

# Compensation of Load-Dependent Position Error for A Hybrid Stepper Motor

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**Abstract** – Load-dependent position error and its compensation approach in a hybrid stepper motor has been studied in this paper. With stepper motor modelling and load angle concept, low position stiffness feature for a stepper motor with open loop operation has been explained. A specific field oriented control has been proposed to maximize stepper motor position stiffness. Experimental verification shows the effectiveness of the proposed method. Results also show that stepper motor efficiency can be improved significantly by the proposed method.

**Index Terms** – Field-oriented control, vector control, hybrid stepper motor, position stiffness.

## I. INTRODUCTION

Stepper motors are commonly used in a wide variety of applications and continue to become more powerful and less expensive. Stepper motor has many advantages, e.g. low cost, simple structure, robust and high torque density<sup>[1]</sup>. Hybrid stepper motors are one type of most popular commercial stepper motors. Magnetic flux for a hybrid stepper motor is excited by a combination of windings and a permanent magnet<sup>[1]</sup>. Stepper motors are often used in digital control systems where the motor receives open loop commands in the form of a train of pulses to turn a shaft. Hybrid stepper motors are often a preferred choice when low cost positioning is required within an industrial application. The functionality and dynamic modelling of stepper motors are described in [2]-[5].

However, to meet relative high performance applications, stepper motor performance needs to be further improved. Due to stepper motors' specific design, they can reach relatively high linearity even at open loop operation. The angular position error is the difference between the reference angular position and the angular position of the rotor. As shown in Fig. 1, position error in experiment is below 0.1 degree. However, when an external torque is applied to the motor, the position error increases dramatically, as shown in Fig. 2. The position error is to a large degree due to the so-called load angle when the motor is positioned by an open-loop controller. The load angle is necessary for the stepper motor to produce electromagnetic torque. Therefore, applying an external torque to the stepper motor causes the magnetic rotor to be out of phase with the electrical field. The ability to maintain the position when subject to an external load is referred to as the

dynamic position stiffness. The test result shows that a stepper motor has low dynamic position stiffness, which makes the relatively high linearity meaningless for many industry applications.

Another problem for stepper motor is low efficiency<sup>[6]</sup>. Since maximum phase currents are always applied in open loop operation to avoid losing step, stepper motor has low efficiency and its related problems, e.g. thermal issue. This paper investigates these problems and proposes to solve them from control approach.

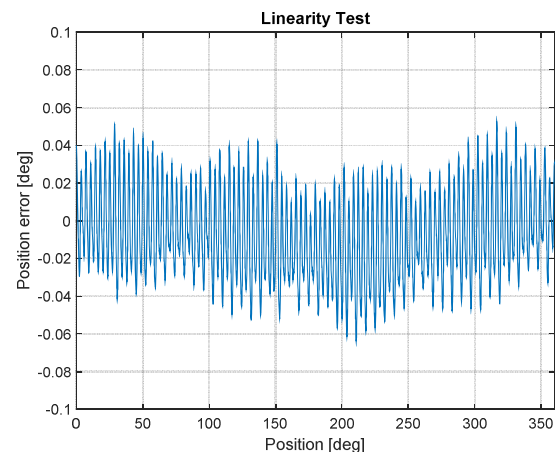


Fig. 1 Stepper motor linearity test result.

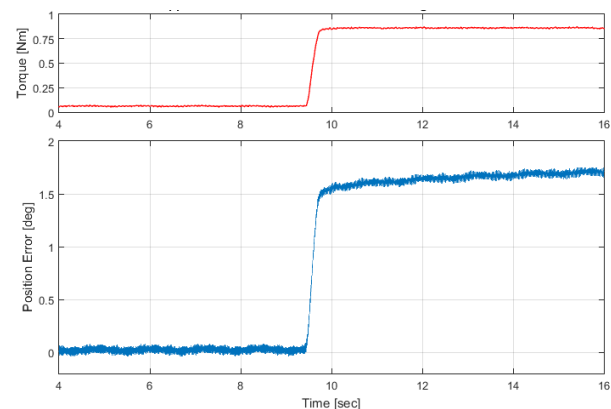


Fig. 2 Position error when an external torque applied to the motor.

## II. MODELLING AND PROBLEM FORMULATION

### A. Hybrid Stepper Motor Principle

Schematic of a two-phase hybrid stepper motor is shown in Fig. 3. The hybrid stepper motor is made up of certain distinguishable components arranged in a specific layout. The outer layer of the stepper typically contains eight electromagnets spread out evenly around the central rotor wheel. The central rotor is a solid metal piece with several teeth. There are usually 50 teeth in total. The motor works by attracting and repulsing teeth using the electromagnets. In a hybrid stepper motor, the rotor is a permanent magnet and is moved by exciting a single electromagnet (one phase) or a pair of electromagnets (two phase) in turn. The effect is complemented by a minimal reluctance effect, where the rotor is attracted to a position where the space between the teeth and the electromagnet is minimized. The rotor does not line up against all of the electromagnets at the same time, as is shown in Fig. 3. By varying the magnitude and direction of the winding currents, the rotor is continuously attracted in the desired direction. A "step" occurs whenever a rotor tooth moves slightly to align itself to an electromagnet tooth.

It is possible to decrease the step size of a hybrid stepper motor by using a control logic called micro stepping<sup>[7]</sup>. Micro stepping involves transitioning between each phase shift. That is, the current references are defined by sinusoidal signals displaced 90 electrical degrees from each other.

### B. Hybrid Stepper Motor Modelling

The dynamic model of a stepper motor is formed by the following equations<sup>[2]</sup>:

$$u_a(t) = Ri_a(t) + L\{i_a(t), i_b(t), \theta_e(t)\} \cdot \frac{di_a(t)}{dt} + e_a\{i_a(t), i_b(t)\} \quad (1)$$

$$u_b(t) = Ri_b(t) + L\{i_a(t), i_b(t), \theta_e(t)\} \cdot \frac{di_b(t)}{dt} + e_b\{i_a(t), i_b(t)\} \quad (2)$$

$$\frac{d\omega_r(t)}{dt} = \frac{1}{J} \{ \tau_e(t) - \tau_l(t) \} - K_d \sin[N_r \theta_r(t)] - \beta \omega_r \quad (3)$$

$$\tau_e(t) = -K_m i_a(t) \sin(N_r \theta_r(t)) + K_m i_b(t) \cos(N_r \theta_r(t)) \quad (4)$$

$$\frac{d\theta_r(t)}{dt} = \omega_r(t) \quad (5)$$

The constants in the model are the winding resistances  $R$ , the motor torque constant  $K_m$ , the magnitude of the detent torque  $K_d$ , the rotor inertia  $J$ , the viscous friction constant  $\beta$  and the number of motor pole pairs  $N_r$ . The variables  $u_a(t)$  and  $u_b(t)$  are the terminal voltages. The alternating currents  $i_a(t)$  and  $i_b(t)$  are the phase currents which give rise to the torque and position change produced by the stepper motor. The electromagnetic

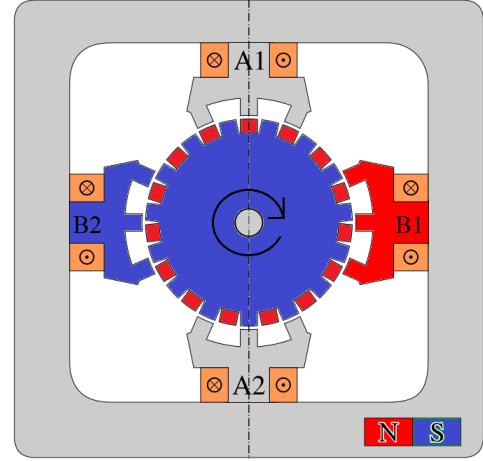


Fig. 3 Schematic of a two-phase hybrid stepper motor.

torque given by  $\tau_e(t)$  is the torque produced by the motor. The load torque,  $\tau_l(t)$ , is the torque applied by an external system. Both  $\tau_e(t)$  and  $\tau_l(t)$  are considered to be time-dependent in order to properly describe a dynamic physical system.

The self-inductance of the windings,  $L$ , has some slight position and current dependencies. The position dependency surfaces due to a varying air gap distance in the motor and is of less importance in hybrid stepper motors. The current dependency is caused by the relation between the flux linkage and the reluctance of the magnetic circuit, which is current dependent.

The detent torque is normally not considered in dynamic model equations for hybrid stepper motors as the magnitude of the detent torque  $K_d$  is generally less than 10% of the holding torque.

The angle  $\theta_r(t)$  denotes the rotor angle, and the phase of the electrical field is denoted by  $\theta_e(t)$ . These angles are related by the number of rotor pole pairs

$$\theta_e(t) = N_r \theta_r(t) \quad (6)$$

Furthermore, the electrical angular velocity is given by

$$\frac{d\theta_e(t)}{dt} = \omega_e(t) = N_r \omega_r(t) \quad (7)$$

The back-EMF  $e_a(i_a(t), i_b(t))$  and  $e_b(i_a(t), i_b(t))$  are given by

$$\begin{cases} e_a\{i_a(t), i_b(t)\} = -K_m \omega_r(t) \sin[N_r \theta_r(t)] \\ e_b\{i_a(t), i_b(t)\} = K_m \omega_r(t) \cos[N_r \theta_r(t)] \end{cases} \quad (8)$$

The dq-transformation for a hybrid stepper motor is shown as follows:

$$\begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta_e(t)) & \sin(\theta_e(t)) \\ -\sin(\theta_e(t)) & \cos(\theta_e(t)) \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \end{bmatrix} \quad (9)$$

This transformation and its inverse transformation are often used when applying field oriented control.

### C. Load Angle

The load angle in any synchronous machine, such as the hybrid stepper motor, is necessary for the electric machine to produce electromagnetic torque in order to meet the torque

requirements of the application<sup>[8]</sup>. The load angle can however significantly contribute to the motor's angular position error and will potentially decrease the accuracy and precision of the final application. For example, a robot with several stepper motor joints will have poor precision at the point of the final tool if this error is not compensated. The position error needs to be either measured or estimated, or both, in order to be compensated properly. It is therefore vital to understand the relationship between the load angle and the torque of a hybrid stepper motor in order to be able to control and compensate the position error.

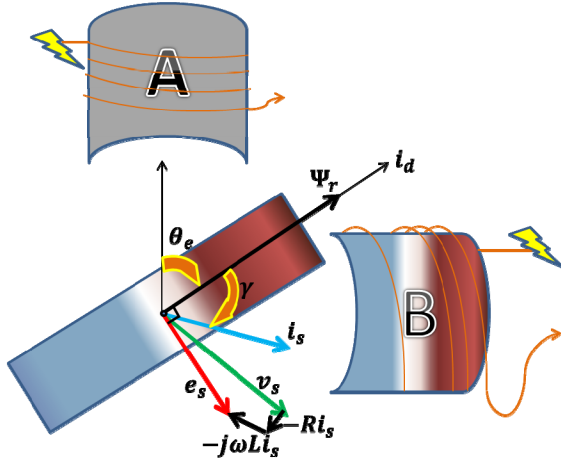


Fig. 4 Rotor magnetic flux and current vector in dq-reference.

Rotor flux vector  $\Psi_r$  and current vector  $i_s$  in the rotor fixed dq-reference frame are shown in Fig. 4. Load angle  $\gamma$  is in inertial reference frame. The working principle is illustrated with two stator poles and two rotor teeth respectively. The produced torque from a stepper motor is given by the cross product between the stator flux linkage space vector  $\Psi_s$  and the stator current resultant space vector  $i_s$  by

$$\tau_e = \Psi_s \times i_s \quad (10)$$

If saturation is neglected, the stator flux linkage space vector  $\Psi_s$  is formed by the sum of the permanent-magnet rotor flux  $\Psi_r$  and the two stator flux linkages within dq-reference frame

$$\tau_e = (\Psi_r + i_d L_d + i_q L_q) \times i_s \quad (11)$$

By performing the cross-product, the electromagnetic torque  $\tau_e$  can be expressed as:

$$\tau_e = \Psi_r i_s \sin(\gamma) + \frac{L_d - L_q}{2} i_s^2 \sin(2\gamma) \quad (12)$$

The load angle  $\gamma$  is defined as the angle between  $i_s$  and  $\Psi_r$ . The first term is dominating the second term when the phase inductances are assumed to be constant and equally large for the two phases. Thus the electromagnetic torque can be written as follows:

$$\tau_e \cong \Psi_r i_s \sin(\gamma) \quad (13)$$

Stepper motors are primarily used to realize position demands but can also be used to meet speed demands in an

application. To control the position of the hybrid stepper motor's rotor and the attached application, the stepper motor torque is controlled. Since the stepper motor torque is a function of the winding currents according to (13), utilizing an inverter, i.e. a current controller, to control the winding currents is necessary. The desired reference position is thus translated into desired winding current according to

$$\begin{cases} i_a = I_0 \cos(N_r \theta_{ref}) \\ i_b = I_0 \sin(N_r \theta_{ref}) \end{cases} \quad (14)$$

Inserting these currents into the electromagnetic torque equation gives

$$\begin{aligned} \tau_e &= -K_m I_0 \cos(N_r \theta_{ref}) \sin(\theta_e) \\ &\quad + K_m I_0 \sin(N_r \theta_{ref}) \cos(\theta_e) \\ &= K_m I_0 \sin(N_r \theta_{ref} - \theta_e) \end{aligned} \quad (15)$$

According to previous discussion,

$$\gamma = N_r \theta_{ref} - \theta_e \quad (16)$$

by setting the current references according to Equation (14), the produced torque will turn the unloaded stepper motor's rotor to the electrical reference position. Since sine has its smallest value for a zero angle, this position is also stable. This is how an open-loop controller can select the appropriate winding currents. When a motor is subject to an external load, the controller also has to account and be able to compensate for that as well.

### III. LOAD ANGLE COMPENSATION

Field Oriented Control (foc), sometimes also known as Vector Control, applies a zero direct current and a controlled quadrature current, considering the dq-reference frame described in [9]. The direct and quadrature currents are then transformed into reference currents  $i_{a,ref}$  and  $i_{b,ref}$  for the two phases using the inverse dq-transform. Field oriented controller may vary both the load angle and the amplitude of the current, thus it can run the motor with larger load angle and reduce the current - making the motor be more energy efficient.

Various control strategies exists for hybrid stepper motors. The concept of field oriented control<sup>[10][11]</sup> is especially appealing. By applying field oriented control, the motor is driven with a large load angle and smaller currents than during open loop control. The field oriented control suggested by [10] use PID control with additional velocity feed-forward to first derive a desired torque

$$\begin{aligned} \tau_{e,ref} &= k_p (\theta_{r,ref} - \theta_r) + k_i \int (\theta_{r,ref} - \theta_r) d\tau \\ &\quad + k_D (\dot{\theta}_{r,ref} - \dot{\theta}_r) + B \omega_{ref} \end{aligned} \quad (17)$$

The desired torque is then fulfilled by setting the phase currents according to

$$\begin{cases} i_{a,ref} = \frac{\tau_{e,ref}}{K_m} \cos(N_r \theta_{r,ref}) \\ i_{b,ref} = \frac{\tau_{e,ref}}{K_m} \sin(N_r \theta_{r,ref}) \end{cases} \quad (18)$$

Since  $N_r = 50$  is the number of pole pairs in the machine,  $N_r \theta_{r,ref}$  is the desired electrical angle of the electric field. This control strategy will properly vary the winding currents, and therefore the electric field, until the mechanical position of the rotor is in the desired reference position. This choice of phase currents  $i_a$  and  $i_b$  are the same as applying a desired direct and quadrature current:

$$\begin{cases} i_{d,ref} = 0 \\ i_{q,ref} = \frac{\tau_{e,ref}}{K_m} \end{cases} \quad (19)$$

To maximize dynamic position stiffness, according to equation (13), the load angle  $\gamma$  should be controlled to  $\pi/2$ . In field oriented control equation (18), if the  $\theta_{r,ref}$  is chosen as follows, the stepper motor dynamic position stiffness can be maximized

$$\theta_{r,ref} = \frac{\left(\theta_e + \frac{\pi}{2}\right)}{N_r} \quad (20)$$

$\theta_e$  can be calculated from rotor angle according to equation (6).

$$\tau_e = \tau_{e,ref} \sin\left(\frac{\pi}{2}\right) \quad (21)$$

#### IV. EXPERIENTIAL VERIFICATION

In order to verify the proposed approach, experimental verification has been done in a stepper motor control and test platform as shown in Fig. 5 and Fig. 6. The stepper motor test rig has a stepper motor, an encoder, a torque sensor and a hysteresis brake. The brake can effectively apply torque to the mechanical shaft at any normal speed. The stepper motor is controlled by a real time computer.

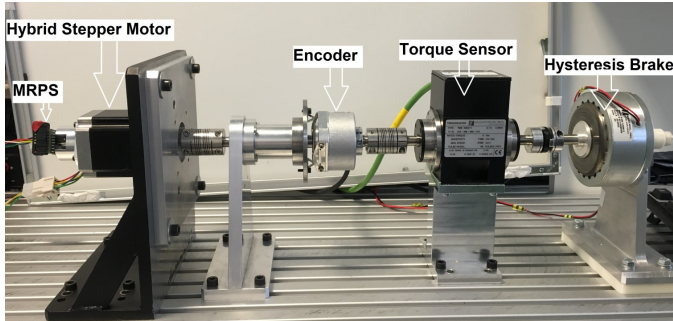


Fig. 5 Stepper motor test rig.

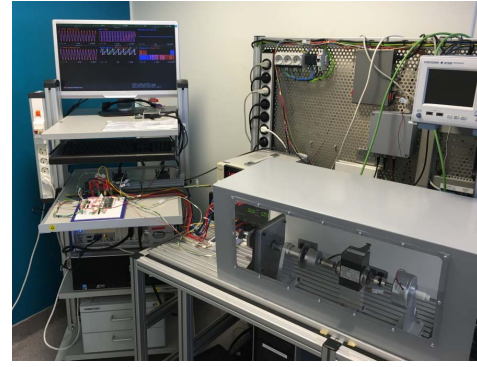
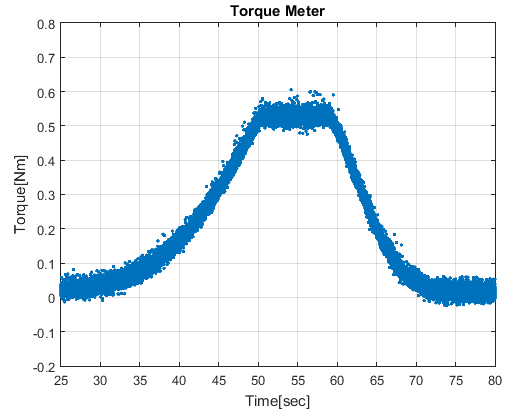
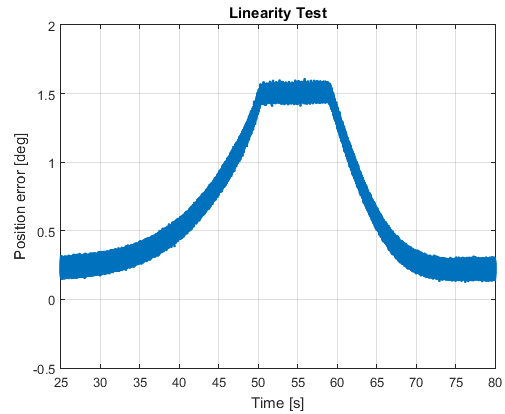


Fig. 6 Stepper motor control & test platform.

In the first test case, the stepper motor is controlled by open loop micro stepping approach. An external torque is applied on the motor and both external torque and motor position error have been recorded. As shown in Fig. 7, when the external torque gradually increases from 0.05Nm to 0.55Nm, the stepper motor position error increases from 0.25 degree to 1.5 degree. The position error is 2.5 degree per Nm.

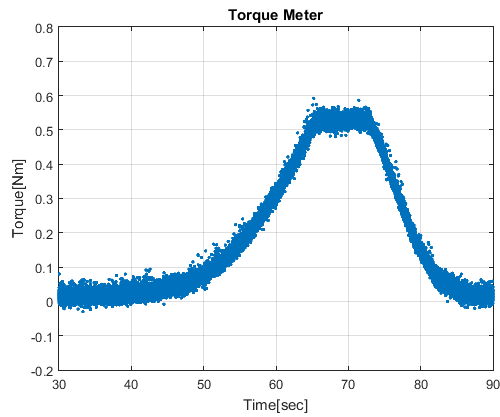


(a) External load

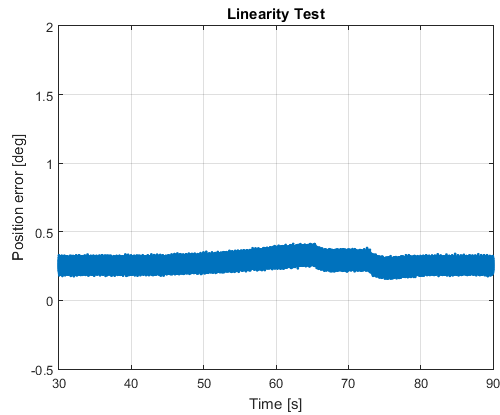


(b) Position error

Fig. 7 Position stiffness test: traditional control.



(a) External load



(b) Position error

Fig. 8 Position stiffness test: maximum position stiffness field oriented control.

In the second test case, the stepper motor is controlled by proposed maximum position stiffness field oriented control approach. The same external torque is applied on the motor and both external torque and motor position error have been recorded. As shown in Fig. 8, when the external torque gradually increases from 0.05Nm to 0.55Nm, the stepper motor position error increases from 0.25 degree to 0.35 degree. The position error in this case is 0.2 degree per Nm.

The transient phase currents from open loop micro stepping control to maximum position stiffness field oriented control are given in Fig. 9. At open loop control, maximum phase currents are always applied to the motor avoid losing step. Phase currents will not change according to load. After switching into the field oriented control, only the required currents are realized in the phase windings. Efficiency of the stepper motor therefore increased significantly. For many cases, maximum position stiffness field oriented control could reduce stepper motor power consumption by about 50% compared to open loop micro stepping operation.

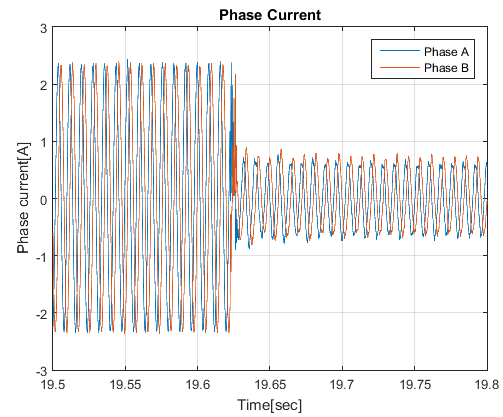


Fig. 9 Transient phase currents from open loop control to maximum position stiffness field oriented control.

## V. CONCLUSION

Load-dependent position error in a hybrid stepper motor has been studied and compensated. With stepper motor modelling and load angle concept, low position stiffness feature for a stepper motor with open loop operation has been explained. A specific field oriented control has been proposed to maximize stepper motor position stiffness. Experimental verification shows that the proposed approach could improve the position stiffness significantly (position error decreased from 2.5 degrees per Nm to 0.2 degrees per Nm). Results also show that stepper motor efficiency can be improved significantly by the proposed method.

Position signal from an encoder has been used in the field oriented control at this stage. Next step, low cost position sensors and sensorless estimation will be further investigated to replace the encoder signal.

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