Portfolio Credit Risk

University of Chicago Masters in Financial Mathematics 36702

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Lecture 2

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Simulation and non-simulation models of portfolio default

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Last week and this week

We want a model of the portfolio default rate.

- We'll add LGD and loss later.

We assume banks have good estimates of PDs.

Data show that the defaults of firms are ***not*** independent.

There are very few copulas that might connect defaults.

- With the *t*-copula, independent variables do ***not*** have zero correlation. This complicates the analysis that we perform.
- The Gauss copula is more familiar. It keeps the focus on learning the portfolio model, rather than learning a new copula.

This week we introduce the simplest Gauss copula.

- It is called the single risk factor model.

Tonight's list of topics

The standard portfolio simulation

Modeling without simulation

The single risk factor (SRF) model

The Vasicek distribution

Basel minimum capital requirement

The standard portfolio simulation

The standard portfolio simulation

A simulation run has inputs:

- There are k firms in the portfolio.
- Each firm in the portfolio has a probability of default, PD_i.
- Each firm has a latent variable, Z_i.
  - Firm i is in default if the percentile of Z_i is less than PD_i.
- The latent variables have a joint standard normal distribution.
  - Gauss copula.
- Each pair of latent variables has a correlation, $\rho_{i,j}$.
  - The off-diagonal correlations are not all zero.

In each simulation run,

- Draw values of the correlated latent variables $\{Z_i\}_{i=1}^{k}$.
- Calculate the set of default indicators $D_i = I[z_i < \Phi^{-1}[PD_i]]$.
- Keep track of the defaults.
  - Often the main item of interest is the default rate, $DR = \frac{\sum_{i=1}^{k} D_i}{k}$.

Last week's optional reading

The standard portfolio simulation was first articulated for a wide audience in the CreditMetrics manual, 1998.

It required a lender to know whether a loan had defaulted, and to know the probability with which a loan would default within a year.

- **The event of default had to be *defined*.**
- **The probabilities had to be estimated.**

And the model required estimates of the $\{\rho_{i,j}\}$.

- **Most banks used asset correlations or equity correlations.**
  - **Again, data vendors supply the estimates.**

One run with a 4-firm portfolio

Firm	PD_i	Correlation Matrix $\rho_{i,j}$				Simulated Z_i	$\Phi^{-1}[PD_i]$	D_i
1	0.1	1	0.1	0.2	0.3	-1.3559	-1.2816	1
2	0.2	0.1	1	0.4	0.5	-0.6171	-0.8416	0
3	0.3	0.2	0.4	1	0.6	-0.4817	-0.5244	0
4	0.4	0.3	0.5	0.6	1	-0.0562	-0.2533	0
		Number of defaults in this simulation run =						1

Keeping track over many runs, you could estimate the distribution of the default rate, its expectation, and so forth.

You could judge the sensitivity of the results to the inputs.

Time and risk

This is a one-period model.

- It estimates the distribution of the default rate in the next year.
- It says nothing about the world after that.

But real life goes on, and it usually resembles today.

- If the current quarter is weak, next quarter is likely to be weak.
- If the current quarter is not weak, next quarter is likely to be not weak.

The serial dependence of credit loss data is a channel of inquiry that we touch on in Week 5.

Portfolio ***loss*** simulation

Simulation can handle portfolio *loss* as well as default.

More inputs are needed:

- **The dollar exposure of each loan to each firm**
- **The distribution of LGD for each loan**

If a loan defaults in a simulation run:

- **Draw a value of LGD from its distribution.**
- **Multiply LGD by exposure to find dollar loss.**

The loss portfolio rate: $\boldsymbol{Loss = \sum_i \$Loss_i \,/\, \sum_i \$Exposure_i}$**.**

- **We simplify when we assume all the exposures are equal.**
- **We model conditional LGD in Week 4.**

Questions? Comments?

Something like this simulation is run by most banks.

Modeling without simulation

To simulate or not to simulate?

Simulation keeps track of all the relevant details, but it is easy to get lost in the details.

Suppose you want to know what kind of loan would produce the least risk when added to your portfolio.

- **The simulation answer depends on all the PDs, ρs, etc.**
- **A clear picture might not appear.**

But given explicit formulas for loss distributions, the answers are easier to see and to comprehend.

- **And the results appear close enough for some purposes.**

To not simulate, simplify

The default rate simulation has two major inputs:

- **$\{PD_i\}$, the set of PD's for the k firms in the portfolio**
- **$\{\rho_{i,j}\}$, the set of $\frac{(k-1)k}{2}$ correlations between latent variables.**
  - **The correlation matrix is the target of the simplification.**

The Gauss copula allows any correlation matrix that is positive definite.

The *single factor model* restricts the choice of matrix.

- **Every firm depends on a single random factor, Z.**
  - **Think of it as an overall level of credit stress.**
  - **When Z is elevated, *every* firm becomes more likely to default.**
- **There is no other variable affecting more than one firm.**
  - **There is no source of correlation between firms other than Z.**
  - **Some firms are affected more strongly by Z than others.**

Single risk factor (SRF) model

Each firm's latent variable, Z_i, depends on a portfolio-wide (or economy-wide) factor, Z.

- This is called the "systematic" factor.

The model also contains k other factors, $\{X_i\}$.

- X_i affects Z_i and affects nothing other than Z_i.
- The X_i are called "idiosyncratic" factors.

Since Z affects every Z_i, the Z_i's are correlated.

The next slide expresses this in linear math.

$$Z_i = -\sqrt{\rho_i}\, Z + \sqrt{1 - \rho_i}\, X_i$$

Z and $\{X_i\}$ are assumed independent standard normal.

- **Therefore, Z and $\{X_i\}$ are *jointly* normal.**
- **Therefore, $\{Z_i\}$ are *jointly* normal.**
  - **That is, they are normal variables connected by a Gauss copula.**

$$E[Z_i] = E[-\sqrt{\rho_i}\, Z] + E\left[\sqrt{1-\rho_i}\, X_i\right] = 0 + 0 = 0.$$

$$\begin{aligned} Var[Z_i] &= Var[-\sqrt{\rho_i}\, Z] + Var\left[\sqrt{1-\rho_i}\, X_i\right] + 2\, Cov[-\sqrt{\rho_i}\, Z, \sqrt{1-\rho_i}\, X_i] \\ &= \rho_i + 1 - \rho_i + 0 \\ &= 1 \end{aligned}$$

Therefore, $\{Z_i\}$ are jointly *standard* normal.

- **They always were, and still are, jointly standard normal.**

Correlations galore

When we had a portfolio with two firms, we used ρ to represent the correlation between their latent variables.

We articulated the two firms in the symbol $\rho_{1,2}$.

The expression for the single risk factor model contains a new symbol, ρ_i.

- **Each firm has its own value of ρ_i.**
- **We assume that $0 \le \rho_i \le 1$ for all i.**
  - **Remember, correlation is in the range 5% to 15% for most firms.**

Next, we calculate $Corr[Z_i, Z_j]$ in the single factor model.

$Corr[Z_i, Z_j]$

$$Corr[Z_i, Z_j] = Cov[Z_i, Z_j]$$

$$= Cov[-\sqrt{\rho_i}\, Z + \sqrt{1-\rho_i}\, X_i, -\sqrt{\rho_j}\, Z + \sqrt{1-\rho_j}\ X_j]$$

$$= (-\sqrt{\rho_i})\left(-\sqrt{\rho_j}\right) Cov[Z, Z]$$

$$+ -\sqrt{\rho_i}\sqrt{1-\rho_j}\, Cov[Z, X_j]$$

$$+ \sqrt{1-\rho_i}\left(-\sqrt{\rho_j}\, Z\right) Cov[X_i, Z]$$

$$+ \sqrt{1-\rho_i}\sqrt{1-\rho_j}\, Cov[X_i, X_j]$$

$$= \sqrt{\rho_i\, \rho_j}$$

If $\rho_i = \rho_j = \rho$, then $Corr[Z_i, Z_j] = \rho$. We refer to the value of ρ_i as "the correlation of Firm i."

The correlation matrix

In a single factor model, the correlation between two firms is the geometric average of ρ_i and ρ_j:

$$Corr[Z_i, Z_j] = \sqrt{\rho_i \, \rho_j}$$

The k values $\{\rho_i\}$ imply the $k(k-1)/2$ values of $\rho_{i,j}$.

This greatly restricts the set of positive definite matrices.

The correlation matrix of *<u>any</u>* single factor model is positive definite.

- **I don't have a proof handy. If you find one, please send it to me.**

Interpretations

When Z is elevated, each Z_i tends to be depressed.

- **Therefore, each firm is more likely to default than otherwise.**

A firm with a greater value of ρ_i is more affected by Z.

- **Such a firm is said to be a "cyclical" firm, such as an airline.**
  - **Airlines have high fixed costs and revenues that are highly dependent on the overall economy. They are bailout magnets.**
- **A firm with low ρ_i is effected less, like Proctor and Gamble.**
  - **Consumer nondurable firm P&G is a noncyclical firm.**

Questions? Comments?

The single risk factor model is the main topic for today.
And it is the main topic for the course.

Conditional expectations

PD is the probability of default in the next year.

- It can be viewed as an average of what is expected if future conditions are favorable, unfavorable, or neutral to the firm.
  - In favorable conditions, the probability of default would be low.
  - In unfavorable conditions, the probability of default would high.

Next we use the single factor model to find the probability of default given Z.

- This is called “conditional PD” and symbolized cPD.
  - If the economy has a bad year indicating that Z is elevated, then conditional PD is greater than PD.

Later we find the ***distribution*** of conditional PD.

Conditional PD: cPD

$$cPD_i = Pr\left[\, Z_i < \ \Phi^{-1}[PD_i] \mid Z = z\right]$$

$$= Pr\left[\, -\sqrt{\rho_i}\, z + \sqrt{1-\rho_i}\, X_i < \Phi^{-1}\,[PD_i]\right]$$

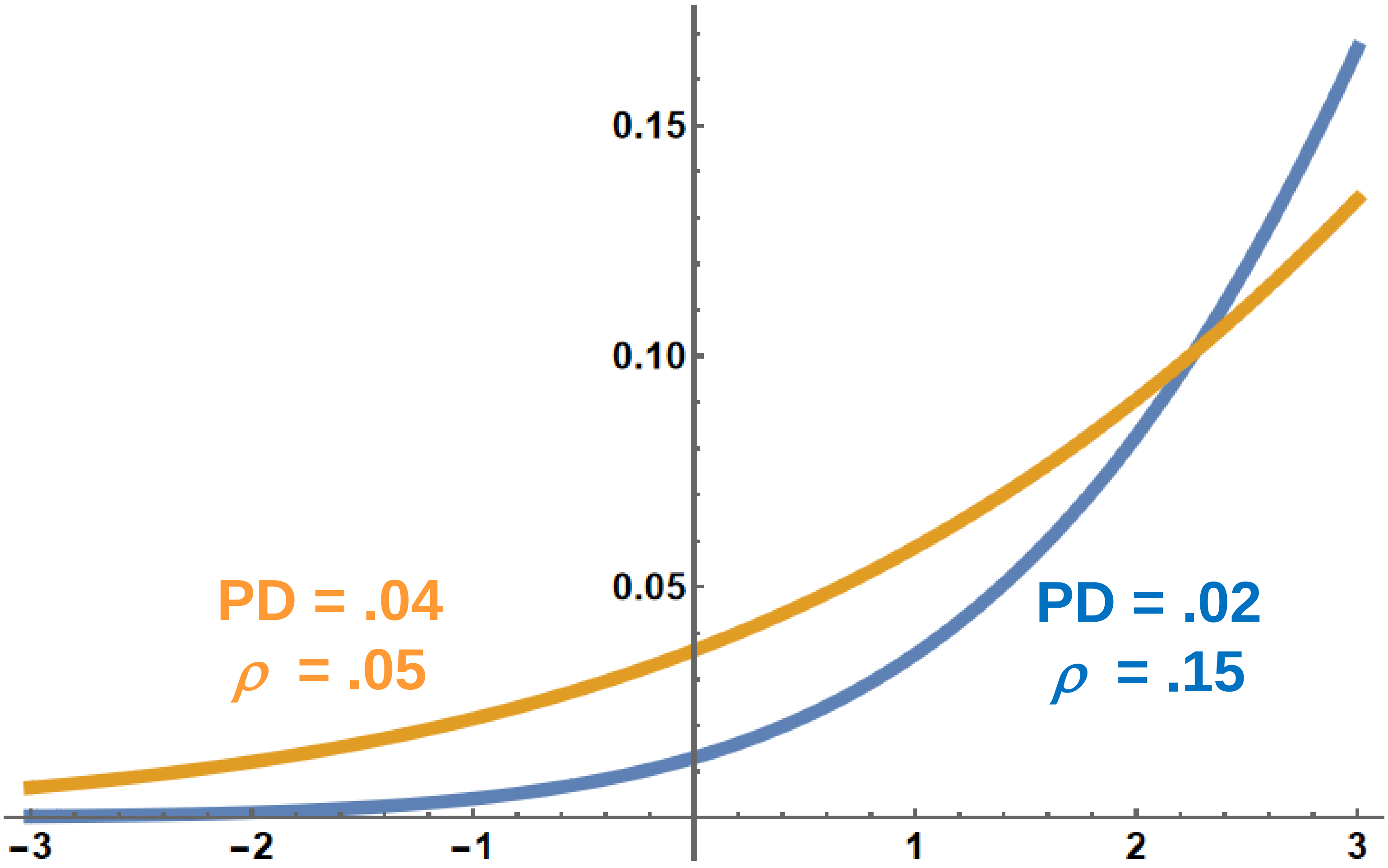
$$= Pr\left[X_i < \frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\, z}{\sqrt{1-\rho_i}}\right] \ = \ \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\, z}{\sqrt{1-\rho_i}}\right]$$

This applies to each firm in a single risk factor model.

- **If every firm had the same PD and the same value of ρ, this would be the conditionally expected PD of the portfolio.**
- **If in addition the number of firms in the portfolio were very large, then the law of large numbers assures that the default rate would equal the conditionally expected rate.**

This expression is called the Vasicek formula.

cPD is monotonic in Z



Conditional independence

$Z = z$ **fully specifies conditions in a SRF model.**

- **A firm's default depends only on X_i, which is independent.**
  - **Given z, the defaults of firms are *conditionally independent*.**

This provides a second way to perform simulation.

- **Instead of drawing k correlated values $\{Z_i\}_{i=1}^{k}$, draw k independent values $\{X_i\}_{i=1}^{k}$ and an independent value of Z.**
- **Then calculate each Z_i : $Z_i = -\sqrt{\rho_i}\, Z + \sqrt{1-\rho_i}\, X_i$.**
- **Check for defaults and keep track as before…**

Instead of simulating, we now find a compact expression for the distribution of the number of defaults.

- **$\sum D = \sum_{i=1}^{k} D_i$, the number of defaults among k firms.**
- **"PMF" = "probability mass function." Think: "Discrete PDF".**

Conditional distribution of $\sum_{i=1}^{k} D_i$

**Suppose that there are k firms in the portfolio,
that the firms are affected by a single factor model, and
that all firms have the same PD and same ρ.**

In conditions $Z = z$, the k firms default independently at rate $cPD[z]$. The number of defaults conditioned on Z is has the Binomial$\left[k, cPD[z]\right]$ distribution:

$$\left(\sum_{i=1}^{k} D_i \middle| Z = z\right) \sim Bin\left[k, cPD[z]\right].$$

$$PMF[\Sigma D|z] = \binom{k}{\Sigma D}\left(\Phi\left[\frac{\Phi^{-1}[PD]+\sqrt{\rho}\,z}{\sqrt{1-\rho}}\right]\right)^{\Sigma D}\left(1-\Phi\left[\frac{\Phi^{-1}[PD]+\sqrt{\rho}\,z}{\sqrt{1-\rho}}\right]\right)^{k-\Sigma D}$$

Remove the conditioning

We have the conditional distribution, $PMF[\Sigma D|z]$.

The joint distribution is $PMF[\Sigma D, z] = PMF[\Sigma D|z]\ PDF[z]$.

The desired distribution is $PMF[\Sigma D] = \int_{-\infty}^{\infty} PMF[\Sigma D, z]\, dz$.

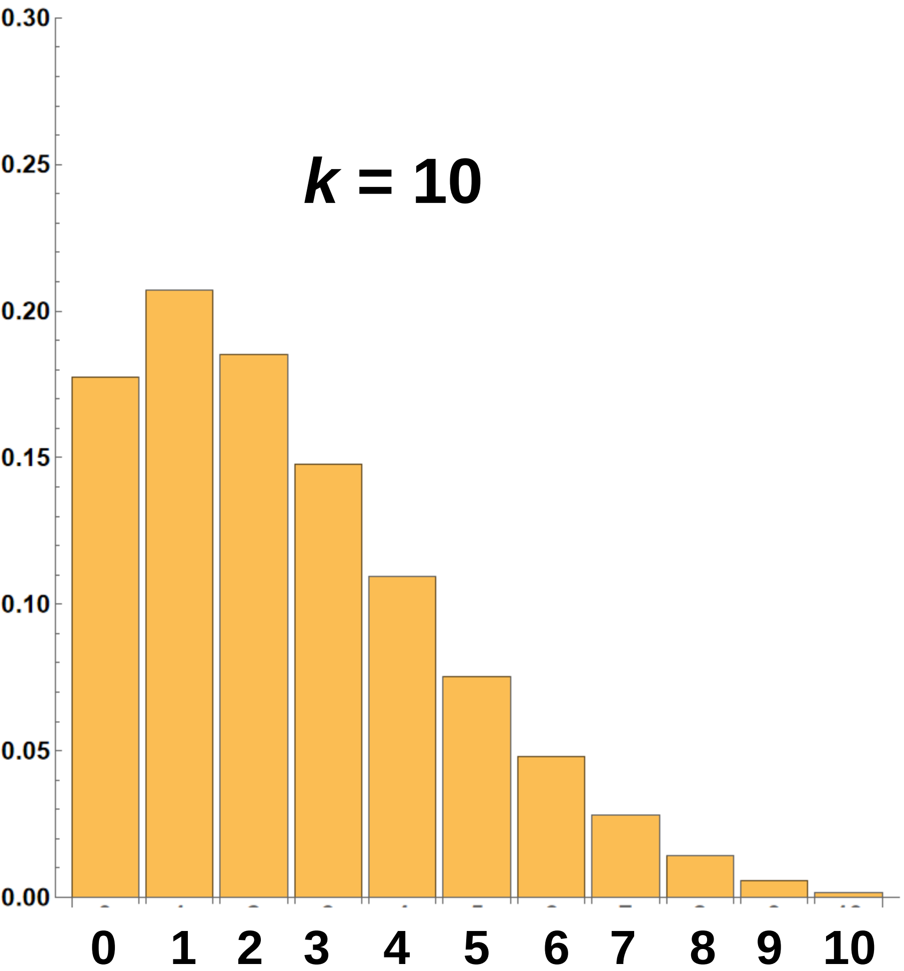
$$= \int_{-\infty}^{\infty} \binom{k}{\Sigma D} \left(\Phi\left[\frac{\Phi^{-1}[PD] + \sqrt{\rho}\, z}{\sqrt{1-\rho}} \right] \right)^{\Sigma D} \left(1 - \Phi\left[\frac{\Phi^{-1}[PD] + \sqrt{\rho}\, z}{\sqrt{1-\rho}} \right] \right)^{k-\Sigma D} \phi[z]\ dz$$

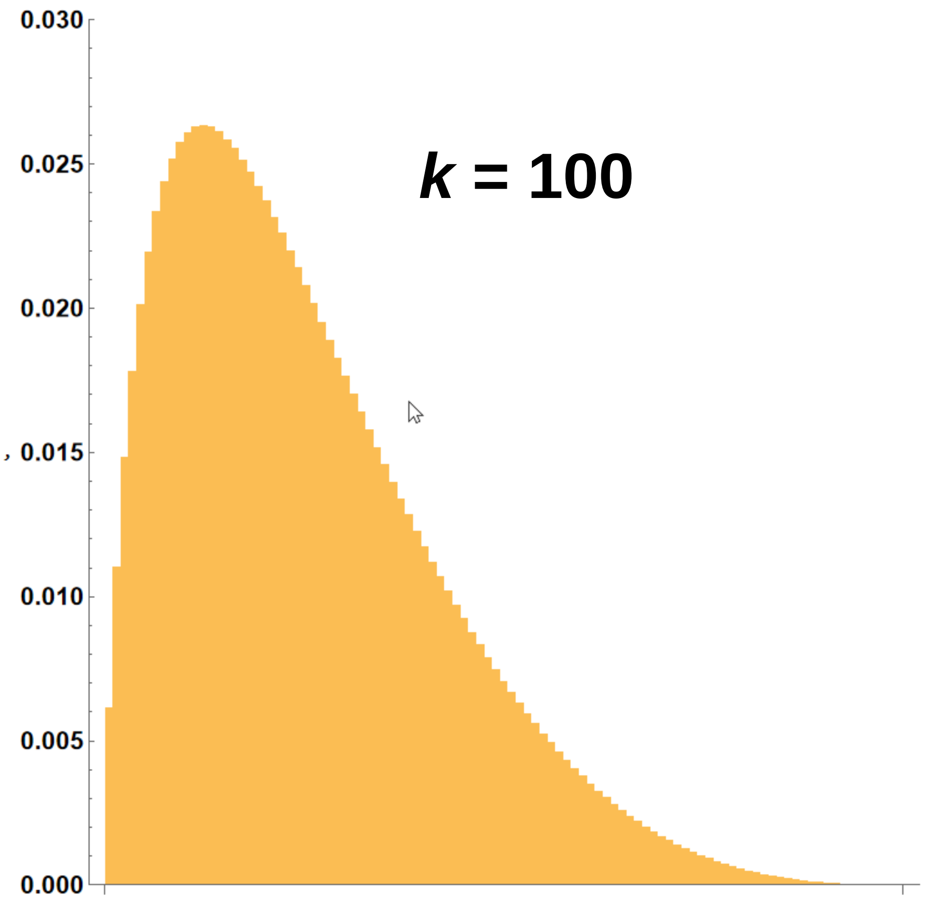
where $PDF[z] = \phi[z]$, the standard normal density bell curve.

This is the distribution of ΣD defaults among k firms where

- Each firm has the same value of PD,
- Firms respond to a single risk factor and
- Each firm has the same value of correlation.

Two PMFs: PD = 0.25, ρ = 0.25





Questions? Comments?

This stuff is important.

You should be able to derive the Vasicek formula for cPD.

Letting $k \to \infty$?

- **As k, the number of firms in the portfolio, increases, the PMF approaches a continuous distribution.**
  - **An easier route to finding the same distribution is to find the distribution of the cPD of a *single firm*.**

- **To derive the PDF of cPD:**
  - **Find the inverse CDF of cPD from the Vasicek function.**
  - **Find the CDF of cPD by inverting the inverse CDF.**
  - **Find the PDF by differentiating the CDF.**

- **The resulting distribution is the Vasicek Distribution.**
  - **Same guy who did the interest rate model.**

The inverse CDF

$$cPD_i[z] = \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\, z}{\sqrt{1-\rho_i}}\right]$$

Evaluate the Vasicek function at $q = \Phi[z]$:

$$invCDFcPD_i[q] = \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\ \Phi^{-1}[q]}{\sqrt{1-\rho_i}}\right]$$

q is the quantile of Z; therefore, q is the quantile of cPD.

Given q, this gives the associated value of cPD.

- **It is the inverse CDF of the random variable cPD.**

Invert the inverse CDF

The inverse CDF from the previous slide is:

$$cPD_i = \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\ \Phi^{-1}[q]}{\sqrt{1-\rho_i}}\right]$$

Solve for q :

$$q = \Phi\left[\frac{\sqrt{1-\rho_i}\ \Phi^{-1}[cPD_i] - \Phi^{-1}[PD_i]}{\sqrt{\rho_i}}\right] = CDF_i[cPD_i]$$

For a value of cPD_i, this gives its quantile.

- **It is the CDF of the random variable cPD.**

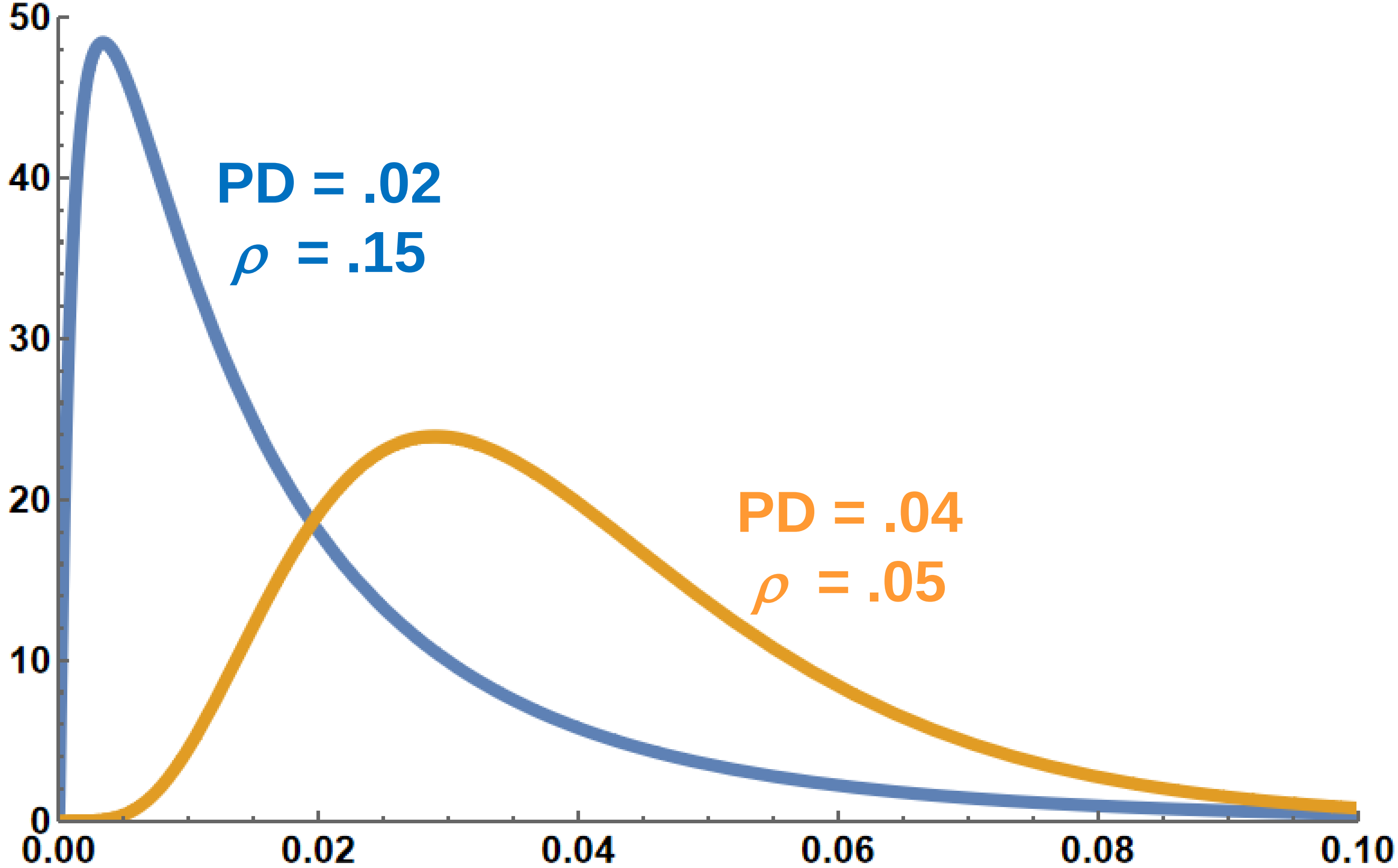
Find the PDF by differentiation

$$CDF_i[cPD_i] = q = \Phi\left[\frac{\sqrt{1-\rho_i}\,\Phi^{-1}[cPD_i] - \Phi^{-1}[PD_i]}{\sqrt{\rho_i}}\right]$$

$$PDF_i[cPD_i] = \frac{\sqrt{1-\rho_i}}{\sqrt{\rho_i}\,\phi[\Phi^{-1}[cPD_i]]}\,\phi\left[\frac{\sqrt{1-\rho_i}\,\Phi^{-1}[cPD_i] - \Phi^{-1}[PD_i]}{\sqrt{\rho_i}}\right]$$

This is the Vasicek distribution. It gives the probability density of cPD. Parameters are PD and ρ.

Vasicek PDFs



Vasicek Distributions

Inverse CDF: $CDF^{-1}[q] = \Phi\left[\frac{\Phi^{-1}[PD]+\sqrt{\rho}\,\Phi^{-1}[q]}{\sqrt{1-\rho}}\right]$

CDF: $CDF[cPD] = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[cPD]-\Phi^{-1}[PD]}{\sqrt{\rho}}\right]$

PDF: $PDF[cPD] = \frac{\sqrt{1-\rho}}{\sqrt{\rho}\,\phi\left[\Phi^{-1}[cPD]\right]}\,\phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[cPD]-\Phi^{-1}[PD]}{\sqrt{\rho}}\right]$

You should know how to derive these formulas.

You don’t need to memorize them.

Questions? Comments?

PDF by change-of-variable

- Suppose the distribution of Z is known, and we want the distribution of R = g[Z], where g is monotonic.
  - When these conditions are met, it is possible to find the distribution of R by "change-of-variable."
    - It is the same as the derivation just completed.
- cPD is a monotonic function of Z, which has a standard normal distribution. We're good to go…

Change-of-variable technique

Suppose the distribution of Z is known, and we want the distribution of R = g[Z], where g is monotonic.

$$CDF_R[r] = \Pr[R < r] = \Pr[g[Z] < r] = \Pr\left[Z < g^{-1}[r]\right] = CDF_Z[g^{-1}[r]]$$

$$PDF_R[r] = \frac{\partial\, CDF_Z[g^{-1}[r]]}{\partial\, r} = \left|\frac{\partial g^{-1}[r]}{\partial r}\right| PDF_z[g^{-1}[r]]$$

So, "change-of-variable" is also called "the chain rule".

$$PDF_R[r] = \left|\frac{\partial g^{-1}[r]}{\partial r}\right| PDF_Z[g^{-1}[r]]$$

Know this cold. When applied to $cPD[Z]$ with $Z \sim N0, 1]$, it produces the PDF of the Vasicek distribution.

Nice properties of Vasicek dist.

- It has support on [0,1].
  - The other common distribution with support on [0,1] is Beta.
- The first parameter is the expected value.
  - Expected cPD is PD.
- The second parameter takes a limited range of values.
  - Most estimated values of ρ are between 5% and 15%.
    - The range reflects the difference between cyclical and non-cyclical firms.
- The PDF, CDF, and $\text{CDF}^{-1}[\cdot]$ can all be stated compactly.
  - This is uncommon among statistical distributions.
- The Vasicek distribution is a plank in a shipwreck.
  - Otherwise, we are clueless. Like with the Central Limit Theorem.

Vasicek summary

The simplest Gauss copula is the independence copula.

- **But the data say that firms are not independent.**
- **The next-simplest Gauss copula is the single risk factor model.**
  - **Simpler models have fewer things that can go wrong. That's good!**

If a firm responds to a single risk factor, then its cPD is a monotonic function of the risk factor.

- **The distribution of its cPD is Vasicek.**

If a *portfolio* contains firms with uniform values of PD and ρ, each has the same cPD.

- **The default rate of a large portfolio is distributed like Vasicek.**

Questions? Comments?

You must know the change of variable formula,

and you must be able to use it.

You should be able to derive it if you forget it.

The Basel capital requirement

The Basel Committee

- **The Bank for International Settlements is in Basel, CH.**
  - **There is a Basel Committee on Bank Supervision, BCBS.**
- **The BCBS drafted legislation requiring banks to have minimum *capital*. "Basel II", "Basel III", etc.**
  - **A similar law was adopted by each developed country.**
    - **The US has other requirements that tend to be more binding than Basel.**
- ***Capital* is the money a bank has available to lose.**
  - **But it is a muddy accounting concept, hard to define.**
- **The capital *requirement* is like a margin requirement.**
  - **To make a given loan, a bank must have minimum capital.**
    - **Capital lets the bank survive if it has some credit loss.**
    - **This protects bank depositors and the public.**

One rule to ring them all

The BCBS wanted a function that would give minimum capital that a bank regulator would require for a loan.

- **The characteristics of the loan would imply minimum capital.**
- **The rest of the portfolio would not matter. Two implications:**
  - **Minimum capital for a loan would be the same for every bank.**
  - **Portfolio required capital would be the sum of loan required capital.**
    That is: there is no opportunity to diversify risk; risk is additive.

Risk additivity implies a single factor model.

- **It is impossible to diversify risk with a single factor model.**
- **The characteristics of the loan are its PD, ρ, and ELGD.**
  - **The bank estimates PD and ELGD; BCBS specifies the value of ρ.**
- **Minimum capital would be loss at quantile 0.999.**

Basel formula and cPD

Per dollar of "wholesale" loans, Basel requires a bank to have capital (K) equal to a fraction of the loan amount.

- **"R" is Basel notation for correlation (ρ)**
- **"N" is the standard normal CDF (Φ), and**
- **"M" is the maturity of the loan in years ranging from 1 to 5.**

$$K = \left[LGD \times \underbrace{N\left(\frac{N^{-1}(PD) + \sqrt{R} \times N^{-1}(0.999)}{\sqrt{1-R}} \right)}_{\text{This is cPD at } q = 0.999.} - \left(LGD \times PD \right) \right] \times \left(\frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b} \right)$$

Three main differences

1. Capital is required for ***loss***, not just for ***default***.
   - The formula multiplies by LGD to take care of this.

2. Capital is required only for "unexpected" loss.
   - Reserves should handle expected loss.
   - Expected loss, LGD x PD, is subtracted from the risk.

3. Loans might deteriorate but not default.
   - Basel adjusts by a maturity adjustment factor.
     - Loans with longer maturity require perhaps 3 times more capital.

$$K = \left[LGD \times N\left(\frac{N^{-1}(PD) + \sqrt{R} \times N^{-1}(0.999)}{\sqrt{1-R}} \right) - \left(LGD \times PD \right) \right] \times \left(\frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b} \right)$$

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$

More Basel calibration

Basel specifies these parameters in the formula:

- R (correlation) = 0.12 + .12 * Exp[-50 PD]
  - Note: This is monotonic decreasing.
  - There is no evidence that correlation and PD are related this way.
  - A value of PD implies approximately nothing for correlation.
- b (in the maturity adjustment) = $(0.11852 - 0.05478 \text{ Log[PD]})^2$
  - Don't ask. Someone fit some data, probably on an emergency basis.

A bank might estimate Basel parameters like this:

- A loan's PD equals the average annual default rate
  - within the rating grade that the bank assigns to the loan.
- LGD is the average LGD in historical "downturn" conditions,
  - taken among loans with similar seniority and security.
- M is maturity in years, bounded between 1 and 5.
- All estimates are subject to supervisory oversight.

Basel formula summary

Basel requires banks to have minimum capital.

- Banks treat capital as if it were expensive, so there are games galore surrounding the input estimates.
- On top of the capital required for credit loss, more capital is required for other reasons.

Minimum capital is a high percentile of the cPD formula. Minimum capital for the portfolio is the sum of minimum capital for each loan because there's only one risk factor.

The formula depends on estimates of PD and LGD.

- These must be estimated by the bank.
- The estimation process is overseen by bank supervisors.

Questions? Comments?

Multistate simulation models

Simulating rating transitions

So far, we have simulated a two-state model.

- **State 1: Default.   State 0: No default.**

One could model transitions to other states.

- **Usually, the other states are internal rating grades.**
- **This requires**
  - **the probability that a firm with a given rating experiences transition to a new rating**

    The probabilities are given in a ratings transition matrix like the next slide
  - **the cost or benefit to the lender of the rating transition**

    Downgrades of borrowers hurt the lender; upgrades of borrowers help.

A rating transition matrix

High grade

Rat'g	Rating at year end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60
D	0	0	0	0	0	0	0	100

High yield

The numbers are outdated, but this gives an idea.

The most likely thing is no change of rating.

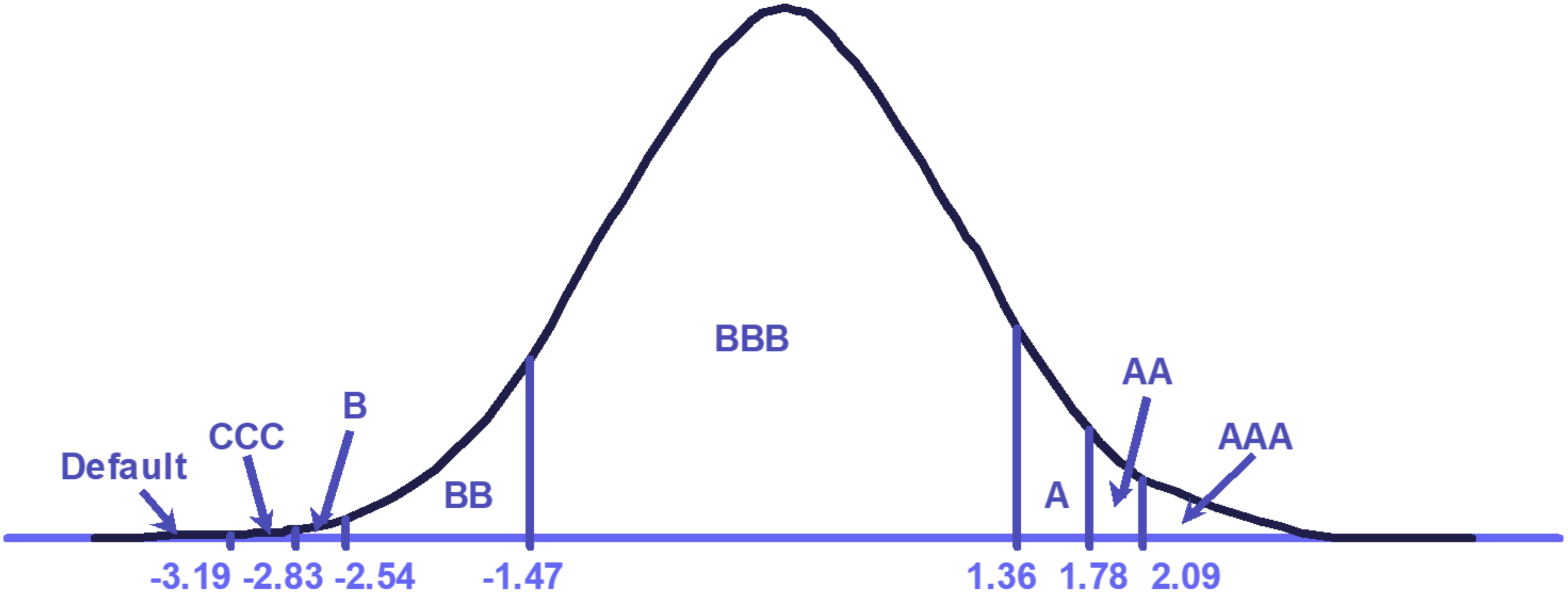
Consider Firm i, rated BBB

Assume that Z_i controls all transitions.

- **According to the previous slide, a firm rated BBB defaults if Z_i is in the worst 0.07% of its range.**
- **The firm transitions from BBB to CCC if Z_i is very low but above the 0.07 percentile.**
  - **Specifically, the BBB firm is downgraded to CCC if its value of Z is between the 0.07% quantile and the 0.16% quantile.**
- **And so forth, right up through upgrades to AAA.**

Partition the range of Z_i according to the transition probabilities…

Transitions for a firm rated BBB



The latent variable Z_i controls all the transitions.

Thoughts on transition matrices

- A model of rating transitions requires a cost matrix.
  - In a default-only model, the cost is LGD.
  - With a multistate model, you need the cost of transition from every initial state to every other state.
- The set-up is too rich for a non-simulation approach.
- The rest of the course studies the default-only model.
  - You can always simulate the multi-state model if needed.

Questions? Comments?

Don't forget

Homework Set 2 is due by 6PM Thursday April 6.

Lisheng's TA session will be 6PM Sunday April 2.