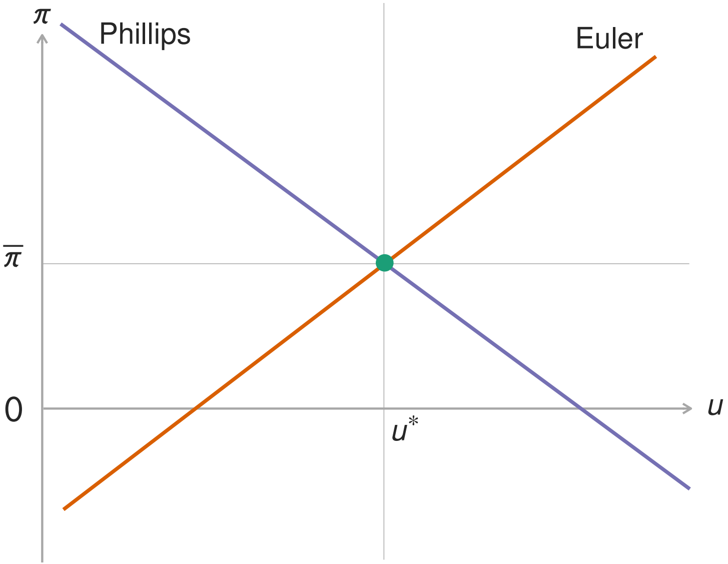


$$\boxed{\dot{u} = (1 - u^*)\left[\sigma / \hat{u} + (1 - \phi)\hat{\pi}\right]}$$

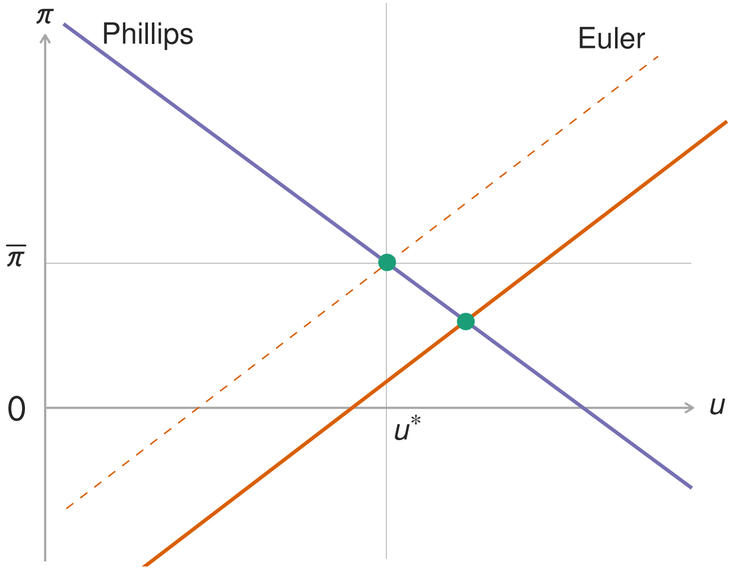
PHASE DIAGRAM OF THE EARTHLY MODEL (PEG)



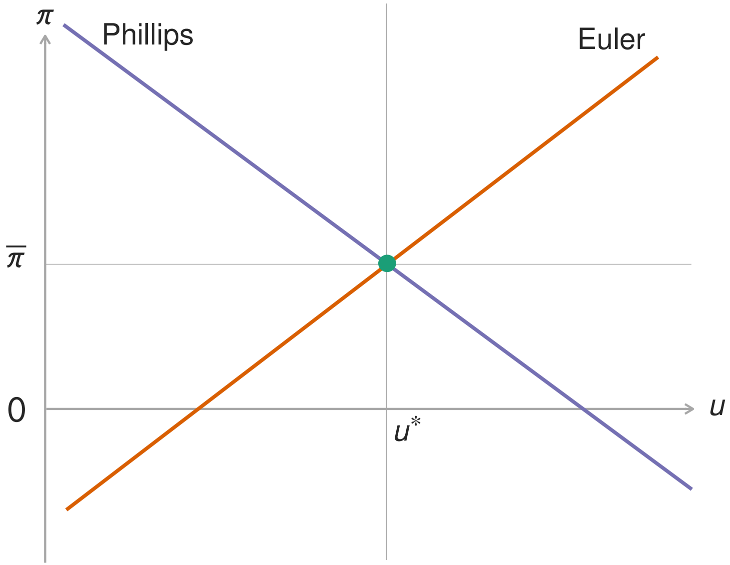
NEGATIVE AD OR MONETARY-POLICY SHOCK



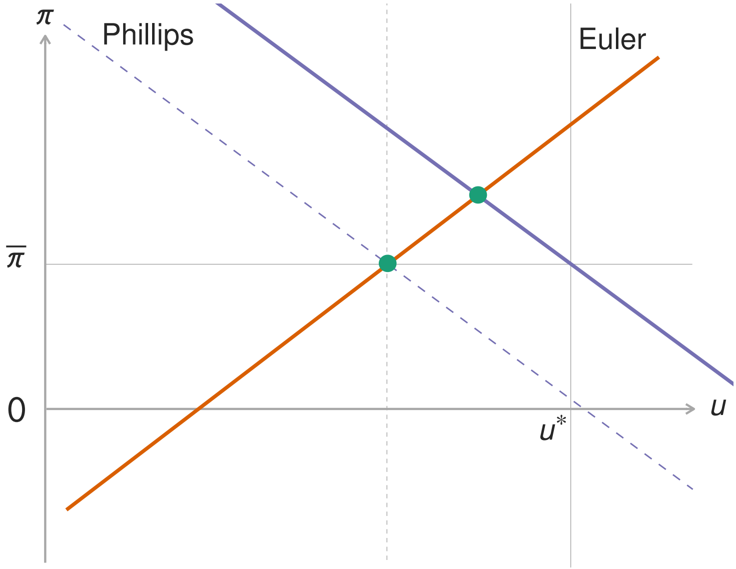
NEGATIVE AD OR MONETARY-POLICY SHOCK



OUTWARD SHIFT OF THE BEVERIDGE CURVE (PANDEMIC)



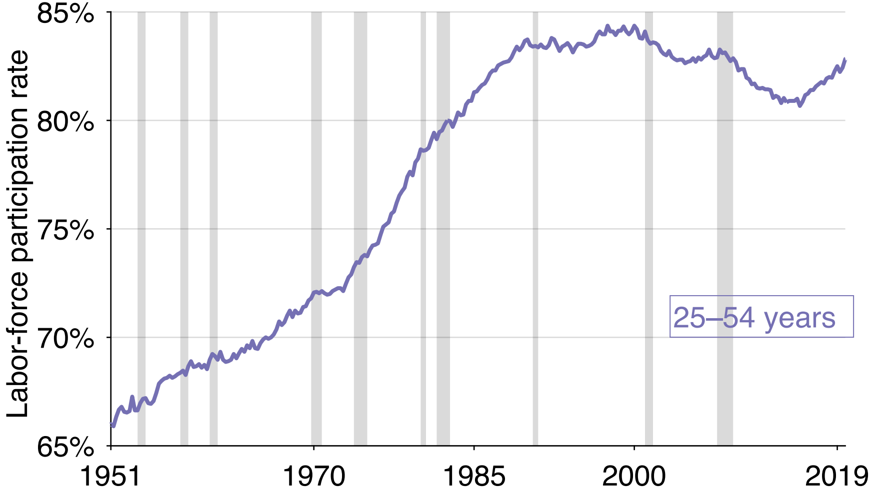
OUTWARD SHIFT OF THE BEVERIDGE CURVE (PANDEMIC)



SIMILARITIES WITH NEW KEYNESIAN DYNAMICS

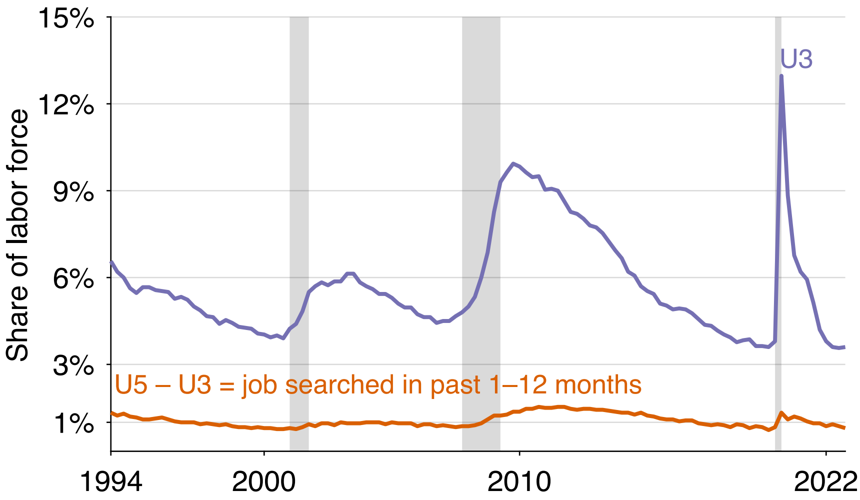
- dynamical properties of the models are similar
  - dynamical system is a source with sufficient concerns for wealth
  - even with interest-rate peg and at the ZLB
  - required concerns for wealth are lower with higher price rigidity
- when dynamical system is a source:
  - economy jumps to steady state and remains there
  - no anomalies at the ZLB (Michaillat, Saez 2021)
- fluctuations in output caused by fluctuations in unemployment
  - instead of fluctuations in markups
- fluctuations in inflation caused by fluctuations in customer queues
  - instead of fluctuations in marginal costs

US LABOR-FORCE PARTICIPATION ≈ ACYCLICAL



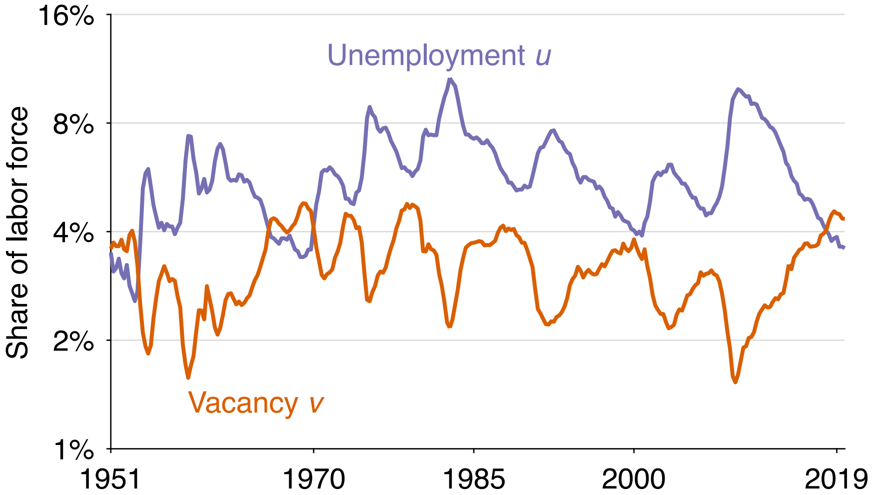
‣ Return to theory

US MARGINAL ATTACHMENT RATE ≈ 1% LABOR FORCE



Return to theory

LOG UNEMPLOYMENT AND VACANCY RATES



‣ Return to Beveridge curve

HOUSEHOLD UTILITY

- household $j \in [0, 1]$ maximizes utility

$$\int_0^{\infty} e^{-\delta t} \left\{ \ln\left(c_j(t)\right) + \sigma \cdot \left[\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)} \right] \right\} dt$$

- $\delta > 0$: time discount rate
- $\sigma > 0$: status concerns
- $c_j(t) = \int_0^1 c_{jk}(t)\, dk$: consumption of chauffeur services
- $b_j(t)$: saving in government bonds
- $b(t) = \int_0^1 b_j(t)\, dj$: aggregate wealth

MATCHING BETWEEN CHAUFFEURS AND CUSTOMERS

- household $k \in [0,1]$ has d_k chauffeurs
  - y_{jk} chauffeurs work for household j
  - $y_k = \int_0^1 y_{jk}(t)\, dk$ chauffeurs are employed
  - $u_k = d_k - y_k$ chauffeurs are unemployed
- household j sends v_{jk} customers to parking lot k to hire chauffeurs
  - $v_k = \int_0^1 v_{jk}(t)\, dj$ customers are hiring chauffeurs
- matching function determines flow of new matches on parking lot k:

$$h_k = \omega \cdot \sqrt{u_k \cdot v_k}$$

- market tightness $\theta_k = v_k/u_k$ determines trading rates
  - customer-finding rate: $f(\theta_k) = h_k/u_k = \omega \cdot \sqrt{\theta_k}$
  - chauffer-finding rate: $q(\theta_k) = h_k/v_k = \omega/\sqrt{\theta_k}$

COST OF UNEMPLOYMENT AND HIRING

- unemployed chauffeurs wait in their parking lot
  - no home production
  - no income
- each v_{jk} customer looking for a chauffeur is driven to parking lot k by one of the y_{jk} chauffeurs from household k working for household j
  - consumption < output: $c_{jk} = y_{jk} - v_{jk}$

BALANCED FLOWS AND UNEMPLOYMENT

- chauffeur-customer relationships separate at rate $s > 0$
- number of employed chauffeurs in household k:

$$\dot{y}_k = f(\theta_k) \cdot u_k - s \cdot y_k = f(\theta_k) \cdot u_k - s \cdot [d_k - u_k]$$

- US flows are always approximately balanced (Michaillat, Saez 2021)
  - assume that flows are balanced in all (j, k) cells
  - in particular flows are balanced in all household k: $\dot{y}_k = 0$
- tightness determines unemployment:

$$u_k = \frac{s}{s + f(\theta_k)} \cdot d_k = u(\theta_k) \cdot d_k$$

BALANCED FLOWS AND MATCHING WEDGE

- number of employed chauffeurs from household j in household k:

$$\dot{y}_{jk} = q(\theta_k) \cdot v_{jk} - s \cdot y_{jk} = q(\theta_k) \cdot [y_{jk} - c_{jk}] - s \cdot y_{jk}$$

- flows are balanced in all (j, k) cells: $\dot{y}_{jk} = 0$
- tightness determines the wedge between consumption and output:

$$y_{jk} = \frac{q(\theta_k)}{q(\theta_k) - s} \cdot c_{jk} = [1 + \tau(\theta_k)] \cdot c_{jk}$$

PRODUCTIVE EFFICIENCY ON PARKING LOT k

- efficient allocation maximizes chauffeur services consumed

$$c_k = \frac{y_k}{1+\tau(\theta_k)} = \frac{1-u(\theta_k)}{1+\tau(\theta_k)} \cdot d_k$$

- efficient tightness θ_k^* maximizes $[1-u(\theta_k)]/[1+\tau(\theta_k)]$
- efficiency condition: share of unemployed workers $u(\theta_k^*)$ = share of consumption devoted to matching $\tau(\theta_k^*)$
- up to a first-order approximation: vacancy ≈ unemployment
  - just like in sufficient-statistic analysis

DIRECTED SEARCH AND PRICE/TIGHTNESS COMPETITION

- all chauffeurs from household k charge price p_k per unit time
- expenditure by household j on chauffeurs k is

$$p_k \cdot y_{jk} = p_k \cdot [1 + \tau(\theta_k)] \cdot c_{jk}$$

- all chauffeurs are perfectly substitutable
- $p_k \cdot [1 + \tau(\theta_k)]$ must be the same across sellers (Moen 1997)
  - if not, there are cheaper chauffeurs available
  - or chauffeurs that can be hired with less wait

$\rightsquigarrow$ for all k, $p_k \cdot [1 + \tau(\theta_k)] = p \cdot [1 + \tau(\theta)]$

EFFECT OF SELLING PRICE ON LOCAL TIGHTNESS

- price chosen by household j determines the tightness θ_j it faces:

$$p_j[1 + \tau(\theta_j)] = p[1 + \tau(\theta)] \;\Rightarrow\; \theta_j = \tau^{-1}\left(\frac{p}{p_j}[1 + \tau(\theta)] - 1\right)$$

- the function τ^{-1} is increasing, so θ_j is decreasing in p_j
  - a high price leads to low tightness, high unemployment
  - a low price leads to high tightness, low unemployment

PRICE RIGIDITY

- inflation for household k: $\pi_k(t) = \dot{p}_k(t)/p_k(t)$
- changing prices is costly (Rotemberg 1982)
  - $\rho(\pi_k) = \frac{\kappa}{2} \cdot [\pi_k - \bar{\pi}]^2$: share of workers devoted to pricing
  - $\kappa > 0$: price-adjustment cost
  - unexpected price changes require communication with customers (Zbaracki et al 2004)
- l_k: labor-force participants from household k
- because of price-adjustment costs:

$$d_k = [1 - \rho(\pi_k)] \cdot l_k$$

HOUSEHOLD BUDGET CONSTRAINT

- law of motion of government bond holdings for household j:

$$\dot{b}_j = i \cdot b_j - \int_0^1 p_k y_{jk}\, dk + p_j y_j$$

- because of matching and directed search, expenditure becomes:

$$\int_0^1 p_k y_{jk}\, dk = \int_0^1 p_k [1 + \tau(\theta_k)] c_{jk}\, dk$$
$$= p \cdot [1 + \tau(\theta)] \cdot \int_0^1 c_{jk}\, dk$$

- because of matching and price rigidity, income becomes:

$$p_j \cdot y_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot d_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot [1 - \rho(\pi_j)] \cdot l_j$$

SOLUTION TO HOUSEHOLD MAXIMIZATION BY HAMILTONIAN

- Hamiltonian of household j's maximization is

$$\mathcal{H}_j = \ln\left(c_j\right) + \sigma \cdot \left[\frac{b_j}{p} - \frac{b}{p}\right]$$
$$+ \mathcal{A}_j \cdot \left[i \cdot b_j - p \cdot [1 + \tau(\theta)] \cdot c_j + p_j \cdot [1 - u(\theta_j(p_j))] \cdot [1 - \rho(\pi_j)] \cdot l_j\right]$$
$$+ \mathcal{B}_j \cdot \pi_j \cdot p_j$$

- control variables: c_j, π_j
- state variables: b_j, p_j
- costate variables: $\mathcal{A}_j, \mathcal{B}_j$
- we focus on symmetric solution of model
  - all households behave the same, can drop j

FIRST-ORDER CONDITION WITH RESPECT TO CONSUMPTION

- $d\mathcal{H}_j/dc_j = 0$

$\Leftrightarrow$ $1/c_j = \mathcal{A}_j \cdot p \cdot \left[1 + \tau(\theta)\right]$

$\Leftrightarrow$ $1/\mathcal{A} = p \cdot \left[1 + \tau(\theta)\right] \cdot c$

$\Leftrightarrow$ $1/\mathcal{A} = p \cdot y$

$\Leftrightarrow$ $-\ln(\mathcal{A}) = \ln(p) + \ln(y)$

- taking time derivative yields:

$$-\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \pi + \frac{\dot{y}}{y}$$

FIRST-ORDER CONDITION WITH RESPECT TO INFLATION

- $d\mathcal{H}_j/d\pi_j = 0$

$\Leftrightarrow \quad \mathcal{B}_j \cdot p_j = \mathcal{A}_j \cdot p_j \cdot [1 - u(\theta_j(p_j))] \cdot \rho'(\pi_j) \cdot l_j$

$\Leftrightarrow \quad \mathcal{B} = \mathcal{A} \cdot [1 - u(\theta)] \cdot \rho'(\pi) \cdot l$

$\Leftrightarrow \quad \mathcal{B} = \mathcal{A} \cdot [1 - u(\theta)] \cdot \kappa \cdot (\pi - \overline{\pi}) \cdot l$

$\Leftrightarrow \quad \mathcal{B} = \mathcal{A} \cdot \frac{\kappa[\pi - \overline{\pi}]}{1 - \rho(\pi)} \cdot y$

$\Leftrightarrow \quad \ln(\mathcal{B}) = \ln(\mathcal{A}) + \ln(y) + \ln(\kappa) + \ln(\pi - \overline{\pi}) - \ln(1 - \rho(\pi))$

- taking time derivative yields:

$$\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \frac{\dot{\mathcal{A}}}{\mathcal{A}} + \frac{\dot{y}}{y} + \frac{\dot{\pi}}{\pi - \overline{\pi}} \cdot \frac{1 + \rho(\pi)}{1 - \rho(\pi)}$$

FIRST-ORDER CONDITION WITH RESPECT TO SAVING

- $d\mathcal{H}_j/db_j = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j$

$\Leftrightarrow$ $\frac{\sigma}{p} + \mathcal{A}_j \cdot i = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j$

- reshuffling terms yields:

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - i - \frac{\sigma}{p \cdot \mathcal{A}}$$

- using $1/\mathcal{A} = p \cdot y$ finally gives:

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - (i + \sigma \cdot y)$$

FIRST-ORDER CONDITION WITH RESPECT TO PRICE [1]

- $d\mathcal{H}_j/dp_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j$

$\Leftrightarrow$ $\mathcal{A}_j \cdot (1 - u_j) \cdot (1 - \rho(\pi_j)) \cdot l_j - \mathcal{A}_j \cdot p_j \cdot (1 - \rho(\pi_j)) \cdot l_j \cdot u'(\theta_j) \cdot \theta'(p_j) + \mathcal{B}_j \cdot \pi_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j$

- we have the following derivatives:

$$u'(\theta_j) = -\frac{[1 - u(\theta_j)] \cdot u(\theta_j)}{2 \cdot \theta_j}, \qquad \theta'(p_j) = -\frac{2 \cdot \theta_j}{\tau(\theta_j) \cdot p_j}$$

- hence, $p_j \cdot u'(\theta_j) \cdot \theta'(p_j) = (1 - u(\theta_j)) \cdot$

FIRST-ORDER CONDITION WITH RESPECT TO PRICE [2]

- reshuffling terms gives:

$$
\begin{aligned}
(\delta - \pi_j) \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j &= \mathcal{A}_j \cdot y_j \cdot \left[1 - \frac{u(\theta_j)}{\tau(\theta_j)}\right] \\
-\frac{\dot{\mathcal{B}}_j}{\mathcal{B}_j} &= \pi_j - \delta + \frac{\mathcal{A}_j \cdot y_j}{\mathcal{B}_j} \cdot \left[1 - \frac{u(\theta_j)}{\tau(\theta_j)}\right]
\end{aligned}
$$

- using $[1 - \rho(\pi_j)]/[\kappa(\pi_j - \overline{\pi})] = \mathcal{A}_j \cdot y_j/\mathcal{B}_j$, we finally get:

$$
-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1 - \rho(\pi)}{\pi - \overline{\pi}} \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]
$$

LINEARIZED SYSTEM

- set $i = i^* + \phi(\pi - \overline{\pi})$ and linearize differential equations around $(u^*, \overline{\pi})$
- deviations from efficient steady state: $\hat{u} = u - u^*$, $\hat{\pi} = \pi - \overline{\pi}$
- linearized AD curve: $\dot{u} = (1 - u^*) \cdot [\sigma \cdot l \cdot \hat{u} + (1 - \phi)\hat{\pi}]$
- steady-state linearized AD curve: $\hat{\pi} = -[\sigma/(1 - \phi)] \cdot l \cdot \hat{u}$
  - slope depends on policy rule
  - under Taylor principle ($\phi > 1$), standard increasing AD curve
- linearized AS curve: $\dot{\pi} = \delta\hat{\pi} + 2/[\kappa u^*(1 - u^*)]\hat{u}$
- steady state linearized AS curve: $\hat{\pi} = -2/[\delta\kappa u^*(1 - u^*)]\hat{u}$
  - recover downward-sloping Phillips curve in (unemployment, inflation) plane

SPECIAL CASES

- consider AD and AS curves in (y, π) plane
- AD curve without wealth in the utility ($\sigma = 0$)
  - horizontal: $\pi = i - \delta$
  - real rate = discount rate
  - inflation is determined one-for-one by policy rate (Fisher effect)
  - degenerate: does not involve output
- AS curve without price rigidity ($\kappa = 0$)
  - vertical: $y = y^* = (1 - u^*) \cdot l$
  - unemployment is always efficient, irrespective of inflation

AGGREGATE DEMAND: DISCOUNTED EULER EQUATION

- from optimal consumption and saving:

$$\frac{\dot{y}}{y} = (i - \pi + \sigma \cdot y) - \delta$$

- $i - \pi$: real interest rate, financial return on saving
- $\sigma \cdot y$: MRS between wealth & consumption, hedonic return on saving
  - discounted Euler equation (McKay, Nakamura, Steinsson 2017)
  - from wealth in the utility function (Michaillat, Saez 2021)
- in steady state ($\dot{y} = 0$), nondegenerate AD curve:

$$y = \frac{\delta - i + \pi}{\sigma}$$

AGGREGATE SUPPLY: PHILLIPS CURVE

- from optimal pricing:

$$\dot{\pi} \cdot \frac{1+\rho(\pi)}{1-\rho(\pi)} = \delta \cdot (\pi - \overline{\pi}) - \frac{1}{\kappa} \cdot [1-\rho(\pi)] \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]$$

- κ: price-adjustment cost
- $1 - \frac{u(\theta)}{\tau(\theta)}$: tightness gap
  - $u(\theta)$: share of idle drivers waiting for a job
  - $\tau(\theta)$: share of idle drivers waiting for a match
  - zero iff $\theta = \theta^*$
  - positive iff $\theta > \theta^*$
- in steady state ($\dot{\pi} = 0$), nonlinear AS curve:

$$\delta \cdot (\pi - \overline{\pi}) = \frac{1}{\kappa} \cdot [1-\rho(\pi)] \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]$$

DIVINE COINCIDENCE APPEARS IN PHILLIPS CURVE

- recall steady-state Phillips curve:

$$\kappa \cdot \delta \cdot (\pi - \overline{\pi}) = [1 - \rho(\pi)] \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]$$

- inflation is on target ($\pi = \overline{\pi}$) iff
  - $1 - \frac{u(\theta)}{\tau(\theta)} = 0$

$\Leftrightarrow$ $u(\theta) = \tau(\theta)$

$\Leftrightarrow$ tightness and unemployment are efficient ($\theta = \theta^*$, $u = u^*$)

OPTIMAL MONETARY POLICY

- optimal nominal interest rate i^* ensures:
  - inflation is on target: $\pi = \overline{\pi}$
  - unemployment is efficient: $u = u^*$
- optimal policy can take different forms:
  - interest-rate peg: $i = i^*$
  - Taylor rule with $\phi > 0$: $i = i^* + \phi \cdot (\pi - \overline{\pi})$
- from AD curve:
  - $\delta - i^* + \overline{\pi} = \sigma \cdot y^* = \sigma \cdot (1 - u^*) \cdot l$

  $\Leftrightarrow$ $i^* = \overline{\pi} + \delta - \sigma \cdot (1 - u^*) \cdot l$
- by divine coincidence, just need to target efficient unemployment
  - no need to compute i^*

DYNAMICAL PROPERTIES OF THE LINEARIZED MODEL

$$\begin{bmatrix} \dot{u}(t) \\ \dot{\pi}(t) \end{bmatrix} = \begin{bmatrix} \sigma y^* & (1-\phi)(1-u^*) \\ \frac{2}{\kappa u^*(1-u^*)} & \delta \end{bmatrix} \begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix}$$

- solution is unique iff dynamical system is a source

  $\Leftrightarrow$ trace > 0 and determinant > 0
- trace: $\delta + \sigma y^* > 0$
  - trace > 0 for any $\sigma \geq 0$
- determinant: $\delta\sigma y^* - 2(1-\phi)/(\kappa u^*)$
  - no wealth in utility ($\sigma = 0$): determinant > 0 iff $\phi > 1$ (Taylor)
  - interest-rate peg ($\phi = 0$): determinant > 0 iff $\sigma > 2/(\kappa\delta u^* y^*)$
  - fixed inflation ($\kappa \to \infty$): determinant > 0 iff $\sigma > 0$