Assignment 1 – Prescriptive Modelling with Gurobi

The due date for this assignment is **Wednesday, 02.11.2022, 23:59**.

The assignment should be done **individually**, and you should submit a **one** pdf document with all your answers. This document should contain your Python code (as an appendix). The assignment is worth **25 points**. The points for each sub-task are given in the task descriptions below. **Please pay attention to the readability and presentation of your solutions.** Poor readability and presentation lead to point reductions.

Motivation

Prescriptive analytics is about 'giving prescriptions' i.e. advice on what decisions to make. These decisions can be for example choosing one out of many options or selecting a value for a parameter. The problems can be made mathematically solvable when they are formulated as mathematical programming problems, e.g. linear programming (LP), binary programming (BLP), and mixed-integer linear (MILP) problems.

The problems are then solved using dedicated solver algorithms or software consisting of several algorithms and user interface. There are many alternatives with different capabilities, some proprietary, some open and free to use, and one can also build an algorithm from scratch. On previous courses you may have used Excel's Solver to solve these problems but for very large problems, Excel is perhaps not the best option.

In this assignment, and on this course in general, we will use one of the most sophisticated solvers available, Gurobi. It offers very strong solving capabilities combined with flexible interfacing for many different programming languages. Gurobi is proprietary software but offers free student and academic licences. Learning Gurobi gives you 1) experience on a widely used strong tool for solving demanding problems and 2) a good experience on writing code for solving mathematical optimisation problems. As said, there are alternatives available, and for different situations, other options might be better.

In the following two tasks, you will implement and solve two simple mathematical optimisation models in Gurobi. We encourage you to think about how these problems could be constructed in a way that could be easily scaled to much larger models, as that is where the use of sophisticated solvers really adds value.

Problem 1: Deterministic optimisation

For the winter season 2022/2023 the *Data Science café* in Helsinki has estimated the demand for glögi. It will be 500 litres in November, 600 litres in December and 400 litres in January. The café needs to buy red wine to make glögi on its own. For simplicity, let's assume that for 1 litre of glögi we need 1 litre of red wine, so it does not evaporate.

	November	December	January
Liquor Oy	4.3 EUR	5.2 EUR	4.2 EUR
Booze Oy	4.9 EUR	4.4 EUR	4.9 EUR

Table 1: Prices per litre of red wine from different suppliers in different months

Red wine can be supplied by two companies: *Liquor Oy* and *Booze Oy*. Each company can provide a maximum of 350 litres of red wine per month and a maximum of 800 litres in total over 3 months. The red wine provided by the suppliers is interchangeable. The prices per litre of wine for every supplier in every month are given in the Table 1. To optimise costs, the café can purchase more wine in cheaper periods and store the surplus. The cost of storage is 0.5 EUR per litre per month, and the cost applies only to surplus.

Your task is to help the Data Science café find the cost-minimising purchase plan for red wine. To fulfil this task, do the following:

Task a) Write a mathematical formulation for the given problem (6 pt)
NB! You can write this one by hand and include a photo/screenshot into the final report, however, your handwriting should be easily recognisable. Otherwise, you will receive 0 for this part.

Task b) Implement the mathematical programming model in Gurobi for Python (6 pt)

Task c) Write down the solution you receive in the form of a report to the manager of the café (3 pt)

Problem 2: Stochastic optimisation

In many cases, we are not sure of the outcomes of decisions or circumstances that affect them. These situations call for stochastic optimisation. In this problem, your task is to code and solve a stochastic model using Gurobi. The mathematical formulation of this model is in *A1.2-draft-for-students.ipynb*. After solving the model, your job is to interpret the results and compare to another option.

	Evaluation stage	Implementation stage
Wind power project	€2B	€2B
Geothermal heat project	€3B	€2B

Table 2: Investments for each project at each stage

The government of the country Energyland is looking for an opportunity to invest in two projects: a wind power station and a geothermal heat station. Each project is realised in two stages: evaluation and implementation. After evaluation there are two equally possible scenarios: initial assumptions are confirmed or initial assumptions are not confirmed. Based on the scenarios, the decision is made whether to implement the project further or to terminate it. The investment necessary for each of the stages for each project is given in Table 2.

The initial budget for all investments is $9B. Furthermore, surplus cash can be deposited with risk-free interest rate of 4%. The resulting cash flows from the projects depend on which of two possible

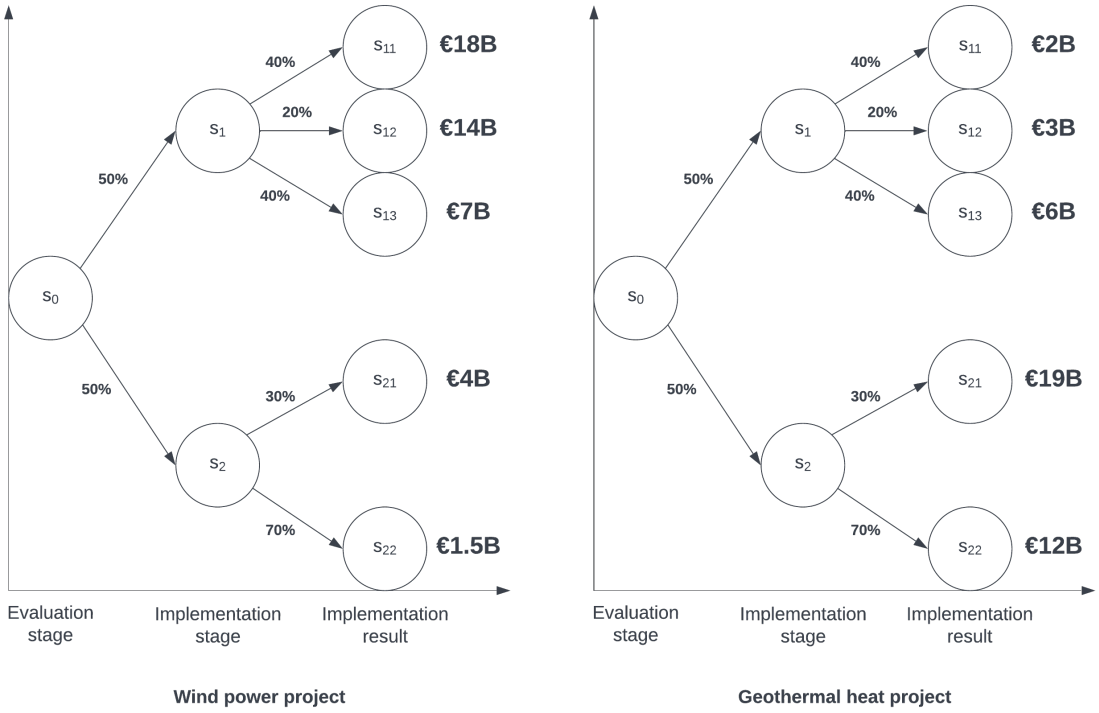

Figure 1: Scenarios and outcomes for projects given investments at evaluation and implementation stages

scenarios realize in each period. You can see the decision tree presented in Figure 1. Note, that the government knows the scenario realised on evaluation stage, when it makes the decision about whether to implement or terminate the project. If a project is terminated, the resulting cash flow from this project is equal to 0.

Task d) Implement the mathematical programming model for Gurobi in Python (mathematical formulation is given in the A1.2-draft-for-students.ipynb file). Explain the meaning of the results in a short managerial report (5 pt)

Task e) The government found new companies, which will do the projects. Now, the investment needed for the evaluation of wind power project is 1.5 Bln, for the evaluation of geothermal heat project is 2 Bln and for the implementation of wind power project is 3.5 Bln. Make the necessary adjustments to the code, find the new solution, report it and compare to the one in **Task d)** (5 pt)