UChicago MSFM Project Lab (2023 spring) with Mizuho Securities USA LLC

Time scaling of rating transition matrix
Kick-off meeting: project introduction (4/4/2023)

1. Introduction

The primary responsibility for our desk is to manage and hedge additional risks (known as xVA[1] family) to Mark to Market price from counterparty portfolio associated with their OTC derivatives contracts. This is known as xVA family, where x denotes various initial alphabets to describe a type of adjustments.
In the xVA evaluation, "rating" plays an important role. For example,

- CVA (Credit Valuation Adjustment) is a function of exposure and rating.
- FVA (Funding Valuation Adjustment) is a function of exposure and rating.
- ColVA (Collateral Valuation Adjustment) is a function of exposure.
- KVA (Capital Valuation Adjustment) is a function of exposure and rating.

where exposure itself may be also a function of rating through various exotic collateral agreement (e.g. rating triggered CSA). That is, all xVAs exemplified here may depend on ratings. However, there is no common standard or theory how to consider the risk of rating changes. One of useful reference is to use rating transition data or rating transition matrix that some rating agencies publish. However, the rating transition matrices are historically estimated on annual basis. That implies that the matrix may not have power root with real numbers, or may contain negative probabilities even available in real numbers.

2. Objective

The goal of this project are

- To find a reasonable adjustments/statistical inference of the matrices even if roofs of the matrix includes complex/negative numbers as probabilities.
- To explain "how" reasonable. If we found flooring/taking the real part is the most reasonable, it is fine. However, I believe that these approaches are something assuming the given rating transition matrices are "correct", and our estimate (e.g. rooting here) is somehow noise. But we know that the matrices would be "dirty" as historically estimated with limited data. We should better start an estimate with right assumptions and calibrate it to the "dirty" data.

3. Sample Case

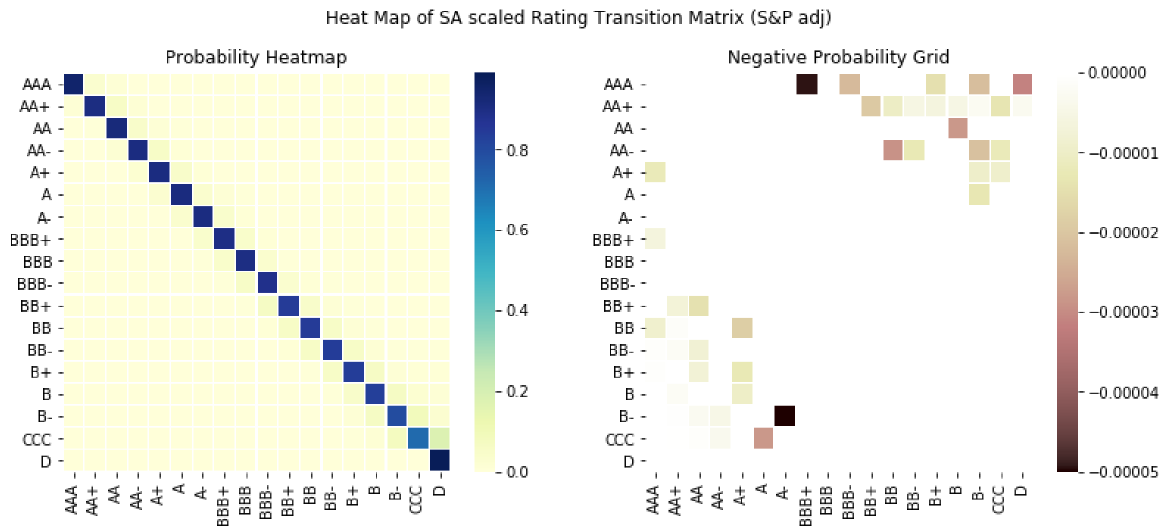
Let's define M as a rating transition matrix (RTM), and we are looking for a weekly scaled matrix for a simulation purpose. An easiest estimate is to take a 52th power root of the RTM, Q, subject to:

$$\mathrm{M} = \mathrm{Q}^{52}$$

That can be computationally obtainable, but this approach works if the M is mathematically fruitful; however, the M is typically estimated from historical data and does not guarantee that the power roots of the matrix have features as probability.
The following heat map denotes semi-annually scaled RTM from the RTM published by S&P Global. We can observe that some components have negative floating numbers that are illegal as probabilities. This is because RTMs published by rating agencies are mathematically dirty.

[1] xVA stands for various Valuation Adjustments where x denotes collective types.

Given that, we would look for <u>another approach</u> for scaling RTM in different time duration: hopefully starting from a variable $\widetilde{Q}$ that is defined as probability matrix then make it optimized/calibrated by some statistical methods.



4. **<u>Considerable steps in the project</u>**
   a) 1) Review a jupyter notebook we provide and find the approach there NOT suitable for us. Also simultaneously, 2) try to find possible method/theories/papers that can be applicable for this project, as well as 3) try to get some sample data of RTMs so as to test your possible approach found in this quarter. [1 week]
   b) Implement some possible methods you find. [3-5 week, hopefully reviewing bi-weekly]
   c) Validate/analyze the approaches you implemented here. [3-4 weeks]

<u>Possible further approaches (out of scope this quarter)</u>

a) Find further advanced way to deal with the same problem.
b) Apply weekly scaled RTMs estimated in the project to CVA/FVA calculations.

<u>Reference</u>

[1] Kreinin, A., & Sidelnikova, M. (2001). Regularization algorithms for transition matrices. Algo Research Quarterly, 4(1/2), 23-40.
[2] Dos Reis, G., & Smith, G. (2018). Robust and consistent estimation of generators in credit risk. Quantitative Finance, 18(6), 983-1001.
[3] S&P Global Ratings, *2021 Annual Global Corporate Default And Rating Transition Study*, https://www.maalot.co.il/Publications/TS20220424121828.PDF
[4] Fitch Ratings, *2022 Transition and Default Studies*, https://www.fitchratings.com/research/fund-asset-managers/2022-transition-default-studies-29-03-2023