64018 Assignment 1

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Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a longterm contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

Product 1: Collegiate - 3sq ft of nylon fabric, can sell 1000/week, 45min of labor, $32 marginal profit Product 2: Mini - 2sq ft of nylon fabric, can sell 1200/week, 40min of labor, $24 marginal profit

Definition of the problem: Determine what the production rates should be for the two products in order to maximize their total profit, subject to the restrictions imposed by the limited production capacities given the constraints listed below

Clearly define the decision variables

A decision variable is one that "if there are n related quantifiable decisions to be made, they are represented as decision variables (say, x1, x2, . . . , xn) whose respective values are to be determined" (pg 36)

x1 : quantity of product 1 (Collegiate) to produce each week x2: quantity of product 2 (Mini) to product each week

What is the objective function?

The objective function is a "mathematical function of these decision variables (for example, P 3x1 2x2 + 5xn)." (pg 36)

Z = 32(x1) + 24 (x2) The 32 and 24 represent the respective marginal profits for x1 and x2. The objective is to choose the values of x1 and x2 so as to maximize Z = 32 (x1) + 24(x2), subject to the restrictions imposed on their values by the constraints below.

What are the constraints?

"Any restrictions on the values that can be assigned to these decision variables are also expressed mathematically, typically by means of inequalities or equations (for example, x1 3x1x2 2x2 10). Such mathematical expressions for the restrictions often are called constraints" (p. 36)

1. The first constraint is the amount of nylon material. We have 5000 sq ft/week. Therefore, 3(x1) + 2(x2) must be less than or equal to 5000
2. Next is a labor constraint. We have 35 workers who are available 40hrs/week. This means we have a total of 1400 hours or 84,000 minutes to keep consistent units. Therefore, 45(x1) + 40(x2) must be less than or equal to 84,0000
3. Since production rates can not be negative, we also have x1 and x2 must be greater than or equal to 0

Write down the full mathematical formulation for the LP model

Step 1:

I will begin by installing and calling the lpSolve package

Step 2:

Next, I will input the coefficients of my objective function established above

```
f.obj <- c(32, 24)
```

Step 3:

Next I will input the two constrains established above. I will not input the third as it is assumed the answer must be positive

```
f.con <- matrix(c(3, 2,
                  45, 40), nrow = 2, byrow = TRUE)

f.dir <- c("<=",
           "<=")

f.rhs <- c(5000,
           84000)

lp("max", f.obj, f.con, f.dir, f.rhs)

## Success: the objective function is 55733.33
```

Step 4

Next we will interpret what we just did. First we set the left side of our constraints, then the inequalities, and finally the right side. This then gave us an objective function of $55,733.33

meaning this is the maximum profit the company can earn. We next need to determine what proportion of each product will produce this.

```
lp("max", f.obj, f.con, f.dir, f.rhs)$solution

## [1] 1066.667  900.000
```

Step 5:

From this, we can see that we should produce 1066.667 units of x1 and 900 units of x2 to obtain a maximum profit of $55,733.33. Therefore the full formula is 55,733.33 = (1066.66 x 32) + (900 x 24). When we fact check our restraints, we first will see that 83,970 minutes are used which is less the 84,000. 4998 sq ft of product is used which is less than the 5000 constraint. Finally, both outputs are a positive number.

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates ofthe new product.

Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

Plant Capacity Plant 1: 750Units/day, 13000 sq/ft Plant 2: 900Units/day, 10000sq/ft Plant 3: 450Units/day, 5000sq/ft

Product Capacity Small: 20sq/ft, 900 sold Medium: 15sq/ft, 1200sold Large: 12sq/ft, 750sold

Clearly define the decision variables

A decision variable is one that "if there are n related quantifiable decisions to be made, they are represented as decision variables (say, x1, x2, . . . , xn) whose respective values are to be determined" (pg 36)

Z = 300x1 + 360x4 + 420x7 + 300x2 + 360x5 + 420x8 + 300x3 + 360x6 + 420x9 where: Z = maximum profit x1 = small product produced at plant 1 x2 = small product produced at plant 2 x3= small product produced at plant 3 x4 = medium product produced at plant 1 x5 = medium product produced at plant 2 x6 = medium product produced at plant 3 x7 = large product produced at plant 1 x8 = large product produced at plant 2 x9 = large product produced at plant 3

subject to the following constraints:

Excess Capacity Constraints x1 + x4 + x7 less than or equal to 750units x2 + x5 + x8 less than or equal to 900units x3 + x6 + x9 less than or equal to 450units

sq ft capacity 20x1 + 15x4 + 12x7 less than or equal to 13000sq ft 20x2 + 15x5 + 12x8 less than or equal to 100000sq ft 20x3 + 15x6 + 12x9 less than or equal to 5000sq ft

sales forecast capacity x1 + x2 + x3 less than or equal to 750units x4 + x5 + x6 less than or equal to 1200units x7 + x8 + x9 less than or equal to 900units

percentage capacity 1/750 (x1 + x4 + x7) - 1/900 (x3 + x6 + x9) = 0 1/900 (x3 + x6 + x9) - 1/450 (x2 + x5 + x8) = 0

Common sense constraint x1, x2, x3, x4, x5, x6, x7, x8, x9 >= 0

Write down the full mathematical formulation for the LP model

```
f.obj2 <- c(300, 360, 420, 300, 360, 420, 300, 360, 400)

f.con2 <- matrix(c(1, 1, 1, 0, 0, 0, 0, 0, 0,
                  0, 0, 0, 1, 1, 1, 0, 0, 0,
                  0, 0, 0, 0, 0, 0, 1, 1, 1,
                  20, 15, 12,0, 0, 0, 0, 0, 0,
                  0, 0, 0, 20, 15, 12, 0, 0, 0,
                  0, 0, 0, 0, 0, 0, 20, 15, 12,
                  1, 0, 0, 1, 0, 0, 1, 0, 0,
                  0, 1, 0, 0, 1, 0, 0, 1, 0,
                  0, 0, 1, 0, 0, 1, 0, 0, 1,
                  0.001333, 0.001333, 0.001333, 0, 0, 0, -0.00111, -0.00111,
-0.00111,
                  0, 0, 0, 0.00111, 0.00111, 0.00111, -0.00222, -0.00222, -
0.00222), nrow = 11, byrow = TRUE)

f.dir2 <- c("<=",
            "<=",
            "<=",
```

```
           "<=",
           "<=",
           "<=",
            "<=",
           "<=",
           "<=",
           "=",
           "=")

f.rhs2 <- c(750,
           900,
           450,
           13000,
           10000,
           5000,
           750,
           1200,
           900,
           0,
           0)

lp("max", f.obj2, f.con2, f.dir2, f.rhs2)

## Success: the objective function is 590711.5

lp("max", f.obj2, f.con2, f.dir2, f.rhs2)$solution

## [1]   0.00000 327.53188   0.00000   0.00000 186.66667 600.00000   0.00000
## [8]  93.33333 300.00000
```

Interpretation

Based on the output above, the maximum profit the company can obtain is $590,711.5. The can be obtained with the following input variables: x1:0 units x4:328 units x7:0 units x2:0 units x5:187 units x8:600 units x3:0 units x6:93 units x9:300 units

This means that at plant 1 we want to produce 328 medium. At plant 2 we will want to produce 187 large. Finally at plant 3 we will want to produce 93 medium and 300 large. These numbers are rounded since its not feasible to produce a fraction of a product.