Pre-equilibrium nuclear reactions: An introduction to classical and quantum-mechanical models

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I. INTRODUCTION

A. Classification of nuclear reactions

The reaction of an incident particle with an atomic nucleus can take place in many ways. In studies of light-ion induced nuclear reactions one distinguishes between three different mechanisms: direct, compound and pre-equilibrium nuclear reactions. These reaction processes can be subdivided according to time scales or, equivalently, the number of intranuclear collisions taking place before emission. Furthermore, each mechanism preferentially excites certain parts of the nuclear level spectrum and is characterized by different types of angular distributions. The overall situation is depicted in fig. 1.

Direct processes are characterized by short reaction times ($\sim 10^{-22}$ seconds, which is roughly equal to the time it takes the incident particle to traverse the target nucleus) and a strong correlation between the initial and final channel of the reaction. For incident energies above about 10 MeV, the low-lying, discrete states of the residual nucleus are almost completely excited by direct reaction processes. The associated angular distributions are strongly peaked in the forward direction and exhibit markedly oscillatory behaviour. This specific oscillatory shape enables the determination of the spin and parity of the residual nuclear state. The analysis of direct reactions is almost invariably performed with either a DWBA (Distorted Wave Born Approximation) or a coupled channels approach [1,2]. *Compound processes* involve long reaction times ($\sim 10^{-18}$ seconds) and are predominant at low energies (below 10 MeV). In contrast with direct reactions, which are mostly one-step processes, the compound reaction mechanism is known to proceed by many intranuclear collisions. The incident particle is captured by the target nucleus to form a compound nucleus. Subsequently, the incident energy is shared among the other nucleons and after a long time a sufficient amount of energy may be accumulated for one nucleon (or group of nucleons) to escape. Apart from conservation of total energy and total angular momentum, the outgoing and incident channel are completely uncorrelated.

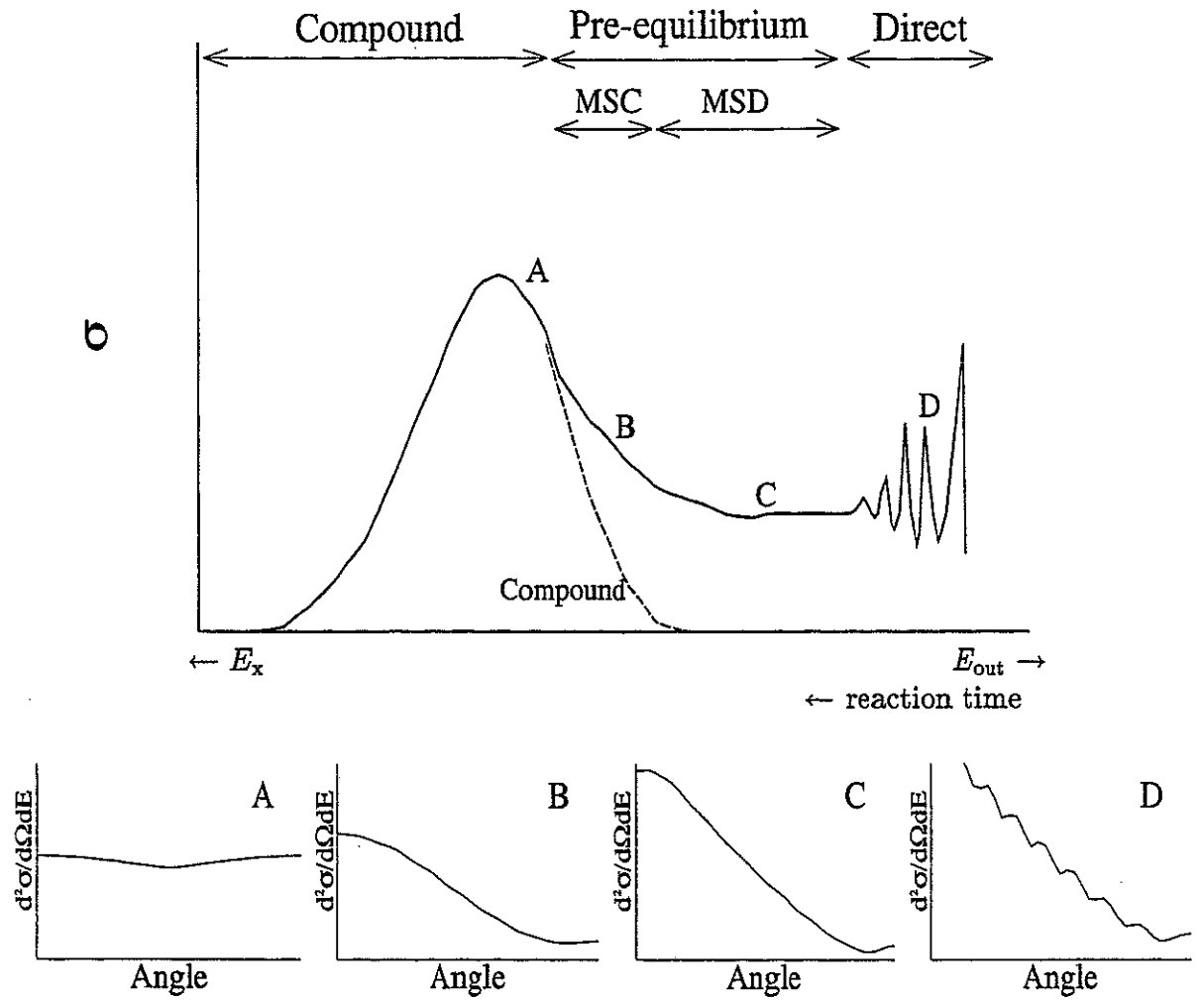

FIG. 1. Typical energy spectrum of a reaction A(a,b)B with an incident energy of several tens of MeV. The associated angular distributions show a gradual transition to isotropy for decreasing outgoing energies.

This is sometimes referred to as Bohr's amnesia hypothesis [3]. The memory loss of the compound nucleus is confirmed by the associated symmetric angular distribution. The nuclear theories that describe compound reactions in the continuum (see [4] for a review), are of a statistical nature.

As indicated in 1961 by Weisskopf [5] the separation of nuclear reaction mechanisms into direct and compound is too simplistic. The cross section as predicted by the pure compound process is too small with respect to the measured spectrum (see fig. 1). Furthermore, the measured angular distributions in this intermediate region are anisotropic, indicating the existence of a memory-preserving, direct-like reaction process. Apparently, as an intermediate between the aforementioned two extremes, there exists a reaction type that

embodies both direct- and compound-like features. These reactions are referred to as *pre-equilibrium, precompound* or, usually in a quantummechanical context, *multi-step processes.* Pre-equilibrium emission takes place after the first stage of the reaction but long before statistical equilibrium of the compound nucleus is attained. It is imagined that the incident particle step-by-step creates more complex states of the compound system and gradually loses its memory of the initial energy and direction. Pre-equilibrium processes provide a sizeable part of the reaction cross section for incident energies between 10 and (at least) 200 MeV.

B. Classical vs. quantum-mechanical pre-equilibrium reaction theory

Most widely used in explaining the characteristics of pre-equilibrium reactions are the phenomenological models (see [6,7] for extensive reviews). These models, which are of an essentially classical nature, embody the stepwise creation (and annihilation) of particle-hole pairs. The bookkeeping parameter of this process is the exciton number (number of particles + holes). The statistical aspect of the model comes in because many particle-hole states are involved in the process. A time-dependent master equation describes the probability of transitions to more and less complex particle-hole states as well as transitions to the continuum (emission). Upon integration over time the energy-averaged emission spectrum is obtained. A salient feature of classical pre-equilibrium theory is that the state transitions are imagined to be energy conserving. This aspect is mainly responsible for the classical flavour of the phenomenological models. Furthermore, it causes the typical and attractive convolution-type mathematical structure of these models [8]. In its original form [9,10], the exciton model was only able to predict angle-integrated spectra. This deficiency was removed in [11] with the introduction of the generalized exciton model. In this model, an angle-dependent scattering kernel is included in the master equation. Hence, besides the energy of the leading particle, its direction is also followed throughout the reaction process. In [12], this model was put in a suitable mathematical form, enabling simple but effective practical calculations of both energy spectra and angular distributions. Furthermore, in [13] it is demonstrated that in the equilibrium limit the Weisskopf-Ewing model and the Hauser-Feshbach model emerge as special cases of certain classes of exciton models.

Despite its general success, the exciton model is subject to some criticism. The most important one is that it is classical. This is unsatisfactory since the related theories (direct and compound) are quantum-mechanical. This calls for a quantum-mechanical pre-equilibrium theory that provides a better founded and more natural link between the theories governing the various regions of the energy spectrum. Such a quantum-mechanical theory is not only of theoretical interest. One of the practical problems of the exciton model is the systematical underestimation of the double-differential cross section at backward angles [6,12]. It is expected, and demonstrated, that quantum-mechanical pre-equilibrium models provide a better prediction of the angular distributions.

Quantum-mechanical descriptions of pre-equilibrium reactions have been presented in [14–16]. The distinction between multi-step compound (MSC) and multi-step direct (MSD) processes in the continuum was first proposed in [16]. These two complementary processes are depicted in fig. 1. *Multi-step compound* reactions prevail at somewhat higher energies than those characteristic of compound nucleus decay. As with compound reactions, it is

imagined that the incident particle is captured by the target nucleus. Subsequently, the stepwise reaction proceeds exclusively by the *bound* configurations of the composite system (target nucleons + incident particle). Emission takes place before the attainment of statistical equilibrium. This picture is very reminiscent of that of the exciton model. The main difference is that MSC models are constructed by combining quantum-mechanical perturbation theory (with respect to the compound system) with statistical postulates. This problem was basically solved as early as 1975 [14]. Based upon a random-matrix model of the Hamiltonian, a probability-balance equation was derived that could naturally be interpreted as the time-integrated master equation of the exciton model. Similar models have been proposed in [16,17] and more recently in [18]. The MSC mechanism yields symmetrical angular distributions.

If the reaction proceeds by the unbound configurations of the composite system, we speak of *multi-step direct* reactions. A crucial feature of the MSD process is that one can make a meaningful distinction between a leading fast particle and the residual nucleus. In contrast with MSC reactions, the MSD process generates the smooth forward peaked angular distributions that are characteristic for pre-equilibrium decay. Furthermore, since the MSC reaction mechanism is restricted to energies between 10 and 20 MeV [19], MSD reactions generally provide the larger part of the pre-equilibrium cross section. MSD models have been proposed simultaneously by Feshbach, Kerman and Koonin (FKK) [16] and Tamura, Udagawa and Lenske (TUL) [15] and later on by Nishioka, Weidenmüller and Yoshida (NWY) [20]. Without exception, these approaches embody the extrapolation to the continuum of the DWBA approach for discrete direct reactions. The high level density in the pre-equilibrium region inevitably necessitates statistical assumptions.

C. Practical applications

As already mentioned, the significance of pre-equilibrium reactions extends from 10 MeV to several hundred MeV. This implies that the associated models are relevant for a number of practical applications. *How* to apply them will be discussed in another lecture. Here, we will mention a couple of important examples.

At the low energy end of this energy region (between 10 and 50 MeV) pre-equilibrium models provide nuclear data that are necessary for fusion reactor-design calculations (activation, neutron transport, radiation damage, etc.). The required data include double-differential cross sections for structural materials [21], *e.g.* 14 MeV neutron reactions (originating from the (d,t) step in the fusion chain) on a variety of nuclides. Since only a limited fraction of these data can be measured, the experimental data are to be supplemented by model calculations. The cross sections as calculated by e.g. GNASH, as done during this workshop, contribute to this.

In the region above 10 MeV, pre-equilibrium models can play a significant role in the analysis of applications that require accelerators, such as the transmutation of radioactive waste where, with the help of high-energy protons, long-lived radioactive actinides are spallated into radioactive materials with a shorter half-life. At energies above about 150 MeV, intranuclear cascade models will presumably be sufficient for the prediction of the relevant nuclear data. Below this energy however, exciton and quantum-mechanical models are still necessary for a proper analysis of these reactions.

II. CLASSICAL MODELS: THE EXCITON MODEL

A. Introduction

Since the basic work of Griffin [9] in 1966, a variety of different pre-equilibrium models have been developed. The most widely used models are the intranuclear cascade model and, particularly, the exciton and hybrid models.

A classical approach to precompound decay is provided by the intranuclear cascade model [22,23]. The classical trajectories of the particles inside the nucleus are followed in coordinate space by means of Monte Carlo methods. When incident nucleon wavelengths are short relative to internucleon distances (i.e. energies in excess of 100 MeV) the collisions can be treated as quasi-free scattering processes. In the cascade model, two-body scattering is assumed and the numerical simulation of the scattering process is based on experimental free nucleon-nucleon scattering cross sections and angular distributions. Until two decades ago, the cascade model was the only pre-equilibrium model able to predict angular distributions of emitted particles. However, emission into the backward hemisphere is underestimated by several orders of magnitude.

In the exciton and hybrid models, the nuclear state is characterized by the excitation energy E and the total number n of particles above and holes below the Fermi surface. Particles and holes are indiscriminately referred to as excitons. Furthermore, it is assumed that all possible ways of sharing the excitation energy between different particle-hole configurations with the same exciton number n have equal a-priori probability.

To keep track of the evolution of the scattering process, one merely traces the temporal development of the exciton number, which changes in time as a result of intranuclear two-body collisions. This assumptions makes the exciton model amenable for practical calculations. The price to be paid, however, is the introduction of a free parameter, viz. the average matrix element of the residual two-body interaction, occurring in the transition rates between two exciton states.

Qualitatively, the equilibration process of the excited nucleus is imagined to proceed as follows (see figure 2). After entering the target nucleus, the incident particle collides with one of the nucleons of the Fermi sea. Thus, an initial state with $n_0 = 3$ is formed (in the case of nucleon-induced reactions). Subsequent interactions result in changes in the number of excitons, characterized by $\Delta n = +2$ (a new particle-hole pair) or $\Delta n = -2$ (annihilation of a particle-hole pair) or $\Delta n = 0$ (creation of a different configuration with the same exciton number). In the first stage of the process, corresponding to low exciton numbers, the $\Delta n = +2$ transitions are predominant. However, at any stage there is a non-zero probability that a particle is emitted. Should this happen at an early stage, it is intuitively clear that the emitted particle retains some "memory" of the incident energy and direction: Bohr's amnesia hypothesis is not valid. This phase is called the pre-equilibrium phase, which is expected to be responsible for the experimentally observed high-energy tails and forward-peaked angular distributions. If emission does not occur at an early stage, the system eventually reaches a (quasi-) equilibrium. The equilibrium situation, corresponding to high exciton numbers, is established after a large number of interactions, i.e. after a long lapse of time, and the system has "forgotten" about the initial state. Accordingly, this stage may be called the compound or evaporation stage. Hence, in principle the exciton

model enables to compute the emission cross sections in an unified way, without introducing arbitrary adjustments between equilibrium and pre-equilibrium contributions.

The equilibration process is most completely described quantitatively by the master-equation exciton model. The other types of exciton and hybrid models may be considered simplifications or variants of this model. The master-equation model, which predicts energy distributions of emitted particles, is discussed in the next subsection.

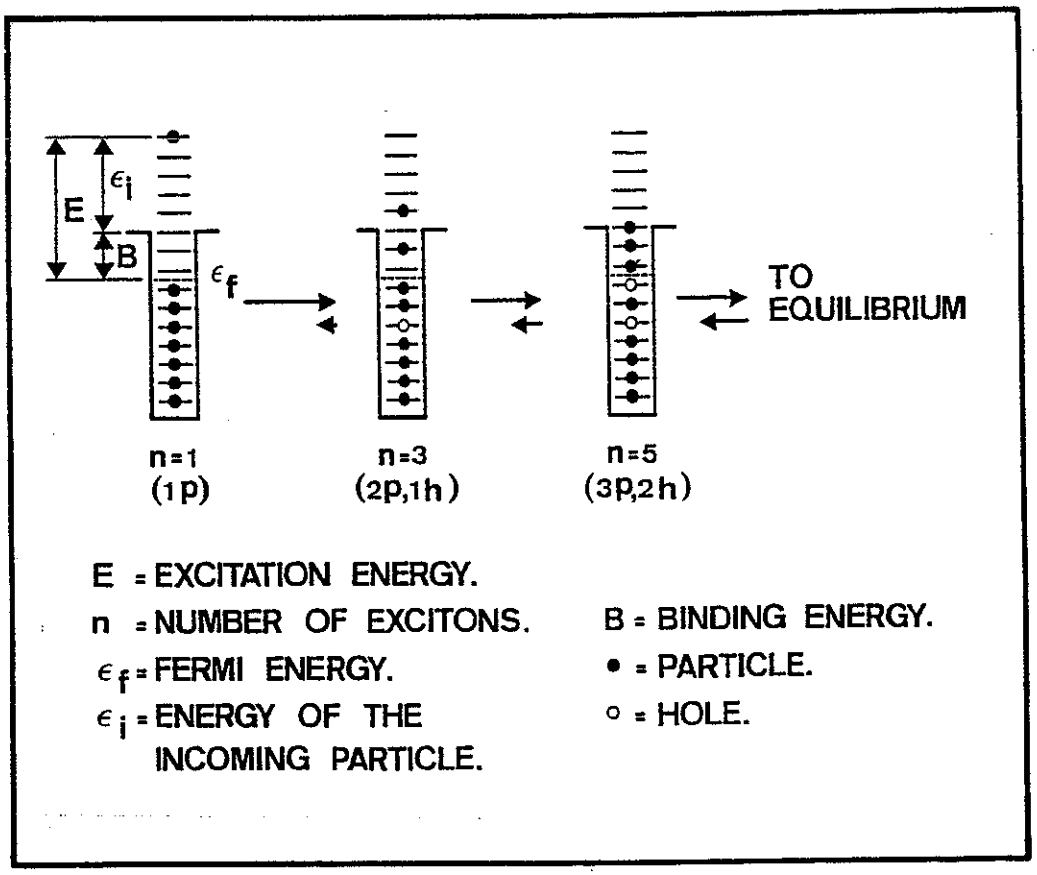

FIG. 2. A schematical representation of the equilibration process as formulated in the exciton models. The relative probability and the direction of the transitions is roughly indicated by the length and direction of the arrows

B. The master equation exciton model

The pre-equilibrium master equation was first proposed by Cline and Blann [10]. Let $q(n,t)$ be the probability that the nuclear system is in the exciton state n at time t. For notational simplicity, we do not indicate the dependence of the excitation energy. Also, we

note in advance that in subsequent years the master equation has been generalized to include a dependence on angle (generalized exciton model), angular momentum, parity and isospin. For a clear picture, we here restrict ourselves to the original version. The time evolution of the exciton probability distribution is given by the master equation:

$$\frac{dq(n,t)}{dt} = \lambda^+(n-2)q(n-2;t) + \lambda^-(n+2)q(n+2,t) - [\lambda^+(n) + \lambda^-(n) + w(n)]q(n,t). \tag{2.1}$$

Here, $\lambda^+(n)$ and $\lambda^-(n)$ are the internal transition rates from state n with $\Delta n = +2$ and $\Delta n = -2$, respectively, whereas $w(n)$ indicates the total emission rate from state n summed over all outgoing particles and energies. Eq. (2.1) is a gain-loss equation, the positive terms representing the feeding to a particular state n and the negative terms describing the loss to other states. The term with w is a sink term describing the decay of the nucleus. To solve Eq. (2.1) it should be supplemented by an initial condition. For nucleon-induced reactions the most obvious choice is $q_0(n) \equiv q(n, t=0) = \delta_{n,n_0}$ with $n_0 = 3$. We note that in Eq. (2.1) the transitions with $\Delta n = 0$, having transitions rates $\lambda^0(n)$, cancel and therefore do not contribute. In the generalized exciton model, where angular distributions are taken into account, $\lambda^0(n)$ does play a role.

To calculate the emission rates, Cline and Blann [10] derived expressions from the principle of microreversibility. The emission rate from exciton state n for a particle b with energy ϵ, relative mass μ_b, spin s_b and total excitation energy E is

$$w_b(n,\epsilon) = \frac{2s_b+1}{\pi^2\hbar^3}\mu_b\epsilon\sigma_{b,inv}(\epsilon)\frac{\omega(p-p_b,h,U)}{\omega(p,h,E)}Q_b(n)\phi_b, \tag{2.2}$$

where $\sigma_{b,inv}(\epsilon)$ is the inverse reaction cross section e.g. as calculated from an optical model; $\omega(p,h,E)$ is the particle-hole state density with p and h being the number of particles and holes, respectively ($n = p+h$); U is the excitation energy of the residual nucleus; $Q_b(n)$ is a factor accounting for the distinguishability of neutrons and protons and ϕ_b is a formation factor for complex particles, which is unity for neutrons, protons and gammas. We note that $w(n)$ of Eq. (2.1) is obtained by integrating Eq. (2.2) over all outgoing energies and by summing over all outgoing particles.

Much effort has been devoted to investigating the particle-hole state densities. On the assumption on equidistant level spacing, Ericson [24] derived an expression, subsequently corrected by Böhning [25], Williams [26] and Betak and Dobes [27] for the effect of the Pauli exclusion principle and for the finite depth of the potential well (see also Ref. [28]). The form for the particle-hole state density is

$$\omega(p,h,E) = \frac{g^n}{p!h!(n-1)!}\sum_{j=0}^{h}(-1)^j\binom{h}{j} \times (E - A_{p,h} - jF)^{n-1}\Theta(E - E_{PP} - jF). \tag{2.3}$$

Here, F is the Fermi energy, B is the binding energy, $E_{PP} = [p^2 + h^2 + p - h]/2g$ is the minimum energy required to excite p particles and h holes satisfying the Pauli principle, Θ is the unit step function (which takes care of the finite hole depth), $g = \frac{6a}{\pi^2}$ is the single-particle

state density related to the level density parameter a, $n = p + h$ is the exciton number and $A_{p,h} = [p(p-1) + h(h-1)]/4g$ is the Pauli correction factor.

The internal transition rates are calculated [9] with the aid of Fermi's golden rule of time-dependent pertubation theory: $\lambda(n \rightarrow n') = 2\pi \langle M^2 \rangle \omega_f / \hbar$, with $\langle M^2 \rangle$ the average squared matrix element of the residual interaction and ω_f the density of the accessible final states. Expressions for the final state densities are discussed in refs. [26,27]. The transition rates are

$$\lambda^-(n) = \frac{2\pi}{\hbar} \left\langle M^2 \right\rangle \frac{1}{2} gph(n-2) \left[1 - \left(\frac{C(p-1,h-1)}{gE - A(p-1,h-1)} \right)^{n-3} \right],$$

$$\lambda^+(n) = \frac{2\pi}{\hbar} \left\langle M^2 \right\rangle \frac{1}{2} \frac{g}{n+1} \frac{[gE - A(p+1,h+1)]^{n+1} - [C(p+1,h+1)]^{n+1}}{[gE - A(p,h)]^{n-1}}, \tag{2.4}$$

where $C(p,h) = gh(E-F) - hA(p,h)$ corrects for the finite depth.

It has been pointed out [29] that in the investigation of the angular distributions $\Delta n = 0$ transitions do play a role. Therefore we give its transition rate for completeness

$$\lambda^0(n) = \frac{2\pi}{\hbar} \left\langle M^2 \right\rangle \frac{1}{2} \frac{g}{n} \left[gE - \frac{1}{2}(p^2 + h^2) \right] [p(p-1) + h(h-1) + 4ph], \tag{2.5}$$

although we do not discuss the generalized exciton model in this paper.

In practical calculations, the energy E of the aforementioned formulae is usually replaced by $E - P$, where P is an overall pairing correction.

As discussed before, the average squared matrix element $\langle M^2 \rangle$ is considered to be a free parameter. This problem stimulated the development of a variety of exciton models, in particular of the hybrid model. For the exciton model, the following semi-empirical expression [30] has become very popular and successful in practical applications

$$\left\langle M^2 \right\rangle = \frac{c}{A^3 E} \tag{2.6}$$

where A is the mass number of the composite nucleus and c is a constant to be adjusted. Eq. (2.6) can be approximately justified on the basis of the properties of nucleon-nucleon collisions in nuclear matter, on the assumption of $g \sim A$.

From Eq. (2.4) we see that $\lambda^+(n) \sim g^3 E^2 / n$, whereas $\lambda^-(n) \sim g n^3$. Accordingly, at low n we have $\lambda^+ >> \lambda^-$, and the equilibration process will proceed mainly in the direction of increasing complexity in the initial stages. At high n we have $\lambda^+ << \lambda^-$. Thus, at later times an equilibrium is established with an exciton distribution centered around an equilibrium exciton number $\overline{n} \sim \sqrt{gE}$, defined by $\lambda^+(\overline{n}) = \lambda^-(\overline{n})$. This is in accordance with the intuitive notions about the pre-equilibrium and equilibrium processes as explained in the previous section.

Now on the basis of Eqs. (2.2-2.6), the master equation (2.1) may be solved for the exciton probability distribution $q(n,t)$. Integration gives

$$\begin{aligned} -q_0(n) = {} & \lambda^+(n-2)t(n-2) + \lambda^-(n+2)t(n+2) \\ & - \left[\lambda^+(n) + \lambda^-(n) + w(n) \right] t(n-2) \end{aligned} \tag{2.7}$$

where $q_0(n)$ is the initial condition for the process and we used the fact that the exciton probability distribution will eventually tend to zero by emission (i.e. $q(n, t = \infty) = 0$). Here, the mean lifetime of an exciton state $t(n)$ is defined by

$$t(n) = \int_0^\infty q(n,t)dt. \tag{2.8}$$

which is the quantity required for the calculation of emission cross sections. The average cross section for the emission of particle b with energy ϵ is now given by

$$\frac{d\sigma}{d\epsilon}^{PE}(a,b) = \sigma_a \sum_n w_b(n,\epsilon)t(n) \tag{2.9}$$

where a denotes the type of incident particle, and σ_a is the composite-nucleus formation cross section. The way of determining the mean lifetimes has given rise to a lot of debate and provided a second reason for many different formulations of the exciton model. We will restrict ourselves to two cases, the full solution and the never-come-back approximation.

1. The exact solution

An exact solution of the time-integrated master equation (2.7) has been provided by Akkermans [29]. Using straightforward matrix algebra one finds for the mean lifetime for the maximum exciton number N:

$$t(N) = \tau_N h_N \sum_{j=n_0,\Delta j=2} q_0(j) \left(\prod_{i=j,\Delta i=2}^{N-2} \lambda^+(i)\tau_i h_i \right) \tag{2.10}$$

where

$$\tau_i = [\lambda^+(i) + \lambda^-(i) + w(i)]^{-1} \tag{2.11}$$

and

$$\begin{aligned} h_{i+1} &= [1 - h_i\lambda^+(i)\tau_i\lambda^-(i+1)\tau_{i+1}]^{-1} \\ h_1 &= 1 \end{aligned} \tag{2.12}$$

For $j = N$ the product within the parentheses is to be replaced by a factor one. The remaining mean lifetimes can successively be obtained from the time-integrated master equation

$$\lambda_n^+ t(n) = (\tau_{n+2})^{-1} t(n+2) - \lambda^-(n+4)t(n+4) - q_0(n+2) \tag{2.13}$$

where $n_0 \leq n \leq N-2$ and $t(N+2) = 0$.

For primary pre-equilibrium emission, the common initial situation of projectile + target is specified by $q_0(n) = \delta_{n,n_0}$. In this case the mean lifetime of the n-th exciton state is

$$\begin{aligned} t(n) = \tau_n h_n &\left(\prod_{m=n_0,\Delta m=2}^{n-2} \lambda^+(m)\tau_m h_m \right) \\ &\times \left(1 + \sum_{s=n+2,\Delta s=2} \left[\prod_{k=n,\Delta k=2}^{s-2} \lambda^+(k)\tau_k h_k \lambda^-(k+2)\tau_{k+2} h_{k+2} \right] \right) \end{aligned} \tag{2.14}$$

where for $n = n_0$ the first product is to be replaced by a factor one, and for $n = N$ the summation equals zero. The summation clearly arises as a consequence of the competition of $\Delta n = -2$ transitions with $\Delta n = +2$ transitions in the higher states throughout the equilibration process, whereas the first product describes the depletion in the levels below n. The quantities denoted by h are, in general, not equal to one (see Eq. (2.12)). They are related to the fact that the system may reach a certain exciton level along many different paths, which leads to a change in the lifetimes. Insertion in Eq. (2.9) leads to an easy calculation of the pre-equilibrium spectrum.

2. The never-come-back approximation

Before the abovementioned exact solution, the so-called never-come-back approximation was frequently used to come to a simple analytical solution of the master equation. It is based on the assumption that at the beginning of the cascade one neglect the interactions that decrease the exciton number, i.e. $\lambda^-(n) = 0$. With this assumption we obtain

$$\begin{aligned} h_n &= 1 \\ \tau_n &= (\lambda^+(n) + w(n))^{-1} \end{aligned} \tag{2.15}$$

and Eq. (2.14) simplifies to

$$t(n) = \tau_n \left(\prod_{m=n_0,\Delta m=2}^{n-2} \lambda^+(m)\tau_m \right) \tag{2.16}$$

The expression for the pre-equilibrium cross section then becomes very simple:

$$\frac{d\sigma}{d\epsilon}^{PE}(a,b) = \sigma_a \left[\frac{w_b(n,\epsilon)}{w(n)+\lambda^+(n)} + \sum_{n=n_0+2,\Delta n=2}^{\overline{n}} \left(\prod_{j=n_0,\Delta j=2}^{n-2} \frac{\lambda^+(j+2)}{w(j)+\lambda^+(j+2)} \right) \frac{w_b(n,\epsilon)}{w(n)+\lambda^+(n+2)} \right] \tag{2.17}$$

where $\overline{n}$ is the exciton number for equilibrium. In practice the sum over n is cut off after several terms, since from there on one can assume full equilibration and emission can be described by standard evaporation models.

C. The Kalbach systematics

A valuable tool for continuum nuclear reactions is the Kalbach systematics. This is a purely phenomenological approach based on fitting a large number of experimental continuum angular distributions [31]. This parametrization generally provides an adequate fit to the shape of the pre-equilibrium angular distributions and is therefore frequently employed in nuclear data applications. It gives a better representation of the double-differential data than that obtained with classical exciton methods. Kalbach obtained the following formula for the continuum double-differential cross section:

$$\frac{d^2\sigma}{d\Omega d\epsilon_b} = \frac{1}{4\pi}\frac{d\sigma}{d\epsilon_b}\frac{a}{\sinh(a)}[\cosh(a\cos\Theta) + f_{PE}\sinh(a\cos\Theta)] \tag{2.18}$$

with,

$$\begin{aligned} a(e'_b, e'_a) &= 0.04\frac{E_1 e'_b}{e'_a} + 1.8\times 10^{-6}(\frac{E_1 e'_b}{e'_a})^3 + 6.7\times 10^{-7} M_a m_b (\frac{E_3 e'_b}{e'_a})^4, \\ E_1 &= \min(e'_a, 130 \pm 10\text{MeV}) \\ E_3 &= \min(e'_a, 41 \pm 5\text{MeV}) \\ e'_b &= \epsilon_b + S_b \\ e'_a &= \epsilon_a + S_a. \end{aligned} \tag{2.19}$$

Here, ϵ_a and ϵ_b are the incident and the outgoing energy, respectively. The number M_a (representing the incident particle) is 1 for neutrons, protons and deuterons and 0 for alpha particles, while m_b (representing the outgoing particle) is 1 for protons, deuterons, tritons and ^{3}He particles, $\frac{1}{2}$ for neutrons and 2 for alpha particles. The Myers and Swiatecki mass formula for the separation energy S can be found in Ref. [31]. The fraction f_{PE} represents the contribution of the pre-equilibrium process.

The continuum angle–integrated cross section $\frac{d\sigma}{d\epsilon_b}$ of Eq. (2.18) can be written as the sum of an equilibrium (EQ) part and an pre-equilibrium (PE) part:

$$\frac{d\sigma}{d\epsilon_b} = \frac{d\sigma_{PE}}{d\epsilon_b} + \frac{d\sigma_{EQ}}{d\epsilon_b}. \tag{2.20}$$

When applied in a quantummechanical context, f_{PE} is usually replaced by a factor f_{MSD} since the multi-step direct component of the pre-equilibrium region is the only non-symmetric part.

D. Semi-classical models: Some selected results

The exciton model has been tested extensively against experimental data over the past 30 years. Here we restrict ourselves to a few illustrations. Figure 3 shows the predictions by several exciton models that took part in a code intercomparison [6] for ^{93}Nb(n,xn) at 14.6 MeV. Clearly, the exciton + evaporation model is very succesful in predicting continuum spectra. The main deficiencies are usually at high outgoing energies, where collective effects need to be included. Figure 4 gives an impression of the impact of extending the exciton model with angular momentum and parity conservation. The main conclusion is that the standard precompound model results are close to those of the angular-momentum conserving model [13], implying that the former are quite reliable from a practical point of view.

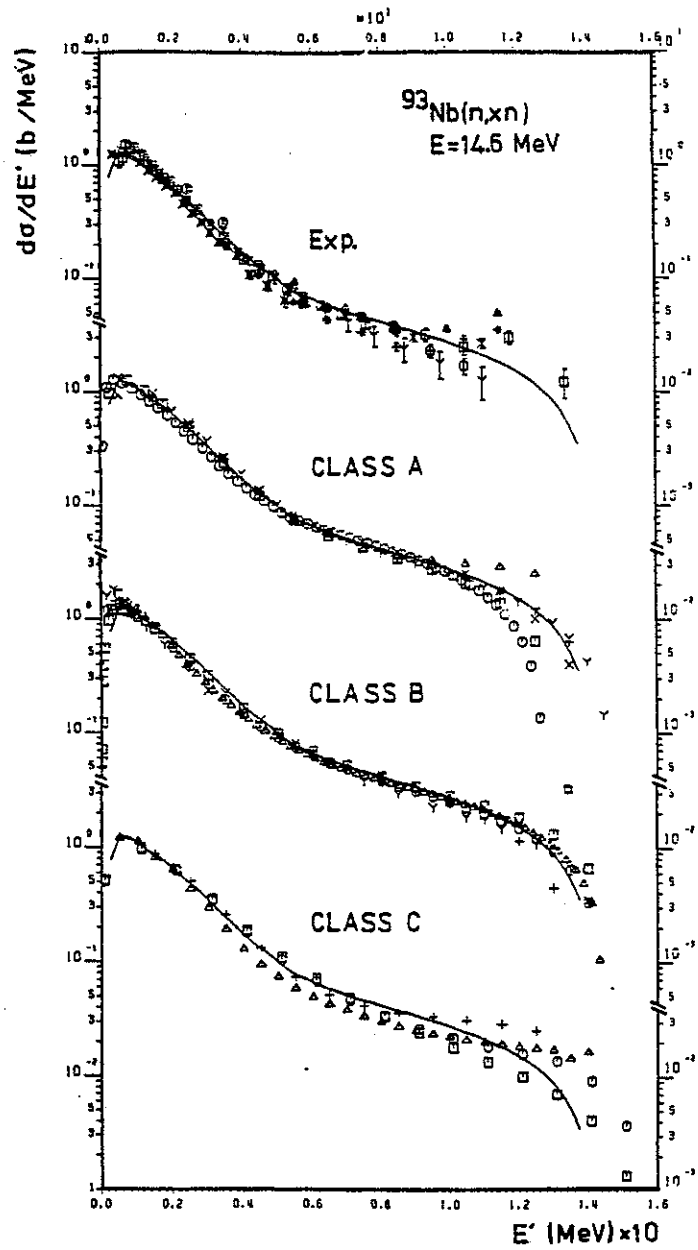

FIG. 3. Intercomparison of experimental and calculated neutron emission spectra (angle-integrated) of the reaction ^{93}Nb(n,xn) at 14.6 MeV. The experimental data are given at the top of the figure. The other symbols are model calculations by several classes of exciton and hybrid model codes. [6]

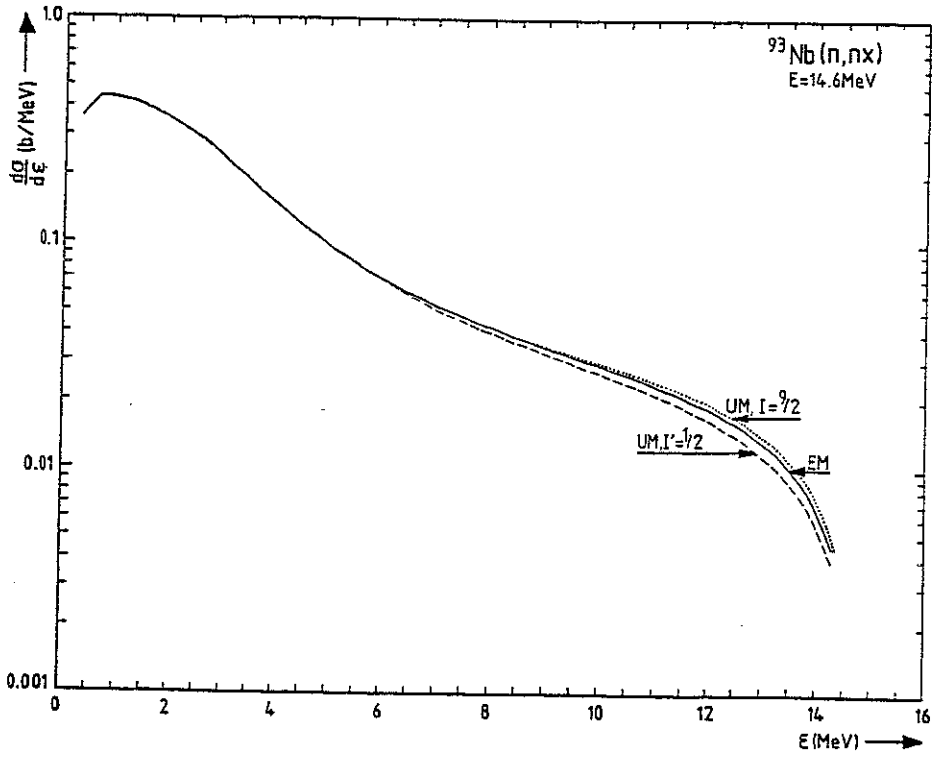

FIG. 4. Comparison between the unified model (UM, i.e. including angular-momentum conservation) and the exciton model (EM) for the reaction ^{93}Nb(n,n'x) at 14.6 MeV. The full curve represents the EM spectrum, the dotted line represents the UM with the target spin $I = \frac{9}{2}$, and the dashed one corresponds to the UM with a fictitious target spin $I' = \frac{1}{2}$ [13].

III. QUANTUM-MECHANICAL MODELS

A. Introduction

In recent years quantum mechanical theories have been developed to describe the preequilibrium nuclear reaction mechanism [7,32,33], and the advent of fast computers has enabled numerical computations of these cross sections. Although some controversies regarding the underlying quantum statistics in multistep reactions still exist (such as causality issues in the MSD theory of Feshbach, Kerman, and Koonin [16,34]), quantum mechanical preequilibrium theories tend to account for experimental angle-integrated emission spectra with an

accuracy comparable to that found in the semiclassical models, and with a higher accuracy for angular distributions, competing with the phenomenological experiment-based systematics of Kalbach [31]. After pioneering calculations by Tamura *et al.* [15] and Bonetti *et al.* [35,36], several independent and more sophisticated computer codes for both multistep direct (MSD) and multistep compound (MSC) processes have emerged, enabling a better insight into the contributions of each reaction mechanism to the spectrum.

Due to space limitations, we refer to Ref. [38] for a recent extensive analysis of continuum data by the MSD model.

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