H_∞ Feedback Control of a Permanent Magnet Stepper Motor

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Abstract-**The aim of this paper is to propose H_∞ feedback control of a permanent magnet (PM) stepper motor. Firstly, a simple linear model is derived. Then, this model is used to design a controller. The control technique consists in using a robust control H_∞. It is shown that our technique guaranties good robustness properties to overcome the effect of nonlinearities, parameter variations and changes in load torque. Simulation results have shown the usefulness and the advantages of the proposed controller.**

I. INTRODUCTION

Permanent Magnet (PM) stepper motors have been employed extensively in open-loop controls, and the classical method of analysis and application has been covered in the literature ([12]-[15]). Originally, stepper motors were designed to provide precise positioning control within an integer number of steps without using sensors. That is, they are open-loop stable to any step position, and consequently no feedback is needed to control them. But, using the stepper motor in an open-loop configuration results in very low performance. In particular, the PM stepper motor has a step response with a significant overshoot and a long setting time. Due to these problems, one is motivated to go ahead and consider feedback for the stepper motor.

In past decade the feedback control problem of PM stepper motors has received some attention ([12]-[15]). One can find in the literature some control laws which have been shown effective. However, we strongly feel that previously proposed procedures will not meet significant success in practice mainly because of their complexity.

The PM stepper motor dynamical model includes nonlinearities and contains some physical parameters. The values of these latters are not exactly known and can be subject to some variations, so the model is not very easy to handle for control synthesis.

In this paper, the feedback control problem of PM stepper motors with all nonlinearities and uncertainties is investigated. First, a very simple linear model under some sharp assumptions is presented. Next, a classical two-degree-of-freedom (TDF) structure is used to deal with the feedback controller synthesis. This strategy consists in designing a robust α-stabilizing H_∞ controller. This approach, which is appealing when the parameters of the system and the load torques are submitted to variations, is illustrated through a position control of a PM stepper motor. Simulation results are given to show the advantages of the proposed controller.

II. MODEL OF PM STEPPER MOTORS

A. The complete model

Basically, the model of a Permanent Magnet (PM) stepper motor consists of two part, the electrical and mechanical parts. The mathematical model can be expresssed in state-space form as follows ([14], [15]).

$$\begin{cases} L\frac{di_a}{dt} &= v_a - Ri_a + K_m w \sin(N\theta) \\ L\frac{di_b}{dt} &= v_b - Ri_b - K_m w \cos(N\theta) \\ J\frac{dw}{dt} &= -K_m i_a \sin(N\theta) + K_m i_b \cos(N\theta) - \\ & \quad Bw - C\text{sign}(w) - K_d \sin(4N\theta) - \tau_l \\ d\frac{\theta}{dt} &= w \end{cases} \quad (1)$$

where v_a, v_b and i_a, i_b are respectively the voltages and currents in phases a and b respectively; w, θ are the rotor speed and angle position.

B and C are the viscous and Coulomb friction coefficients respectively. Further, R and L are the resistance

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and the inductance of the phase winding, N is the number of rotor teeth, J is the rotor inertia, K_m is the motor torque constant, K_d denotes the detent torque constant and τ_l is the load torque.

The complete model given by (1) will be used for all simulations with all non linearities and without any approximation. However, for the purpose of the control design, we will present an approximate linear model as discussed in the next section.

B. A simplified model for control

We now derive a simplified model of the PM stepper motor as a basis for control synthesis. To this end, the following assumptions are required

- $K_d \simeq 0$, so that the detent torque is negligeable; this is true for high enough values of speed;
- $C \simeq 0$ so that the Coulomb frictions torque is negligeable;
- $L \simeq 0$ so that one can use singular perturbations approach.

Thus the above nonlinear model can be simplified and reduced to

- *a fast part (electrical):*

$$\begin{cases} i_a &= \frac{1}{R}\left(v_a + K_m w \sin(N\theta) \right) \\ i_b &= \frac{1}{R}\left(v_b - K_m w \cos(N\theta) \right) \end{cases}$$

- *a slow part (mechanical)*

$$\begin{cases} \dot{\theta} &= w \\ \dot{w} &= -\frac{1}{J}\left(B + \frac{K_m^2}{R}\right) w - \frac{K_m}{JR}\sin(N\theta) v_a \\ & \quad + \frac{K_m}{JR}\cos(N\theta) v_b - \frac{\tau_l}{J} \end{cases}$$

Next one can make use of the so-called direct-quadrature (DQ) transformation

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos(N\theta) & \sin(N\theta) \\ -\sin(N\theta) & \cos(N\theta) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} \qquad (2)$$

hence the slow part can be rewritten as

$$\dot{X} = AX + Bv_q + G \qquad (3)$$

with

$$X = \begin{bmatrix} \theta \\ w \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \beta \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ g \end{bmatrix}.$$

where

$$\beta = \frac{K_m}{JR} \quad , \quad g = -\frac{\tau_c}{J}$$

and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix}, \qquad \alpha = \frac{1}{J}(B + \frac{K_m^2}{R}) \qquad (4)$$

System (3) can be viewed as a linear system subjected to the perturbation G. Its transfer function is

$$\theta(s) = \frac{\beta v_q(s)}{s(s+\alpha)} + \frac{g}{s(s+\alpha)} \qquad (5)$$

We see that the perturbation g acts with the same dynamics as the input v_q itself.

The simplified model (3) is single-input while the complete model (1) has two inputs v_a and v_b.

From (2) we obtain

$$v_a = \cos(N\theta)v_d - \sin(N\theta)v_q$$

and

$$v_b = \sin(N\theta)v_d + \cos(N\theta)v_q$$

By means, v_d can be make zero, we then put $v_d = 0$ resulting in the following relations

$$\begin{cases} v_a &= -\sin(N\theta)v_q \\ v_b &= \cos(N\theta)v_q \end{cases} \qquad (6)$$

We are interested in designing a controller which ensures a specified dynamics for rotor position θ and which eliminates the effect of the perturbation g. We present two methods to cope this problem

- a robust model-matching H_∞ controller which can deal with changes in the system's parameters

III. ROBUST H_∞ CONTROLLER DESIGN

In this section, a brief summary of robust normalized left coprime factors stabilization and the loop-shaping design will be given. This H_∞ design technique will be used for the design of robust linear controllers for a PM stepper motor.

The basis of the loop-shaping design procedure is a special case of the general H_∞ stavilization problem: the normalized left coprime factors stabilization problem. This problem is based on the configuration as depicted in Fig. 1, where $(N, M) \in RH_\infty$, the space of stable transfer function matrices, is a normalized left coprime factorisation of the nominal plant G. That is, $G = M^{-1}N$, and there exists $V, U \in RH_\infty$ such that $MV + NU = I$, and $NN^* + MM^* = I$, where for real rational function of s, X^* denotes $X^t(-s)$.

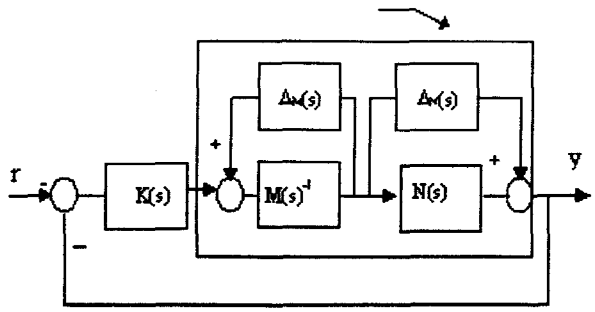

Figure 1: Robust stabilisation of normalized coprime factor plant descriptions

For a minimal realization of $G(s)$

$$G(s) = C(sI - A)^{-1}B + D$$

a normalized coprime factorisation of G can be given by [8]

$$M := \begin{bmatrix} A+BF & BS^{-\frac{1}{2}} \\ F & S^{-\frac{1}{2}} \end{bmatrix}$$
$$\text{and } N := \begin{bmatrix} A+BF & BS^{-\frac{1}{2}} \\ C+DF & BS^{-\frac{1}{2}} \end{bmatrix}$$

where

$$S := I + D^T D, \quad R := I + DD^T$$
$$\text{and } F := -S^{-1}(D^T C + B^T X)$$

and the matrix X is the unique symmetric positive definite solution of the algebraic Riccati equation (ARE) :

$$(A - BS^{-1}D^T C)^T X + X(A - BS^{-1}D^T C) \\ -XBS^{-1}B^T X + C^T R^{-1} C = 0$$

A perturbed model G_p of G is defined as

$$\text{where } G_P = (M - \Delta_M)^{-1}(N + \Delta_N)$$

where $\Delta_M, \Delta_N \in RH_\infty$.

The problem consists of computing a single, linear, real-rational feedback controller K, which internally stabilizes the set of plant D_γ given as

$$D_\gamma = \{G_P \mid \|[\Delta_M \ \ \Delta_N]\|_\infty < \gamma^{-1}\}$$

To maximize the class of perturbed models such that the closed-loop system in Fig. 1 is stabilized by a controller $K(s)$, the latter must stabilize the nominal plant G and minimize γ [8]

From the small gain theorem, the closed-loop will remain stable if

$$\|[\Delta_M \ \ \Delta_N]\|_\infty < \gamma^{-1}$$

The minimum value of $\gamma, (\gamma_{opt})$, for all stabilizing controllers is given by

$$\gamma_{opt} = [1 + \lambda_{max}(XZ)]^{1/2}$$

where $\lambda_{max}(.)$ represents the maximum eigenvalue, and Z is the unique stabilizing solution of the Riccati equation

$$(A - BD^T R^{-1} C)Z + Z(A - BD^T R^{-1} C)^T \\ -ZC^T R^{-1} CZ + B(I_m - D^T R^{-1} D)B^T = 0$$

A controller which achieves a $\gamma > \gamma_{opt}$ is given in [8] by

$$K = \begin{bmatrix} A + BF + \gamma^2 W_1^{T-1} Z C^T (C + DF) & \gamma^2 W_1^{T-1} Z C^T \\ B^T X & -D^T \end{bmatrix}$$

where $W_1 = I + (XZ - \gamma^2 I)$. A descriptor system approach may be used to synthesize an optimal controller such that the minimum value γ_{opt} is achieved.

In pratical designs, the plant needs to be weight to meet closed-loop performance requirement. In a first step, the singular value plot of the nominal, linear plant G is inspected, and Design a pre-filter W_1 and/or a post-filter W_2, the shape of the open loop is formed to meet some design specification. In a second step, the shaped plant $G_s = W_1 G W_2$ is formed and the feedback controller K_s, is synthesized which robustly stablizes the normalized left coprime factorisation of G_s with a stability margin γ^{-1}. In addition to the stability robustness properties discussed above, this controller possesses the advantage of creating only a limited deterioration of the specified loop shape. Furthermore, a number of standard closed-loop design objectives, such as sensitivity and input sensitivity, are minimized. Finally, the complete controller is $K = W_1 K_s W_2$

IV. CONTROLLER DESIGN

The above design procedure can be developed further in two-degree-of-freedom(TDF) scheme as shown in Fig. 2.

The philosophy of the TDF scheme is to use the feedback controller $K(s)$ to meet the requirements of internal stability, disturbance rejection, measuremant noise attenuation, and sensitivity minimization. The precompensateur $T(s)$ is then applied to the reference signal, which optimizes the response of the overall system to the command input.

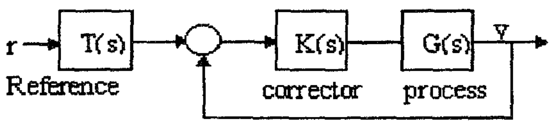

Figure 2: Two-Degree-of-Freedom Structure

Our main contribution is in the way to design $K(s)$ and $T(s)$ in order to ensure stability and robust performance for all the family of transfer functions described by $G(s)$ ([4], [5]).

In fact, our synthesis method is based on the following requirements :

- the model-reference $T(s)$ is assumed to be choosen by the user with respect to desired dynamics and steady-state values. We denote by p the smallest (i.e. the dominant) pole of $T(s)$.
- the controller $K(s)$ must be designed such that the closed-loop part of the TDF is α-stable and has unity-gain for all the plants $G(s)$. The parameter α is choosen such that

$$-\alpha \ll p$$

Hence, we see that only $K(s)$ has to be designed in such a way that the transient behavior of the closed part of the TDF is dominated by that of $T(s)$.

In what follows, we discuss a method to solve our design requirements for $K(s)$. This will be done using the concept of α-stability in H^{∞}-approach [1] [2] [8].

A. Robust α-stabilizing H^{∞} controller

One has the following result:

Theorem (Saeki) [6]

Assume that a transfer function $G(s)$ is regular inside a domain on the left of a vertical line and consider the change of variables $s = u - \alpha$ to obtain $Clp_t(u) = Clp(u - \alpha)$. If $\alpha > 0$ and there exists a controller $K_t(u)$ which verifies:

$$\left\| \begin{bmatrix} I \\ K_t \end{bmatrix} (I - G_t K_t)^{-1} M_t^{-1} \right\|_{\infty} \leq \gamma$$

then the controller defined by $K(s) = K_t(s+\alpha)$ satisfies

$$\left\| \begin{bmatrix} I \\ K \end{bmatrix} (I - GK)^{-1} M^{-1} \right\|_{\infty} \leq \gamma$$

and all the poles of closed loop $clp(s)$ lie to the left of the vertical line $s = -\alpha$ on the complex plane.

•••

B. Robust α-stabilizing H_{∞} controller ensuring unity gain

It is required that the controller must ensure a unity gain for the closed-loop for all the plants described by $G(s)$. This can be very achieved by including an integrator in the controller. To satisfy this constraint, one can simply replace the family plants $G(s)$ by $W(s)G(s)$ where $W(s)$ is a weighting functions that has the form

$$W(s) = \frac{w}{s}$$

where w is a design parameter.

The aim is to design a controller in order to meet an ideal closed loop which we specify by the following reference model $T(s)$

Consequently, we fix α for the design procedure; hence, we require the closed loop to be 10 times faster than the reference model.

V. SIMULATION RESULTS

The proposed control strategies are compared from the robustness point of view. In particular, we study the effect of the load torque and of variations on the motor mechanical and electrical parameters (J, R, L, K_m, ...)

For all cases, the desired rotor motion trajectory is specified as

$$T(s) = \frac{1}{0.1s + 1}$$

A. Case 1: Parameters variations with $\tau_l = 0$

Fig 3, 4, 5 and 6 show respectively, the actual position, the speed and currents of the PM motor in the case of variations in the parameters α and β with $\tau_l = 0$. One can see that, we have good tracking and the currents sufficiently smaller than the nominal currents.

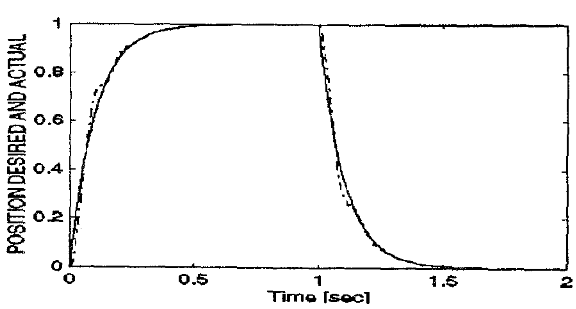

Figure 3: Position: actual(–) and desired

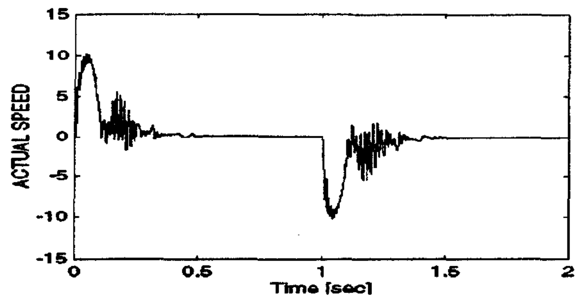

Figure 4: Actual speed

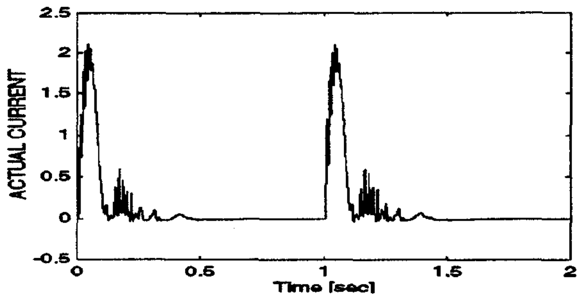

Figure 5:Actual current in phase a

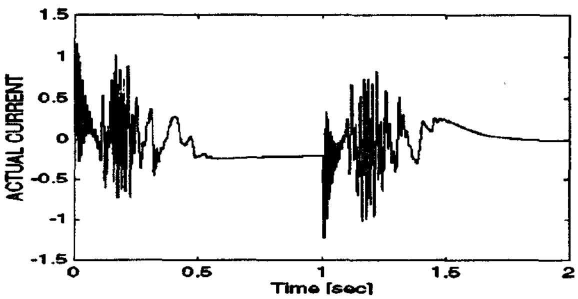

Figure 6: Actual current in phase b

B. Case 2: Parameters variations with sinusoidal load torque

Fig 7, 8, 9 and 10 show respectively, the actual position, the speed and currents of the PM motor in the case of variations in the parameters α and β with $\tau_l = 0.2(1 - \cos(\theta))$. One can see that, we have good tracking and the currents sufficiently smaller than the nominal currents.

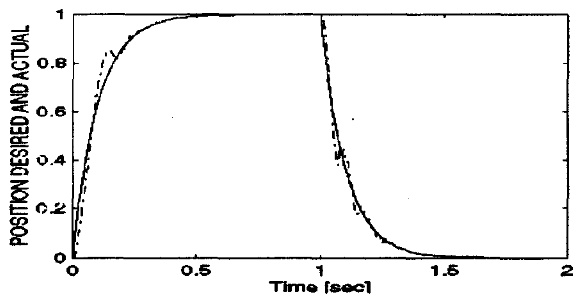

Figure 7: Position: actual(–) and desired

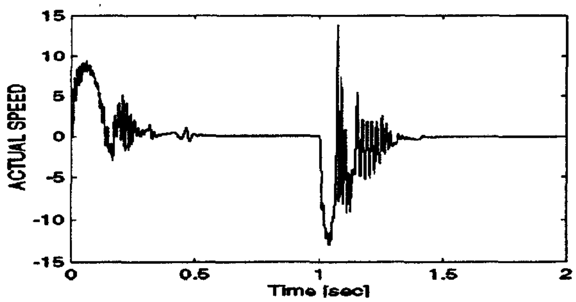

Figure 8: Actual speed

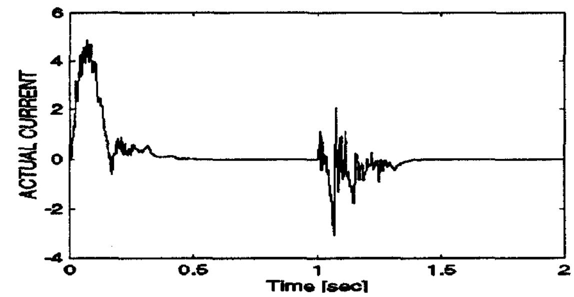

Figure 9: Actual current in phase a

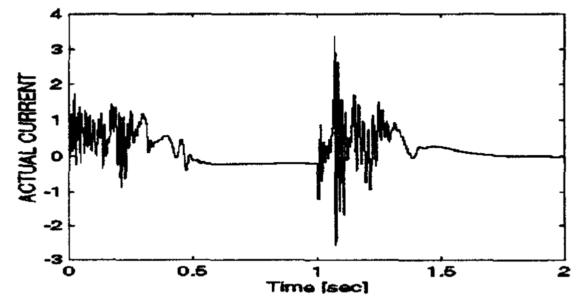

Figure 10: Actual current in phase b

These simulation results show that controller ensures very satisfactory transient position tracking performances despite time varying load perturbations and parameters variations.

The above figures illustrate the usefulness of our approach.

VI. CONCLUSION

In this paper, we have presented a new solution to the feedback control problem of a permanent magnet stepper motor. A control scheme has been performed on the basis of an approximate linear model. We have proposed :

A TDF controller method has been discussed to deal with robust performance problems of the model nonlinearities and uncertainties. the proposed procedure consists in designing a robust α-stabilizing H_∞ controller.

Appendix

All numerical computations are performed using a PM stepper motor with 200 steps/rotation with parameters defined as follows

$$L = 0.0046\ H, R = 0.28\ \Omega, K_m = 0.464\ N.m/A,$$

$$N = 50, J = 3.65\ kg.cm^2, K_d = 0.12\ N.m,$$

$$B = 0.011\ N.m.s/rad, C = 0.008\ N.m,$$

The nominal current is 5.6 A.

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