

Implementation of the exponential function

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Abstract

A "quick-and-dirty" implementation of the exponential function introduced and explained. A test is also included.

1 Code implementation

The code is implemented in the following way:

```
double ex(double x)
if(x<0)return 1/ex(-x);
if(x>1./8)return pow(ex(x/2),2);
return 1+x*(1+x/2*(1+x/3*(1+x/4*(1+x/5*(1+x/6*(1+x/7*(1+x/8*(1+x/9*(1+x/10)))))))));
```

The code takes a double value, x, and returns an approximate value of:

$$f(x) = e^x . \quad (1)$$

2 Explanation

The first line inside the function makes sure no values of negative signs are included in the summation. The second line returns the square of $e^{x/2}$ if the x value is larger than 0.125. This ensures that only small values of x are calculated with the Taylor expansion in the third line. The Taylor expansion is defined as [1],

$$\exp x := \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots . \quad (2)$$

The expansion is implemented to the 10th order which should make the determination of $\exp(x)$ very accurate.

3 Test

The validity of the implemented function is tested via plotting the function together with the $\exp(x)$ function from the math library. These two functions are shown in figure (3) below. The figure shows that the two functions are very similar and thereby it is shown that the approximation gives very accurate values.

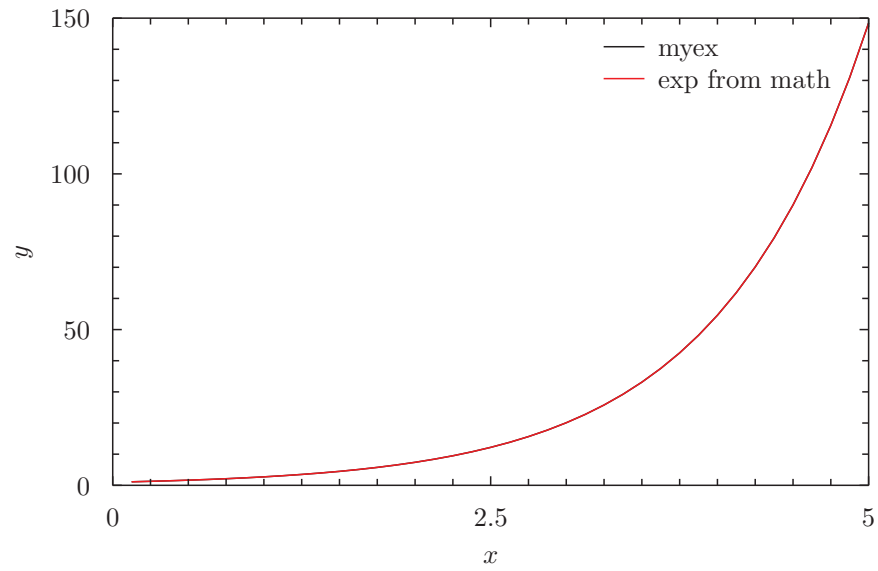


Figure 1: The implemented ex-function and the $\exp(x)$ function from math

References

- [1] Rudin, Walter (1987). Real and complex analysis (3rd ed.). New York: McGraw-Hill. p. 1. ISBN 978-0-07-054234-1.