**Instructions to Candidates:**

Answer any **FOUR** of the seven questions.

**Question 1**

**(a)** The share price and dividend history in pence of Beta Plus are as follows. Dividends are paid on 31 December in each year

**Year 1 January Price Dividend**

2013 480 10

2014 520 10

2015 480 12

2016 560 15

2017 590 20

What are the arithmetic and geometric average time-weighted rates of return for the investor? **(8 marks)**

1. The table below shows the expected return and risk (standard deviation) for eight shares.

|  |  |  |
| --- | --- | --- |
|  | **Expected**  **Return** | **Risk** |
| Share A | 13% | 8% |
| Share B | 7% | 4% |
| Share C | 17% | 12% |
| Share D | 13% | 12% |
| Share E | 8% | 10% |
| Share F | 18% | 15% |
| Share G | 5% | 5% |
| Share H | 16% | 15% |

Use the concept of an Efficient Frontier to identify four shares that would be selected for the optimal portfolio. Explain why you have chosen each one and why the remaining shares were not selected. Draw the efficient frontier portfolio on a suitable diagram. **(12 marks)**

**(c)** Dividends for Blackwood plc were as follows.

|  |  |
| --- | --- |
| **Year** | **Dividend (p)** |
| 2012 | 29.3 |
| 2013 | 31.6 |
| 2014 | 33.8 |
| 2015 | 35.5 |
| 2016 | 38.7 |

If the required rate of return were 10% and dividend growth from 2012 to 2016 is 7.2% per year what is the value of a share in Blackwood? **(5 marks)**

**Total: 25 marks**

**Question 2**

Your client has a choice between two investment portfolios:

(i) An actively managed portfolio with an expected rate of return of 8% and a standard deviation of 15%.

(ii) A passive portfolio tracking the FTSE All Share index which yields an expected rate of return of 6% with a standard deviation of 12%.

The risk free interest rate is 2%.

**(a)** Use the concept of capital allocation line (CAL) and capital market line (CML) to explain to the client which of the two portfolios provides a better choice of investment. You should support your explanation with drawings of CAL and CML and numerical calculations. **(12 marks)**

**(b)** What further advice might you offer the client if he believes that the market is efficient? **(3 marks)**

**(c)** Fund A produced a return of 14% and had a beta of 1.6 in 2017. In contrast Fund B produced a return of only 9% with a beta of 0.9.

If the risk free rate was 3% and the market return during the period was 9% which Fund produced the better performance? **(4 marks)**

**(d)** Explain briefly the concept of the Separation Theorem. **(6 marks)**

**Total: 25 marks**

**Question 3**

Based on current yields the expected rates of return on portfolios A and B are 5% and 10% respectively. The beta of A is 0.8 while that of B is 1.4. The risk free rate is currently 2%, while the expected rate of return on the FTSE 100 Index is 9%. The standard deviation of portfolio A is 10% annually, while that of B is 24%, and for the FTSE 100 index is 20%.

**(a)** If you currently hold a market index portfolio, would you choose to add either of these portfolios to your holdings? Explain your choice of method for evaluating the performance of the portfolios. **(9 marks)**

**(b)** If instead you could invest only in the risk free asset and one of these portfolios, which would you choose? Explain your choice of criterion for deciding between the two portfolios. **(9 marks)**

**(c)** In the Capital Asset Pricing model, discuss the relationship between systematic risk, unsystematic risk and beta. **(7 marks)**

**Total: 25 marks**

**Question 4**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Consensus estimates** | | |
| **Year end** | **December**  **2018** | **December**  **2019** | **December**  **2020** |
| Dividend per share (pence) | 20 | 24 | 28 |
| Earnings per share (pence) | 40 | 50 | 56 |

It is now March 2018. The above figures are consensus estimates for a FTSE 250 quoted company which is in a mature sector with stable growth in sales. The shares of the company are currently trading in the market for £10.00 per share. Its beta is 0.9, the risk free rate is 2.0% and market risk premium is 6%.

**(a)** Given that the consensus forecast for the firm’s long term dividend growth rate is 7%, estimate the intrinsic value of the firm using the multistage growth DDM model. **(6 marks)**

**(b)** Obtain another estimate of the firm’s intrinsic value using the multistage growth DDM model based on the information that the firm’s ROE has stabilised at 12%. **(7 marks)**

**(c)** Discuss which of the two intrinsic values you obtain in (a) and (b) would be more realistic in practice. **(5 marks)**

**(d)** Explain whether the following statements are true, false or uncertain according to the constant growth DDM model.

With all else held constant, a firm will have a higher P/E ratio if its beta is higher. **(2 marks)**

P/E ratio will tend to be higher when the growth rate of dividends is higher. **(2 marks)**

P/E ratio will tend to be higher when the plowback or retention ratio is higher. **(3 marks)**

**Total: 25 marks**

**Question 5**

**(a)** Details of two £ sterling bonds with par value of £100 issued by the same company and paying annual interest are provided below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Maturity** | **Coupon** | **YTM** | **Price** | **Duration** |
| Bond A | 5 years | 3% | 4% | £95.55 |  |
| Bond B | 3 years | 4% | 3% |  | 2.89 years |

What is the current price of Bond B? **(4 marks)**

What is the duration of Bond A? **(4 marks)**

Explain why the coupon rates on the two bonds are different **(2 marks)**

How would the price of Bond B change if the Yield to Maturity (YTM) decreased from 3% to 2%? **(3 marks)**

**(b)** The share price of Boring has been very close to £40 for a year and you think this will not change in the next few months.

The price of a three month put option with exercise price £40 is £4 and a call option for the same expiry date and exercise price sells for £7.

Define a simple strategy using a put and a call to produce a profit if your view on the future share price is correct. **(6 marks)**

What is the maximum profit that can be made with this strategy? **(2 marks)**

Show the profit or loss from this strategy if the share price of Boring in three months’ time is a range from £20 to £60 **(4 marks)**

**Total: 25 marks**

**Question 6**

**(a)** An investor considers buying a bond which pays a coupon rate of 8% per year semi-annually. The bond has five years until maturity and its par value is £1,000. The current market price for the bond is £950.

Calculate the yield to maturity of the bond and explain to the investor why, in practice, the yield to maturity may not be realised. **(15 marks)**

**(b)** The investor is also curious why the Treasury yield curve can have the two different shapes shown below. Identify the likely reasons underlying the two different shapes of yield curve and the information with regard to the future level of short rates and the state of the economy. **(10 marks)**

 **Total: 25 marks**

**Question 7**

**(a)** Use the Black-Scholes formula to find the exercise value and time value of a call option on the following share:

Time to expiration = 9 months

Standard deviation = 35% per year

Exercise price = £90

Share price = £100

Interest rate per annum = 3%

**(9 marks)**

**(b)** Provide a brief explanation for your answers to the following questions.

1. All else being equal, is a call option on a share with a lot of firm-specific risk worth more than one on a stock with little firm-specific risk? The betas of the shares are equal. **(3 marks)**
2. Would you expect a £1 increase in a call option’s exercise price to lead to a decrease in the option’s value of more or less than £1? **(3 marks)**

**(c)** Describe how the following financial activity may be viewed as transacting an option.

**i)** Buying shares in a company with assets of £A and debt £D. **(5 marks)**

**ii)** Making a loan of £B to the company. **(5 marks**)

**Total: 25 marks**

**(Paper Total: 100 marks)**

**END OF QUESTION PAPER**

**PLEASE SEE PAGES SEVEN TO TEN FOR FORMULAS AND TABLES**

**Formulas**

**Expected returns** 

**Variance **

**Standard Deviation **

**Reward-to-volatility (Sharpe ratio) **

**Utility **

**Real rate of return **

**Expected returns for combinations of risky asset and risk free asset**

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**Standard Deviation for combinations of risky asset and risk free asset**

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**Covariance **

**Correlation coefficient **

**Expected return for two risky assets **

**Variance for two risky assets **

**Single factor model **

**CAPM expected return-beta relationship **

**Bond Pricing (equivalent formulas)**







**Current Yield** Coupon/(Bond Price)

**Realised Compound Yield**  *Semi-annual coupon bond*

**Forward interest rate formula** 

**Dividend discount model: No Growth **

**Dividend discount model: Constant Growth **

**where** *g = b\*ROE, b = 1 – D1E1*

**Dividend discount model: Multistage Growth **

**Present value of growth opportunities **

**Price-earnings ratio: No Growth **

**Price-earnings ratio: Constant Growth **

**Black-Scholes formula (Call)** 

**where**  and 

**Call hedge delta** 

**Black-Scholes formula (Put)** 

**Put hedge delta** 

**Put-Call Parity** 

**Spot-Futures parity** 

**where** *d* is the dividend yield (), and D is dividend

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard normal distribution function *P*(*Z* < *z*)** | | | | | | | | | | | |
| *z* | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| -3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| -2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| -2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| -2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| -2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| -2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| -2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| -2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| -2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| -2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| -2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| -1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| -1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| -1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| -1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| -1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| -1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| -1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| -1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| -1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| -1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| -0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| -0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| -0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| -0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| -0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| -0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| -0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| -0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| -0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Present value factor = PVF(*r*, *T*) = 1 / (1 + r)T** | | | | | | | | | | | | | | | | | | | | |
| Interest rate *r* | | | | | | | | | | | | | | | | | | | | |
| *T* | 1% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% | 11% | 12% | 13% | 14% | 15% | 16% | 17% | 18% | 19% | 20% |
| 1 | 0.9901 | 0.9804 | 0.9709 | 0.9615 | 0.9524 | 0.9434 | 0.9346 | 0.9259 | 0.9174 | 0.9091 | 0.9009 | 0.8929 | 0.8850 | 0.8772 | 0.8696 | 0.8621 | 0.8547 | 0.8475 | 0.8403 | 0.8333 |
| 2 | 0.9803 | 0.9612 | 0.9426 | 0.9246 | 0.9070 | 0.8900 | 0.8734 | 0.8573 | 0.8417 | 0.8264 | 0.8116 | 0.7972 | 0.7831 | 0.7695 | 0.7561 | 0.7432 | 0.7305 | 0.7182 | 0.7062 | 0.6944 |
| 3 | 0.9706 | 0.9423 | 0.9151 | 0.8890 | 0.8638 | 0.8396 | 0.8163 | 0.7938 | 0.7722 | 0.7513 | 0.7312 | 0.7118 | 0.6931 | 0.6750 | 0.6575 | 0.6407 | 0.6244 | 0.6086 | 0.5934 | 0.5787 |
| 4 | 0.9610 | 0.9238 | 0.8885 | 0.8548 | 0.8227 | 0.7921 | 0.7629 | 0.7350 | 0.7084 | 0.6830 | 0.6587 | 0.6355 | 0.6133 | 0.5921 | 0.5718 | 0.5523 | 0.5337 | 0.5158 | 0.4987 | 0.4823 |
| 5 | 0.9515 | 0.9057 | 0.8626 | 0.8219 | 0.7835 | 0.7473 | 0.7130 | 0.6806 | 0.6499 | 0.6209 | 0.5935 | 0.5674 | 0.5428 | 0.5194 | 0.4972 | 0.4761 | 0.4561 | 0.4371 | 0.4190 | 0.4019 |
| 6 | 0.9420 | 0.8880 | 0.8375 | 0.7903 | 0.7462 | 0.7050 | 0.6663 | 0.6302 | 0.5963 | 0.5645 | 0.5346 | 0.5066 | 0.4803 | 0.4556 | 0.4323 | 0.4104 | 0.3898 | 0.3704 | 0.3521 | 0.3349 |
| 7 | 0.9327 | 0.8706 | 0.8131 | 0.7599 | 0.7107 | 0.6651 | 0.6227 | 0.5835 | 0.5470 | 0.5132 | 0.4817 | 0.4523 | 0.4251 | 0.3996 | 0.3759 | 0.3538 | 0.3332 | 0.3139 | 0.2959 | 0.2791 |
| 8 | 0.9235 | 0.8535 | 0.7894 | 0.7307 | 0.6768 | 0.6274 | 0.5820 | 0.5403 | 0.5019 | 0.4665 | 0.4339 | 0.4039 | 0.3762 | 0.3506 | 0.3269 | 0.3050 | 0.2848 | 0.2660 | 0.2487 | 0.2326 |
| 9 | 0.9143 | 0.8368 | 0.7664 | 0.7026 | 0.6446 | 0.5919 | 0.5439 | 0.5002 | 0.4604 | 0.4241 | 0.3909 | 0.3606 | 0.3329 | 0.3075 | 0.2843 | 0.2630 | 0.2434 | 0.2255 | 0.2090 | 0.1938 |
| 10 | 0.9053 | 0.8203 | 0.7441 | 0.6756 | 0.6139 | 0.5584 | 0.5083 | 0.4632 | 0.4224 | 0.3855 | 0.3522 | 0.3220 | 0.2946 | 0.2697 | 0.2472 | 0.2267 | 0.2080 | 0.1911 | 0.1756 | 0.1615 |
| 11 | 0.8963 | 0.8043 | 0.7224 | 0.6496 | 0.5847 | 0.5268 | 0.4751 | 0.4289 | 0.3875 | 0.3505 | 0.3173 | 0.2875 | 0.2607 | 0.2366 | 0.2149 | 0.1954 | 0.1778 | 0.1619 | 0.1476 | 0.1346 |
| 12 | 0.8874 | 0.7885 | 0.7014 | 0.6246 | 0.5568 | 0.4970 | 0.4440 | 0.3971 | 0.3555 | 0.3186 | 0.2858 | 0.2567 | 0.2307 | 0.2076 | 0.1869 | 0.1685 | 0.1520 | 0.1372 | 0.1240 | 0.1122 |
| 13 | 0.8787 | 0.7730 | 0.6810 | 0.6006 | 0.5303 | 0.4688 | 0.4150 | 0.3677 | 0.3262 | 0.2897 | 0.2575 | 0.2292 | 0.2042 | 0.1821 | 0.1625 | 0.1452 | 0.1299 | 0.1163 | 0.1042 | 0.0935 |
| 14 | 0.8700 | 0.7579 | 0.6611 | 0.5775 | 0.5051 | 0.4423 | 0.3878 | 0.3405 | 0.2992 | 0.2633 | 0.2320 | 0.2046 | 0.1807 | 0.1597 | 0.1413 | 0.1252 | 0.1110 | 0.0985 | 0.0876 | 0.0779 |
| 15 | 0.8613 | 0.7430 | 0.6419 | 0.5553 | 0.4810 | 0.4173 | 0.3624 | 0.3152 | 0.2745 | 0.2394 | 0.2090 | 0.1827 | 0.1599 | 0.1401 | 0.1229 | 0.1079 | 0.0949 | 0.0835 | 0.0736 | 0.0649 |
| 16 | 0.8528 | 0.7284 | 0.6232 | 0.5339 | 0.4581 | 0.3936 | 0.3387 | 0.2919 | 0.2519 | 0.2176 | 0.1883 | 0.1631 | 0.1415 | 0.1229 | 0.1069 | 0.0930 | 0.0811 | 0.0708 | 0.0618 | 0.0541 |
| 17 | 0.8444 | 0.7142 | 0.6050 | 0.5134 | 0.4363 | 0.3714 | 0.3166 | 0.2703 | 0.2311 | 0.1978 | 0.1696 | 0.1456 | 0.1252 | 0.1078 | 0.0929 | 0.0802 | 0.0693 | 0.0600 | 0.0520 | 0.0451 |
| 18 | 0.8360 | 0.7002 | 0.5874 | 0.4936 | 0.4155 | 0.3503 | 0.2959 | 0.2502 | 0.2120 | 0.1799 | 0.1528 | 0.1300 | 0.1108 | 0.0946 | 0.0808 | 0.0691 | 0.0592 | 0.0508 | 0.0437 | 0.0376 |
| 19 | 0.8277 | 0.6864 | 0.5703 | 0.4746 | 0.3957 | 0.3305 | 0.2765 | 0.2317 | 0.1945 | 0.1635 | 0.1377 | 0.1161 | 0.0981 | 0.0829 | 0.0703 | 0.0596 | 0.0506 | 0.0431 | 0.0367 | 0.0313 |
| 20 | 0.8195 | 0.6730 | 0.5537 | 0.4564 | 0.3769 | 0.3118 | 0.2584 | 0.2145 | 0.1784 | 0.1486 | 0.1240 | 0.1037 | 0.0868 | 0.0728 | 0.0611 | 0.0514 | 0.0433 | 0.0365 | 0.0308 | 0.0261 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Annuity Factors AF(*r*, *T*) = 1/*r* - 1/[*r*(1 + r)T]** | | | | | | | | | | | | | | | | | | | | |
| Interest rate *r* | | | | | | | | | | | | | | | | | | | | |
| *T* | 1% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% | 11% | 12% | 13% | 14% | 15% | 16% | 17% | 18% | 19% | 20% |
| 1 | 0.9901 | 0.9804 | 0.9709 | 0.9615 | 0.9524 | 0.9434 | 0.9346 | 0.9259 | 0.9174 | 0.9091 | 0.9009 | 0.8929 | 0.8850 | 0.8772 | 0.8696 | 0.8621 | 0.8547 | 0.8475 | 0.8403 | 0.8333 |
| 2 | 1.9704 | 1.9416 | 1.9135 | 1.8861 | 1.8594 | 1.8334 | 1.8080 | 1.7833 | 1.7591 | 1.7355 | 1.7125 | 1.6901 | 1.6681 | 1.6467 | 1.6257 | 1.6052 | 1.5852 | 1.5656 | 1.5465 | 1.5278 |
| 3 | 2.9410 | 2.8839 | 2.8286 | 2.7751 | 2.7232 | 2.6730 | 2.6243 | 2.5771 | 2.5313 | 2.4869 | 2.4437 | 2.4018 | 2.3612 | 2.3216 | 2.2832 | 2.2459 | 2.2096 | 2.1743 | 2.1399 | 2.1065 |
| 4 | 3.9020 | 3.8077 | 3.7171 | 3.6299 | 3.5460 | 3.4651 | 3.3872 | 3.3121 | 3.2397 | 3.1699 | 3.1024 | 3.0373 | 2.9745 | 2.9137 | 2.8550 | 2.7982 | 2.7432 | 2.6901 | 2.6386 | 2.5887 |
| 5 | 4.8534 | 4.7135 | 4.5797 | 4.4518 | 4.3295 | 4.2124 | 4.1002 | 3.9927 | 3.8897 | 3.7908 | 3.6959 | 3.6048 | 3.5172 | 3.4331 | 3.3522 | 3.2743 | 3.1993 | 3.1272 | 3.0576 | 2.9906 |
| 6 | 5.7955 | 5.6014 | 5.4172 | 5.2421 | 5.0757 | 4.9173 | 4.7665 | 4.6229 | 4.4859 | 4.3553 | 4.2305 | 4.1114 | 3.9975 | 3.8887 | 3.7845 | 3.6847 | 3.5892 | 3.4976 | 3.4098 | 3.3255 |
| 7 | 6.7282 | 6.4720 | 6.2303 | 6.0021 | 5.7864 | 5.5824 | 5.3893 | 5.2064 | 5.0330 | 4.8684 | 4.7122 | 4.5638 | 4.4226 | 4.2883 | 4.1604 | 4.0386 | 3.9224 | 3.8115 | 3.7057 | 3.6046 |
| 8 | 7.6517 | 7.3255 | 7.0197 | 6.7327 | 6.4632 | 6.2098 | 5.9713 | 5.7466 | 5.5348 | 5.3349 | 5.1461 | 4.9676 | 4.7988 | 4.6389 | 4.4873 | 4.3436 | 4.2072 | 4.0776 | 3.9544 | 3.8372 |
| 9 | 8.5660 | 8.1622 | 7.7861 | 7.4353 | 7.1078 | 6.8017 | 6.5152 | 6.2469 | 5.9952 | 5.7590 | 5.5370 | 5.3282 | 5.1317 | 4.9464 | 4.7716 | 4.6065 | 4.4506 | 4.3030 | 4.1633 | 4.0310 |
| 10 | 9.4713 | 8.9826 | 8.5302 | 8.1109 | 7.7217 | 7.3601 | 7.0236 | 6.7101 | 6.4177 | 6.1446 | 5.8892 | 5.6502 | 5.4262 | 5.2161 | 5.0188 | 4.8332 | 4.6586 | 4.4941 | 4.3389 | 4.1925 |
| 11 | 10.368 | 9.7868 | 9.2526 | 8.7605 | 8.3064 | 7.8869 | 7.4987 | 7.1390 | 6.8052 | 6.4951 | 6.2065 | 5.9377 | 5.6869 | 5.4527 | 5.2337 | 5.0286 | 4.8364 | 4.6560 | 4.4865 | 4.3271 |
| 12 | 11.255 | 10.575 | 9.9540 | 9.3851 | 8.8633 | 8.3838 | 7.9427 | 7.5361 | 7.1607 | 6.8137 | 6.4924 | 6.1944 | 5.9176 | 5.6603 | 5.4206 | 5.1971 | 4.9884 | 4.7932 | 4.6105 | 4.4392 |
| 13 | 12.134 | 11.348 | 10.635 | 9.9856 | 9.3936 | 8.8527 | 8.3577 | 7.9038 | 7.4869 | 7.1034 | 6.7499 | 6.4235 | 6.1218 | 5.8424 | 5.5831 | 5.3423 | 5.1183 | 4.9095 | 4.7147 | 4.5327 |
| 14 | 13.004 | 12.106 | 11.296 | 10.563 | 9.8986 | 9.2950 | 8.7455 | 8.2442 | 7.7862 | 7.3667 | 6.9819 | 6.6282 | 6.3025 | 6.0021 | 5.7245 | 5.4675 | 5.2293 | 5.0081 | 4.8023 | 4.6106 |
| 15 | 13.865 | 12.849 | 11.938 | 11.118 | 10.380 | 9.7122 | 9.1079 | 8.5595 | 8.0607 | 7.6061 | 7.1909 | 6.8109 | 6.4624 | 6.1422 | 5.8474 | 5.5755 | 5.3242 | 5.0916 | 4.8759 | 4.6755 |
| 16 | 14.718 | 13.578 | 12.561 | 11.652 | 10.838 | 10.106 | 9.4466 | 8.8514 | 8.3126 | 7.8237 | 7.3792 | 6.9740 | 6.6039 | 6.2651 | 5.9542 | 5.6685 | 5.4053 | 5.1624 | 4.9377 | 4.7296 |
| 17 | 15.562 | 14.292 | 13.166 | 12.166 | 11.274 | 10.477 | 9.7632 | 9.1216 | 8.5436 | 8.0216 | 7.5488 | 7.1196 | 6.7291 | 6.3729 | 6.0472 | 5.7487 | 5.4746 | 5.2223 | 4.9897 | 4.7746 |
| 18 | 16.398 | 14.992 | 13.754 | 12.659 | 11.690 | 10.828 | 10.059 | 9.3719 | 8.7556 | 8.2014 | 7.7016 | 7.2497 | 6.8399 | 6.4674 | 6.1280 | 5.8178 | 5.5339 | 5.2732 | 5.0333 | 4.8122 |
| 19 | 17.226 | 15.678 | 14.324 | 13.134 | 12.085 | 11.158 | 10.336 | 9.6036 | 8.9501 | 8.3649 | 7.8393 | 7.3658 | 6.9380 | 6.5504 | 6.1982 | 5.8775 | 5.5845 | 5.3162 | 5.0700 | 4.8435 |
| 20 | 18.046 | 16.351 | 14.877 | 13.590 | 12.462 | 11.470 | 10.594 | 9.8181 | 9.1285 | 8.5136 | 7.9633 | 7.4694 | 7.0248 | 6.6231 | 6.2593 | 5.9288 | 5.6278 | 5.3527 | 5.1009 | 4.8696 |