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Pareto Optimal Prediction Intervals of Electricity Price

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***Abstract*—This letter proposes a novel Pareto optimal prediction interval construction approach for electricity price combing extreme learning machine and non-dominated sorting genetic algorithm II (NSGA-II). The Pareto optimal prediction intervals are produced with respect to the formulated two objectives reliability and sharpness. The effectiveness of proposed approach has been verified through the numerical studies on Australia electricity market data.**

***Index Terms*—Electricity price, extreme learning machine, NSGA-II, prediction intervals.**

I. Introduction

Electricity price forecasting is essential to the operation of electricity market which has critical influence on economic operation of modern power systems [1]. However, precise forecasting would be particularly difficult even impossible due to the nonlinearity and heteroscedasticity nature of electricity price, thus probabilistic electricity price forecasting becomes crucial to quantify the uncertainty involved in classical point forecasting [2]. Traditionally, parametric interval forecasting approach based on normal assumption [3] and nonparametric interval forecasting approach [4] for electricity price do not consider the sharpness of electricity price prediction intervals (PIs), though it is very critical and indispensable to the PIs quality and subsequent decision-making. In this letter, a novel Pareto optimal prediction interval construction (POPIC) approach is proposed based on extreme learning machine (ELM) [5] and NSGA-II [6] to generate Pareto optimal PIs of electricity price take into account reliability and sharpness simultaneously. ELM is an extremely efficient algorithm for training a single hidden-layer feedforward neural network and has been used for forecasting and classification [7]–[9]. The multi-objective optimization algorithm NSGA-II has global search capability and evenly distributed nondominated solutions in Pareto front, and has been widely used for power system dispatch and planning [10], [11]. With the proposed genuine Pareto optimality based POPIC approach, optimal multi-confidence PIs produced would benefit decision-making and provide multiple references for electricity market participants.

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II. Methodology

A. Definition and Evaluation of PIs

Different from traditional point forecasting techniques, probabilistic forecasting methods generate PIs with associated confidence levels that can effectively quantify the uncertainties involved in electricity price prediction.

A nonparametric PI $I^{\alpha}_{t+k|t}$ of electricity price addressed at time t for time $t+k$ with nominal coverage probability (NCP) $100(1-\alpha)\%$, $\alpha \in [0, 1]$, can be defined as

$$I^{\alpha}_{t+k|t} = \left[L^{\alpha}_{t+k|t}, U^{\alpha}_{t+k|t}\right] \tag{1}$$

where $L^{\alpha}_{t+k|t}$ and $U^{\alpha}_{t+k|t}$ are the lower and upper bounds of the PI respectively. The future prediction target of electricity price Y_{t+k} is expected to be enclosed by the constructed PI with the coverage probability

$$P(Y_{t+k} \in I^{\alpha}_{t+k|t}) = 100(1-\alpha)\%. \tag{2}$$

The reliability and sharpness are two indispensable indicators for precise assessment of PIs [12].

1) *Reliability:* Reliability is considered as a primary verification indicator for PIs as the lack of reliability can bring systematical bias to decision-making activities.
   The empirical coverage probability (ECP) of generated PIs is a key measure for the reliability of the constructed PIs

$$ECP = \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}\left\{Y_{t+k} \in I^{\alpha}_{t+k|t}\right\} \tag{3}$$

   where T is the size of the test dataset, and $\mathbf{1}\{\cdot\}$ is the indicator function that equals to 1 if the condition inside the brackets is met, and to 0 otherwise.
   Given NCP, the deviation between ECP and NCP can be measured by the average coverage error (ACE), defined as

$$ACE = ECP - 100(1-\alpha)\% \tag{4}$$

   the closer to zero, the better.
2) *Sharpness*: The sharpness is another critical index of PIs quality. It measures the ability of the estimated PIs to concentrate the occurrence probability of a future target [12].

It can be measured by the average width of PIs (AWPI) $\bar{\delta}^{\alpha}_{t,k}$, defined by

$$\bar{\delta}^{\alpha}_{t,k} = \frac{1}{T}\sum_{t=1}^{T}\delta^{\alpha}_{t+k|t} = \frac{1}{T}\sum_{t=1}^{T}\left[U^{\alpha}_{t+k|t} - L^{\alpha}_{t+k|t}\right]. \tag{5}$$

Given similar reliability, the sharper PIs with the smaller AWPI are to be preferred by decision makers. Actually, high reliability can be easily reached via increasing or decreasing the distance between the upper and lower bounds that respectively have identical values at each time point. Though with high reliability, the obtained PIs cannot provide meaningful reference for any decision making activities since they cannot accurately quantify the uncertainties involved in the processes of electricity price. In general, both reliability and sharpness should be taken into consideration for the evaluation of electricity price PIs.

B. Problem Formulation and Pareto Optimality

Theoretically, the goal of probabilistic forecasting is to maximize the sharpness of the predictive distributions subject to the reliability [12]. Optimal PIs of electricity price should have the best reliability with the maximum ECP and the best sharpness with the minimum AWPI, which can be formulated as two objectives

$$\max f_1(x) = \text{ ECP} \tag{6}$$

$$\min f_2(x) = \text{ AWPI} \tag{7}$$

where ECP and AWPI are defined in (3) and (5) respectively.

If each single objective is optimized alone, it can be resulted that the optimum of the reliability would be 100% (i.e., very wide PIs will be resulted to encapsulate all actual targets), while the optimum of the sharpness would be 0 (i.e., very narrow PIs will be resulted with upper and lower bounds overlapping with each other), which apparently cannot be considered as the genuine optimal PIs of electricity price. As a multi-objective problem, the Pareto optimality is introduced into the study to obtain the Pareto front of optimal decision variables. The optimal decision vector x_l dominates x_m if and only if both the following two conditions are satisfied

$$f_1(x_l) \geq f_1(x_m) \quad \text{and} \quad f_2(x_l) \leq f_2(x_m) \tag{8}$$

$$f_1(x_l) > f_1(x_m) \quad \text{or} \quad f_2(x_l) < f_2(x_m). \tag{9}$$

All decision vectors, which are not dominated by any other vectors, compose the set of Pareto optimal solutions P^* of the proposed multi-objective problem.

C. Optimization Algorithm

The proposed POPIC approach aims to construct ELM-based forecasters to output the bounds of PIs of electricity price with the best quality and independent of distribution assumption. Due to the unique properties of ELM that the input weights and hidden biases are randomly assigned and kept in constant, training the ELM based forecasters would be more simplified comparing with traditional neural networks. The parameters of ELM are optimized with regard to the two objectives defined in (6) and (7). To acquire optimal decision variables, the two objectives should be optimized simultaneously. As one of the most efficient algorithms for multi-objective optimization, NSGA-II is employed to obtain well distributed Pareto optimal solutions of ELM. The detailed procedures of the proposed algorithm are described as shown in the algorithm at the top of the column.

Begin
$g \leftarrow 0$
1) Initialize the parent population P_t (size *N*).
2) Construct ELM with P_t, compute the ECP and AWPI based on (3) and (5).
3) Create an offspring population Q_t (size *N*) by binary tournament selection, recombination and mutation operator of generic algorithm.
while (not termination condition) **do**
begin
$g \leftarrow g + 1$
4) Compute the ECP and AWPI of Q_t based on (3) and (5).
5) Form a combine population $R_{t-1} = P_{t-1} \cup Q_{t-1}$ (size 2*N*).
6) Sort R_{t-1} according to nondomination.
7) Select the new population P_t from R_{t-1} according to the crowded-comparison operator.
Create an offspring population Q_t.
end
end

III. Numerical Study

To verify the effectiveness of the proposed POPIC approach, Victoria (VIC) electricity market of Australian National Electricity Market (ANEM) is used for the implementation of comprehensive numerical experiments. In ANEM, market clearing price (MCP) is calculated based on a half-hour trading interval. Day-ahead MCPs need to be forecasted for decision-making of electricity producers and consumers. The VIC market data from January 2009 to December 2011 are applied in the study.

An unconditional distribution for probabilistic forecasting verification is used as a reference method, of which the forecasted MCPs obey the same normal distribution and the mean and variance can be obtained based on the historical observations. In addition, an advanced model combining bootstrap and ELM (ELM-Bootstrap) is also applied to benchmark the proposed approach [4]. To further validate the effectiveness of the proposed POPIC model, multi-objective optimization differential evolution (MODE) algorithm [13] is utilized to solve the formulated multi-objective optimization problem for comparative analysis, which is termed as MODE-POPIC in the study. Typical peak-load trading interval (11:00–11:30) of VIC electricity market is tested in the study. The VIC market data covering the period from January 2009 to December 2010 are used for model construction, and the rest data are applied for the numerical test.

The Pareto front obtained by the proposed approach is depicted in Fig. 1. It can be seen that optimal forecasters with different NCPs can be reached and determined from the Pareto front. The higher ECPs correspond to the larger AWPIs, consistent with the actual conditions. Note that the AWPIs obtained in the study are based on the normalized values of electricity price data. The evaluation results of PIs with NCPs 80% and 90% generated by different methods are given in Table I.

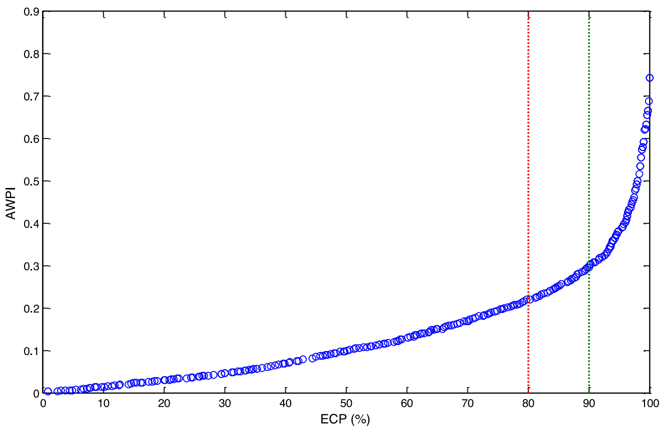

Fig. 1. Pareto front derived by the proposed approach.

TABLE I
PERFORMANCE COMPARISONS OF GENERATED PIS

NCP	Method	ECP	ACE	AWPI
80%	Normal Model	69.51%	−11.49%	0.2267
	ELM-Bootstrap	76.92%	−3.08%	0.2815
	MODE-POPIC	80.94%	0.94%	0.2488
	Proposed POPIC	80.07%	**0.07%**	**0.2147**
90%	Normal Model	79.95%	−10.05%	0.2909
	ELM-Bootstrap	87.09%	−2.91%	0.3614
	MODE-POPIC	91.58%	1.58%	0.3296
	Proposed POPIC	90.71%	**0.71%**	**0.2829**

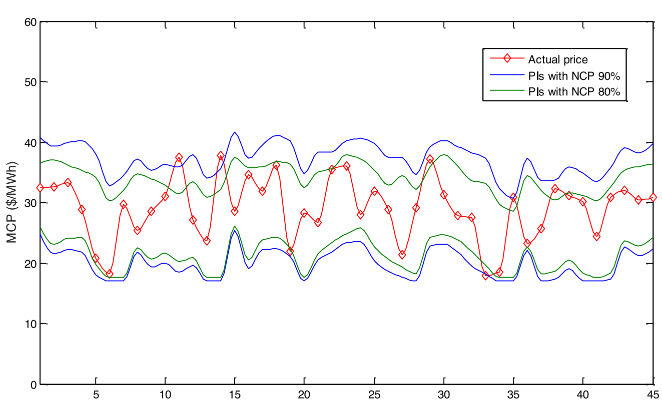

Fig. 2. MCP PIs with NCP 80% and 90% in August/September 2011 obtained by the proposed approach.

From Table I, the proposed POPIC method significantly outperforms the applied three benchmark models. ECPs of the proposed method are close to the corresponding NCPs with the deviation less than 1%, revealing the best reliability in comparison with the three benchmarks. Moreover, the proposed method obtains the smallest AWPI values, indicating a much better sharpness. In particular, the proposed approach has about 25% better sharpness than the ELM-Bootstrap approach at the NCP of 80%. This is due to that the sharpness index is not considered in the ELM-Bootstrap approach. Though the normal model has satisfactory sharpness, its reliability is very low, with absolute ACE larger than 10%. As an unconditional approach, the normal model cannot well ensure the performance. MODE-POPIC has comparable reliability, but it has significantly lower sharpness, verifying that better solutions of the formulated multi-objective optimization problem can be derived by the proposed approach. The study results also solidly prove the importance of both reliability and sharpness in quality assurance of PIs of electricity price. The PIs with NCPs 80% and 90% and the actual price values covering the period from August to September 2011 are displayed in Fig. 2 explicitly demonstrating the excellent performance of the results, where PIs of lower confidence level are well encapsulated by those of higher level.

IV. CONCLUSION

This letter proposes a novel POPIC approach for electricity market price based on ELM and NSGA-II, which generates Pareto optimal PIs with the formulated objectives reliability and sharpness. Numerical experiments have demonstrated the superiority of the proposed POPIC approach. In addition, the study reveals the importance of reliability and sharpness for generating quality PIs. Further work is ongoing to extend and test the developed approach regarding the performance and applications.

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