Part1_network

May 2, 2022

[1]:
```
import networkx as nx
import matplotlib.pyplot as plt
import pandas as pd
from operator import itemgetter
```

[2]:
```
#OK, let us start with the graphml file for London's underground
G = nx.read_graphml('london.graph.xml')
```

[3]:
```
type(G)
```

[3]:
```
networkx.classes.graph.Graph
```

[4]:
```
print(nx.info(G))
```

```
Graph with 401 nodes and 467 edges
```

[5]:
```
# To check node attributes:
list(G.nodes(data = True))[0]
```

[5]:
```
('Wembley Park', {'coords': '(519316.5590174915, 186389.32981656672)'})
```

[6]:
```
station_name=list(G.nodes)
```

[7]:
```
#since coords tuples are stored as string
#need to convert them back to tuples using eval()
for node in G.nodes():
    G.nodes[node]['coords'] = eval(G.nodes[node]['coords'])
```

[8]:
```
# To check node attributes:
list(G.nodes(data = True))[0]
```

[8]:
```
('Wembley Park', {'coords': (519316.5590174915, 186389.32981656672)})
```

[9]:
```
# To check edges attributes:
list(G.edges(data = True))[0]
```

[9]:
```
('Wembley Park',
 'Kingsbury',
 {'length': 2916.7715580506483, 'line_name': 'Jubilee', 'flows': 12356})
```

[10]:
```
# give the 'station name' to the node
station_name = list(G.nodes)
for node in station_name:
    G.nodes[node]['station_name'] = node
```

[11]:
```
# we can also add the stations name to the edge attributes from the nodes␣
↪attributes:

nod_name1={(e1,e2):(G.nodes[e1]['station_name']) for e1, e2 in G.edges()}
nod_name2={(e1,e2):(G.nodes[e2]['station_name']) for e1, e2 in G.edges()}

nx.set_edge_attributes(G,nod_name1,'station_1_')
nx.set_edge_attributes(G,nod_name2,'station_2_')
```

[12]:
```
 #We can print the dataframe from the shapefile to check the data
df = nx.to_pandas_edgelist(G)
df[0:10]
```

[12]:
```
        source         target     line_name       length  flows  \
0  Wembley Park      Kingsbury       Jubilee  2916.771558  12356
1  Wembley Park        Neasden       Jubilee  2353.165938   6744
2  Wembley Park   Preston Road  Metropolitan  1419.735166  36601
3  Wembley Park  Finchley Road  Metropolitan  7266.373927  55216
4     Kingsbury     Queensbury       Jubilee  1245.995234   9419
5    Queensbury    Canons Park       Jubilee  1693.307343   6385
6   Canons Park       Stanmore       Jubilee  1419.669476   3624
7     Stratford       West Ham       Jubilee  1673.509515  91801
8     Stratford       Mile End       Central  2805.001392  12010
9     Stratford         Leyton       Central  2131.342926  56082

     station_1_     station_2_
0  Wembley Park      Kingsbury
1  Wembley Park        Neasden
2  Wembley Park   Preston Road
3  Wembley Park  Finchley Road
4     Kingsbury     Queensbury
5    Queensbury    Canons Park
6   Canons Park       Stanmore
7     Stratford       West Ham
8     Stratford       Mile End
9     Stratford         Leyton
```

0.1 I. Topological network

0.1.1 I.1. Centrality measures:

1)Degree Centrality on nodes:

[13]:
```
#  We can calculate the degree centrality using networkx function:
deg_london =nx.degree_centrality(G)
nx.set_node_attributes(G,dict(deg_london),'degree')
```

[14]:
```
# To dataframe using the nodes as the index
df = pd.DataFrame(index=G.nodes())
df['station_name'] = pd.Series(nx.get_node_attributes(G, 'station_name'))
df['degree'] = pd.Series(nx.get_node_attributes(G, 'degree'))

df_sorted = df.sort_values(["degree"], ascending=False)
df_sorted[0:10]
```

[14]:

	station_name	degree
Stratford	Stratford	0.0225
Bank and Monument	Bank and Monument	0.0200
King's Cross St. Pancras	King's Cross St. Pancras	0.0175
Baker Street	Baker Street	0.0175
Earl's Court	Earl's Court	0.0150
Oxford Circus	Oxford Circus	0.0150
Liverpool Street	Liverpool Street	0.0150
Waterloo	Waterloo	0.0150
Green Park	Green Park	0.0150
Canning Town	Canning Town	0.0150

2)Betweenness Centrality on nodes:

[15]:
```
### Topological betweenness centrality:

#Let us compute the betweenness centrality for the network, without using␣
 ↪weights:
bet_london_t=nx.betweenness_centrality(G, normalized=False)
# We can add these values to the nodes attributes:
nx.set_node_attributes(G,bet_london_t,'betweenness_t')

# To ataframe using the nodes as the index
df = pd.DataFrame(index=G.nodes())
df['station_name'] = pd.Series(nx.get_node_attributes(G, 'station_name'))
df['betweenness_t'] = pd.Series(nx.get_node_attributes(G, 'betweenness_t'))

df_sorted = df.sort_values(["betweenness_t"], ascending=False)
df_sorted[0:10]
```

[15]:

	station_name	betweenness_t
Stratford	Stratford	23768.093434
Bank and Monument	Bank and Monument	23181.058947
Liverpool Street	Liverpool Street	21610.387049
King's Cross St. Pancras	King's Cross St. Pancras	20373.521465
Waterloo	Waterloo	19464.882323
Green Park	Green Park	17223.622114
Euston	Euston	16624.275469
Westminster	Westminster	16226.155916
Baker Street	Baker Street	15287.107612
Finchley Road	Finchley Road	13173.758009

3)Closeness Centrality on nodes

[16]:
```
#topological closeness centrality
clos_t=nx.closeness_centrality(G)
# We can add these values to the nodes attributes:
nx.set_node_attributes(G,clos_t,'closeness_t')

# To dataframe using the nodes as the index
df = pd.DataFrame(index=G.nodes())
df['station_name'] = pd.Series(nx.get_node_attributes(G, 'station_name'))
df['closeness_t'] = pd.Series(nx.get_node_attributes(G, 'closeness_t'))

df_sorted = df.sort_values(["closeness_t"], ascending=False)
df_sorted[0:10]
```

[16]:

	station_name	closeness_t
Green Park	Green Park	0.114778
Bank and Monument	Bank and Monument	0.113572
King's Cross St. Pancras	King's Cross St. Pancras	0.113443
Westminster	Westminster	0.112549
Waterloo	Waterloo	0.112265
Oxford Circus	Oxford Circus	0.111204
Bond Street	Bond Street	0.110988
Farringdon	Farringdon	0.110742
Angel	Angel	0.110742
Moorgate	Moorgate	0.110314

0.1.2 I.3. Impact measures:

A) non-sequential removal:

1)Degree centrality 1a Largest connected component

[17]:
```
G1a=G.copy()
# Number of existing nodes:
```

```
l1a=list(G1a.nodes)
len(l1a)
```

[17]: 401

[18]:
```
degree_df=pd.DataFrame.from_dict(dict(deg_london ),columns=['deg_london␣
 ↪'],orient='index')
node1=list(degree_df.sort_values('deg_london ',axis = 0,ascending = False).
 ↪index)
max_1=node1[0:10]

max_1
```

[18]:
```
['Stratford',
 'Bank and Monument',
 "King's Cross St. Pancras",
 'Baker Street',
 "Earl's Court",
 'Oxford Circus',
 'Liverpool Street',
 'Waterloo',
 'Green Park',
 'Canning Town']
```

[19]:
```
#If we want to remove the node with the max value of degree centrality:
result_no_of_nodes1=[]
result_size1=[]
result_percent1=[]

for i in range(0,11):

# number of connected components
    no_of_nodes=nx.number_connected_components(G1a)
    result_no_of_nodes1.append(no_of_nodes)

# let's subset this graph to the largest connected component
    Gcc1a = sorted(nx.connected_components(G1a), key=len, reverse=True)
    Gsub1a = G1a.subgraph(Gcc1a[0])
    size = Gsub1a.number_of_nodes()
    result_size1.append(size)

# calculate the percentage of nodes present in the largest connected components
    percent_lcc1a = (len(Gsub1a.nodes)/len(G1a.nodes)) * 100
    result_percent1.append(percent_lcc1a)
#remove the node
    G1a.remove_nodes_from([node1[i]])
```

```python
print(result_no_of_nodes1)
print(result_size1)
print(result_percent1)
```

```
[1, 3, 3, 3, 4, 4, 5, 6, 6, 6, 8]
[401, 379, 378, 377, 374, 373, 371, 365, 364, 363, 349]
[100.0, 94.75, 94.73684210526315, 94.72361809045226, 94.20654911838791,
94.1919191919192, 93.92405063291139, 92.63959390862944, 92.6208651399491,
92.60204081632652, 89.25831202046037]
```

1b Global efficiency

[20]:
```python
G1b=G.copy()
# Number of existing nodes:
l1b=list(G1b.nodes)
len(l1b)
```

[20]:
```
401
```

[21]:
```python
#If we want to remove the node with the max value of betweenness centrality:
def global_efficiency(G1b):
        n = len(G1b)
        denom = n * (n - 1)
        if denom != 0:
            lengths = nx.all_pairs_shortest_path_length(G1b)
            g_eff1 = 0
            for source, targets in lengths:
                for target, distance in targets.items():
                    if distance > 0:
                        g_eff1 += 1 / distance
            g_eff1 /= denom
        # g_eff = sum(1 / d for s, tgts in lengths
        #                   for t, d in tgts.items() if d > 0) / denom
        else:
            g_eff1 = 0
    # TODO This can be made more efficient by computing all pairs shortest
    # path lengths in parallel.
        return g_eff1

result_global_eff1=[]
for i in range(11):
    result_global_eff1.append(global_efficiency(G1b))
    G1b.remove_nodes_from([node1[i]])

print(result_global_eff1)
```

```
[0.1012561935972123, 0.08891736066510689, 0.08586164448742485,
0.08028700838265396, 0.07570039409751211, 0.0740361229198828,
```

```
0.07274535237569829, 0.07094844226118287, 0.06898194318071488,
0.06825731584971681, 0.06338447666647608]
```

2)Betweenness centrality 2a Largest connected component

[22]:
```
G2a=G.copy()
```

[23]:
```
between_df=pd.DataFrame.
 ↪from_dict(dict(bet_london_t),columns=['bet_london_t'],orient='index')
node2=list(between_df.sort_values('bet_london_t',axis = 0,ascending = False).
 ↪index)
max_2=node2[0:10]

max_2
```

[23]:
```
['Stratford',
 'Bank and Monument',
 'Liverpool Street',
 "King's Cross St. Pancras",
 'Waterloo',
 'Green Park',
 'Euston',
 'Westminster',
 'Baker Street',
 'Finchley Road']
```

[24]:
```
# Number of existing nodes:
l2a=list(G2a.nodes)
len(l2a)
```

[24]:
```
401
```

[25]:
```
#If we want to remove the node with the max value of between centrality:
result_no_of_nodes2=[]
result_size2=[]
result_percent2=[]

for i in range(0,11):

# number of connected components
    no_of_nodes=nx.number_connected_components(G2a)
    result_no_of_nodes2.append(no_of_nodes)

# let's subset this graph to the largest connected component
    Gcc2a = sorted(nx.connected_components(G2a), key=len, reverse=True)
    Gsub2a = G2a.subgraph(Gcc2a[0])
    size = Gsub2a.number_of_nodes()
```

```
    result_size2.append(size)

# calculate the percentage of nodes present in the largest connected components
    percent_lcc2a = (len(Gsub2a.nodes)/len(G2a.nodes)) * 100
    result_percent2.append(percent_lcc2a)

    G2a.remove_nodes_from([node2[i]])

print(result_no_of_nodes2)
print(result_size2)
print(result_percent2)
```

```
[1, 3, 3, 3, 4, 4, 4, 5, 5, 6, 7]
[401, 379, 378, 377, 371, 370, 369, 346, 345, 342, 339]
[100.0, 94.75, 94.73684210526315, 94.72361809045226, 93.45088161209067,
93.43434343434343, 93.41772151898734, 87.81725888324873, 87.78625954198473,
87.24489795918367, 86.70076726342711]
```

2b Global efficiency

[26]:
```
G2b=G.copy()
```

[27]:
```
# Number of existing nodes:
l2b=list(G2b.nodes)
len(l2b)
```

[27]: 401

[28]:
```
result_global_eff2=[]
for i in range(11):
    result_global_eff2.append(global_efficiency(G2b))
    G2b.remove_nodes_from([node2[i]])

print(result_global_eff2)
```

```
[0.1012561935972123, 0.08891736066510689, 0.08586164448742485,
0.08496349266423939, 0.07849775440713821, 0.07594226578366223,
0.07415154167648695, 0.06820564659789057, 0.06765950327361094,
0.064700058053009, 0.06313903700825897]
```

3)Closeness centrality 3a Largest connected component

[29]:
```
G3a=G.copy()
# Number of existing nodes:
l3a=list(G3a.nodes)
len(l3a)
```

[29]: 401

[30]:
```
closeness_df=pd.DataFrame.
↪from_dict(dict(clos_t),columns=['clos_t'],orient='index')
node3=list(closeness_df.sort_values('clos_t',axis = 0,ascending = False).index)
max_3=node3[0:10]

max_3
```

[30]:
```
['Green Park',
 'Bank and Monument',
 "King's Cross St. Pancras",
 'Westminster',
 'Waterloo',
 'Oxford Circus',
 'Bond Street',
 'Farringdon',
 'Angel',
 'Moorgate']
```

[31]:
```
#If we want to remove the node with the max value of closeness centrality:
result_no_of_nodes3=[]
result_size3=[]
result_percent3=[]

for i in range(0,11):

# number of connected components
    no_of_nodes=nx.number_connected_components(G3a)
    result_no_of_nodes3.append(no_of_nodes)

# let's subset this graph to the largest connected component
    Gcc3a = sorted(nx.connected_components(G3a), key=len, reverse=True)
    Gsub3a = G3a.subgraph(Gcc3a[0])
    size = Gsub3a.number_of_nodes()
    result_size3.append(size)

# calculate the percentage of nodes present in the largest connected components
    percent_lcc3a = (len(Gsub3a.nodes)/len(G3a.nodes)) * 100
    result_percent3.append(percent_lcc3a)

    G3a.remove_nodes_from([node3[i]])

print(result_no_of_nodes3)
print(result_size3)
print(result_percent3)
```

```
[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3]
[401, 400, 399, 398, 397, 396, 395, 394, 393, 392, 389]
```

```
[100.0, 100.0, 100.0, 100.0, 100.0, 100.0, 100.0, 100.0, 100.0, 100.0,
99.48849104859335]
```

3b Global efficiency

[32]:
```python
G3b=G.copy()
```

[33]:
```python
# Number of existing nodes:
l3b=list(G3b.nodes)
len(l3b)
```

[33]: 401

[34]:
```python
result_global_eff3=[]
for i in range(11):
    result_global_eff3.append(global_efficiency(G3b))
    G3b.remove_nodes_from([node3[i]])

print(result_global_eff3)
```

```
[0.1012561935972123, 0.09918991960788402, 0.09487232544791133,
0.08793385149140875, 0.08737164566976727, 0.08485943799789313,
0.08278135073141742, 0.08258086417012774, 0.08260040537396239,
0.08262233108950982, 0.08166991436767818]
```

B) Sequential removal:

1)Degree centrality 1a Largest connected component

[35]:
```python
G4a=G.copy()
# Number of existing nodes:
l4a=list(G4a.nodes)
len(l4a)
```

[35]: 401

[36]:
```python
#If we want to remove the node with the max value of closeness centrality:
remove4=[]
result_no_of_nodes4=[]
result_size4=[]
result_percent4=[]

for i in range(0,11):
    deg_london=nx.degree_centrality(G4a)
    nx.set_node_attributes(G4a,dict(deg_london),'degree')
    degree_df=pd.DataFrame.
 ↪from_dict(dict(deg_london),columns=['degree'],orient='index')
    node4=list(degree_df.sort_values('degree',axis = 0,ascending = False).index)
```

```
    remove4.append(node4[0])

# number of connected components
    no_of_nodes=nx.number_connected_components(G4a)
    result_no_of_nodes4.append(no_of_nodes)

# let's subset this graph to the largest connected component
    Gcc4a = sorted(nx.connected_components(G4a), key=len, reverse=True)
    Gsub4a = G4a.subgraph(Gcc4a[0])
    size = Gsub4a.number_of_nodes()
    result_size4.append(size)

# calculate the percentage of nodes present in the largest connected components
    percent_lcc4a = (len(Gsub4a.nodes)/len(G4a.nodes)) * 100
    result_percent4.append(percent_lcc4a)

    G4a.remove_nodes_from([node4[0]])

print(remove4)
print(result_no_of_nodes4)
print(result_size4)
print(result_percent4)
```

```
['Stratford', 'Bank and Monument', 'Baker Street', "King's Cross St. Pancras",
"Earl's Court", 'Green Park', 'Canning Town', 'Willesden Junction', 'Turnham
Green', 'Oxford Circus', 'Waterloo']
[1, 3, 3, 3, 4, 4, 4, 6, 7, 8, 9]
[401, 379, 378, 377, 374, 373, 372, 358, 344, 338, 336]
[100.0, 94.75, 94.73684210526315, 94.72361809045226, 94.20654911838791,
94.1919191919192, 94.17721518987342, 90.86294416243655, 87.53180661577609,
86.22448979591837, 85.93350383631714]
```

1b Global efficiency

[37]:
```
G4b=G.copy()
# Number of existing nodes:
l4b=list(G4b.nodes)
len(l4b)
```

[37]: 401

[38]:
```
result_global_eff4=[]
remove4b=[]
for i in range(11):
    deg_london=nx.degree_centrality(G4b)
    nx.set_node_attributes(G4b,dict(deg_london),'degree')
    degree_df=pd.DataFrame.
 ↪from_dict(dict(deg_london),columns=['degree'],orient='index')
```

```
    node4=list(degree_df.sort_values('degree',axis = 0,ascending = False).index)
    remove4b.append(node4[0])

    result_global_eff4.append(global_efficiency(G4b))
    G4b.remove_nodes_from([node4[0]])

print(result_global_eff4)
print(remove4b)
```

```
[0.1012561935972123, 0.08891736066510689, 0.08586164448742485,
0.08203328759057034, 0.07570039409751211, 0.0740361229198828,
0.07300367580539921, 0.0677717604548151, 0.06012752519564628,
0.05814535563710196, 0.05685866730429425]
['Stratford', 'Bank and Monument', 'Baker Street', "King's Cross St. Pancras",
"Earl's Court", 'Green Park', 'Canning Town', 'Willesden Junction', 'Turnham
Green', 'Oxford Circus', 'Waterloo']
```

2)Betweenness centrality 2a Largest connected component

[39]:
```
G5a=G.copy()
# Number of existing nodes:
l5a=list(G5a.nodes)
len(l5a)
```

[39]: 401

[40]:
```
#If we want to remove the node with the max value of betweeness centrality:
remove5=[]
result_no_of_nodes5=[]
result_size5=[]
result_percent5=[]


for i in range(0,11):
    bet_london_t=nx.betweenness_centrality(G5a)
    nx.set_node_attributes(G5a,dict(bet_london_t),'betweenness_t')
    between_df=pd.DataFrame.
 ↪from_dict(dict(bet_london_t),columns=['betweenness_t'],orient='index')
    node5=list(between_df.sort_values('betweenness_t',axis = 0,ascending =␣
 ↪False).index)
    remove5.append(node5[0])

# number of connected components
    no_of_nodes=nx.number_connected_components(G5a)
    result_no_of_nodes5.append(no_of_nodes)

# let's subset this graph to the largest connected component
```

```
    Gcc5a = sorted(nx.connected_components(G5a), key=len, reverse=True)
    Gsub5a = G5a.subgraph(Gcc5a[0])
    size = Gsub5a.number_of_nodes()
    result_size5.append(size)

# calculate the percentage of nodes present in the largest connected components
    percent_lcc5a = (len(Gsub5a.nodes)/len(G5a.nodes)) * 100
    result_percent5.append(percent_lcc5a)

    G5a.remove_nodes_from([node5[0]])

print(remove5)
print(result_no_of_nodes5)
print(result_size5)
print(result_percent5)
```

```
['Stratford', "King's Cross St. Pancras", 'Waterloo', 'Bank and Monument',
'Canada Water', 'West Hampstead', "Earl's Court", "Shepherd's Bush", 'Euston',
'Baker Street', 'Acton Town']
[1, 3, 3, 3, 3, 3, 4, 4, 5, 6, 7]
[401, 379, 378, 377, 376, 375, 227, 226, 196, 173, 170]
[100.0, 94.75, 94.73684210526315, 94.72361809045226, 94.7103274559194,
94.6969696969697, 57.46835443037974, 57.360406091370564, 49.87277353689568,
44.13265306122449, 43.47826086956522]
```

2b Global efficiency

[41]:
```
G5b=G.copy()
# Number of existing nodes:
l5b=list(G5b.nodes)
len(l5b)
```

[41]: 401

[42]:
```
result_global_eff5=[]
remove5b=[]
for i in range(11):
    bet_london_t=nx.betweenness_centrality(G5b)
    nx.set_node_attributes(G5b,dict(bet_london_t),'betweenness_t')
    between_df=pd.DataFrame.
 ↪from_dict(dict(bet_london_t),columns=['betweenness_t'],orient='index')
    node5=list(between_df.sort_values('betweenness_t',axis = 0,ascending =␣
 ↪False).index)
    remove5b.append([node5[0]])

    result_global_eff5.append(global_efficiency(G5b))
    G5b.remove_nodes_from([node5[0]])
```

```
print(result_global_eff5)
print(remove5b)
```

```
[0.1012561935972123, 0.08891736066510689, 0.08460293133575152,
0.08182895253292936, 0.07767794342812263, 0.07283234083472483,
0.053210203984026455, 0.05165629952389727, 0.0458442134055722,
0.04163076968121037, 0.0381637040943985]
[['Stratford'], ["King's Cross St. Pancras"], ['Waterloo'], ['Bank and
Monument'], ['Canada Water'], ['West Hampstead'], ["Earl's Court"], ["Shepherd's
Bush"], ['Euston'], ['Baker Street'], ['Acton Town']]
```

3)Closeness centrality 3a Largest connected component

[43]:
```
G6a=G.copy()
# Number of existing nodes:
l6a=list(G6a.nodes)
len(l6a)
```

[43]: 401

[44]:
```
#If we want to remove the node with the max value of closeness centrality:
remove6=[]
result_no_of_nodes6=[]
result_size6=[]
result_percent6=[]

for i in range(0,11):
    clos_t=nx.closeness_centrality(G6a)
    nx.set_node_attributes(G6a,dict(clos_t),'closeness_t')
    closeness_df=pd.DataFrame.
 ↪from_dict(dict(clos_t),columns=['closeness_t'],orient='index')
    node6=list(closeness_df.sort_values('closeness_t',axis = 0,ascending =␣
 ↪False).index)
    remove6.append(node6[0])

# number of connected components
    no_of_nodes=nx.number_connected_components(G6a)
    result_no_of_nodes6.append(no_of_nodes)

# let's subset this graph to the largest connected component
    Gcc6a = sorted(nx.connected_components(G6a), key=len, reverse=True)
    Gsub6a = G6a.subgraph(Gcc6a[0])
    size = Gsub6a.number_of_nodes()
    result_size6.append(size)

# calculate the percentage of nodes present in the largest connected components
    percent_lcc6a = (len(Gsub6a.nodes)/len(G6a.nodes)) * 100
```

```
    result_percent6.append(percent_lcc6a)

    G6a.remove_nodes_from([node6[0]])

print(remove6)
print(result_no_of_nodes6)
print(result_size6)
print(result_percent6)
```

```
['Green Park', "King's Cross St. Pancras", 'Waterloo', 'Bank and Monument',
'West Hampstead', 'Canada Water', 'Stratford', "Earl's Court", "Shepherd's
Bush", 'Oxford Circus', 'Paddington']
[1, 1, 1, 1, 1, 1, 2, 4, 4, 5, 5]
[401, 400, 399, 398, 397, 396, 226, 226, 225, 195, 194]
[100.0, 100.0, 100.0, 100.0, 100.0, 100.0, 57.21518987341773,
57.360406091370564, 57.25190839694656, 49.744897959183675, 49.61636828644501]
```

3b Global efficiency

[45]:
```
G6b=G.copy()
# Number of existing nodes:
l6b=list(G6b.nodes)
len(l6b)
```

[45]:
```
401
```

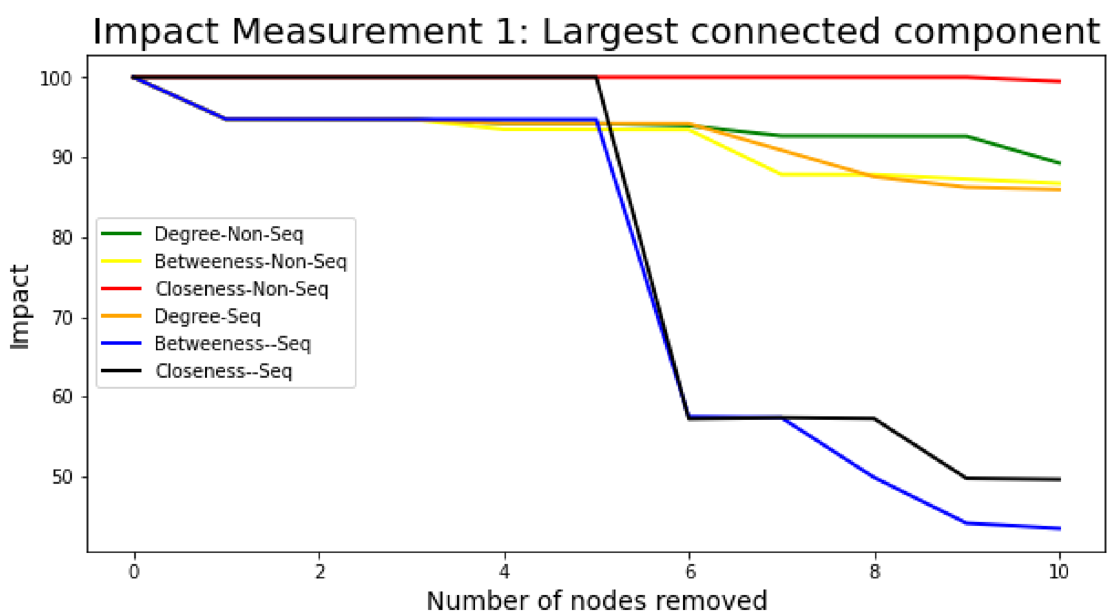
[46]:
```
result_global_eff6=[]
remove6b=[]
for i in range(11):
    clos_t=nx.closeness_centrality(G6b)
    nx.set_node_attributes(G6b,dict(clos_t),'closeness_t')
    closeness_df=pd.DataFrame.
 ↪from_dict(dict(clos_t),columns=['closeness_t'],orient='index')
    node6=list(closeness_df.sort_values('closeness_t',axis = 0,ascending =␣
 ↪False).index)
    remove6b.append([node6[0]])

    result_global_eff6.append(global_efficiency(G6b))
    G6b.remove_nodes_from([node6[0]])

print(result_global_eff6)
print(remove6b)
```

```
[0.1012561935972123, 0.09918991960788402, 0.09443475025566316,
0.09181648060183005, 0.08542563066911478, 0.08054424756502003,
0.05810104159173278, 0.051883620553389555, 0.05035000093626794,
0.04439458727102797, 0.04295771061337044]
[['Green Park'], ["King's Cross St. Pancras"], ['Waterloo'], ['Bank and
```

Monument'], ['West Hampstead'], ['Canada Water'], ['Stratford'], ["Earl's Court"], ["Shepherd's Bush"], ['Oxford Circus'], ['Paddington']]

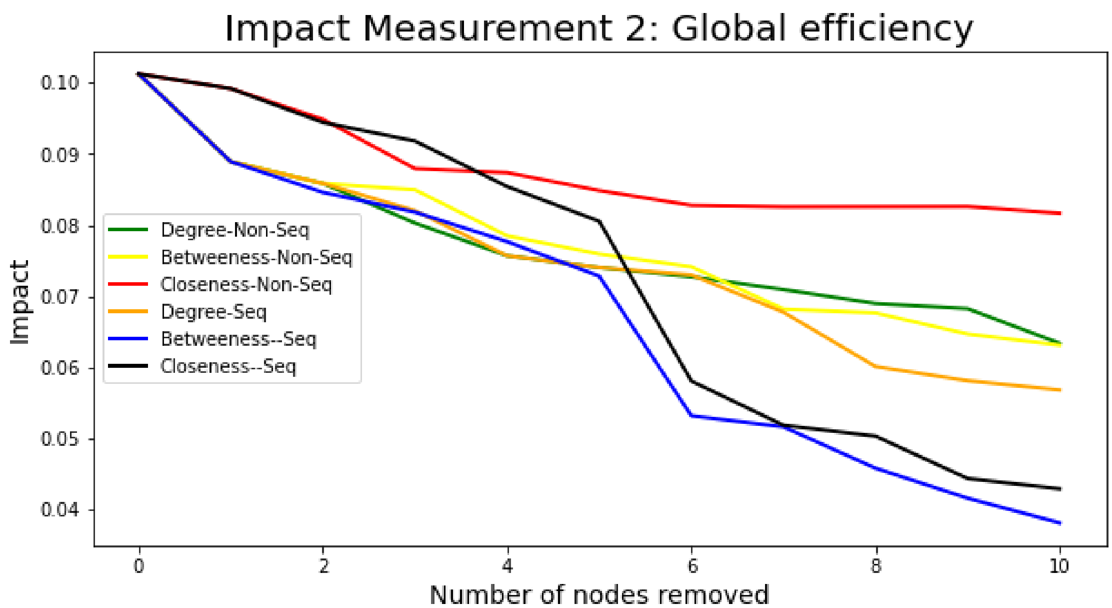
0.2 impact of nodes removal

[47]:
```
# plot the line chart:Non-sequential
plt.figure(figsize=(10,5))
plt.title('Impact Measurement 1: Largest connected component',fontsize=20)
plt.xlabel('Number of nodes removed',fontsize=14)
plt.ylabel('Impact',fontsize=14)

in1, = plt.plot(result_percent1,color="green",linewidth=2)
in2, = plt.plot(result_percent2,color="yellow",linewidth=2)
in3, = plt.plot(result_percent3,color="red",linewidth=2)
in4, = plt.plot(result_percent4,color="orange",linewidth=2)
in5, = plt.plot(result_percent5,color="blue",linewidth=2)
in6, = plt.plot(result_percent6,color="black",linewidth=2)

plt.legend(handles =␣
 ↪[in1,in2,in3,in4,in5,in6],labels=['Degree-Non-Seq','Betweeness-Non-Seq','Closeness-Non-Seq'
plt.show()
```



[48]:
```
# plot the line chart:sequential
plt.figure(figsize=(10,5))
plt.title('Impact Measurement 2: Global efficiency',fontsize=20)
plt.xlabel('Number of nodes removed',fontsize=14)
plt.ylabel('Impact',fontsize=14)
```

```
in1, = plt.plot(result_global_eff1,color="green",linewidth=2)
in2, = plt.plot(result_global_eff2,color="yellow",linewidth=2)
in3, = plt.plot(result_global_eff3,color="red",linewidth=2)
in4, = plt.plot(result_global_eff4,color="orange",linewidth=2)
in5, = plt.plot(result_global_eff5,color="blue",linewidth=2)
in6, = plt.plot(result_global_eff6,color="black",linewidth=2)

plt.legend(handles =␣
 ↪[in1,in2,in3,in4,in5,in6],labels=['Degree-Non-Seq','Betweeness-Non-Seq','Closeness-Non-Seq'
plt.show()
```



0.3 Flows: weighted network

[49]:
```
# Inverse weights:
flows_inv={}

for e1, e2, flow in G.edges(data='flows'):
    if flow != 0:
        flows_inv.update({(e1, e2): round(1./flow,7)})
    else:
        flows_inv.update({(e1, e2): 0})

nx.set_edge_attributes(G,flows_inv,'flows_inv')

list(G.edges(data=True))[0:5]
```

[49]:
```
[('Wembley Park',
  'Kingsbury',
  {'length': 2916.7715580506483,
   'line_name': 'Jubilee',
   'flows': 12356,
   'station_1_': 'Wembley Park',
   'station_2_': 'Kingsbury',
   'flows_inv': 8.09e-05}),
 ('Wembley Park',
  'Neasden',
  {'length': 2353.1659381957816,
   'line_name': 'Jubilee',
   'flows': 6744,
   'station_1_': 'Wembley Park',
   'station_2_': 'Neasden',
   'flows_inv': 0.0001483}),
 ('Wembley Park',
  'Preston Road',
  {'length': 1419.7351657633037,
   'line_name': 'Metropolitan',
   'flows': 36601,
   'station_1_': 'Wembley Park',
   'station_2_': 'Preston Road',
   'flows_inv': 2.73e-05}),
 ('Wembley Park',
  'Finchley Road',
  {'length': 7266.37392749648,
   'line_name': 'Metropolitan',
   'flows': 55216,
   'station_1_': 'Wembley Park',
   'station_2_': 'Finchley Road',
   'flows_inv': 1.81e-05}),
 ('Kingsbury',
  'Queensbury',
  {'length': 1245.9952343630068,
   'line_name': 'Jubilee',
   'flows': 9419,
   'station_1_': 'Kingsbury',
   'station_2_': 'Queensbury',
   'flows_inv': 0.0001062})]
```

[50]:
```
#deg_london =nx.degree_centrality(G,weight='flows_inv')
#nx.set_node_attributes(G,dict(deg_london),'degree')

# To dataframe using the nodes as the index
#df_degree = pd.DataFrame(index=G.nodes())
```

```
#df_degree['station_name'] = pd.Series(nx.get_node_attributes(G,␣
↪'station_name'))
#df_degree['degree'] = pd.Series(nx.get_node_attributes(G, 'degree'))

#df_degree_sorted = df_degree.sort_values(["degree"], ascending=False)
#df_degree_sorted[0:10]
```

[51]:
```
# Recompute according to betweenness centrality
bet_london_w=nx.betweenness_centrality(G,weight='flows_inv',normalized=False)
```

II.1. 10 most important nodes according to weighted betweenness centrality

[52]:
```
between_w_df=pd.DataFrame.
↪from_dict(dict(bet_london_w),columns=['bet_london_w'],orient='index')
node_w=list(between_w_df.sort_values('bet_london_w',axis = 0,ascending = False).
↪index)
max_w=node_w[0:10]

max_w
```

[52]:
```
['Green Park',
 'Bank and Monument',
 'Waterloo',
 'Westminster',
 'Liverpool Street',
 'Stratford',
 'Bond Street',
 'Euston',
 'Oxford Circus',
 'Warren Street']
```

[53]:
```
between_w_df.sort_values('bet_london_w',ascending = False)
```

[53]:
```
                     bet_london_w
Green Park               44892.50
Bank and Monument        39758.50
Waterloo                 31904.25
Westminster              29664.50
Liverpool Street         26530.00
…                             …
Stepney Green                0.00
Bow Road                     0.00
Bromley-by-Bow               0.00
High Barnet                  0.00
Stamford Brook               0.00

[401 rows x 1 columns]
```

[54]:
```
#Weighted closeness centrality:
#clos_w=nx.closeness_centrality(G, distance='flows_inv')
# We can add these values to the nodes attributes:
#nx.set_node_attributes(G,clos_w,'closeness_w')

# To dataframe using the nodes as the index
#df = pd.DataFrame(index=G.nodes())
#df['station_name'] = pd.Series(nx.get_node_attributes(G, 'station_name'))
#df['closeness_w'] = pd.Series(nx.get_node_attributes(G, 'closeness_w'))

#df_sorted = df.sort_values(["closeness_w"], ascending=False)
#df_sorted[0:10]
```

II.3. Node removal

Average shortest path a without flows

[55]:
```
G7a=G.copy()
# Number of existing nodes:
l7a=list(G7a.nodes)
len(l7a)
```

[55]: 401

[56]:
```
Gcc7a = sorted(nx.connected_components(G7a), key=len, reverse=True)
Gsub7a = G7a.subgraph(Gcc7a[0])
betweeness_w_path1=nx.average_shortest_path_length(Gsub7a)
print(betweeness_w_path1)
```

13.545997506234414

[57]:
```
node1[0]
```

[57]: 'Stratford'

[58]:
```
G7a.remove_nodes_from([node1[0]])

Gcc7a = sorted(nx.connected_components(G7a), key=len, reverse=True)
Gsub7a = G7a.subgraph(Gcc7a[0])
betweeness_w_path2=nx.average_shortest_path_length(Gsub7a)
print(betweeness_w_path2)
```

14.496447069006436

[59]:
```
changes_without_flows=(betweeness_w_path2-betweeness_w_path1)/betweeness_w_path1
print(changes_without_flows)
```

```
0.0701646048830724
```

b with flows

[60]:
```python
G7b=G.copy()
# Number of existing nodes:
l7b=list(G7b.nodes)
len(l7b)
```

[60]:
```
401
```

[61]:
```python
list(G7b.edges(data=True))[0:5]
```

[61]:
```
[('Wembley Park',
  'Kingsbury',
  {'length': 2916.7715580506483,
   'line_name': 'Jubilee',
   'flows': 12356,
   'station_1_': 'Wembley Park',
   'station_2_': 'Kingsbury',
   'flows_inv': 8.09e-05}),
 ('Wembley Park',
  'Neasden',
  {'length': 2353.1659381957816,
   'line_name': 'Jubilee',
   'flows': 6744,
   'station_1_': 'Wembley Park',
   'station_2_': 'Neasden',
   'flows_inv': 0.0001483}),
 ('Wembley Park',
  'Preston Road',
  {'length': 1419.7351657633037,
   'line_name': 'Metropolitan',
   'flows': 36601,
   'station_1_': 'Wembley Park',
   'station_2_': 'Preston Road',
   'flows_inv': 2.73e-05}),
 ('Wembley Park',
  'Finchley Road',
  {'length': 7266.37392749648,
   'line_name': 'Metropolitan',
   'flows': 55216,
   'station_1_': 'Wembley Park',
   'station_2_': 'Finchley Road',
   'flows_inv': 1.81e-05}),
 ('Kingsbury',
  'Queensbury',
  {'length': 1245.9952343630068,
```

```
'line_name': 'Jubilee',
'flows': 9419,
'station_1_': 'Kingsbury',
'station_2_': 'Queensbury',
'flows_inv': 0.0001062})]
```

[62]:
```
Gcc7b = sorted(nx.connected_components(G7b), key=len, reverse=True)
Gsub7b = G7b.subgraph(Gcc7b[0])
betweeness_w_path3=nx.average_shortest_path_length(Gsub7b,weight ='flows_inv')
print(betweeness_w_path3)

#components = nx.connected_components(G7b)
# Use the max() command to find the largest one:
#largest_component = max(components, key=len)
# Create a "subgraph" of the largest component
#Largest_subgraph = G7b.subgraph(largest_component)
#betweeness_w_path3 = nx.
↪average_shortest_path_length(Largest_subgraph,weight='flows_inv')
#print(betweeness_w_path3)
```

```
0.0008114665523690714
```

[63]:
```
[node_w[0]]
```

[63]:
```
['Green Park']
```

[64]:
```
G7b.remove_nodes_from([node_w[0]])

Gcc7b = sorted(nx.connected_components(G7b), key=len, reverse=True)
Gsub7b = G7b.subgraph(Gcc7b[0])
betweeness_w_path4=nx.average_shortest_path_length(Gsub7b,weight ='flows_inv')
print(betweeness_w_path4)
```

```
0.0008390851253132848
```

[65]:
```
changes_with_flows=(betweeness_w_path4-betweeness_w_path3)/betweeness_w_path3
print(changes_with_flows)
```

```
0.034035380587876446
```

Largest connected component a) without flows

[66]:
```
G8a=G.copy()
# Number of existing nodes:
l8a=list(G8a.nodes)
len(l8a)
```

[66]: 401

[67]:
```python
# number of connected components
no_of_component8a1=nx.number_connected_components(G8a)

# let's subset this graph to the largest connected component
Gcc8a1 = sorted(nx.connected_components(G8a), key=len, reverse=True)
Gsub8a1 = G8a.subgraph(Gcc8a1[0])
size8a1 = Gsub8a1.number_of_nodes()

# calculate the percentage of nodes present in the largest connected components
percent_lcc8a1 = (len(Gsub8a1.nodes)/len(G8a.nodes)) * 100

print(no_of_component8a1)
print(size8a1)
print(percent_lcc8a1)
```

```
1
401
100.0
```

[68]:
```python
node1[0]
```

[68]: 'Stratford'

[69]:
```python
G8a.remove_nodes_from([node1[0]])

# number of connected components
no_of_component8a2=nx.number_connected_components(G8a)

# let's subset this graph to the largest connected component
Gcc8a2 = sorted(nx.connected_components(G8a), key=len, reverse=True)
Gsub8a2 = G8a.subgraph(Gcc8a2[0])
size8a2 = Gsub8a2.number_of_nodes()

# calculate the percentage of nodes present in the largest connected components
percent_lcc8a2 = (len(Gsub8a2.nodes)/len(G8a.nodes)) * 100

print(no_of_component8a2)
print(size8a2)
print(percent_lcc8a2)
```

```
3
379
94.75
```

[70]:
```
changes_without_flows_lcc1=(no_of_component8a2-no_of_component8a1)/
↪no_of_component8a1
changes_without_flows_lcc2=(size8a2-size8a1)/size8a1
changes_without_flows_lcc3=(percent_lcc8a2-percent_lcc8a1)/percent_lcc8a1
print(changes_without_flows_lcc1)
print(changes_without_flows_lcc2)
print(changes_without_flows_lcc3)
```

```
2.0
-0.05486284289276808
-0.0525
```

b) with flows

[71]:
```
G8b=G.copy()
# Number of existing nodes:
l8b=list(G8b.nodes)
len(l8b)
```

[71]: 401

[72]:
```
G8b.remove_nodes_from([node_w[0]])

# number of connected components
no_of_component8b=nx.number_connected_components(G8b)

# let's subset this graph to the largest connected component
Gcc8b = sorted(nx.connected_components(G8b), key=len, reverse=True)
Gsub8b = G8b.subgraph(Gcc8b[0])
size8b = Gsub8b.number_of_nodes()

# calculate the percentage of nodes present in the largest connected components
percent_lcc8b = (len(Gsub8b.nodes)/len(G8b.nodes)) * 100

print(no_of_component8b)
print(size8b)
print(percent_lcc8b)
```

```
1
400
100.0
```

[73]:
```
changes_without_flows_lcc1=(no_of_component8b-no_of_component8a1)/
↪no_of_component8a1
changes_without_flows_lcc2=(size8b-size8a1)/size8a1
changes_without_flows_lcc3=(percent_lcc8b-percent_lcc8a1)/percent_lcc8a1
print(changes_without_flows_lcc1)
print(changes_without_flows_lcc2)
```

```python
print(changes_without_flows_lcc3)
```

```
0.0
-0.0024937655860349127
0.0
```