

EE320 Assignment

Xiaohan Huang | Reg No. 201920667

Q1.1: Using pen & paper, calculate the squared magnitude response of this system.

Because $H_1(z) = 1 + z^{-1}$ $H(z)|_{z=e^{j\Omega}} = H(e^{j\Omega})$

So the squared magnitude response is

$$\begin{aligned} |H(e^{j\Omega})|^2 &= H(e^{j\Omega}) \times H^*(e^{j\Omega}) = (1 + e^{-j\Omega})(1 + e^{j\Omega}) \\ &= 2 + e^{j\Omega} + e^{-j\Omega} = 2 + 2\cos\Omega \end{aligned}$$

Q1.2: A new system $H_2(z) = H_1(z^N)$, with $N \in \mathbb{N}$ (given as a return parameter N from the function call `room()`), arises through expansion of $h_1[n]$. Its impulse response $h_2[n]$ is obtained through expansion from $h_1[n]$: $h_2[n] = h_1[Nn]$, i.e. $h_2[n]$ contains the coefficients of $h_1[n]$, but each separated by $(N-1)$ zero coefficients:

```
h1 = [1 1];  
h2 = zeros(1, N*length(h1));  
h2(1:N:end) = h1;
```

Using pen & paper, What is the squared magnitude response of the expanded system

$h_2[n]$?

```
1 — x=audioread('FarEndSignal.wav');  
2 — sound(x, 8000);  
3 — xf=x;  
4 — [y, N]=room(xf, 201920667);  
5 — N
```

```
>> EE320  
  
N =  
  
30  
  
fx >>
```

According to the function call `room()`, we know $N=30$

So the squared magnitude response is

$$\begin{aligned} |H_2(e^{j\Omega})|^2 &= |1 + (e^{-j\Omega})^{30}|^2 = (1 + e^{-30j\Omega})(1 + e^{30j\Omega}) \\ &= 2 + e^{30j\Omega} + e^{-30j\Omega} = 2 + 2\cos 30\Omega \end{aligned}$$

Q1.3: How do the magnitude responses of $h_1[n]$ and $h_2[n]$ related to each other?

① the magnitude responses of $h_1[n]$ is $H_1(e^{j\Omega}) = 1 + e^{-j\Omega}$

② the magnitude responses of $h_2[n]$ is $H_2(e^{j\Omega}) = 1 + e^{-30j\Omega}$

Similar to the $H_1(z^N) = H_2(z) \implies H_2(e^{j\Omega}) = H_1(e^{Nj\Omega})$

$$\therefore H_2(e^{j\Omega}) = H_1(e^{32j\Omega})$$

Q2.1: Design a filter $h_3[n]$ and its N -fold expansion $h[n] = h_3[nN]$.

```
h3 = fir1(23,0.4);
```

```
h = zeros(1,N*length(h3));
```

```
h(1:N:end) = h3;
```

Plot its impulse and magnitude responses.

```
%Design a filter h3 and its N-fold expansion and plot its impulse and
```

```
%magnitude responses
```

```
h3 = fir1(23, 0.4);
```

```
h = zeros(1, N*length(h3));
```

```
h(1:N:end) = h3;
```

```
figure(1);
```

```
impz(h3);
```

```
figure(2);
```

```
fs=8000;
```

```
f=(0:length(h)-1)/length(h)*fs;
```

```
plot(f, abs(fft(h)));
```

```
xlabel('Frequency/[Hz]');
```

```
ylabel('Magnitude');
```

Figure 1

文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)

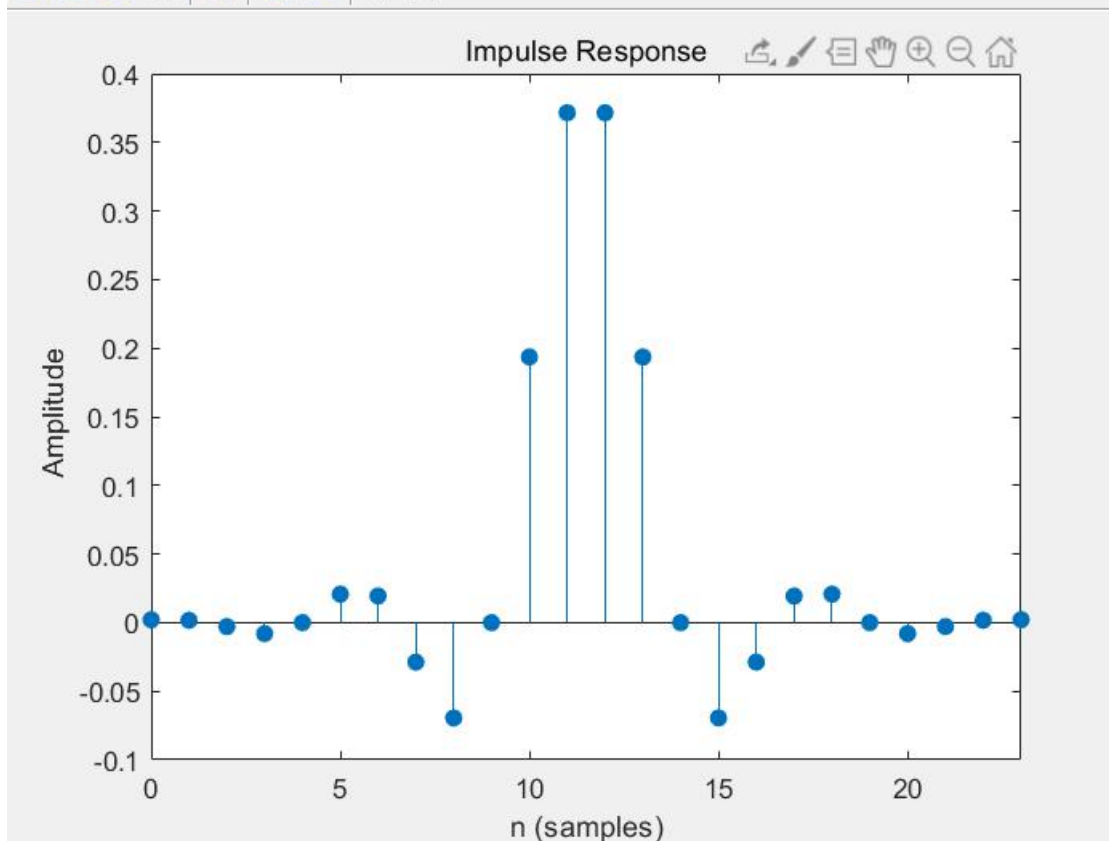
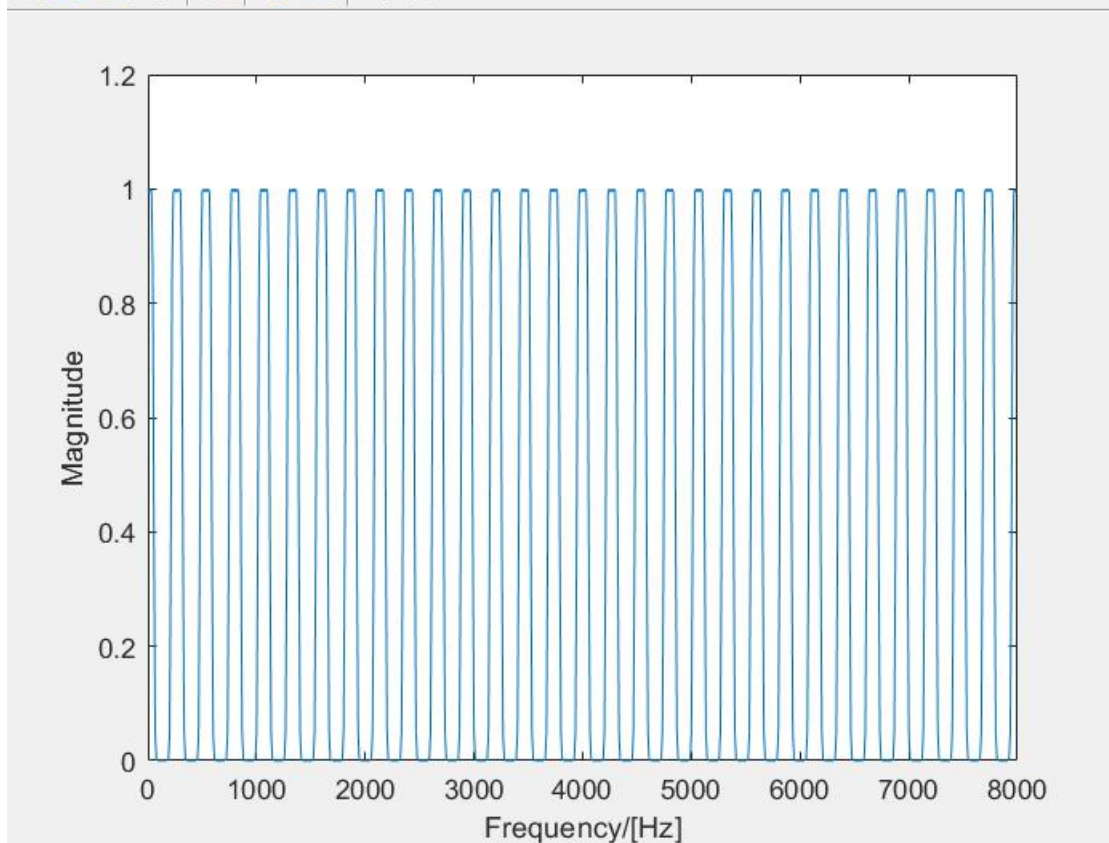


Figure 2

文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)



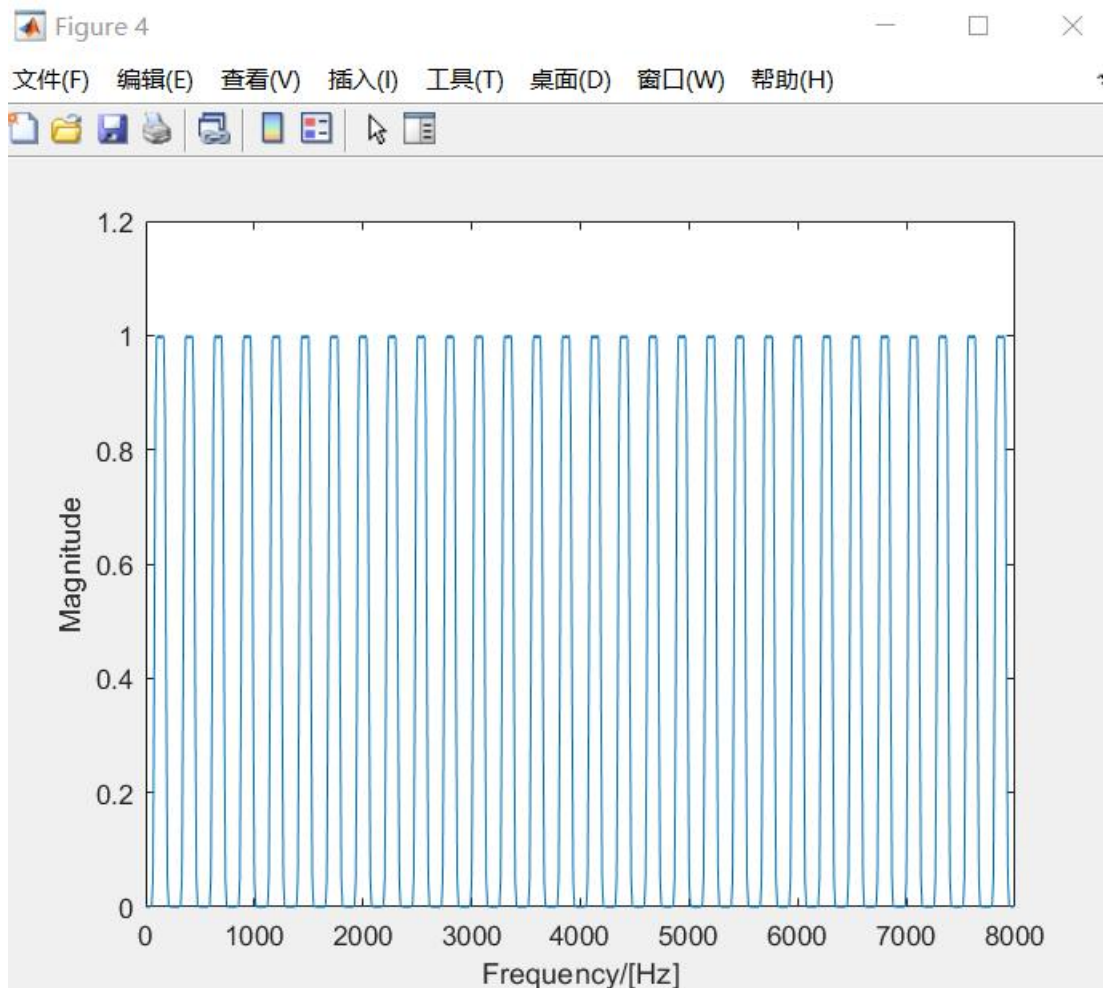
Q2.2: Create a filter $g_3[n] = ((1)nh_3[n]$ and its N-fold expansion $g[n] = g_3[nN]$. Plots its impulse and magnitude responses. How do the magnitude responses of $h[n]$ and $g[n]$ relate to each other?

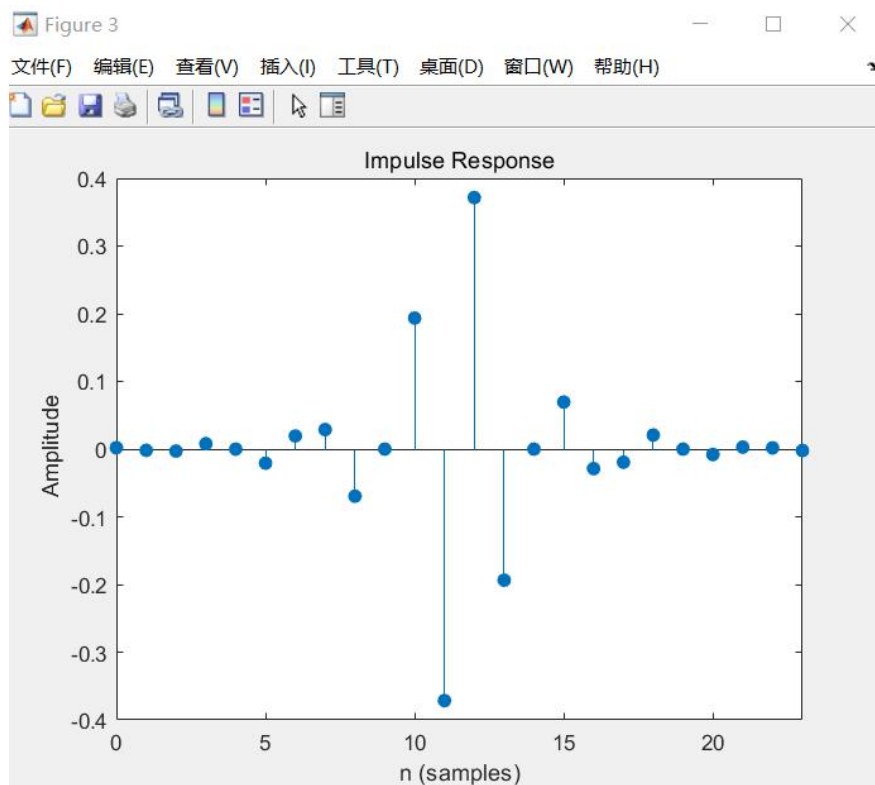
Because the odd terms should be negative so we code like this `g3 = (-1).^(0:23).*h3;`

```
%Design a filter g3[n] = ((1)nh3[n] and its N-fold expansion g[n] = g3[nN]
%Plots its impulse and magnitude responses.
g3=h3;
g3(2:2:end) = -g3(2:2:end);
g3 = (-1).^(0:23).*h3;
g = zeros(1,N*length(g3));
g(1:N:end) = g3;

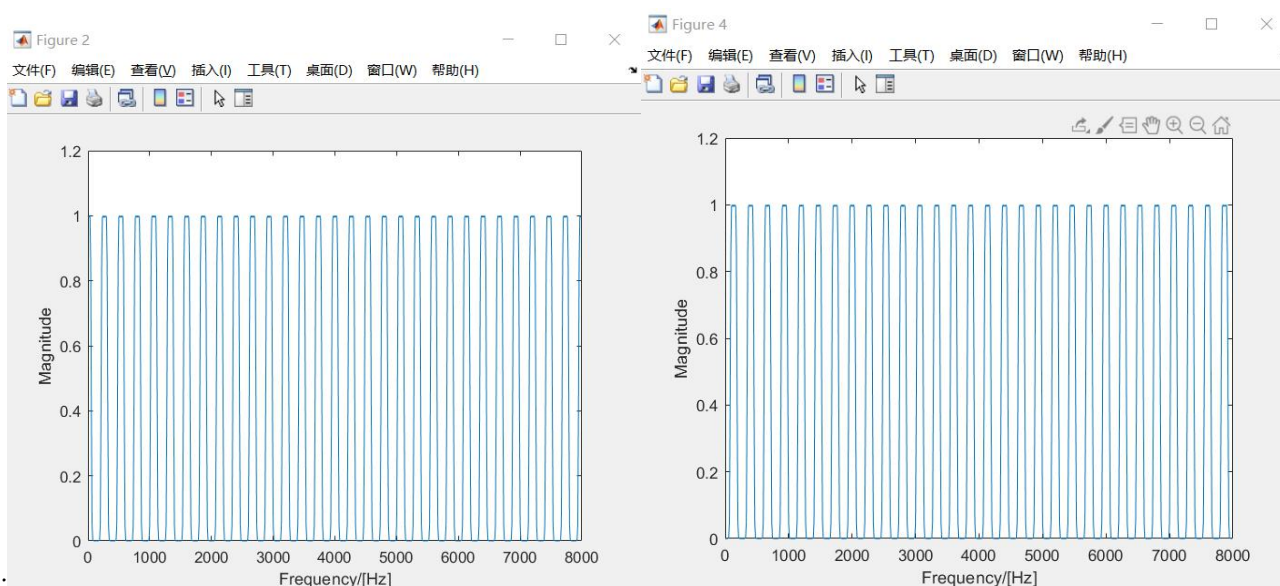
figure(3);
impz(g3);

figure(4);
fs=8000;
f=(0:length(g)-1)/length(g)*fs;
plot(f,abs(fft(g)));
xlabel('Frequency/[Hz]');
ylabel('Magnitude');
```





The relationship between $h[n]$ and $g[n]$: Comparing two magnitude responses figure



below:

we can see that: The Magnitude is same but the Frequency is a little different. As for $h[n]$ when the frequency is 0, the magnitude is 1. As for the $g[n]$, when the frequency is 0, the magnitude is 0.

Q3.1: Listen to the near-end signal $y[n]$ without any processing. What do you hear? By implementing the overall system according to Fig. 1 including the systems $h[n]$ and $g[n]$ as defined in Q2. (you may use Matlab's `filter()` command), what does $y_f[n]$ sound like?

Listen to the near-end signal $y[n]$ without any processing: I can hear the echo and noise.

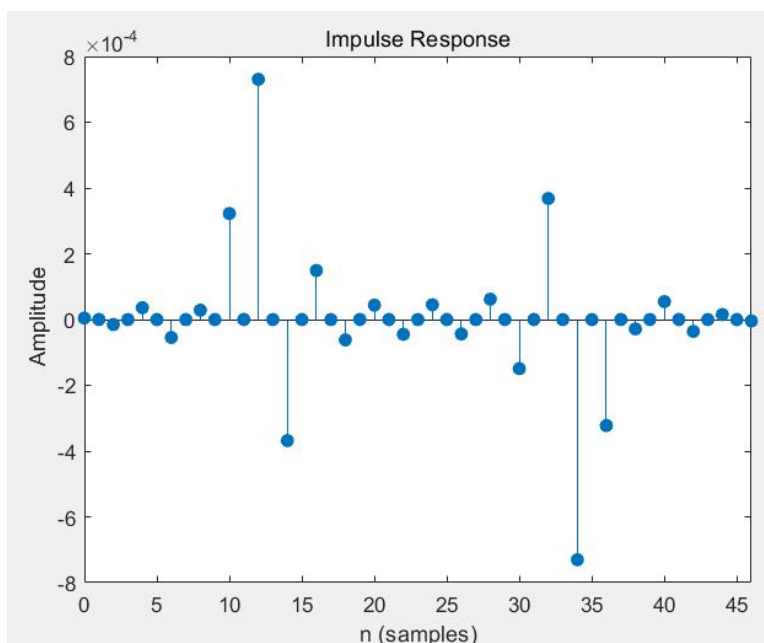
Listen to the signal $yf[n]$: I can hear that the noise is hardly to hear but it exists. As for the echo, it is reduced.

Q3.2: If so, why is the echo of Bob reduced? Also, in case Prof Stewart's voice is distorted, why could this be the case?

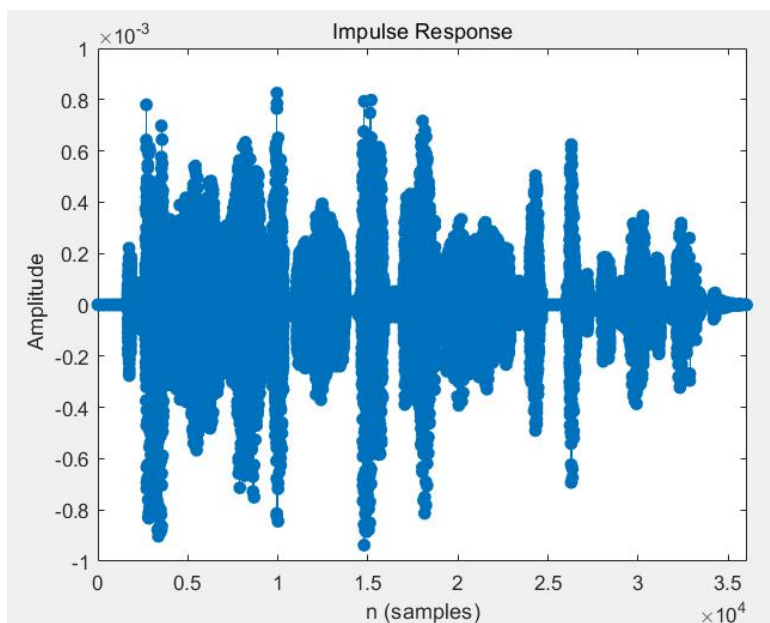
```
% %calculate CONVOLUTION
C1=conv(g3,h3);
figure(5);
impz(C1);

C2=conv(C1,x);
figure(6);
impz(C2)
```

The reason of why the echo of Bob reduced: let $g3[n]$ and $h3[n]$ be convoluted. Then we get a figure C1:



And then convolution of the below figure C1 and x , and we get a figure C2:



We can see that the voice has already been reduced.

The distortion of Prof Stewart's voice : We also can see that there are some times with zeros the voice is disappeared from the figure C2.

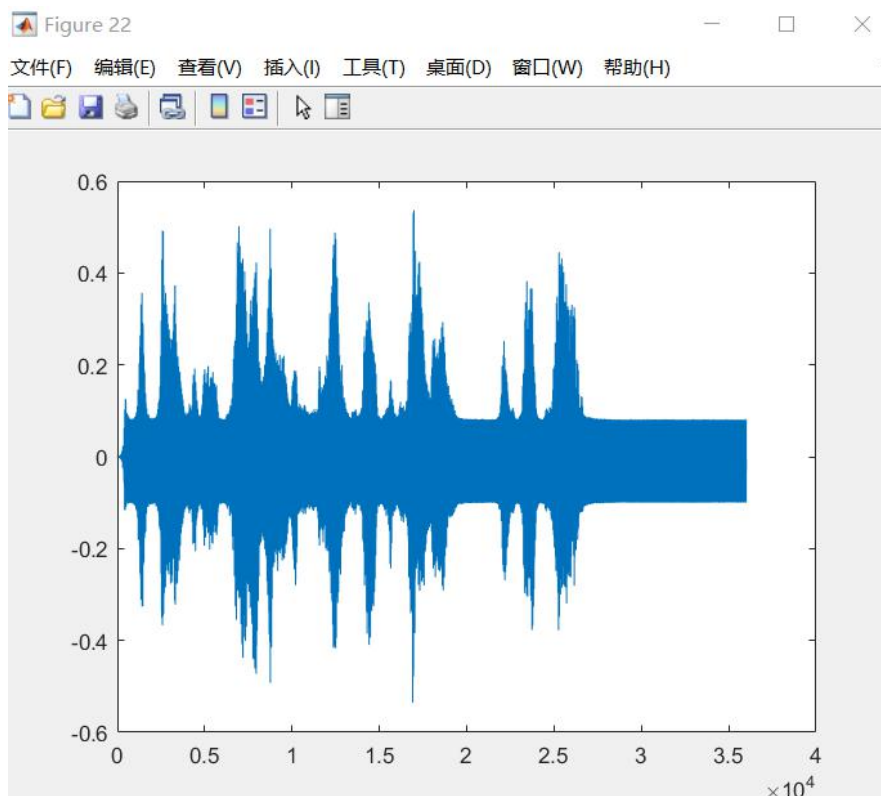
Q3.3: Implement the filter without using any of Matlab's provided functions. How can you minimise the computational complexity of your implementation?

```
%recit the FarEndSignal.wav file and filter it
x=audioread('FarEndSignal.wav');
xf=filter(g,1,x);
[y,N]=room(xf,201920667);
yf=filter(h,1,y);
% sound(yf,8000);

%3.3 NOTE 264/306 FIR Filter Implementation
N1=length(h); % filter length
y_tdl=zeros(N1,1); % TDL vector initialisation
for n=1:length(y) % iteration --- once per sampling period

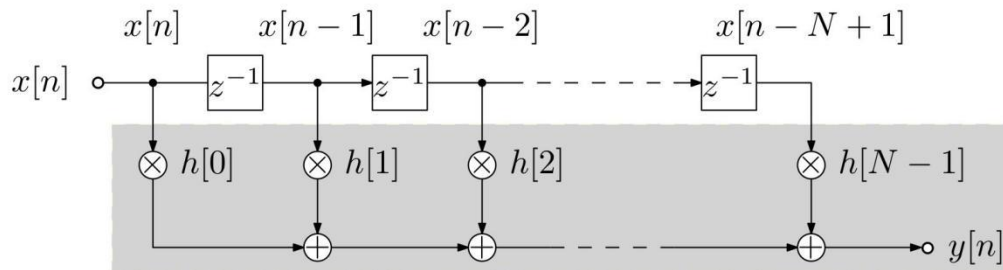
    % step 1: update TDL with latest samples
    y_tdl = [y(n); y_tdl(1:(N1-1))];

    % step 2: calculate output y[n] using scalar product
    yfn(n) = h*y_tdl;
end;
figure(22);
plot(yfn);
```



11.4 FIR Filter Implementation

- To implement the FIR structure below, we require 2 steps per sampling period

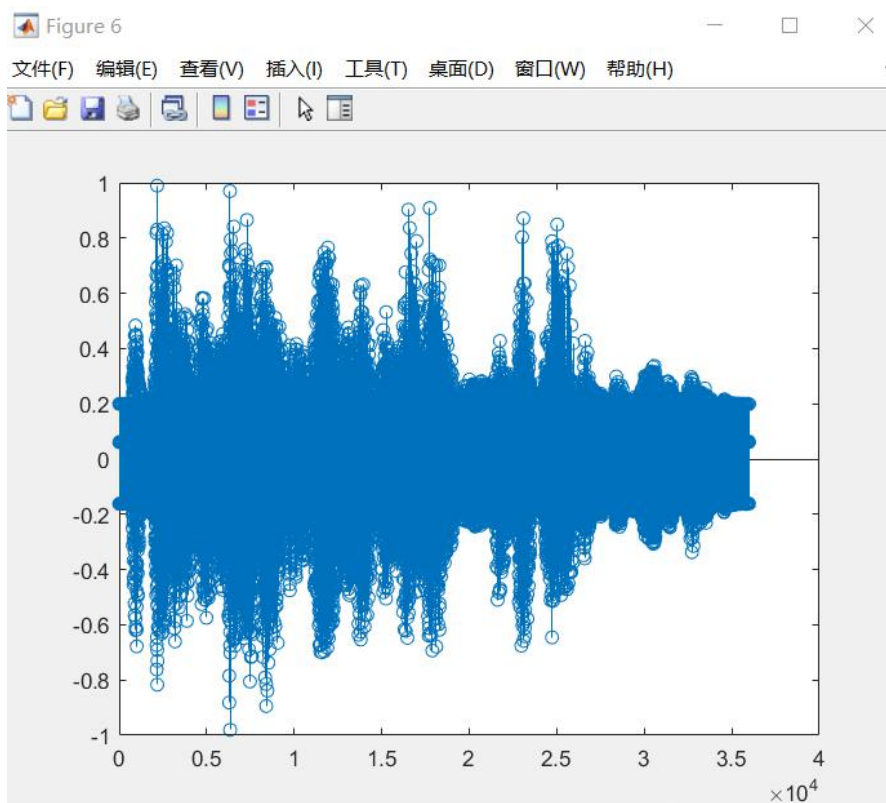


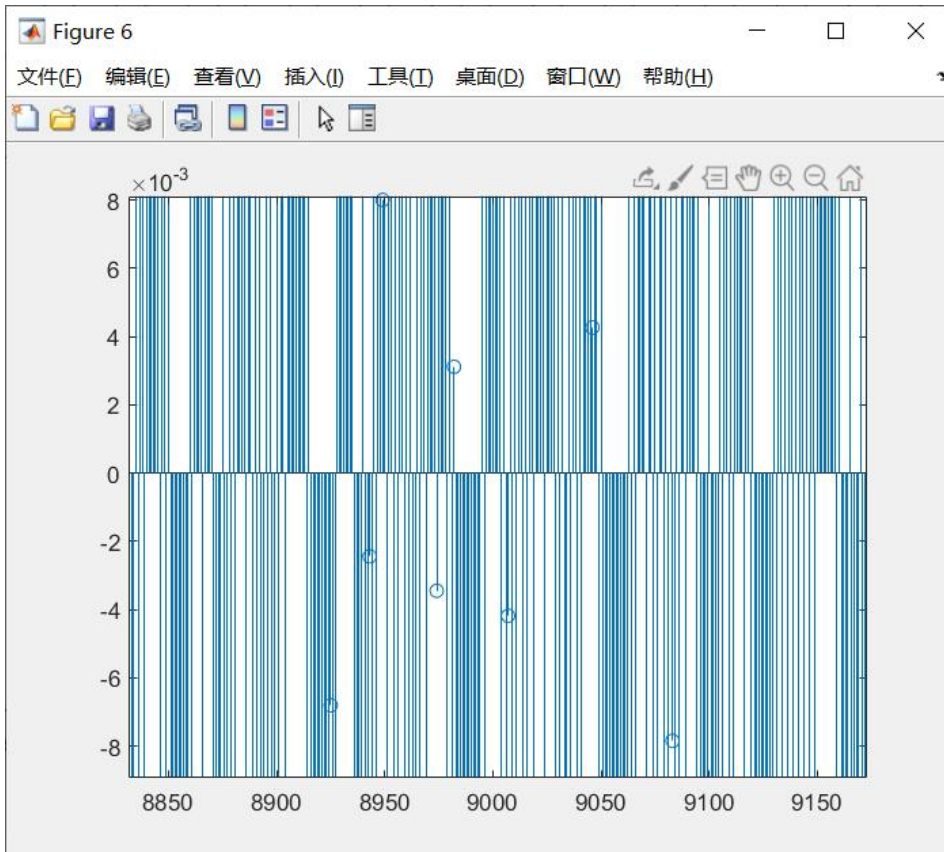
- *Step 1:* update the tapped-delay line with the most recent sample $x[n]$;
- *Step 2:* multiply the data samples with the coefficients, and accumulate the results to determine the output $y[n]$;
- then increment the time index $n \rightarrow n + 1$ and repeat ...

261 / 306

Q4.1: inspecting the time domain waveform $y[n]$ by finding a segment where the signal is otherwise quiet (i.e. interference dominates)

```
%inspecting the time domain waveform y[n]
figure(6);
stem(y);
```





Q4.2: evaluating the discrete-time Fourier transform

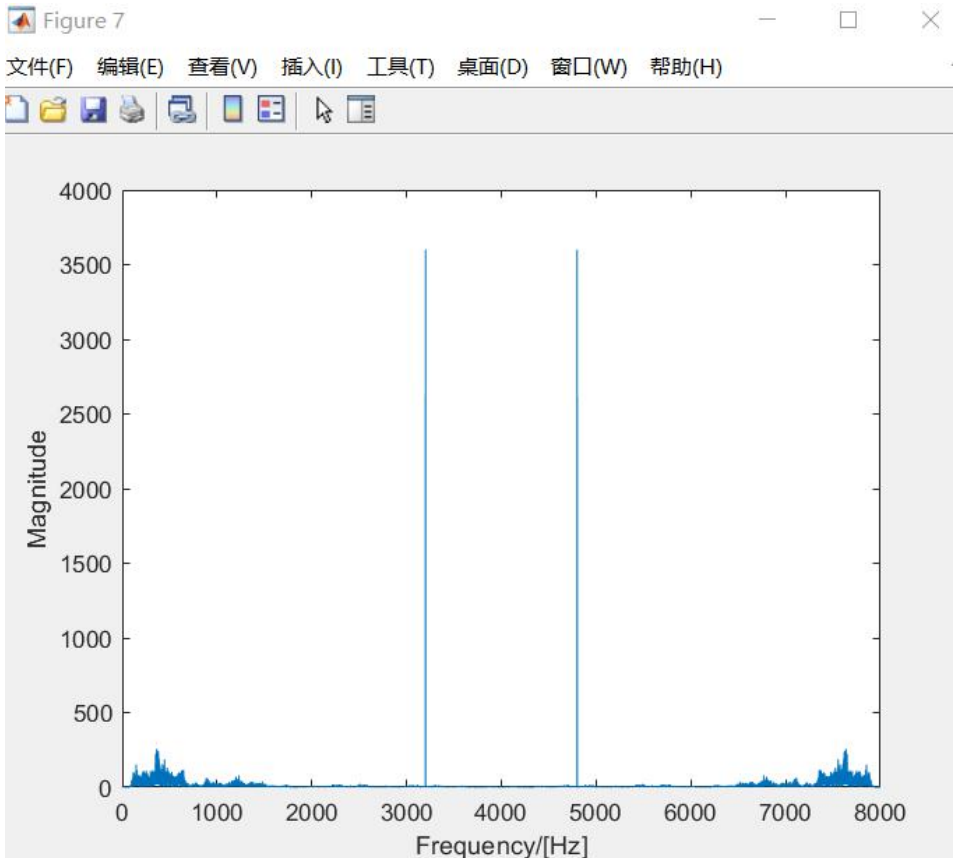
```
fs = 8000; % sampling frequency in Hertz
f = (0:length(y)-1)/length(y)*fs; % frequency scale plot(f,abs(fft(y)));
Plot(f,abs(fft(y))) % discrete Fourier transform of x

xlabel('frequency f / [Hz]'); % label axes

ylabel('magnitude');
```

Provide plots for the time domain segment in Q4.1 and the magnitude spectrum in

```
%Provide plots for the time domain segment in Q4.1 and the magnitude spectrum in Q4.2
figure(7);
fs=8000;
f=(0:length(y)-1)/length(y)*fs;
plot(f, abs(fft(y)));
xlabel('Frequency/[Hz]');
ylabel('Magnitude');
```



X 3200
Y 3600

So we can see that: $f_0 = 3200$

and then calculate the normalized angular frequency: $\Omega_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{3200}{8000} \approx 2.5133$

Fourier Transform



- Fourier transform of $x_s(t)$:

$$\begin{aligned} X_s(j\omega) &= \int_{-\infty}^{\infty} x_s(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[n] \delta(t - nT_s) e^{-j\omega t} dt \\ &= \sum_{n=-\infty}^{\infty} x[n] \int_{-\infty}^{\infty} \delta(t - nT_s) e^{-j\omega t} dt \end{aligned} \quad (161)$$

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega nT_s} \quad (162)$$

with the step from (161) to (162) exploiting the sifting property of $\delta(t)$;

- we define the normalised angular frequency

$$\Omega = \omega T_s = 2\pi \frac{f}{f_s} = 2\pi \frac{\omega}{\omega_s} \quad (163)$$

- note that the normalised angular sampling frequency $\Omega_s = 2\pi$, which defines the periodicity of the spectrum.

Q5.1: Determine the denominator and numerator coefficients a_i and b_i , $i = 0, 1, 2$ when written as

To suppress the sinusoidal interference, we want to utilise an IIR notch filter with transfer function

$$Q(z) = \frac{(1 - e^{j\Omega_0} z^{-1})(1 - e^{-j\Omega_0} z^{-1})}{(1 - \rho e^{j\Omega_0} z^{-1})(1 - \rho e^{-j\Omega_0} z^{-1})}$$

to operate on $y_f[n]$ and generate an output $y_H[n]$. For implementations and analysis below, please assume a value $\rho = 0.9$.

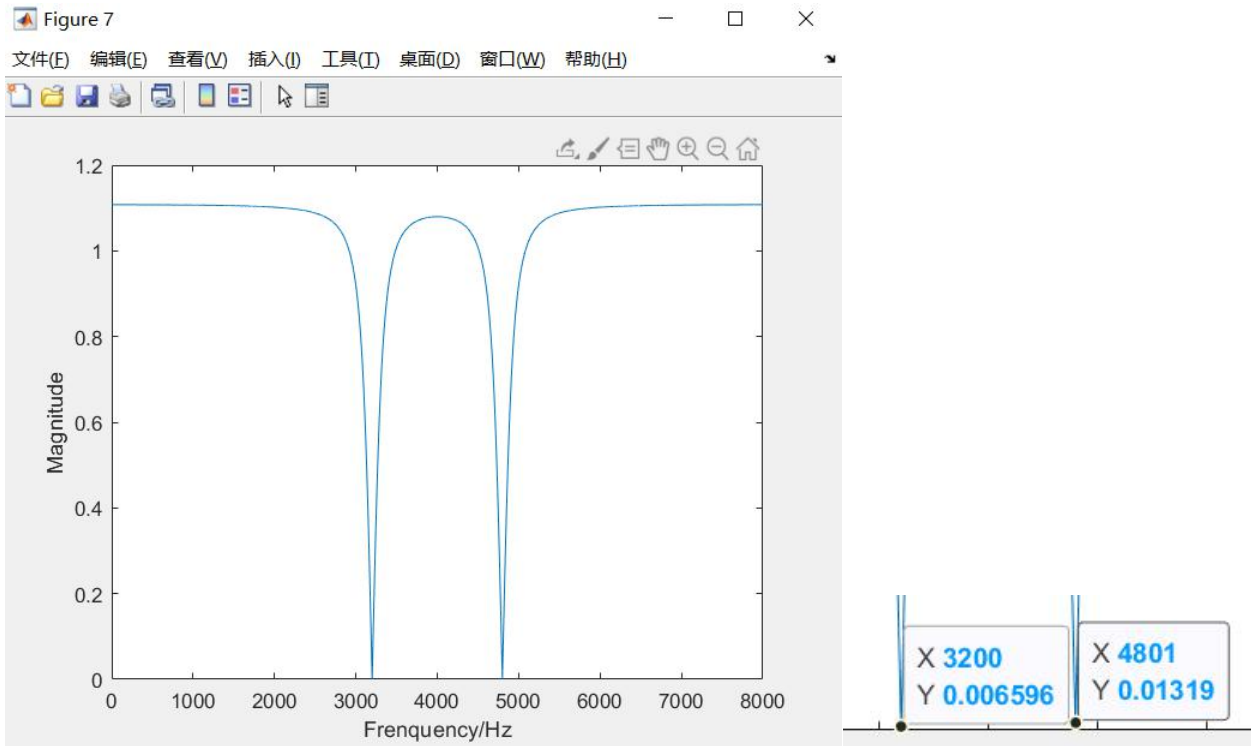
$$Q(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

and plot the magnitude response $|H_2(e^{j\Omega})|$ in Matlab;

$$\begin{aligned} Q(z) &= \frac{(1 - e^{j\Omega_0} z^{-1})(1 - e^{-j\Omega_0} z^{-1})}{(1 - \rho e^{j\Omega_0} z^{-1})(1 - \rho e^{-j\Omega_0} z^{-1})} \quad (\rho = 0.9) \\ &= \frac{1 + z^{-2} - e^{j\Omega_0} z^{-1} - e^{-j\Omega_0} z^{-1}}{1 + \rho^2 z^{-2} - \rho e^{j\Omega_0} z^{-1} - \rho e^{-j\Omega_0} z^{-1}} \\ &= \frac{1 - 2\cos\Omega_0 z^{-1} + z^{-2}}{1 - 1.8\cos\Omega_0 z^{-1} + 0.81z^{-2}} \end{aligned}$$

%Plot the magnitude response $|H_2(e^{j\Omega})|$ in Matlab

```
f1=(0:0.0001:1)*2*pi;
c=cos(3200*2*pi/8000);
H2=(1-2*c*exp(f1*i*(-1))+exp(i*f1*(-2)))/(1-1.8*c*exp(i*(-1)*f1)+0.81*exp(i*f1*(-2)));
f=(0:length(H2)-1)/length(H2)*fs;
plot(f,abs(H2));
xlabel('Frequency/Hz');
ylabel('Magnitude');
```



We can see that when the frequency is 3200 and 4801 the magnitude is zero.

Q5.2: with the Matlab coefficient vectors a and b containing the coefficients determined in (2), filter your music signal y with the IIR notch filter, and listen to it:

```
yff = filter(b,a,yf);      % yf is the input of filter q
sound(yff,8000);          % play back the filtered signal
```

Is the sinusoidal interference suppressed in yff?

We can know that:

$$a = [1 \quad -18 \cdot \cos(2 \cdot \pi \cdot 3200 / 8000) \quad 0.81]$$

$$b = [1 \quad -2 \cdot \cos(2 \cdot \pi \cdot 3200 / 8000) \quad 1]$$

From the yff signal we can see that the sinusoidal interference has been reduced in yff without noise.


```

%filter music signal y with the IIR notch filter
a=[1 -1.8*cos(2*pi*3200/8000) 0.81];
b=[1 -2*cos(2*pi*3200/8000) 1];
yff = filter(b, a, yf);      % yf is the input of filter q
sound(yff,8000);             % play back the filtered signal

```

```

figure(8);
fs=8000;
f=(0:length(yff)-1)/length(yff)*fs;
plot(f, abs(fft(yff)));
xlabel('Frequency/[Hz]');
ylabel('Magnitude');

```

```

figure(9);
fs=8000;
f=(0:length(yf)-1)/length(yf)*fs;
plot(f, abs(fft(yf)));
xlabel('Frequency/[Hz]');
ylabel('Magnitude');

```

```

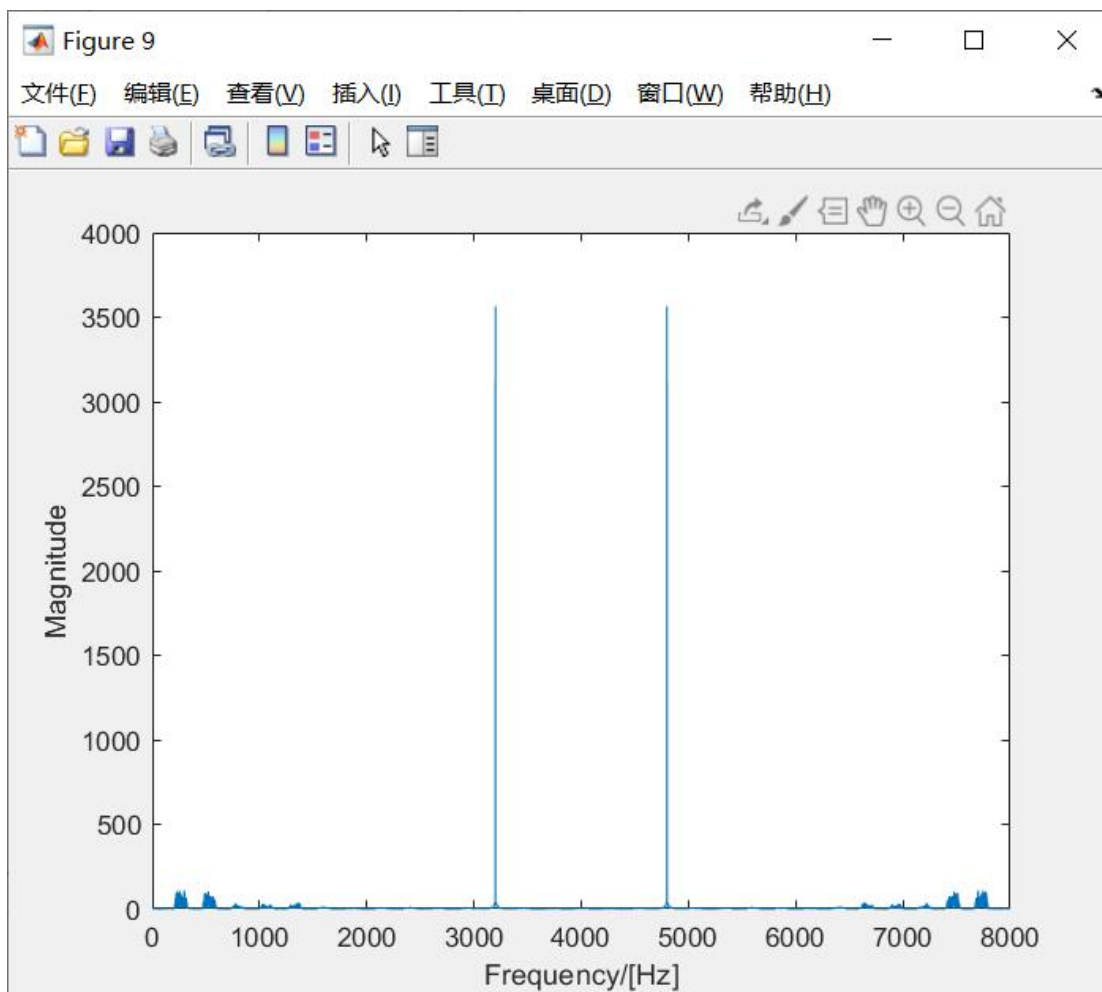
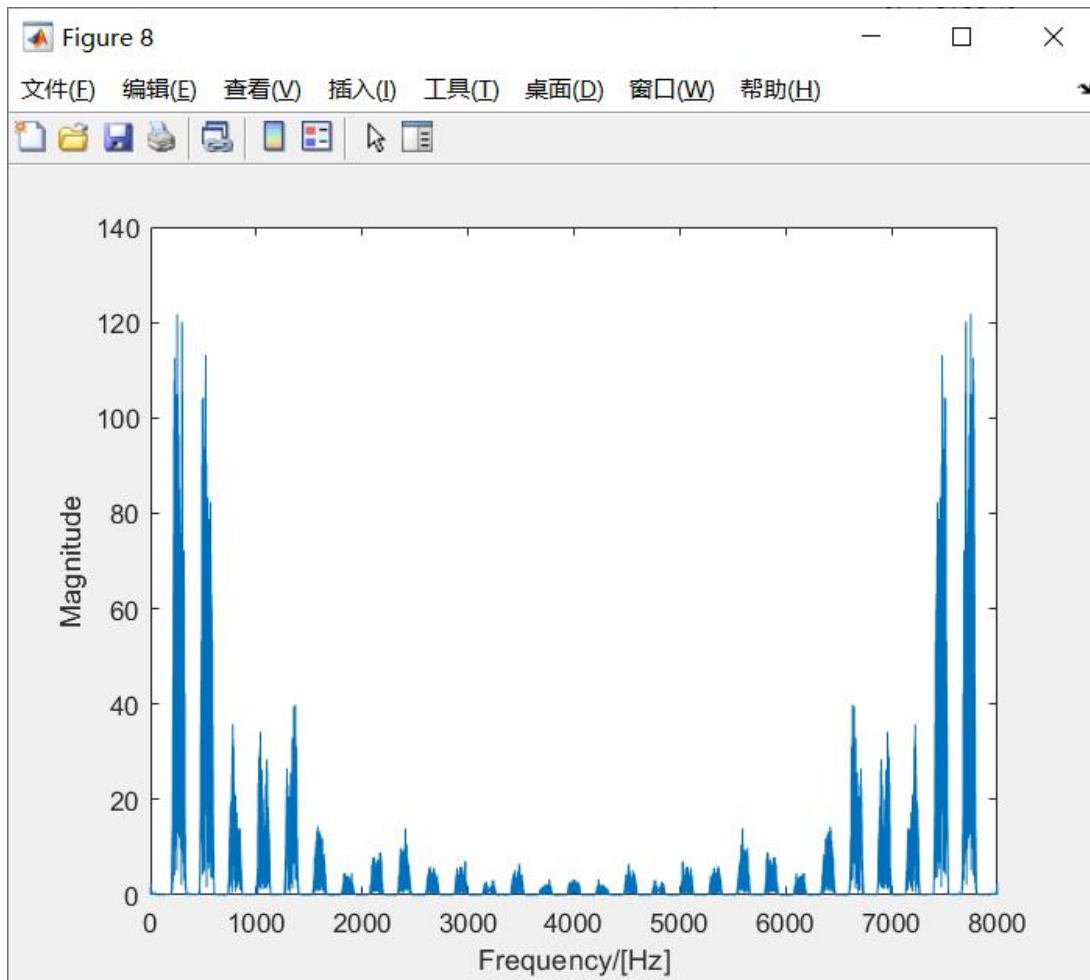
figure(10);
stem(yf);

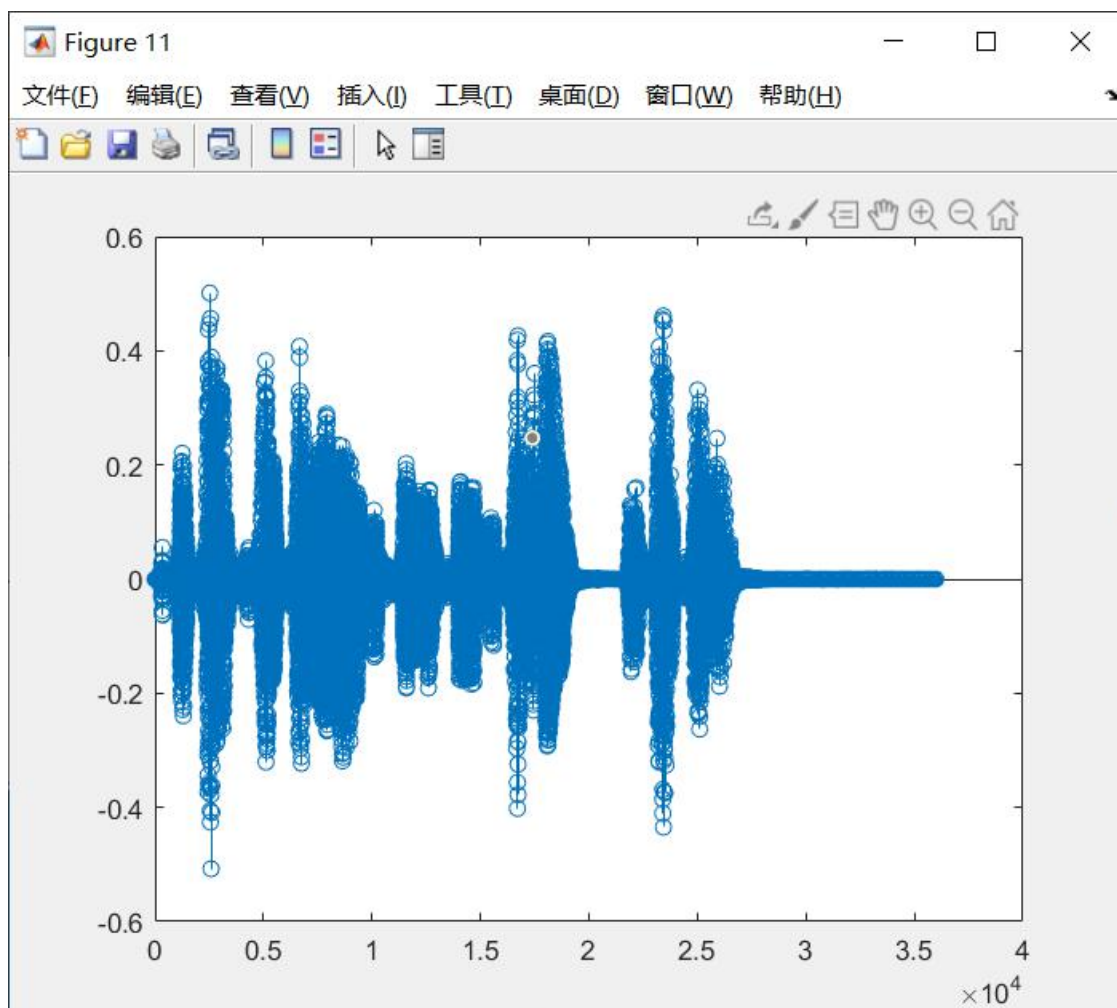
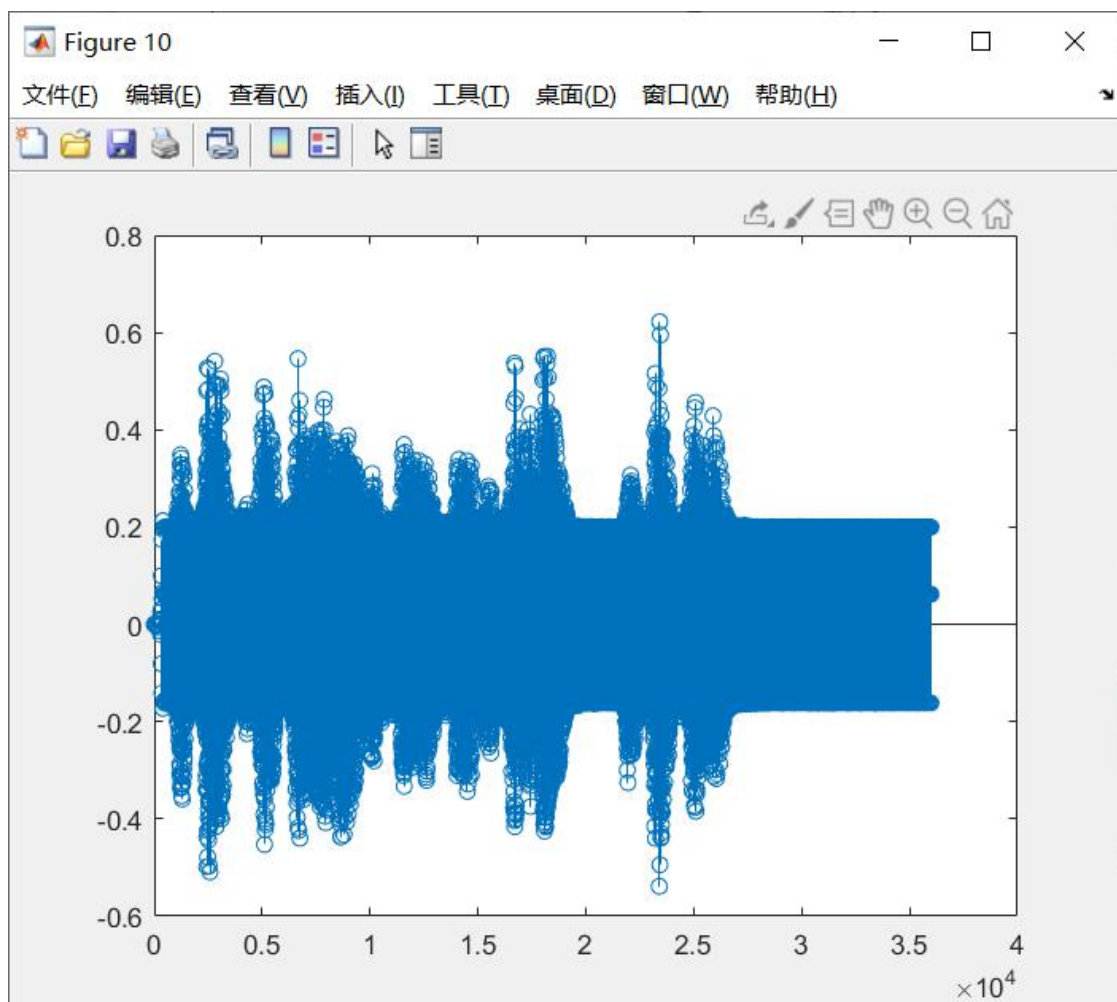
```

```

figure(11);
stem(yff);

```





Q5.3: Can you swap the filters $h[n]$ and $q[n]$ without affecting the overall performance? Is so, why?

Yes, the reason can be shown from the lecture slide.

Commutativity



- ▶ Because $Y(s) = H(s)X(s) = X(s)H(s)$, can we swap arguments of the convolution, e.g. $h(t) = h(t) * x(t) = x(t) * h(t)$?
- ▶ checking:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

- ▶ substitution $\mu = t - \tau$, therefore $d\mu = -d\tau$

$$\begin{aligned} y(t) &= \int_{+\infty}^{-\infty} h(t - \mu)x(\mu)(-d\mu) \\ &= \int_{-\infty}^{+\infty} x(\mu)h(t - \mu)d\mu \\ &= x(t) * h(t) \end{aligned}$$

- ▶ yes we can! Convolution is commutative.

Q5.4: Could you attenuate the interference without using $q[n]$, and without further deteriorating Prof Stewart's voice?

We can know that the $g[n]$ can not allow the noise go. So we can use the filter $g[n]$ to attenuate the noise.

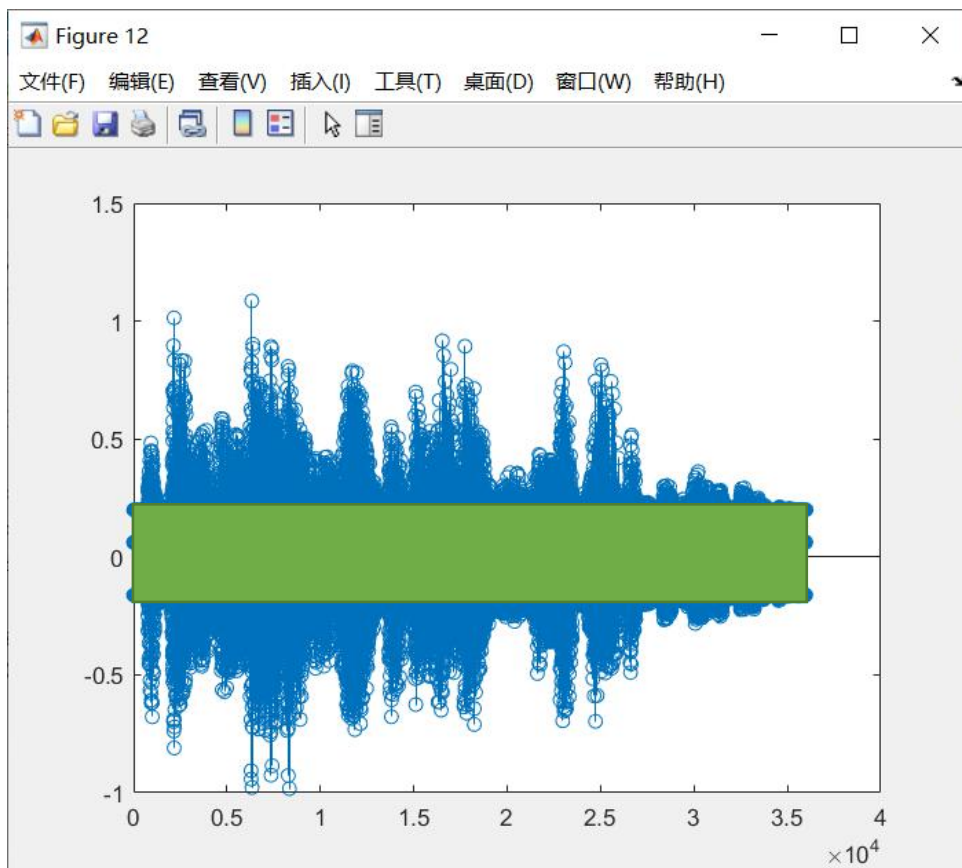
```
%Q5.5 without q[n] to attenuate the noise
```

```
x=audioread('FarEndSignal1.wav');  
xf=filter(h, 1, x);  
[y, N]=room(xf, 201920667);  
yf=filter(g, 1, y);  
sound(yf, 8000);
```

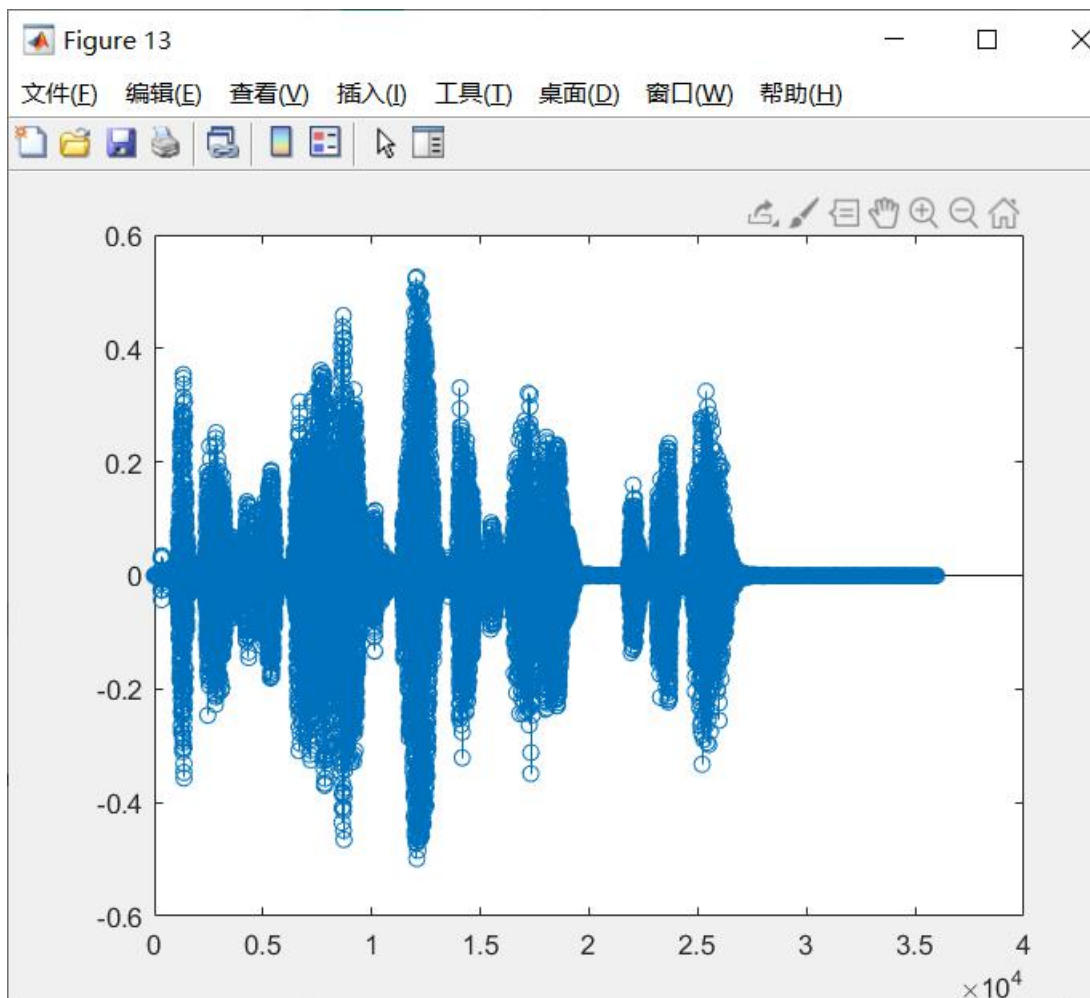
```
figure(12);  
stem(y);
```

```
figure(13);  
stem(yf);
```

And the figure (12) is the $y[n]$ which is not after filter.



We can see that the figure(12) before the filter, has noise which is the be circled by green pen. But after the filter $g[n]$ this changes to the figure(13) :



The noise part is not like the one in figure(12), it becomes smaller. This because when convolution with $g[n]$, when the frequency equal 3200 the magnitude response is zero.

26, Jan 2020