Portfolio Credit Risk

University of Chicago Masters in Financial Mathematics 36702

https://uchicago.instructure.com/courses/48373

Lecture 4

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Conditional LGD risk

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Lecture 3 greatest hits

For any loan, Loss = D * LGD

In any set of conditions, cLoss = cPD * cLGD

For any loan, EL = E [Loss]

= E [D * LGD]

= E [D] * E [LGD] **(this is the mystery step)**

= PD * ELGD

- **Independence because: LGD means "Loss Given Default."**
  - **LGD is the fraction that is lost if there is a default.**
  - **If there is a default, what is the fraction that is lost?**
  - **No mystery! LGD can depend on many things, but it cannot depend on the default of the loan because LGD already assumes the default of the loan.**

Any thoughts before we start?

Questions or comments?

Pep talk

Tonight’s model is used in lots of ways.

- **“Stress tests" are required of big banks.**
- **“Current expected credit loss" (CECL) is required of all banks.**
- **Credit default swaps, and OTC derivatives in general, are also subject changing values of LGD in changing conditions.**

You might get involved in risks where data is sparce.

- **With sparce data, a simpler model is less likely to be overfit.**
  - **Tonight, you’ll see a model that is simpler than linear regression.**
  - **It is based on something obvious that no one had noticed.**
  - **Some day, you might take advantage of something like this.**

Week 4 topics

Four ways to model conditional LGD

LGD functions

The Frye-Jacobs LGD function

Testing the Frye-Jacobs LGD function

Four ways to model cLGD

Four ways to model cLGD

1. Ignore it.

2. Pretend to not ignore it, then ignore it.

3. Model cLGD separately from cPD.
   - The product, cLoss, has nonsensical behaviors that give banks nonsensical incentives.

4. Start fresh.
   - cPD and cLoss are comonotonic.
   - Therefore, every loan has an LGD function.
   - Frye and Jacobs make a particular choice.
   - The Frye-Jacobs LGD function is ***testable***,
     - and it has survived testing so far.

1. Ignore, ignore, ignore

Of the approaches to systematic LGD risk, ignoring it has the longest history and greatest popularity.

This approach makes the simulation model easy:

- **Simulate the defaults as usual.**
- **In each run, Loss = DR * ELGD; ELGD is a fixed number.**
- **Done.**

CreditMetrics© makes this slightly more sophisticated:

- **In it, LGD is random, but the distribution of LGD *<u>does</u>* *<u>not</u>* *<u>depend</u>* on conditions in the simulation run.**
  - **The only LGD risk comes from the randomness of a small portfolio.**

2. Pretend to not ignore

To pretend to not ignore systematic LGD risk, do this:

- **Test H_0: LGD does not respond to economic variables.**
- **Assemble data of such poor quality that H_0 is not rejected.**
- **Conclude that *<u>there</u>* *<u>is</u>* *<u>no</u>* systematic LGD risk.**

Note the sequence of steps:

- **H_0 is implausible.**
  - **LGD is an *<u>economic</u>* variable. Why is it independent of the economy?**
- **Be sure to use a short, poor-quality data set.**
- **Then, conclude that the implausible hypothesis is *<u>true</u>*.**

As you can imagine, the people who do this have PhDs.

Few believe it anymore

- **When I wrote Collateral Damage (2000), there was one carefully observed downturn, 1990-91.**
  - **I found that LGD went up significantly in 1990-91.**
    - **Skeptics still believed that LGD is independent of other variables.**

- **In the tech recession (2001), LGD went up again.**
  - **Basel II acknowledged that LGD goes up and down.**
    - **LGD in Basel II became the confusing mess that you saw earlier.**

- **In the 2008 crisis, LGD went up again.**
  - **You already saw the LGD chart with the three spikes.**

- **It is now agreed that LGD goes up in times of stress.**
    - **I hoped that Covid 19 would produce a high default period, but no.**

3. Model LGD naively

The naïve approach handles LGD and default separately.

An example of this is the Basel formula.

- **It uses the Vasicek formula for cPD times a value for LGD.**

Banks compared two calculations.

- **Default was defined as a loan that produced credit loss.**
- **Default was defined as a loan that was ever 90 days late.**

The second definition required *<u>less</u>* capital.

- **Historical default went up, but historical LGD went down.**
- **The Basel formula is more sensitive to LGD than PD.**

Banks added covenant defaults to the definition…

$$K = \left[LGD \times N\left(\frac{N^{-1}(PD) + \sqrt{R} \times N^{-1}(0.999)}{\sqrt{1 - R}} \right) - (LGD \times PD) \right] \times \left(\frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b} \right)$$

		Definition 1			Definition 2	
Loan		**Default**	**Loss**		**Default**	**Loss**
1		0	0		0	0
2		0	0		0	0
3		0	0		0	0
4		0	0		1	0
5		1	0.5		1	0.5

EL = .5/5 = 0.1 | **EL = .5/5 = 0.1**

PD = 1/5 = 0.2 | **PD = 2/5 = 0.4**

ELGD = .1/.2 = 0.5 | **ELGD = .1/.4 = 0.25**

Basel formula:

$$.5 * \Phi\left[\frac{\Phi^{-1}[.2] + \sqrt{.1}\,\Phi^{-1}[.999]}{\sqrt{1-.1}}\right] - .5 * .2$$

= 0.28

$$.25 * \Phi\left[\frac{\Phi^{-1}[.4] + \sqrt{.1}\,\Phi^{-1}[.999]}{\sqrt{1-.1}}\right] - .25 * .4$$

= 0.19

4. Start fresh modeling LGD

The definition of default should *not* affect loss.

- **If there is a change in the definition of default, then the value of LGD should *change* to offset it: cLGD = cLoss / cPD.**

We have the Vasicek PDF for cPD.

If there is a nice expression for cLoss, we might be able to back out an expression for cLGD.

- **This would take care of the problem of definitional arbitrage.**
  - **There's more to it, so hold on.**

Questions? Comments?

LGD functions

What would an LGD function do?

Every loan has an LGD function

An LGD function has three inputs

PD: The probability of default in the next 12 months

- **PD depends on the firm, its financial condition, etc.**

ELGD: The expected LGD of the loan

- **ELGD depends on seniority, security, guarantees, etc.**

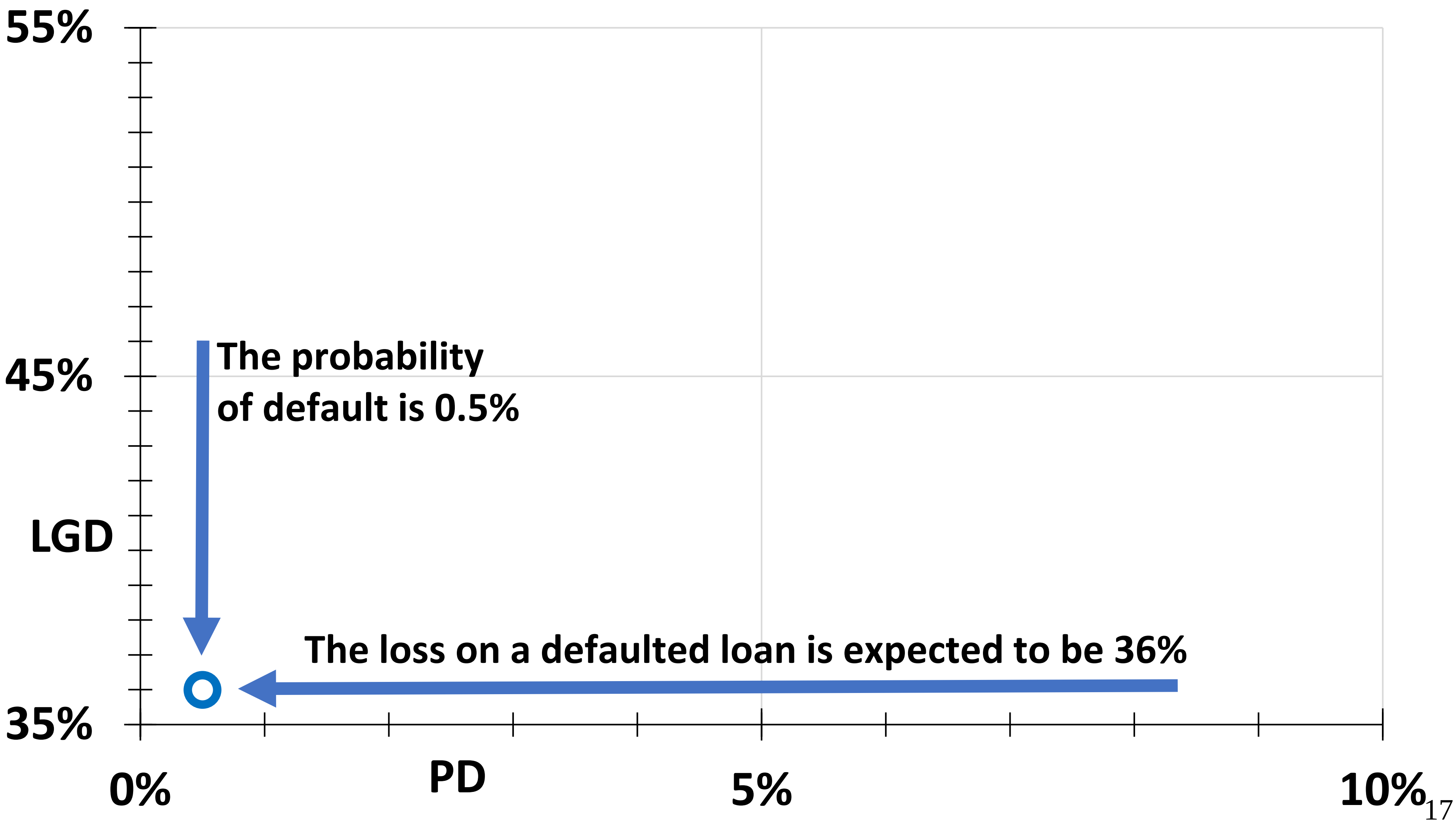
cPD: The *conditional* probability of default.

- **The conditions are those present in some scenario.**
  - **In our single-factor models, conditions are determined by Z.**
  - **In a FR stress test, conditions are defined by hypothetical values of several macroeconomic variables. cPD is the "stress default rate."**

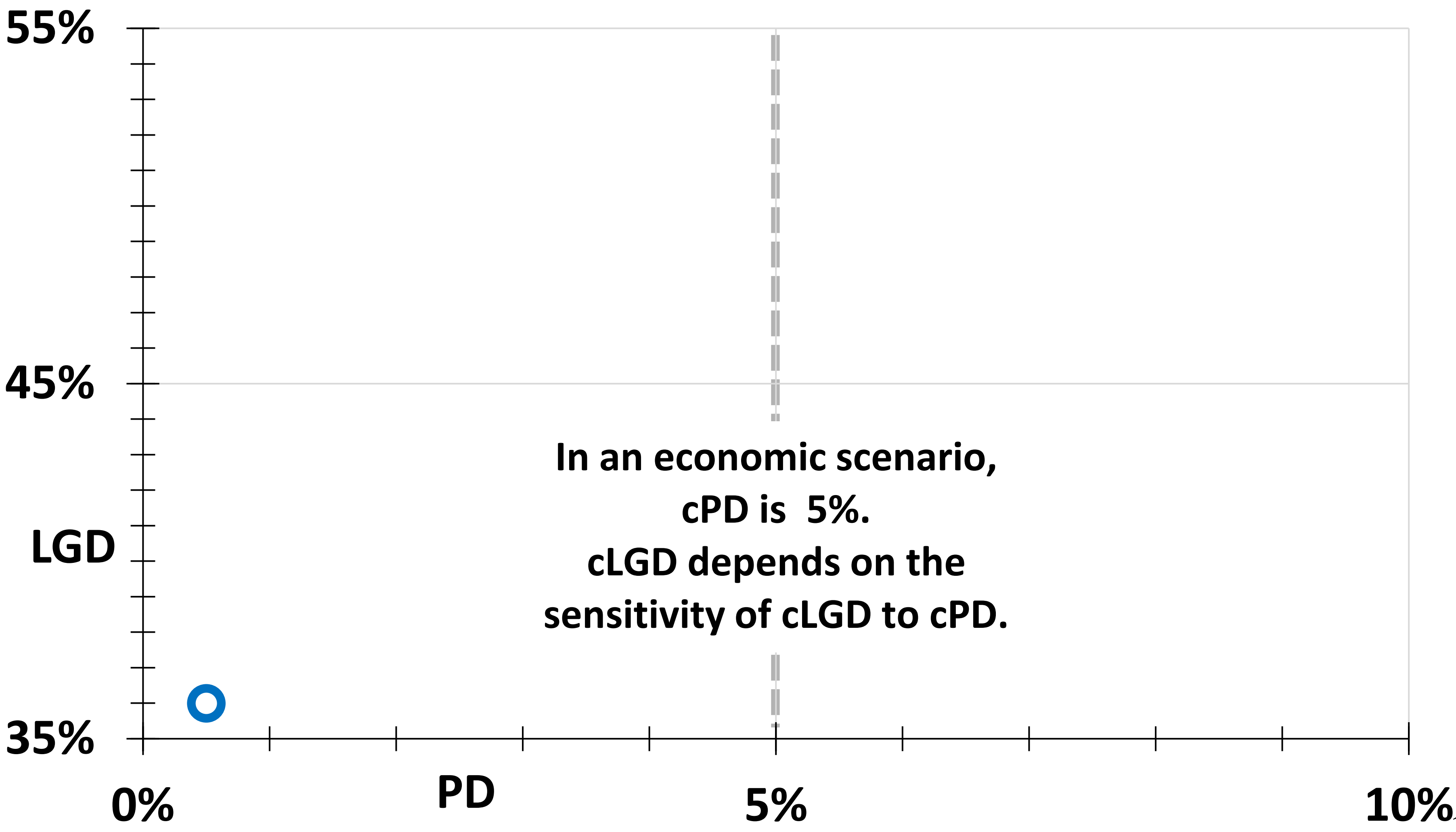
The output is *conditional* LGD, cLGD.

- **cLGD is the LGD to be expected in the conditions that produce cPD; sometimes referred to as "stress LGD."**

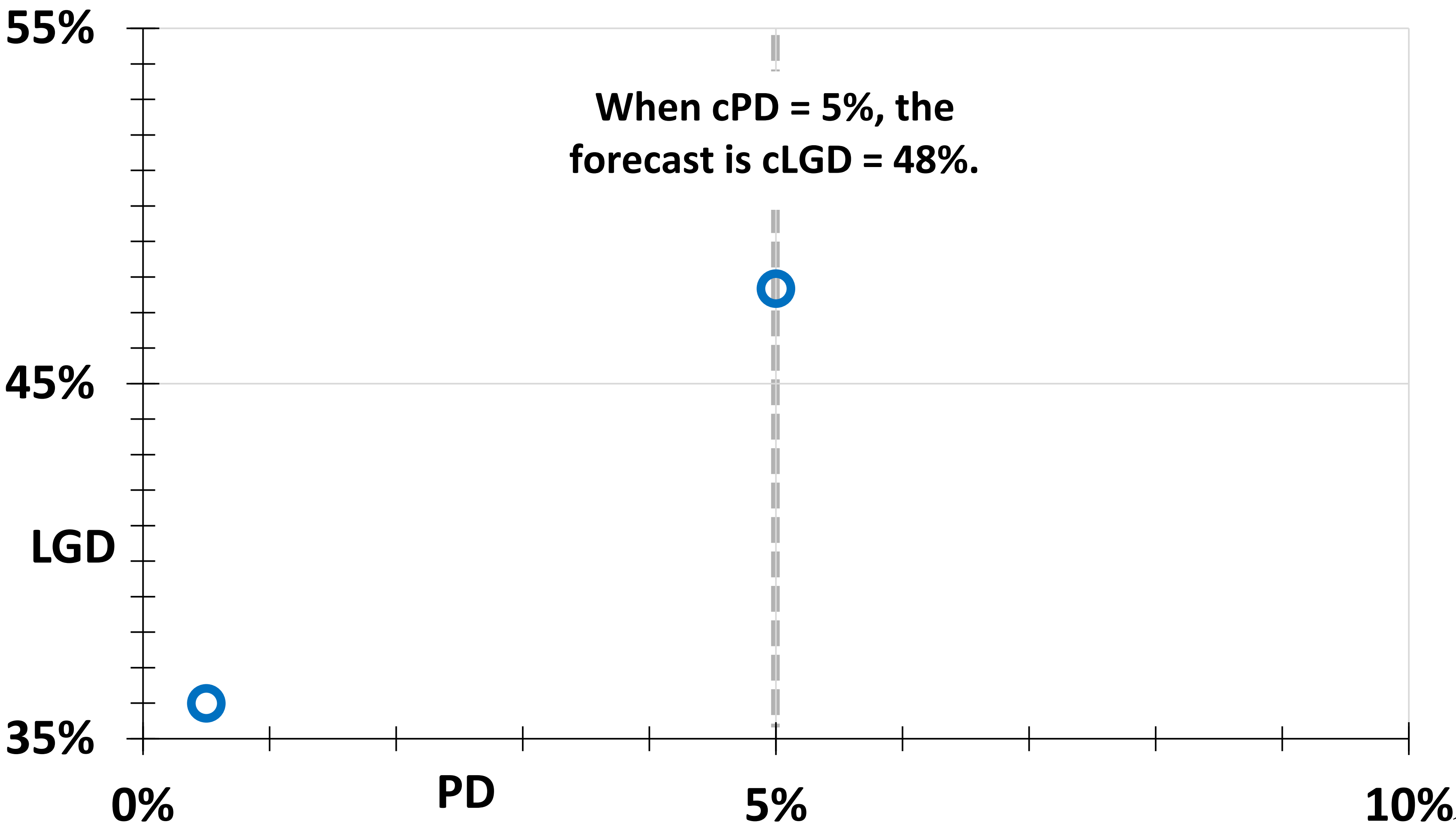
The first two inputs



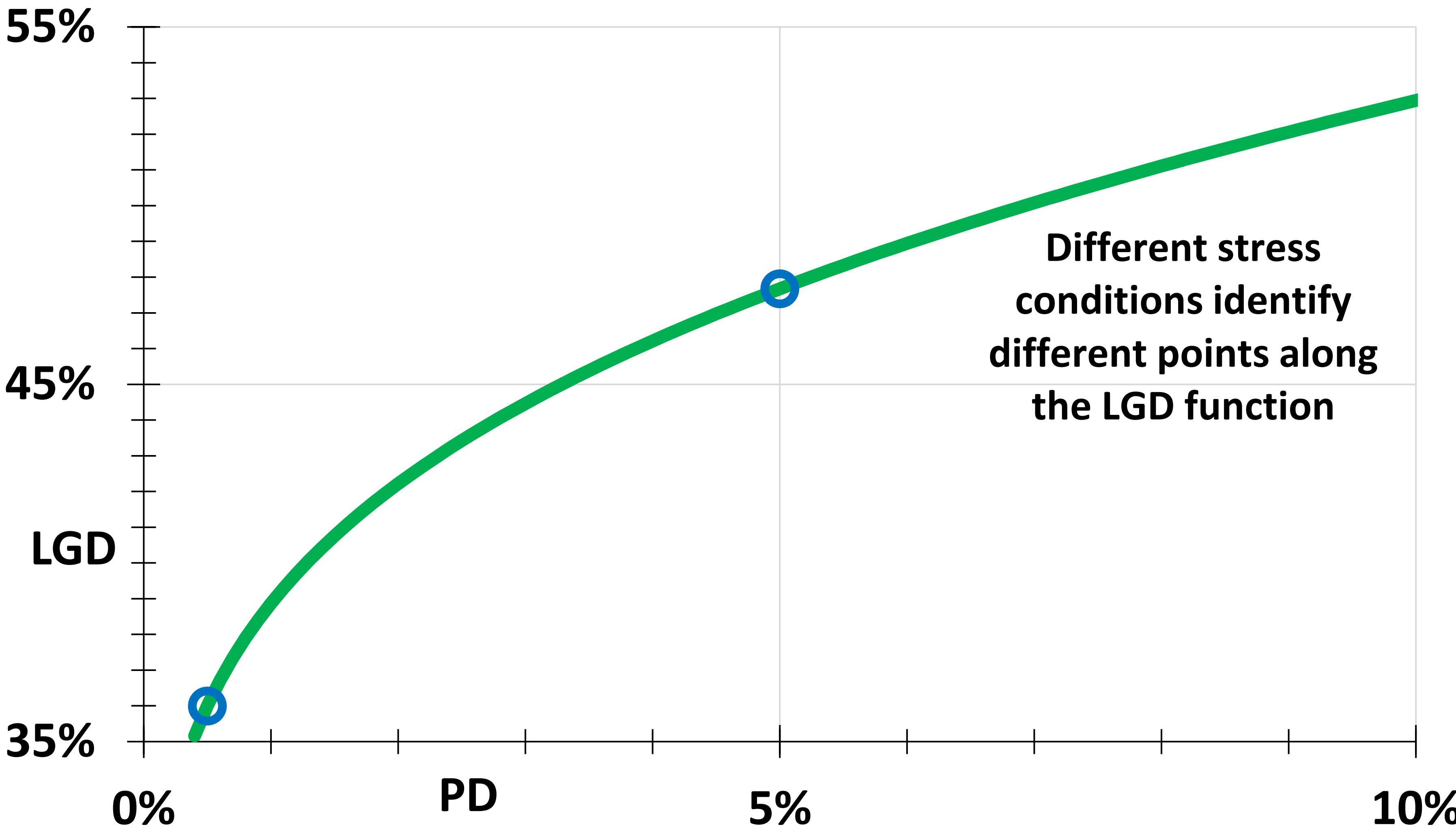
The third input



The value of the LGD function



An LGD function



Questions? Discussion?

An LGD function forecasts cLGD for a loan,
given the stress default rate (cPD)
and given the loan-specific inputs PD, ELGD.

Every loan has an LGD function

The key assumption

If a given loan is compared in two sets of conditions, the conditions producing greater default produce greater loss.

- **"If the default rate goes up, the loss rate goes up."**
  - **The conditionally expected default rate and the conditionally expected loss are comonotonic.**

Comonotonic variables go up and down in lockstep.

- **Two jointly normal variables are comonotonic if and only if they are perfectly correlated.**
- **Comonotonicity generalizes this; the line can be curved.**
  - **Its slope must be either everywhere positive or everywhere negative.**

For two comonotonic random variables X and Y:

If X is at its q^{th} quantile, then Y is at its q^{th} quantile.

cPD and cLoss are ***comonotonic***

Comonotonicity of cPD and cLoss is the key assumption.

If the assumption is violated, then this could occur:

	cPD	cLoss
Scenario A	10%	4%
Scenario B	<10%	> 4%

In Scenario B, the default rate goes down and the loss rate goes up. cLGD would need to go up a *lot*.

I'm still waiting for someone to describe conditions that would make this happen.

An LGD function

Here's how to calculate cLGD given that cPD and cLoss are comonotonic:

- **Begin with a value of cPD.**
- **Find its quantile within the distribution of cPD.**
- **Find the value of cLoss at the *same* quantile.**
  - **Same quantile because the variables are *comonotonic*.**
- **cLGD = cLoss / cPD.**

Therefore, an LGD function has this form:

$$cLGD\,[\,cPD\,] = CDF^{-1}_{cLoss}\,[\,CDF_{cPD}\,[\,cPD\,]\,]\,/\,cPD$$

The same logic applies to every loan.

Every loan has an LGD function.

$$cLGD[cPD] = CDF_{cLoss}^{-1}\big[CDF_{cPD}[cPD]\big]/cPD$$

Many things affect the distributions of cPD and cLoss.

- **The distribution of cPD depends on its mean, PD.**
  - **The PD of a high-rated firm is *<u>less</u>* than the PD of a low-rated firm.**
  - **The distribution of cPD for a high-rated firm is *<u>left</u>*.**
- **And the distribution of cLoss depends on ELoss and much else.**

But an LGD function takes a single argument, cPD.

- **If there are other things in an LGD model, certain values of them would produce non-comonotonic cPD and cLoss.**
  - **There's no need for an ad-hoc search for correlations than can be spurious.**

Variables can affect cLGD *<u>only</u>* if they affect cPD or cLoss.

Every loan has an LGD function.

Questions? Comments?

The Frye-Jacobs LGD function

Derivation

Alternative hypotheses

Finite portfolios

The tests

The Frye-Jacobs LGD function

So far:

- **Every loan has an LGD function.**
- **The function itself depends entirely on two CDFs.**
- **The function has a single argument, cPD.**

Frye and Jacobs make assumptions that appear to lead to the simplest LGD function.

- **cPD ~ Vasicek distribution [PD, ρ].**
- **cLoss ~ Vasicek distribution [EL, ρ].**
  - **Note: the value of the second parameter is the same.**
  - **In practice, one estimates ρ from default data and uses the same estimate in the cLoss distribution.**

The F-J LGD function is easy to derive…

Frye-Jacobs derivation

cPD ~ Vasicek [PD, ρ]; $F_{cPD}[cPD] = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[cPD] - \Phi^{-1}[PD]}{\sqrt{\rho}}\right]$,

cLoss ~ Vasicek [EL, ρ]; $F^{-1}_{cLoss}[q] = \Phi\left[\frac{\Phi^{-1}[EL]+\sqrt{\rho}\,\Phi^{-1}[q]}{\sqrt{1-\rho}}\right]$

$$cLGD\,[\,cPD\,] = F^{-1}_{cLoss}\,[\,F_{cPD}\,[\,cPD\,]\,]\;/\;cPD$$

$$= \Phi\left[\frac{\Phi^{-1}[EL] + \sqrt{\rho}\;\Phi^{-1}\left[\Phi\left[\frac{\sqrt{1-\rho}\;\Phi^{-1}[cPD] - \Phi^{-1}[PD]}{\sqrt{\rho}}\right]\right]}{\sqrt{1-\rho}}\right] / \; cPD$$

$$\underbrace{= \Phi\left[\Phi^{-1}[cPD] - k\right]/cPD}_{\textbf{LGD function}};\quad \underbrace{k = \left(\Phi^{-1}[PD] - \Phi^{-1}[EL]\right)/\sqrt{1-\rho}}_{\textbf{Definition of } k}$$

LGD function ; Definition of k

LGD function properties

$$\mathbf{cLGD} = \Phi\left[\Phi^{-1}[cPD] - k\right]/cPD$$

This function is monotonic increasing in cPD.

- **I found this difficult to prove.**
  - **But I found the graphs on the next slide to be persuasive.**

$\boldsymbol{k}$ **summarizes the effects of parameters PD, EL, and** ρ**:**

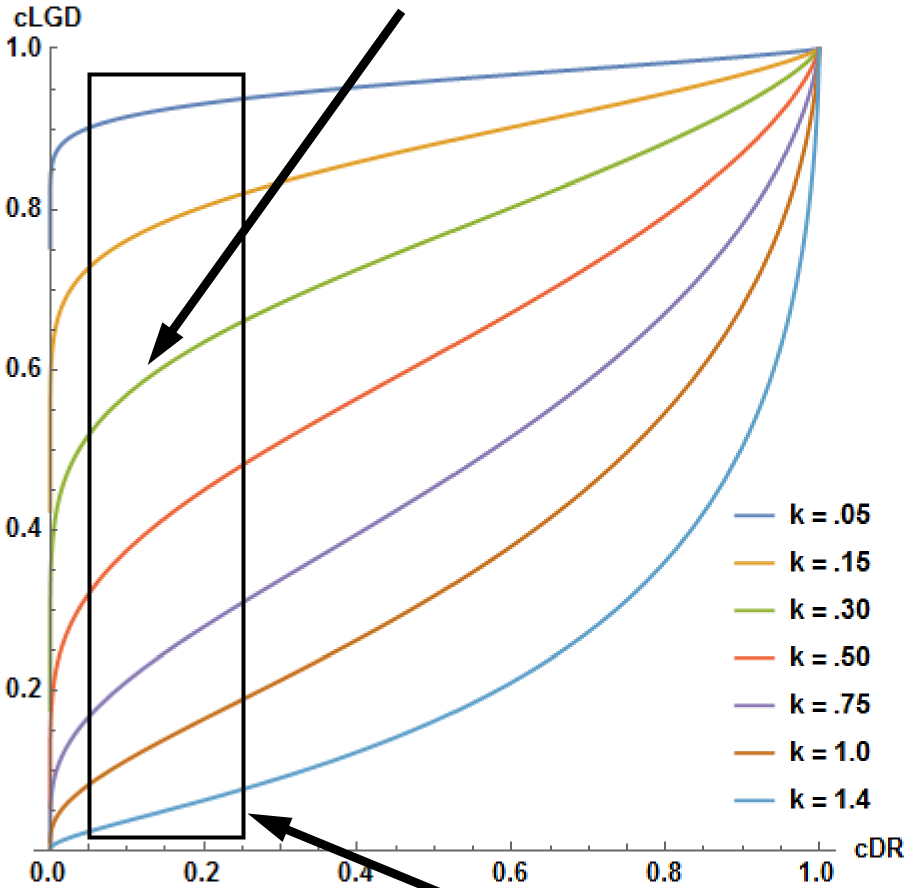
$$\boldsymbol{k} = \frac{\Phi^{-1}[PD] - \Phi^{-1}[EL]}{\sqrt{1-\rho}}$$

- **It is worth noting that** ρ **has _little_ _effect_.**
  - **E.g., if** $\rho = 0.19$**, then the denominator is 0.90.**

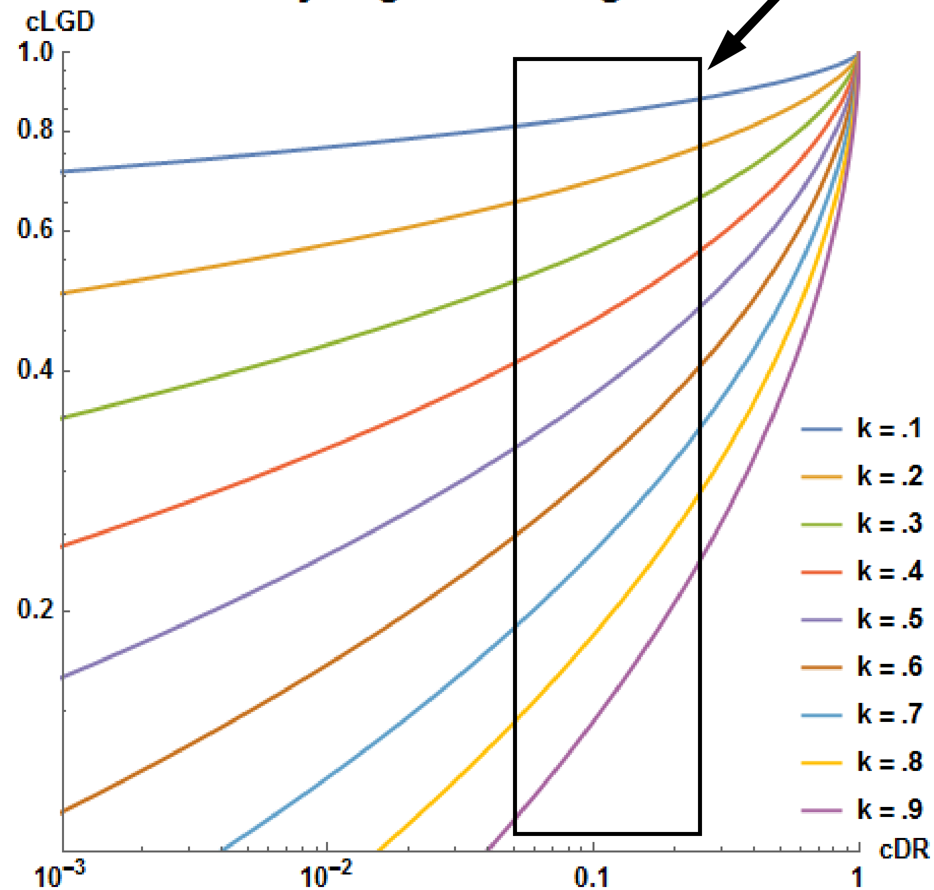
The properties are broadly consistent with observation…

1: Bounded on unit square and monotonic

2: Risk is moderate at low cDR



4: Elasticity is greater for greater k



3. Slope is *similar* for all loans on 5% < cPD < 25%.

Questions? Comments?

Toward testing the LGD function

Most of the Frye-Jacobs paper is an attempt to reject the Frye-Jacobs LGD function in a hypothesis test.

- **We couldn't do it, and no one else has tried that we know.**

To perform the test requires that we assemble:

- **an *alternative* hypothesis that uses**
- ***finite* portfolios that contain firms in**
- ***diverse* rating grades and loans with**
- ***diverse* seniority and security.**

We go through these points in order.

Alternative hypotheses

Alternative Hypothesis

The Alternative LGD function has an additional parameter. We wanted three nice properties.

- **The parameter controls the sensitivity of cLGD to cPD.**
  - **Sensitivity is the only thing that matters to the function, as you saw.**
- **It controls *<u>only</u>* the sensitivity of cLGD to cPD.**
  - **Neither expected loss (EL) nor the distribution of cPD are affected by the value of the additional parameter. Just the LGD function.**
- **It must *<u>nest</u>*.**
  - **At some value of the parameter, the Alternative equals F-J.**

You will see that when the extra parameter is fit by MLE, its value does not differ significantly from zero.

- **The Null hypothesis of the F-J LGD function is not rejected.**
- **That's why it is considered useful.**

Alternative: Steps 0 and 1

Step 0: Let r symbolize cPD. Let $cLGD[\cdot]$ be the Frye-Jacobs LGD function. The mathematical expectation of cLoss is EL:

$$EL = \int_0^1 r\, cLGD[r]\, pdf_{cPD}[r, PD, \rho] dr$$

$$= \int_0^1 \Phi\left[\Phi^{-1}[r] - \frac{\Phi^{-1}[PD] - \Phi^{-1}[EL]}{\sqrt{1-\rho}}\right] pdf_{cPD}[r, PD, \rho] dr$$

Step 1: That equation holds for any value of EL, such as ψ:

$$\psi = \int_0^1 \Phi\left[\Phi^{-1}[r] - \frac{\Phi^{-1}[PD] - \Phi^{-1}[\psi]}{\sqrt{1-\rho}}\right] pdf_{cPD}[r, PD, \rho] dr$$

Alternative: Steps 2-4

Step 2: Multiply by ELGD*ᵃ*, where *a* is a real number:

$$\psi \, ELGD^{a} = \int_{0}^{1} ELGD^{a} \, \Phi\left[\Phi^{-1}[r] - \frac{\Phi^{-1}[PD] - \Phi^{-1}[\psi]}{\sqrt{1-\rho}}\right] pdf_{cPD}[r, PD, \rho] dr$$

Step 3: Set ψ = EL / ELGD^{a}; the left side is now EL:

$$EL = \int_{0}^{1} \underbrace{ELGD^{a} \, \Phi\left[\Phi^{-1}[r] - \frac{\Phi^{-1}[PD] - \Phi^{-1}[EL/ELGD^{a}]}{\sqrt{1-\rho}}\right]} pdf_{cPD}[r, PD, \rho] dr$$

Step 4: The expectation of this is EL! This must be cLoss!

$$cLGD[cPD, a] = ELGD^{a} \, \Phi\left[\Phi^{-1}[cPD] - \frac{\Phi^{-1}[PD] - \Phi^{-1}[EL/ELGD^{a}]}{\sqrt{1-\rho}}\right] / cPD$$

$$\mathbf{c}LGD = ELGD^{a}\,\Phi\left[\Phi^{-1}[cPD] - \frac{\Phi^{-1}[PD] - \Phi^{-1}[EL/ELGD^{a}]}{\sqrt{1-\rho}}\right]/cPD$$

We just showed that   E [this expression * cPD] = EL.

Therefore, this expression is an LGD function.

The next slide shows that the parameter "a" controls the sensitivity of cLGD to cPD…

$$\mathbf{c}LGD = ELGD^{a}\,\Phi\left[\Phi^{-1}[cPD] - \frac{\Phi^{-1}[PD] - \Phi^{-1}[EL/ELGD^{a}]}{\sqrt{1-\rho}}\right]/cPD$$

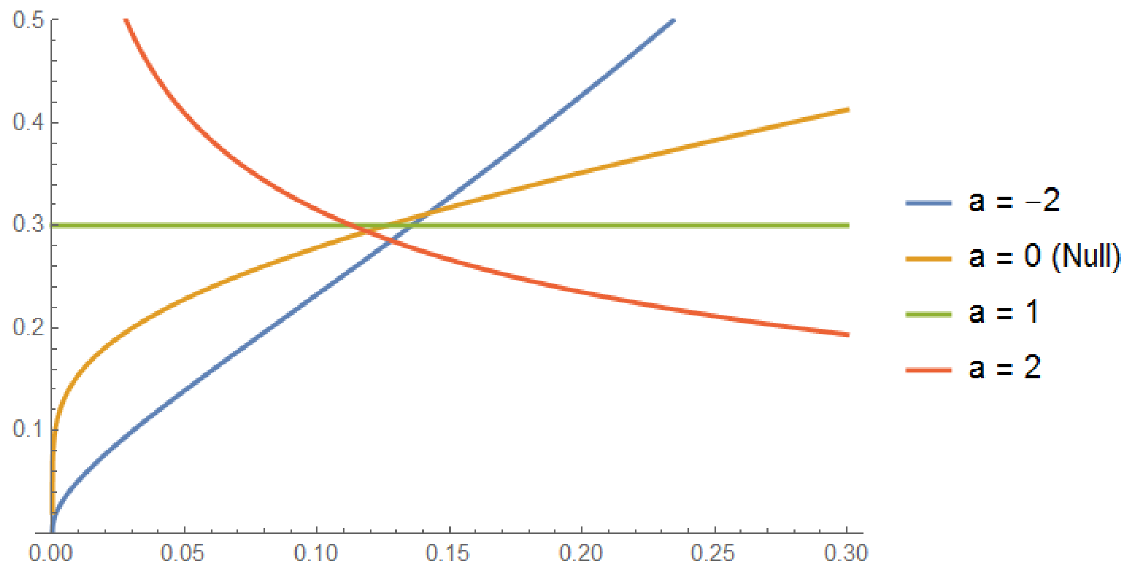
If a = 0, this is the Frye-Jacobs LGD function.

- **The Null Hypothesis nests with the Alternative.**

If a = 1, this is cLGD = ELGD.

- **cLGD can be a constant function and independent from cPD.**

Other values give a monotonic-looking function. →→→→



Summary: Alternative A

We have an alternative LGD function with the new parameter, a, which controls the sensitivity of cLGD.

- **a has no effect on EL or on the distribution cPD.**
- **It affects only the relationship between cPD and cLGD.**

If a significantly different from 0, we reject Frye-Jacobs.

- **(It isn’t and we don’t.)**

If significantly different from 1, we reject fixed cLGD.

- **(It is and we do.)**
  - **One can't assume that an implausible hypothesis is true, simply because the available data don’t allow rejection in a particular model framework.**
    I can’t reject the null hypothesis that vaccines are worthless, but it hardly proves that they are.
  - **The data reject $\{H_0\colon\ a = 0$ and cLGD does not depend on conditions.$\}$**

Questions?

Finite portfolios

Finite portfolio

A finite portfolio introduces randomness into the portfolio default rate and into average LGD.

We assume that the finite portfolio is *uniform*:

- **All loans in the portfolio have equal PD and equal ρ.**

The number of defaults is *Binomial* with mean equal *cPD*.

  - **Same as when we derived the PMF of the number of defaults on Week 1.**

We assume each LGD is normally distributed around cLGD:

- $LGD_i \sim N\left[\, cLGD[cPD, a], \sigma^2 \right]$ **using the Alternative LGD function.**
- **We assume σ = 20%.**
- **Normality is convenient because we take averages.**
  - **The average of two normal variables is a normal variable.**
  - **Among useful distributions, only the normal has this property.**

Symbols

We are deriving the distribution of portfolio credit loss. (Given the name of this course, it is about time.)

We revise the definitions of these symbols:

- **N: The number of firms in the portfolio.**
- **D: The number of defaults among the N firms.**
- **LGD: The *average* LGD rate among the D defaults.**
- **Loss: The portfolio loss rate.**
  - **Loss = LGD * D / N.**
- **cLGD and cPD are conditional expectations as always.**

If there is no default, there is no loss…

Point mass at zero loss

The probability of zero defaults among N loans is:

$$\int_0^1 (1 - cPD)^N \, pdf_{cPD}[cPD] \, dcPD$$

where $pdf_{cPD}[cPD]$ is the Vasicek PDF.

Example calculation: If N = 10, PD = 0.1, ρ = 0.15, then the probability of zero defaults is 0.431.

- **Try it and see!**

Loss when D > 0

We seek the distribution of Loss for a portfolio of N loans that has D > 0 defaults. We assume that the LGD of each defaulted loan is normal:

$$\boldsymbol{LGD_i} \sim \textbf{IID}\ \boldsymbol{N}\ [\ \boldsymbol{cLGD}[\ \boldsymbol{cPD, a}\ ]\ , \boldsymbol{\sigma^2}]$$

Then, portfolio average LGD is also normal:

$$\boldsymbol{LGD} \sim \boldsymbol{N}\ [\ \boldsymbol{cLGD}[\ \boldsymbol{cPD, a}\ ]\ , \boldsymbol{\sigma^2/D}]$$

This is the distribution of portfolio average LGD using the Alternative to F-J and

- ***conditioned*** **on the random number of defaults, D, and**
- ***conditioned*** **on random cPD.**

Loss conditioned on D and cPD

Infer the distribution of Loss from the distribution of LGD:

$$LGD|cPD, D \sim N\,[\,cLGD[\,cPD, a\,]\,, \sigma^2/D]$$

$$LGD = cLGD[cPD, a] + \frac{\sigma}{\sqrt{D}}Y, \quad Y \sim N[0, 1]$$

$$Loss = \frac{D}{N}LGD = \frac{D\;cLGD[cPD, a] + \sqrt{D}\,\sigma\,Y}{N}$$

Invert: $$Y = \frac{N\;Loss - D\;cLGD[cPD, a]}{\sigma\sqrt{D}}\,;\;\frac{\partial\,Y}{\partial\;Loss} = \frac{N}{\sigma\sqrt{D}}$$

$$pdf_{Loss|D,cPD}[Loss] = \frac{N}{\sigma\sqrt{D}}\phi\left[\frac{N\;Loss - D\;cLGD[cPD, a]}{\sigma\sqrt{D}}\right]$$

Remove the conditioning

The next step resembles finding the PMF of the number of defaults in a finite portfolio, as in Week 2.

The derivation expresses the PDF of Loss in terms of

- **the PDF of Loss given D and cPD (found on the last slide),**
- **the PDF of D (the number of defaults) given cPD, and**
- **the PDF of cPD, which is the Vasicek PDF.**

$$pdf[Loss]$$

$$= \int pdf[Loss, cPD]\, dcPD$$

$$= \int pdf[cPD]\; pdf[Loss|cPD]\, dcPD$$

$$= \int pdf[cPD] \sum_{D=1}^{N} pdf[Loss, D|cPD]\; dcPD$$

$$= \int pdf[cPD] \sum_{D=1}^{N} pdf[Loss|D, cPD]\, pmf[D|cPD]\; dcPD$$

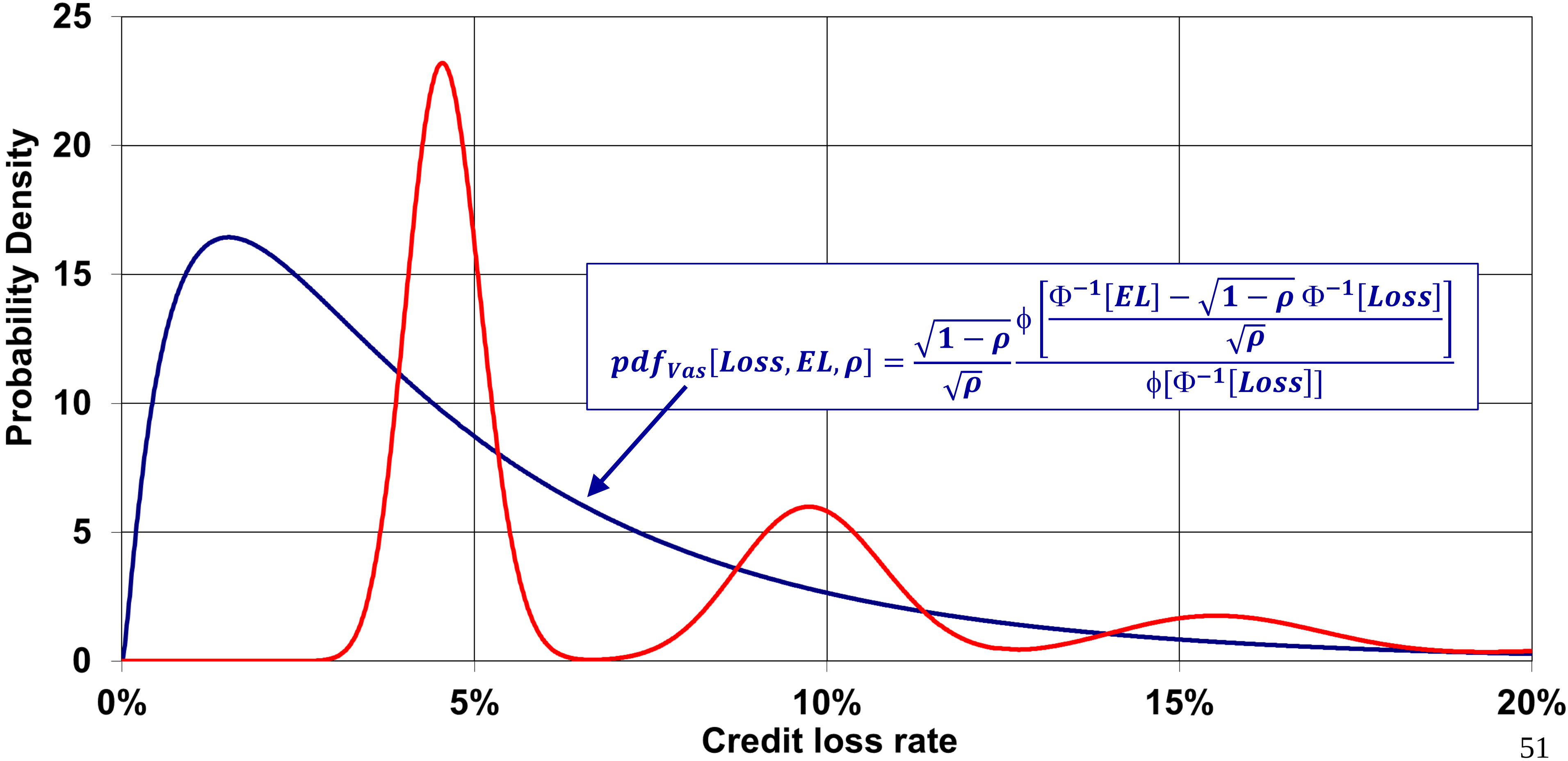
$$= \int pdf_{Vas}[cPD] \sum_{D=1}^{N} \frac{N}{\sigma\sqrt{D}}\; \phi\left[\frac{N\, Loss - D\; cLGD[cPD, a]}{\sigma\sqrt{D}}\right] \binom{N}{D} cPD^{D}\, (1 - cPD)^{N-D}\; dcPD$$

This the distribution of loss each period of one year.

Distribution of loss in a finite portfolio

N=10, PD=10%, EL=5%, ρ=15%, σ=1%, a=0; Pr[D=0]=0.431

$$\int pdf_{Vas}[cPD] \sum_{D=1}^{N} \frac{N}{\sigma\sqrt{D}} \; \phi\left[\frac{N\,Loss - D\; cLGD[cPD, a]}{\sigma\sqrt{D}}\right] \binom{N}{D} cPD^{D} \, (1 - cPD)^{N-D} \;\; dcPD$$



$$pdf_{Vas}[Loss, EL, \rho] = \frac{\sqrt{1-\rho}}{\sqrt{\rho}} \frac{\phi\left[\frac{\Phi^{-1}[EL] - \sqrt{1-\rho}\,\Phi^{-1}[Loss]}{\sqrt{\rho}}\right]}{\phi[\Phi^{-1}[Loss]]}$$

Summary: Finite portfolio

In a finite portfolio, the default rate has expectation equal to cPD and the LGD rate has expectation equal to cLGD.

- **The number of defaults has a Binomial distribution.**
- **LGDs are assumed normally distributed around cLGD.**

The resulting distribution of Loss in a finite portfolio is quite a lot more complicated than the Vasicek distribution.

The most difficult step was to find an alternative hypothesis that changed the steepness of the LGD function without affecting EL.

Like all models, this model is wrong. So far, it appears that there isn't enough data to show that it is wrong.

Questions?

The tests and summary

Multiple grades and classes

There are 5 rating grades and 5 seniority classes.

We assume a single risk factor, Z.

- **As before, the losses within different grades and classes are *<u>conditionally</u>* *<u>independent</u>*.**
  - **We perform change-of-variable to get all losses as functions of Z.**
  - **We integrate over Z rather than cPD.**
  - **We multiply PDFs of the sub-portfolios to produce the integrand.**
  - **This is messy to write down on a slide, but you get the idea.**

Doing this lets us analyze all loans together, all bonds together, or all instruments together.

One test

For both the Null and the Alternative:

- **PD and ρ are set to MLE's based on *default* data.**
- **ELGD equals average LGD in each grade–class combo.**
- **σ is set to 20%**
  - **This is on the low side. I am *not* packing the model with false noise.**

H_0: $a = 0$; Frye-Jacobs explains the data.

H_1: a equals its MLE based on the loss data.

- **The sensitivity of cLGD to cPD is different from Frye-Jacobs.**

Result: MLE [a] = 0.01

MLE [a] = 0.01 for all loans taken together.

- **a = 0.01 is not significantly different from a = 0.**
  - **The Frye-Jacobs LGD function is not rejected.**
- **a = 0.01 is significantly different from a = 1.**
  - **The idea that LGD is fixed is rejected**
  - **LGD varies with the default rate.**

There is no value of a that does a significantly better job modeling cLGD than $a = 0$.

- **We conclude that F-J is consistent with Moody's.**

Very hard to get this published!

- **Referees expect a paper to display statistical significance and to reject the null hypothesis.**

Summary: ***An*** LGD function

Assumption: If a set of conditions are expected to make the default rate go up, they should be expected to make the Loss rate go up.

- **cPD and cLoss are comonotonic.**

Implication: Every loan has an LGD function that maps its cPD to its cLGD.

- **The function depends on two distributions.**
  - **There is no limit to the complexity of either one.**

Summary: ***The*** LGD function

Frye and Jacobs find an LGD function that is strictly monotonic for all values of PD, EL, and ρ.

An Alternative LGD function contains a parameter that controls the sensitivity of cLGD to cPD.

The additional sensitivity parameter is *not* significantly different from zero.

The Fed uses F-J in stress testing:
https://www.federalreserve.gov/publications/files/2022-march-supervisory-stress-test-methodology.pdf

References

Jon.Frye@chi.frb.org

Frye, Modest Means, *Risk*, January 2010.

- **A two-parameter credit loss distribution is adequate, given Altman's data. The LGD function is later inferred from this idea.**

Frye and Jacobs, Credit loss and systematic loss given default, *Journal of Credit Risk*, Spring 2012

- **The sensitivity of cLGD to cPD calibrated to Moody's data is nearly equal to the sensitivity built into the LGD function.**

Frye, The link from default to LGD, *Risk*, March 2014

- **Tail LGD is better predicted by the LGD function than by linear regression using simulated data from a linear model.**

Questions?

Don’t forget

Homework 4 is due next week at 6PM.

Lisheng’s TA session will be Sunday at 6PM.