A Threshold Selection Method from Gray-Level Histograms

一种基于灰度直方图的阈值选择方法

玉晓东
954767762@qq.com

计算学部
哈尔滨工业大学

12th April 2022

[A threshold selection method from gray level histograms]

背景介绍

- N. Otsu, “A threshold selection method from gray-level histogram,” IEEE Transactions on System Man Cybernetics, Vol. SMC-9, No. 1, 1979, pp. 62-66.

- Paper citations:32470

- Paper structure:

  Ⅰ.Introduction
  Ⅱ.Formulation
  Ⅲ.Discussion
    A. Analysis of further important aspects
    B. Extension to Multithresholding
    C. Experimental Results
    D. Unimodality of the object function
  Ⅳ.Conclusion

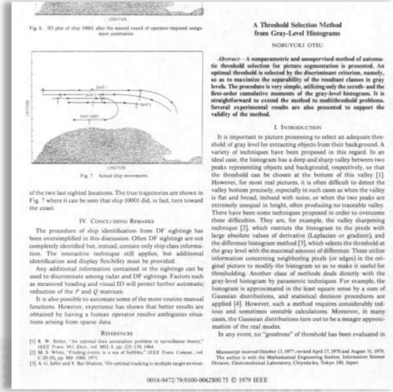


Fig. 6. ID plot of ship 10001 after the second round of operator-imposed assignment constraints.

Fig. 7. Actual ship movements.

of the two last sighted locations. The true trajectories are shown in Fig. 7 where it can be seen that ship 10001 did, in fact, turn toward the coast.

IV. Concluding Remarks

The procedure of ship identification from DF sightings has been oversimplified in this discussion. Often DF sightings are not completely identified but, instead, contain only ship class information. The interactive technique still applies, but additional identification and display flexibility must be provided.

Any additional information contained in the sightings can be used to discriminate among radar and DF sightings. Factors such as measured heading and visual ID will permit further automatic reduction of the P and Q matrices.

It is also possible to automate some of the more routine manual functions. However, experience has shown that better results are obtained by having a human operator resolve ambiguous situations arising from sparse data.

References

[1] R. W. Sittler, "An optimal data association problem in surveillance theory," IEEE Trans. Mil. Elect., vol. MIL-8, pp. 125-139, 1964.
[2] M. S. White, "Finding events in a sea of bubbles," IEEE Trans. Comput., vol. C-20 (9), pp. 988-1006, 1971.
[3] A. G. Jaffer and Y. Bar-Shalom, "On optimal tracking in multiple-target environ-

A Threshold Selection Method from Gray-Level Histograms

NOBUYUKI OTSU

Abstract—A nonparametric and unsupervised method of automatic threshold selection for picture segmentation is presented. An optimal threshold is selected by the discriminant criterion, namely, so as to maximize the separability of the resultant classes in gray levels. The procedure is very simple, utilizing only the zeroth- and the first-order cumulative moments of the gray-level histogram. It is straightforward to extend the method to multithreshold problems. Several experimental results are also presented to support the validity of the method.

I. Introduction

It is important in picture processing to select an adequate threshold of gray level for extracting objects from their background. A variety of techniques have been proposed in this regard. In an ideal case, the histogram has a deep and sharp valley between two peaks representing objects and background, respectively, so that the threshold can be chosen at the bottom of this valley [1]. However, for most real pictures, it is often difficult to detect the valley bottom precisely, especially in such cases as when the valley is flat and broad, imbued with noise, or when the two peaks are extremely unequal in height, often producing no traceable valley. There have been some techniques proposed in order to overcome these difficulties. They are, for example, the valley sharpening technique [2], which restricts the histogram to the pixels with large absolute values of derivative (Laplacian or gradient), and the difference histogram method [3], which selects the threshold at the gray level with the maximal amount of difference. These utilize information concerning neighboring pixels (or edges) in the original picture to modify the histogram so as to make it useful for thresholding. Another class of methods deals directly with the gray-level histogram by parametric techniques. For example, the histogram is approximated in the least square sense by a sum of Gaussian distributions, and statistical decision procedures are applied [4]. However, such a method requires considerably tedious and sometimes unstable calculations. Moreover, in many cases, the Gaussian distributions turn out to be a meager approximation of the real modes.

In any event, no "goodness" of threshold has been evaluated in

Manuscript received October 13, 1977; revised April 17, 1978 and August 31, 1978.
The author is with the Mathematical Engineering Section, Information Science Division, Electrotechnical Laboratory, Chiyoda-ku, Tokyo 100, Japan.

0018-9472/79/0100-0062$00.75 © 1979 IEEE

背景介绍

灰度图二值化

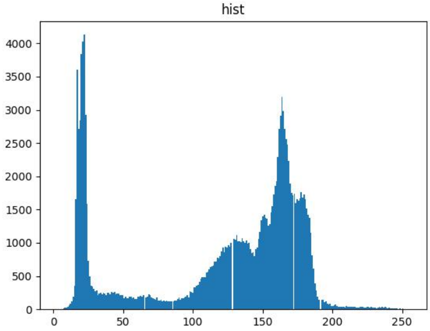
背景介绍

不同阈值,不同效果

背景介绍

灰度直方图：

灰度直方图就是图像的像素点在不同灰度级的分布情况



背景介绍

人工阈值：大致的阈值区间

原图

图一：threshold=120

图二：threshold=75

Formulation

基本原理：

以最佳阈值将图像的灰度直方图**分割成多部分**，使各个部分之间的**方差取最大值**，即分离性最大。

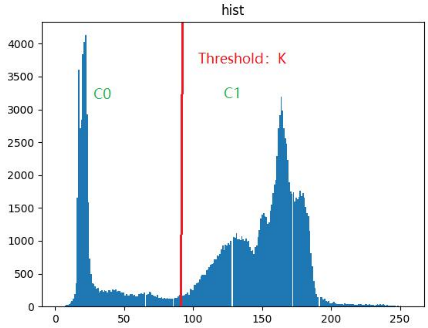
在灰度直方图的基础上具有统计意义上的最佳分割阈值

Formulation

- 一副具有L级灰度级的图像，其中第i级像素为Ni个，其中i的值在0到L-1之间。
- 图像的总像素点个数： $N = \sum_{i=0}^{L-1} N_i$
- 第i级出现的概率为： $P_i = \frac{N_i}{N}$
- 以**阈值**K将所有像素分为两类

  C_0 denotes pixels with levels [0,...,k-1]

  C_1 denotes pixels with levels [k,...,L-1]



Formulation

- 图像的总平均灰度级： $\mu = \sum_{i=0}^{L-1} iP_i$
- C_0类像素所占面积的比例为： $w_0 = \sum_{i=0}^{k-1} P_i$
- C_1类像素所占面积的比例为： $w_1 = 1 - w_0$
- C_0类像素的平均灰度： $\mu_0 = \mu_0(k) / w_0$
- C_1类像素的平均灰度： $\mu_1 = \mu_1(k) / w_1$
- 则类间方差公式为： $\delta^2(k) = w_0(\mu - \mu_0)^2 + w_1(\mu - \mu_1)^2$
- 最佳的阈值K： $\delta^2(k^*) = \max_{1 \le k < L} \delta^2(k)$

其中：

$$\mu_0(k) = \sum_{i=0}^{k-1} iP_i$$

$$\mu_1(k) = \sum_{i=k}^{L-1} iP_i = 1 - \mu_0(k)$$

Formulation

补充：

1. 类间方差公式化简

$$\because \quad w_0 + w_1 = 1 \, , \; w_0\mu_0 + w_1\mu_1 = \mu$$

$$\therefore \quad \delta^2(k) = w_0(\mu - \mu_0)^2 + w_1(\mu - \mu_1)^2$$

$$= w_0((w_0\mu_0 + w_1\mu_1) - \mu_0)^2 + w_1((w_0\mu_0 + w_1\mu_1) - \mu_1)^2$$

$$= w_0 w_1(\mu_1 - \mu_0)^2$$

Formulation

补充：

2. 总方差、类间方差、类内方差的关系

$$\delta^2 = \delta_B^2 + \delta_W^2$$

δ^2 :总方差 total variance

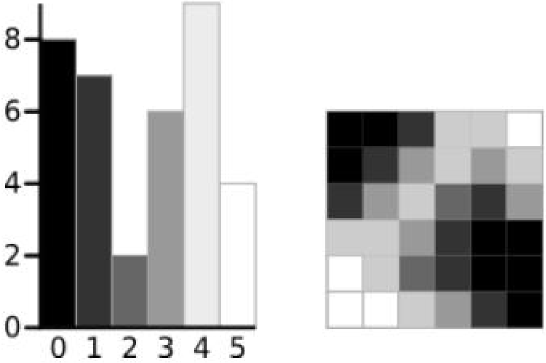
δ_B^2 :类间方差 the between-class variance

δ_W^2:类内方差 the within-class variance

相关证明：https://blog.csdn.net/qq_42164483/article/details/119064535

Formulation

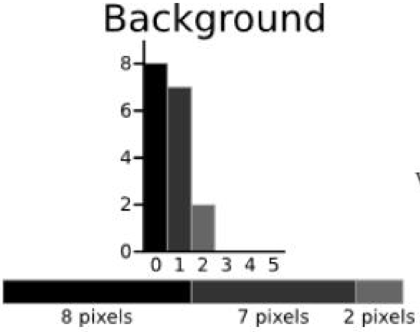
举例： A 6-level greyscale image and its histogram



Formulation

举例：

以阈值为3的情况，即K=3，分别计算前景物体和背景的类内方差



$$\text{Weight} \quad W_b = \frac{8+7+2}{36} = 0.4722$$

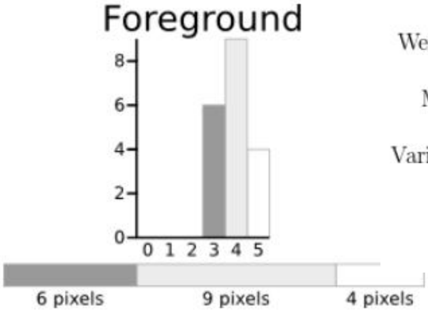
$$\text{Mean} \quad \mu_b = \frac{(0\times 8)+(1\times 7)+(2\times 2)}{17} = 0.6471$$

$$\begin{aligned}\text{Variance} \quad \sigma_b^2 &= \frac{((0-0.6471)^2\times 8)+((1-0.6471)^2\times 7)+((2-0.6471)^2\times 2)}{17} \\ &= \frac{(0.4187\times 8)+(0.1246\times 7)+(1.8304\times 2)}{17} \\ &= 0.4637\end{aligned}$$

Formulation

举例：

计算阈值为3的情况，即T=3，分别计算前景物体和背景的类内方差



$$\text{Weight} \quad W_f = \frac{6+9+4}{36} = 0.5278$$

$$\text{Mean} \quad \mu_f = \frac{(3\times 6)+(4\times 9)+(5\times 4)}{19} = 3.8947$$

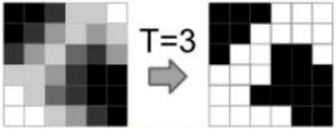
$$\begin{aligned}\text{Variance} \quad \sigma_f^2 &= \frac{((3-3.8947)^2\times 6)+((4-3.8947)^2\times 9)+((5-3.8947)^2\times 4)}{19}\\ &= \frac{(4.8033\times 6)+(0.0997\times 9)+(4.8864\times 4)}{19}\\ &= 0.5152\end{aligned}$$

Formulation

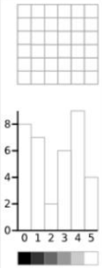
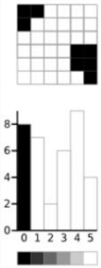
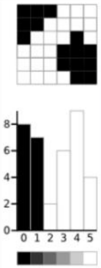
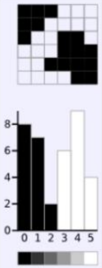
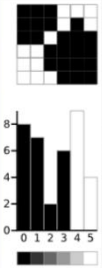
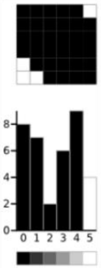
举例：

计算图像的类内方差

$$\sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2$$
$$= 0.4722 * 0.4637 + 0.5278 * 0.5152$$
$$= 0.4909$$



Result of Otsu's Method

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
						
Weight, Background	$W_b = 0$	$W_b = 0.222$	$W_b = 0.4167$	$W_b = 0.4722$	$W_b = 0.6389$	$W_b = 0.8889$
Mean, Background	$M_b = 0$	$M_b = 0$	$M_b = 0.4667$	$M_b = 0.6471$	$M_b = 1.2609$	$M_b = 2.0313$
Variance, Background	$\sigma^2_b = 0$	$\sigma^2_b = 0$	$\sigma^2_b = 0.2489$	$\sigma^2_b = 0.4637$	$\sigma^2_b = 1.4102$	$\sigma^2_b = 2.5303$
Weight, Foreground	$W_f = 1$	$W_f = 0.7778$	$W_f = 0.5833$	$W_f = 0.5278$	$W_f = 0.3611$	$W_f = 0.1111$
Mean, Foreground	$M_f = 2.3611$	$M_f = 3.0357$	$M_f = 3.7143$	$M_f = 3.8947$	$M_f = 4.3077$	$M_f = 5.000$
Variance, Foreground	$\sigma^2_f = 3.1196$	$\sigma^2_f = 1.9639$	$\sigma^2_f = 0.7755$	$\sigma^2_f = 0.5152$	$\sigma^2_f = 0.2130$	$\sigma^2_f = 0$
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\boldsymbol{\sigma^2_W = 0.4909}$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$

Formulation

举例：

A Faster Approach：使用类间方差计算

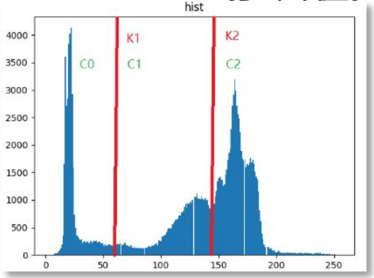
Within Class Variance $\sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2$ (as seen above)

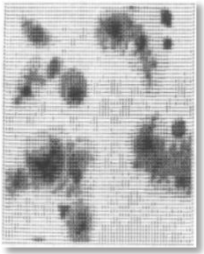
Between Class Variance
$$\sigma_B^2 = \sigma^2 - \sigma_W^2$$
$$= W_b(\mu_b - \mu)^2 + W_f(\mu_f - \mu)^2 \quad \text{(where } \mu = W_b\,\mu_b + W_f\,\mu_f\text{)}$$
$$= W_b\,W_f\,(\mu_b - \mu_f)^2$$

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Within Class Variance	σ^2_W = 3.1196	σ^2_W = 1.5268	σ^2_W = 0.5561	**σ^2_W = 0.4909**	σ^2_W = 0.9779	σ^2_W = 2.2491
Between Class Variance	σ^2_B = 0	σ^2_B = 1.5928	σ^2_B = 2.5635	**σ^2_B = 2.6287**	σ^2_B = 2.1417	σ^2_B = 0.8705

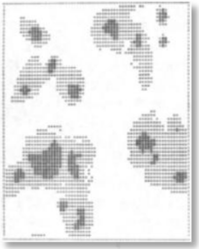
Expansion

Multithreshold(多阈值):





原图



三值化

$$\delta^2(k) = w_0(\mu - \mu_0)^2 + w_1(\mu - \mu_1)^2$$

M class

$$\delta_B^2 = \sum_{k=1}^{M} w_k(\mu - \mu_k)^2$$

Optimal threshold k^*

$$k^* = [k_1^*, \ldots, k_{M-1}^*]$$

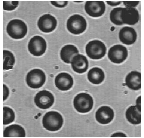
Expansion

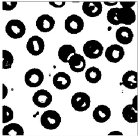
Multithreshod(多阈值):

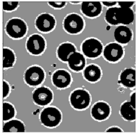
如何计算最佳的 $k^* = [k_1^*, \ldots, k_{M-1}^*]$?

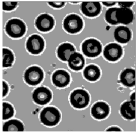
排列组合问题?

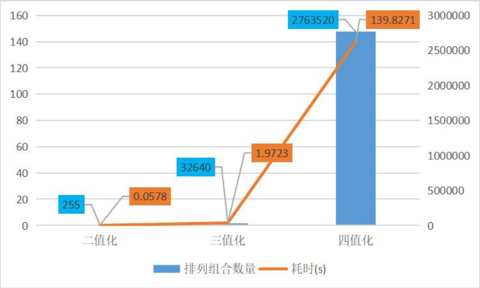
$$C_{256}^{n} = \frac{256!}{(256-n)! * n!}$$


a.原图


b.二值化


c.三值化


d.四值化



Expansion

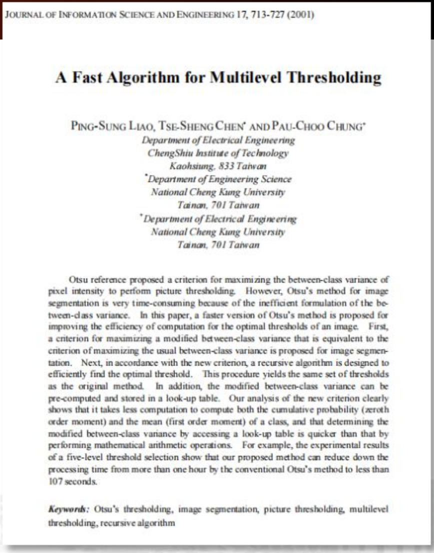
Multithreshod(多阈值):

1. Simplify the formulate

$$\sigma_B^{\,2} = \sum_{k=1}^{M} \omega_k (\mu_k-\mu_T)^2 \Longrightarrow (\sigma_B')^2 = \sum_{k=1}^{M} \omega_k \mu_k^{\,2}$$

2. With recursion

 Efficiently find the optimal threshold.


JOURNAL OF INFORMATION SCIENCE AND ENGINEERING 17, 713-727 (2001)

A Fast Algorithm for Multilevel Thresholding

PING-SUNG LIAO, TSE-SHENG CHEN* AND PAU-CHOO CHUNG+
Department of Electrical Engineering
ChengShiu Institute of Technology
Kaohsiung, 833 Taiwan
*Department of Engineering Science
National Cheng Kung University
Tainan, 701 Taiwan
+Department of Electrical Engineering
National Cheng Kung University
Tainan, 701 Taiwan

Otsu reference proposed a criterion for maximizing the between-class variance of pixel intensity to perform picture thresholding. However, Otsu's method for image segmentation is very time-consuming because of the inefficient formulation of the between-class variance. In this paper, a faster version of Otsu's method is proposed for improving the efficiency of computation for the optimal thresholds of an image. First, a criterion for maximizing a modified between-class variance that is equivalent to the criterion of maximizing the usual between-class variance is proposed for image segmentation. Next, in accordance with the new criterion, a recursive algorithm is designed to efficiently find the optimal threshold. This procedure yields the same set of thresholds as the original method. In addition, the modified between-class variance can be pre-computed and stored in a look-up table. Our analysis of the new criterion clearly shows that it takes less computation to compute both the cumulative probability (zeroth order moment) and the mean (first order moment) of a class, and that determining the modified between-class variance by accessing a look-up table is quicker than that by performing mathematical arithmetic operations. For example, the experimental results of a five-level threshold selection show that our proposed method can reduce down the processing time from more than one hour by the conventional Otsu's method to less than 107 seconds.

Keywords: Otsu's thresholding, image segmentation, picture thresholding, multilevel thresholding, recursive algorithm

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A Fast Algorithm for Multilevel Thresholding ,citations： 893

Expansion

Multithreshod(多阈值):

(a) bi-level (b) tri-level

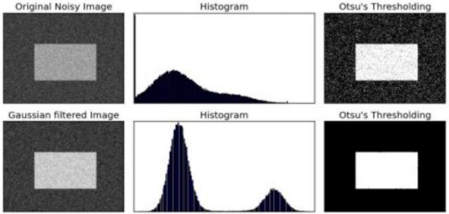
(c) four-level (d) five-level

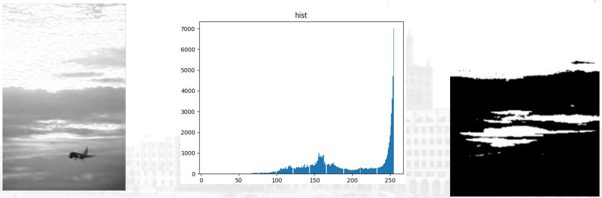
Table 5. Thresholds and computation times for the test images.

Images	thresholds				computation time: Otsu's method (with recursion without recursion)				computation time: proposed method			
	2	3	4	5	2	3	4	5	2	3	4	5
F16 Jet	156	111 172	96 149 191	86 130 171 202	<1s <1s	<1s 1s	5s 70s	6m 1h	<1s	<1s	1s	37s
House	147	88 154	86 130 177	64 92 131 178	<1s <1s	<1s 1s	6s 91s	7.5m 1. 5h	<1s	<1s	1s	68s
Lena	101	77 145	56 106 159	46 83 119 164	<1s <1s	<1s 2s	9s 166s	12.0m 2.5h	<1s	<1s	1s	107s
Peppers	102	81 142	43 98 152	40 88 134 173	<1s <1s	<1s 1s	7s 105s	8.5m 1. 7h	<1s	<1s	1s	77s

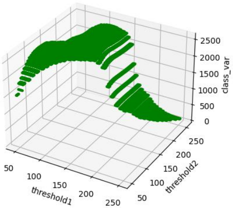
Conclusion

- 应用：求图像全局阈值的最佳方法
- 优点：计算简单快速，不受图像亮度和对比度的影响
- 缺点：1. 对图像噪声敏感
  2. 当目标和背景大小比例悬殊、类间方差函数可能呈现双峰或者多峰，这个时候效果不好。





Thanks for your listening



[A threshold selection method from gray level histograms]