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RESEARCH ARTICLE

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Bayesian prediction interval for a constant-stress partially accelerated life test model under censored data

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ABSTRACT

The present communication develops the tools for Bayesian prediction of the Gompertz distribution based on CSPALT. The Metropolis-Hastings algorithm is applied to evaluate the BPIs for a censored sample based on unified hybrid censoring scheme. In order to investigate the impact of methodologies adopted, two numerical examples are performed. The simulated results show that reducing the censoring percentages causes smaller BPIs. The flexibility of the UHCS in evaluating the BPIs can be helped to overcome many difficulties in engineering problems.

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1. Introduction

In many experiments of life tests for units or products, it is important to use past information to predict future information to improve product efficiency. Prediction is one of the interesting topics in practical reliability problems. Different prediction methods have a high degree of discussion recently. One of the important methods is BPIs for future observations which have been studied by various authors, see for example, Abdel-Aty et al. [1], Kundu and Howlader [2], Balakrishnan and Shafay [3], Ghazal and Hasaballah [4] and Ahmad et al. [5]. In life-testing tests, observing the failure time for all units often takes a long time, resulting in a substantial increase in test time and expenses. As a consequence of considering efficiency and cost, life-testing experiments schemes have been cleverly designed to collect samples. The two most common schemes are Type I censoring, in which the experiment ends when the experimental period exceeds the prescribed time, as well as Type II censoring, in which the experiment ends when it collects a specified amount of data [6]. The combination of Type I and Type II censoring schemes is called HCS which was firstly introduced by Epstein [7]. Later, generalizations of the Type I and Type II HCSs, respectively known as generalized Type I and Type II HCSs were proposed by Chandrasekar et al. [8]. Although these two censoring schemes are improvements over the HCSs, they still have some problems. So, Balakrishnan et al. [9] proposed the unified hybrid censoring scheme which is a mixture of Type I generalized hybrid and Type II generalized hybrid. This censoring scheme bears some special cases to other censoring schemes as:

- If $T_1 \to 0$, then UHCS becomes generalized Type I HC scheme.
- If $k \to 0$, then UHCS becomes generalized Type II HC scheme.
- If $T_1 \to 0$ and $k \to 0$, then UHCS reduces to Type I HC scheme.
- If $T_2 \to \infty$ and $k \to n$, then UHCS reduces to Type II HC scheme.
- If $T_1 \to 0, T_2 \to \infty$ and $k \to 0$, then UHCS reduces to Type I scheme.
- If $T_1 \to 0, T_2 \to \infty$ and $k \to n$, then UHCS reduces to Type II scheme.

Due to its compatible features with other censoring plans, this censoring scheme has gained a lot of coverage in literature under multiple scenarios. For example, Panahi and Sayyareh [10] inferred the parameters of the Burr Type XII distribution under UHCS. Panahi [11] explored the MLE and Bayesian estimates for the Burr type III parameters under UHCS. Statistical inference under UHCS was studied by Mohie El-Din et al. [12]. Gwag and Lee [13] discussed UHCS and developed inferences on half logistic distribution. The Rayleigh distribution under UHCS was studied by Jeon and Kang [14].

In UHCS, n units are placed in an experiment, in which two numbers k and r, where $k, r \in \{0, \ldots, n\}$; $k < r < n$ and $T_1 < T_2 \in (0, \infty)$ are decided before hand

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by the experimenter. If the kth data occurs before the pre-fixed T_1, then the experiment will be stopped at $\min\{\max(X_{r:n}, T_1), T_2\}$; if the kth data occurs between T_1 and T_2, the experiment will be terminated at $\min\{X_{r:n}, T_2\}$. Otherwise, the experiment will be terminated at $X_{k:n}$. Figure 1 displays the schematic representation of UHCS. Moreover, in many practical situations, some products or materials are designed to be high reliability and long life time under normal conditions. So, to carry out reliability analysis, the accelerated life tests (ALTs) are the most common ways to overcome these situations. The basic assumption in ALT is that the AC is known and that there is a known statistical distribution to consider the relationship between stress and lifetime. However, this relationship cannot be easily assumed in many situations. Thus, the partially accelerated life tests (PALTs) are a good candidate to overcome this difficulty. There are different models under ALTs, and the most important of them are step-stress model and constant-stress model, respectively. Based on step-stress model, an experimental unit is first use at normal condition and, if the experiment does not fail at the determined time, then it is use at accelerated condition until the experiment stops. In constant-stress model, the experimental items are divided into two groups (group 1 and group 2), group 1 is allocated to a normal condition, and group 2 is allocated to a stress condition. The PALT has become popular and various authors handled it for many studies. For more details, one can refer to Pascual [15], Ding et al. [16], Liu [17], Ismail [18], Lone and Rahman [19], Dey and Nassar [20], Han and Bai [21] and Lone and Ahmed [22].

The rest of this paper is arranged as follows. In Section 2, we build the model and obtain the likelihood function based on UHCS. The likelihood function of the CSPALT model is presented in Section 3. The Bayesian prediction is studied in Section 4. All theoretical outcomes are illustrated with simulation studies under various censoring plans in Section 5. Section 6 contains the real data analysis. At last, some conclusions are provided in Section 7.

Nomenclature:

ALT	Accelerated Life Test	MSE	Mean Square Error
PALT	Partially Accelerated Life Test	G1	Group 1
CSPALT	Constant-Stress PALT	G2	Group 2
CDF	Cumulative Distribution Function	UHCS	Unified Hybrid Censoring Scheme
D_i	The Number of Failures until Time T_i	UH	Unified Hybrid
GOD	Gompertz Distribution	α	Shape Parameter ($\alpha > 0$)
HPD	Highest Posterior Density	γ	Scale Parameter ($\gamma > 0$)
HRF	Hazard Rate Function	λ	Acceleration factor($\lambda > 1$)
LF	Likelihood function	BPI	Bayesian Prediction Interval
SSPALT	Step-Stress PALT	Obs	Observed Sample
PDF	Probability Density Function	HCS	Hybrid censoring scheme
MCMC	Markov chain Monte Carlo	Iff	if and only if
MLEs	Maximum Likelihood Estimates	CPP	Conditional probability posterior
GS	Gibbs Sampling	MH	Metropolis-Hastings
KS	Kolmogrov–Smirnov	AC	Accelerated factor

2. Model description

Suppose that n_1 is the sample size of group 1 (use condition), then the PDF, corresponding CDF and HRF of random variable X with well-known Gompertz distribution are given by

$$f_1(x;\alpha,\gamma) = \gamma \exp\left[\alpha x - \frac{\gamma}{\alpha}(e^{\alpha x} - 1)\right]; \quad x > 0,\ \alpha > 0,\ \gamma > 0 \tag{1}$$

$$F_1(x;\alpha,\gamma) = 1 - \exp\left[-\frac{\gamma}{\alpha}(e^{\alpha x} - 1)\right]; \quad x > 0,\ \alpha > 0,\ \gamma > 0 \tag{2}$$

and

$$h_1(t;\alpha,\gamma) = \gamma e^{\alpha t};\ t > 0,\ \alpha > 0,\ \gamma > 0 \tag{3}$$

respectively. Here, $\alpha > 0$ is the shape parameter and $\gamma > 0$ is the scale parameter. The Gompertz distribution is one of the most important distributions in reliability and life testing. It plays a key role in various practical problems including; reliability theory and clinical trials. It is an extension to the exponential distribution and can be skewed to right and left by adjusting the values of α and γ.

The HRF of an item tested at accelerated condition can be written as $h_2(t) = \lambda h_1(t)$, where λ is an AC. So, the PDF, CDF and HRF of the variable X of an item tested for group 2 are given by:

$$f_2(x;\alpha,\gamma,\lambda) = \gamma\lambda \exp\left[\alpha x - \frac{\gamma\lambda}{\alpha}(e^{\alpha x} - 1)\right]; \quad x > 0,\ \lambda > 1 \tag{4}$$

$$F_2(x;\alpha,\gamma,\lambda) = 1 - \exp\left[-\frac{\gamma\lambda}{\alpha}(e^{\alpha x} - 1)\right]; \quad x > 0,\ \lambda > 1 \tag{5}$$

and

$$h_2(t;\alpha,\gamma,\lambda) = \gamma\lambda e^{\alpha t};\ t > 0,\ \lambda > 1 \tag{6}$$

Suppose $X_{1:n} < X_{2:n} < \ldots < X_{n:n}$ is an ordered sample of $n = n_1 + n_2$ obtained from GOD. According to UH censoring described above, the following cases are observed:

(1) $0 < X_{k:n} < X_{r:n} < T_1 < T_2$ in which case we terminate at T_1,
(2) $0 < X_{k:n} < T_1 < X_{r:n} < T_2$ in which case we terminate at $X_{r:n}$,
(3) $0 < X_{k:n} < T_1 < T_2 < X_{r:n}$ in which case we terminate at T_2,
(4) $0 < T_1 < X_{k:n} < X_{r:n} < T_2$ in which case we terminate at $X_{r:n}$,
(5) $0 < T_1 < X_{k:n} < T_2 < X_{r:n}$ in which case we terminate at T_2,
(6) $0 < T_1 < T_2 < X_{k:n} < X_{r:n}$ in which case we terminate at $X_{k:n}$.

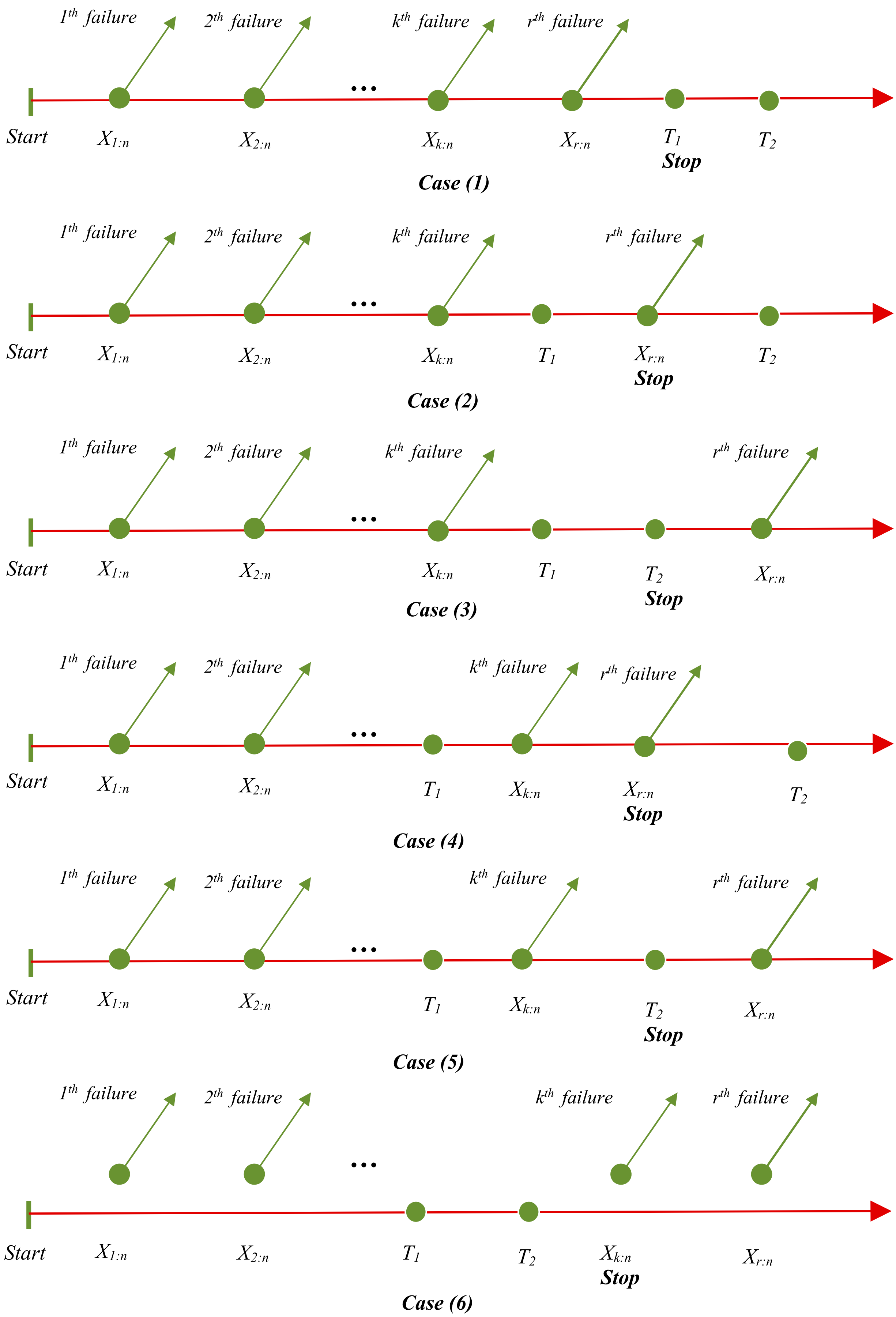

Figure 1. The schematic diagram of the UHCS.

The LF of the UH censored data can be written as:

$$L(\alpha,\gamma,\lambda|data)$$
$$= \begin{cases} \prod_{i=1}^{2} \frac{n_i!}{(n_i-d_i)!} \prod_{j=1}^{d_i} f_i(x_{ij};\alpha,\gamma,\lambda) \times (1-F_i(T_{i1};\alpha,\beta))^{n_i-d_i}, \\ \quad d_{i1}=d_{i2}=d_i=r_i=\ldots n_i, \\ \prod_{i=1}^{2} \frac{n_i!}{(n_i-r_i)!} \prod_{j=1}^{r_i} f_i(x_{ij};\alpha,\gamma,\lambda) \times (1-F_i(x_{ir_i};\alpha,\beta))^{n_i-r_i}; \\ \quad d_{i1}=k_i,\ldots,r_i-1, d_{i2}=r_i, \\ \prod_{i=1}^{2} \frac{n_i!}{(n_i-d_{i2})!} \prod_{j=1}^{d_{i2}} f_i(x_{ij};\alpha,\gamma,\lambda) \times (1-F_i(T_{i2};\alpha,\beta))^{n_i-d_{i2}}; \\ \quad d_{i1}=k_i,\ldots,r_i-1, d_{i2}=k_i,\ldots,r_i-1; d_{i1}\le d_{i2}, \\ \prod_{i=1}^{2} \frac{n_i!}{(n_i-r_i)!} \prod_{j=1}^{r_i} f_i(x_{ij};\alpha,\gamma,\lambda) \times (1-F_i(x_{iri};\alpha,\beta))^{n_i-r_i}; \\ \quad d_{i1}=0,\ldots,k_i-1, d_{i2}=r_i, \\ \prod_{i=1}^{2} \frac{n_i!}{(n_i-d_{i2})!} \prod_{j=1}^{d_{i2}} f_i(x_{ij};\alpha,\gamma,\lambda) \times (1-F_i(T_{i2};\alpha,\beta))^{n_i-d_{i2}}; \\ \quad d_{i1}=0,\ldots,k_i-1, d_{i2}=0,\ldots.,r_i-1, \\ \prod_{i=1}^{2} \frac{n_i!}{(n_i-k_i)!} \prod_{j=1}^{k_i} f_i(x_{ij};\alpha,\gamma,\lambda) \times (1-F_i(x_{ik};\alpha,\gamma,\lambda))^{n_i-k_i}; \\ \quad d_{i2}=0,\ldots,k_i-1. \end{cases} \tag{7}$$

3. LF for cspalt Based on UH censored sample

Based on the Gompertz model and after combined group 1 (g1) and group 2 (g2), the LF for the parameters α,γ and λ can be obtained as:

$$L(\alpha,\gamma,\lambda|data)$$
$$= \prod_{i=1}^{2} \frac{n_i!}{(n_i-D_i)!} \prod_{j=1}^{D_i} \gamma\lambda^{i-1} \exp\left(\alpha x_{ij} - \frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{ij}}-1)\right)$$
$$\times \left(\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha s_i}-1)\right)\right)^{n_i-D_i} ; i=1,2. \tag{8}$$

where

$$(D_i,s_i) = \begin{cases} (d_{i1},T_{i1}) \text{ For Case } (1) \\ (r_i,X_{ir_i}) \text{ For Case } (2) \\ (d_{i2},T_{i2}) \text{ For Case } (3) \\ (r_i,X_{ir_i}) \text{ For Case } (4) \\ (d_{i2},T_{i2}) \text{ For Case } (5) \\ (k_i,X_{ik_i}) \text{ For Case } (6) \end{cases} \tag{1}$$

Also, $X_{ij}, i=1,2; j=1,\ldots,n_i$ are the lifetimes for the tested items allocated from GOD, where $X_{1j}; j=1,\ldots,n_1$ is the lifetime in use condition and $X_{2j}; j=1,\ldots,n_2$ is the lifetime in accelerated condition.

4. Bayesian prediction

The Bayesian strategy takes into consideration both the information from observed sample data and the prior information. It can characterize the problems more rationally and reasonably [23–25]. For Bayesian estimation, the gamma distributions popularized as prior density for the parameters α and γ as:

$$\pi_1(\alpha) = \frac{b_1^{a_1}}{\Gamma(a_1)}\alpha^{a_1-1}e^{-b_1\alpha};\ \alpha>0,\ a_1>0,\ b_1>0, \tag{9}$$

$$\pi_2(\gamma) = \frac{b_2^{a_2}}{\Gamma(a_2)}\gamma^{a_2-1}e^{-b_2\gamma};\ \gamma>0,\ a_2>0,\ b_2>0, \tag{10}$$

where the hyper parameters a_1, b_1, a_2, b_2 are considered as non-negative and known. Various authors have used independent gamma priors for the model parameters in studying Bayes estimators because of its flexibility. On the other hand, the prior for λ is assumed to be the non-informative prior, *i.e.*$\pi_3(\lambda) \propto 1/\lambda$. Therefore, the joint prior PDF of α, γ and λ can be written as:

$$\pi_1(\alpha,\gamma,\lambda) \propto \alpha^{a_1-1}\gamma^{a_2-1}\lambda^{-1}e^{-(b_1\alpha+b_2\gamma)}. \tag{11}$$

Combining the LF (8) and joint prior PDF (11), the posterior PDF of α, γ and λ, and given data can be obtained as:

$$\pi(\alpha,\gamma,\lambda|x_{ij}) \propto L(\alpha,\gamma,\lambda)\pi(\alpha,\gamma,\lambda)$$
$$\propto \alpha^{a_1-1}e^{-(b_1\alpha+b_2\gamma)}\gamma^{a_2+D_1+D_2-1}\lambda^{D_2-1}$$
$$\times \exp\left\{\sum_{j=1}^{D_1}\left(\alpha x_{1j} - \frac{\gamma}{\alpha}(e^{\alpha x_{1j}}-1)\right)\right\}$$
$$\times \exp\left\{\sum_{j=1}^{D_2}\left(\alpha x_{2j} - \frac{\gamma\lambda}{\alpha}(e^{\alpha x_{2j}}-1)\right)\right\}$$
$$\times \left(\exp\left(-\frac{\gamma}{\alpha}(e^{\alpha s_1}-1)\right)\right)^{n_1-D_1}$$
$$\times \left(\exp\left(-\frac{\gamma\lambda}{\alpha}(e^{\alpha s_2}-1)\right)\right)^{n_2-D_2} ; i=1,2. \tag{12}$$

From Equation (12), we evaluate the CPP of α, γ and λ, respectively, as follows:

$$\pi^*(\alpha|\gamma,\lambda,data) \propto \alpha^{a_1-1}e^{-b_1\alpha}$$
$$\times \exp\left\{\sum_{j=1}^{D_1}\left(\alpha x_{1j} - \frac{\gamma}{\alpha}(e^{\alpha x_{1j}}-1)\right)\right\}$$
$$\times \exp\left\{\sum_{j=1}^{D_2}\left(\alpha x_{2j} - \frac{\gamma\lambda}{\alpha}(e^{\alpha x_{2j}}-1)\right)\right\}$$
$$\times \left(exp\left(-\frac{\gamma}{\alpha}(e^{\alpha s_1}-1)\right)\right)^{n_1-D_1}$$
$$\times \left(exp\left(-\frac{\gamma\lambda}{\alpha}(e^{\alpha s_2}-1)\right)\right)^{n_2-D_2}, \tag{13}$$

$$\pi^*(\gamma|\alpha,\lambda,data) \propto e^{-b_2\gamma}\gamma^{a_2+D_1+D_2-1}$$

$$\times \exp\left\{-\frac{\gamma}{\alpha}\sum_{j=1}^{D_1}(e^{\alpha x_{1j}}-1)\right\}$$
$$\times \exp\left\{-\frac{\gamma\lambda}{\alpha}\sum_{j=1}^{D_2}(e^{\alpha x_{2j}}-1)\right\}$$
$$\times \left(\exp\left(-\frac{\gamma}{\alpha}(e^{\alpha s_1}-1)\right)\right)^{n_1-D_1}$$
$$\times \left(\exp\left(-\frac{\gamma\lambda}{\alpha}(e^{\alpha s_2}-1)\right)\right)^{n_2-D_2}, \tag{14}$$

and

$$\pi^*(\lambda|\alpha,\gamma,data) \propto \lambda^{D_2-1} \times \exp\left\{-\frac{\gamma\lambda}{\alpha}\sum_{j=1}^{D_2}(e^{\alpha x_{2j}}-1)\right\}$$
$$\times \left(exp\left(-\frac{\gamma\lambda}{\alpha}(e^{\alpha s_2}-1)\right)\right)^{n_2-D_2}. \tag{15}$$

4.1. One-sample BPI

Suppose that $X_1, X_2, \ldots X_{D_i}, i = 1,2$ are observed sampling items for group i; $i = 1,2$. In this Subsection, we predict the censored data ($X_{i\Im_i:n_i-D_i}$; $\Im_i = D_i+1, D_i+2, \ldots, n_i-D_i$, $i = 1,2$). In UHCS, the conditional PDFs for $X_{\Im_i}$ based on Cases (1)–(6), are given by [4]:

Case (1):

$$f_{i1}(X_{\Im_i:n_i-D_i}|Obs,\alpha,\gamma,\lambda)$$
$$= \sum_{\mathbf{P}=r_i}^{\Im_i-1}\sum_{\mathbf{S}=0}^{\Im_i-\mathbf{P}-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{m}+\mathbf{S}}(n_i-\mathbf{P})!n_i!}{\mathbf{P}!(n_i-\mathbf{P})!\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-\mathbf{P}-1-\mathbf{S})!}$$
$$\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-\mathbf{P}-\mathbf{S}+\mathbf{m}-1}$$
$$\times \frac{\times\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i1}}-1)\right)\right)^{\mathbf{S}+\mathbf{P}}}{\sum_{h=r_i}^{\Im_i-1}\binom{n_i}{h}\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i1}}-1)\right)\right)^h\left(\exp\left(-\frac{\gamma\lambda^{i-1}(n_i-h)}{\alpha}(e^{\alpha T_{i1}}-1)\right)\right)}$$
$$\times \gamma\lambda^{i-1}\exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right), \tag{16}$$

Cases (2) and (4):

$$f_{i2}(X_{\Im_i:n_i-D_i}|Obs,\alpha,\gamma,\lambda)$$
$$= \sum_{\mathbf{S}=0}^{\Im_i-r_i-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{S}+\mathbf{m}}(n_i-\mathbf{r}_i)!}{\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-r_i-\mathbf{S}-1)!}$$
$$\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-r_i-\mathbf{S}+\mathbf{m}-1}$$
$$\times \frac{\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{r_i}}-1)\right)\right)^{\mathbf{S}}}{\exp\left(-\frac{\gamma\lambda^{i-1}(n_i-r_i)}{\alpha}(e^{\alpha x_{r_i}}-1)\right)}$$
$$\times \gamma\lambda^{i-1}\exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right), \tag{17}$$

Cases (3) and (5):

$$f_{i3}(X_{\Im_i:n_i-D_i}|Obs,\alpha,\gamma,\lambda)$$
$$= \sum_{\mathbf{P}=k_i}^{min(r_i,\Im_i)-1}\sum_{\mathbf{S}=0}^{\Im_i-\mathbf{P}-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{m}+\mathbf{S}}(n_i-\mathbf{P})!n_i!}{\mathbf{P}!(n_i-\mathbf{P})!\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-\mathbf{P}-1-\mathbf{S})!}$$
$$\left(1-exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-\mathbf{P}-\mathbf{S}+\mathbf{m}-1}$$
$$\times \frac{\left(1-exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i2}}-1)\right)\right)^{\mathbf{S}+\mathbf{P}}}{\sum_{h=r_i}^{r_i^*-1}\binom{n_i}{h}\left(1-exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i2}}-1)\right)\right)^h\left(exp\left(-\frac{\gamma\lambda^{i-1}(n_i-h)}{\alpha}(e^{\alpha T_{i2}}-1)\right)\right)}$$
$$\times \gamma\lambda^{i-1}exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right), \tag{18}$$

Case (6):

$$f_{i4}(X_{\Im_i:n_i-d_i^*}|Obs,\alpha,\gamma,\lambda)$$
$$= \sum_{\mathbf{S}=0}^{\Im_i-k_i-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{S}+\mathbf{m}}(n_i-k_i)!}{\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-k_i-\mathbf{S}-1)!}$$
$$\left(1-exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-k_i-\mathbf{S}+\mathbf{m}-1}$$
$$\times \frac{\left(1-exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{k_i}}-1)\right)\right)^{\mathbf{S}}}{exp\left(-\frac{\gamma\lambda^{i-1}(n_i-r_i)}{\alpha}(e^{\alpha x_{k_i}}-1)\right)}$$
$$\times \gamma\lambda^{i-1}exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right). \tag{19}$$

The $100(1-\varsigma)\%$ BPI of the $X_{i\Im_i:n_i-D_i}$; $\Im_i = D_i+1, D_i+2, \ldots, n_i-D_i$, $i = 1,2$ can be expressed as:

$$P(L < X_{\Im_i} < U) = 1-\varsigma. \tag{20}$$

where L and U are evaluated by solving the following equations as:

$$P(X_{\Im_i} > L|Obs) = 1-\frac{\varsigma}{2},\ P(X_{\Im_i} > U|Obs) = \frac{\varsigma}{2}. \tag{21}$$

Also, based on UH censored sample, we have

$$P(X_{\Im_i} > K|Obs)$$
$$= \begin{cases} \int_K^\infty F_{i1}^*(x_{\Im_i}|Obs)dx_{\Im_i} & \text{Case (1)} \\ \int_K^\infty F_{i2}^*(x_{\Im_i}|Obs)dx_{\Im_i} & \text{Cases (2) and (4)} \\ \int_K^\infty F_{i3}^*(x_{\Im_i}|Obs)dx_{\Im_i} & \text{Cases (3) and (5)} \\ \int_K^\infty F_{i4}^*(x_{\Im_i}|Obs)dx_{\Im_i} & \text{Case (6)} \end{cases} \tag{22}$$

where

$$F^*_{ij}(x_{\Im_i}|Obs) = \int_\lambda\int_\gamma\int_\alpha f_{ij}(X_{\Im_i:n_i-D_i}|Obs,\alpha,\gamma,\lambda) \times \pi(\alpha,\gamma,\lambda|data)d\alpha d\gamma d\lambda;\ i=1,2,\ j=1,2,3,4. \tag{23}$$

The BPI cannot be gained in explicit form. So, we use the GS with MH algorithm to calculate the BPI, which is presented in the following subsection.

4.2. One-sample BPI using MH algorithm

The MCMC method is one of the important techniques to approximate the Bayesian predictive density functions. The predictive density functions ($F^*_{ij}(x_{\Im_i}|Obs)$) for Cases (1)–(6) can be written as:

Case (1):

$$F^*_{i1}(x_{\Im_i}|Obs) = \sum_{\mathbf{P}=r_i}^{\Im_i-1}\sum_{\mathbf{S}=0}^{\Im_i-\mathbf{P}-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{S}+\mathbf{m}}(n_i-\mathbf{P})!n_i!}{\mathbf{P}!(n_i-\mathbf{P})!\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-\mathbf{P}-1-\mathbf{S})!} \times \int_1^\infty\int_0^\infty\int_0^\infty \gamma\lambda^{i-1} \times \exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-\mathbf{P}-\mathbf{S}+\mathbf{m}-1} \times \frac{\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i1}}-1)\right)\right)^{\mathbf{S}+\mathbf{P}}}{\sum_{h=r_i}^{\Im_i-1}\binom{n_i}{h}\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i1}}-1)\right)\right)^h\left(\exp\left(-\frac{\gamma\lambda^{i-1}(n_i-h)}{\alpha}(e^{\alpha T_{i1}}-1)\right)\right)} \times \pi(\alpha,\gamma,\lambda|Obs)d\alpha d\gamma d\lambda, \tag{24}$$

Cases (2) and (4):

$$F^*_{i2}(x_{\Im_i}|Obs) = \sum_{\mathbf{S}=0}^{\Im_i-r_i-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{S}+\mathbf{m}}(n_i-\mathbf{r}_i)!}{\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-r_i-\mathbf{S}-1)!} \times \int_1^\infty\int_0^\infty\int_0^\infty \gamma\lambda^{i-1} \times \exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-r_i-\mathbf{S}+\mathbf{m}-1} \times \frac{\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{r_i}}-1)\right)\right)^{\mathbf{S}}}{\exp\left(-\frac{\gamma\lambda^{i-1}(n_i-r_i)}{\alpha}(e^{\alpha x_{r_i}}-1)\right)} \times \pi(\alpha,\gamma,\lambda|Obs)d\alpha d\gamma d\lambda, \tag{25}$$

Cases (3) and (5):

$$F^*_{i3}(x_{\Im_i}|Obs) = \sum_{\mathbf{P}=k_i}^{r^*_i-1}\sum_{\mathbf{S}=0}^{\Im_i-\mathbf{P}-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{S}+\mathbf{m}}(n_i-\mathbf{P})!n_i!}{\mathbf{P}!(n_i-\mathbf{P})!\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-\mathbf{P}-1-\mathbf{S})!} \times \int_1^\infty\int_0^\infty\int_0^\infty \gamma\lambda^{i-1} \times \exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-w-1} \times \frac{\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i2}}-1)\right)\right)^{\mathbf{S}+\mathbf{P}}}{\sum_{h=r_i}^{r^*_i-1}\binom{n_i}{h}\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha T_{i2}}-1)\right)\right)^h\left(\exp\left(-\frac{\gamma\lambda^{i-1}(n_i-h)}{\alpha}(e^{\alpha T_{i2}}-1)\right)\right)} \times \pi(\alpha,\gamma,\lambda|Obs)d\alpha d\gamma d\lambda, \tag{26}$$

Case (6):

$$F^*_{i4}(x_{\Im_i}|Obs) = \sum_{\mathbf{S}=0}^{\Im_i-k_i-1}\sum_{\mathbf{m}=0}^{n_i-\Im_i}\frac{(-1)^{\mathbf{S}+\mathbf{m}}(n_i-k_i)!}{\mathbf{S}!\mathbf{m}!(n_i-\Im_i-\mathbf{m})!(\Im_i-k_i-\mathbf{S}-1)!} \times \int_1^\infty\int_0^\infty\int_0^\infty \gamma\lambda^{i-1} \times \exp\left(\alpha x_{\Im_i}-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{\Im_i}}-1)\right)\right)^{\Im_i-k_i-\mathbf{S}+\mathbf{m}-1} \times \frac{\left(1-\exp\left(-\frac{\gamma\lambda^{i-1}}{\alpha}(e^{\alpha x_{k_i}}-1)\right)\right)^{\mathbf{S}}}{\exp\left(-\frac{\gamma\lambda^{i-1}(n_i-r_i)}{\alpha}(e^{\alpha x_{k_i}}-1)\right)} \times \pi(\alpha,\gamma,\lambda|Obs)d\alpha d\gamma d\lambda. \tag{27}$$

Observe that the integrals in Equations (24)–(27) cannot be calculated in closed forms. Therefore, we apply MCMC method to approximate these two integrals. Based on MCMC method, the Bayesian predictive density functions for Case (1), Cases (2) & (4), Cases (3) & (5) and Case (6) can be approximated as (Ahmad et al. [5]):

$$F^*_{i1}(x_{\Im_i}|Obs) = \frac{\sum_{k=1}^{M} f_{i1}(X_{\Im_i}|Obs,\alpha_k,\gamma_k,\lambda_k)}{\sum_{k=1}^{M}\int_{T_{i1}}^\infty f_{i1}(X_{\Im_i}|Obs,\alpha_k,\gamma_k,\lambda_k)dx_{\Im_i}} \tag{28}$$

$$F^*_{i2}(x_{\Im_i}|Obs) = \frac{\sum_{k=1}^{M} f_{i1}(X_{\Im_i}|Obs,\alpha_k,\gamma_k,\lambda_k)}{\sum_{k=1}^{M}\int_{x_{ir_i}}^\infty f_{i1}(X_{\Im_i}|Obs,\alpha_k,\gamma_k,\lambda_k)dx_{\Im_i}} \tag{29}$$

$$F^*_{i3}(x_{\Im_i}|Obs) = \frac{\sum_{k=1}^{M} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k)}{\sum_{k=1}^{M} \int_{T_{i2}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i}} \tag{30}$$

and

$$F^*_{i4}(x_{\Im_i}|Obs) = \frac{\sum_{k=1}^{M} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k)}{\sum_{k=1}^{M} \int_{x_{ik_i}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i}} \tag{31}$$

respectively. Also, α_k, γ_k and λ_k; $k = 1, 2, 3, \ldots, M$, are generated from the posterior PDF. Then, the $100(1 - \varsigma)\%$ BPIs of the censored data can be evaluated as:

Case (1):

$$\sum_{k=1}^{M} \int_{L}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{T_{i1}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = 1 - \frac{\varsigma}{2}$$
$$\sum_{k=1}^{M} \int_{U}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{T_{i1}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = \frac{\varsigma}{2}, \tag{32}$$

Cases (2) and (4):

$$\sum_{k=1}^{M} \int_{L}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{x_{ir_i}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = 1 - \frac{\varsigma}{2}$$
$$\sum_{k=1}^{M} \int_{L}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{x_{ir_i}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = \frac{\varsigma}{2}, \tag{33}$$

Cases (3) and (5):

$$\sum_{k=1}^{M} \int_{L}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{T_{i2}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = 1 - \frac{\varsigma}{2}$$
$$\sum_{k=1}^{M} \int_{U}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{T_{i2}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = \frac{\varsigma}{2}, \tag{34}$$

Case (6):

$$\sum_{k=1}^{M} \int_{L}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{x_{ik_i}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = 1 - \frac{\varsigma}{2}$$
$$\sum_{k=1}^{M} \int_{U}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} / \times \sum_{k=1}^{M} \int_{x_{ik_i}}^{\infty} f_{i1}(X_{\Im_i}|Obs, \alpha_k, \gamma_k, \lambda_k) dx_{\Im_i} = \frac{\varsigma}{2}. \tag{35}$$

5. Numerical results

In this section, we apply a Monte Carlo simulation to evaluate and assess the performance of the BPIs based on UH censoring sample. We consider a systematic algorithm as follows: (Bayarri et al. [26]).

(1) Specified the values of λ, n_1, n_2, r_1, r_2,k_1, k_2,T_{11}, T_{12},T_{21} and T_{22}.
(2) Generate α and γ from the Gamma prior distributions, be starting values $(\alpha^{(0)}, \gamma^{(0)}, \lambda^{(0)})$ of $(\alpha, \gamma, \lambda)$.
(3) Set $k = 1$.
   - Generate $\lambda*$ from $\pi^*(\lambda^{(k)}|\alpha^{(k-1)}, \gamma^{(k-1)}, data)$.
   - Generate $\alpha*$ from $N(\alpha^{(k-1)}, \text{var}(\alpha^{(k-1)}))$.
   - Generate $\gamma*$ from $N(\gamma^{(k-1)}, \text{var}(\gamma^{(k-1)}))$.
   - set $k = k+1$
   - Repeat steps 3–7 up to M times and gain $\alpha^{(k)}$, $\gamma^{(k)}$, $\lambda^{(k)}$; $k = 1, \ldots, M$.
(4) The 95% BPI of $X_{i\Im_i:n_i-D_i}$ are evaluated by substituting $\alpha^{(k)}$, $\gamma^{(k)}$, $\lambda^{(k)}$ in Equations (32)–(35) and then solving them.

By using the values of these generated parameters, the six Cases for the UHCS are considered as:

Case (1):

Let $G_1 : T_{11} = 0.25$, $T_{12} = 0.36$, $k_1 = 5$, $r_1 = 8$ and $G_2 : T_{21} = 0.95$, $T_{22} = 1.15$, $k_2 = 7$, $r_2 = 8$, where, the test terminated at $T_{11}(d_{11} = 8)$ and $T_{21}(d_{21} = 9)$ for use and accelerated conditions respectively. So, the future data from G2 (0.9572, 0.9785, 0.9932, 1.1370, 1.1631, 1.3320) can be predicted using Equation (32).

Cases (2 & 4):

Let $G_1 : T_{11} = 0.19$, $T_{12} = 0.33$, $k_1 = 7\ or\ 8$, $r_1 = 10$ and $G_2 : T_{21} = 0.80$, $T_{22} = 1.17$, $k_2 = 8\ or\ 9$, $r_2 = 10$, where, the test terminated at $x_{r_1}(r_1 = 10)$ and $x_{r_2}(r_2 = 10)$ for use and accelerated conditions respectively. So, the future data from G2 (0.9785, 0.9932, 1.1370, 1.1631, 1.3320) can be predicted by solving Equation (33).

Cases (3 & 5):

Let $G_1 : T_{11} = 0.19$, $T_{12} = 0.33$, $k_1 = 7\ or\ 8$, $r_1 = 12$ and $G_2 : T_{21} = 0.80$, $T_{22} = 0.98$, $k_2 = 8\ or\ 9$, $r_2 = 13$, where, the test terminated at $T_{12}(d_{12} = 11)$ and $T_{22}(d_{22} = 11)$ for use and accelerated conditions respectively. So,

Table 1. The 95% one-sample BPI for $X_{\Im}; \Im = 23, 24, 25, 26, 27, 28$.

Case	$X_{\Im}$	True Value	Lower	Upper	Length
(1)	X_{23}	0.9572	0.8475	1.2422	0.3947
	X_{24}	0.9785	0.8496	1.7037	0.8541
	X_{25}	0.9932	0.8632	1.9308	1.0676
	X_{26}	1.1370	0.9224	2.3795	1.4571
	X_{27}	1.1631	0.9712	2.9985	2.0273
	X_{28}	1.3320	1.1753	3.8976	2.7223

Table 2. The 95% one-sample BPI for $X_{\Im}; \Im = 24, 25, 26, 27, 28$.

Cases	$X_{\Im}$	True value	Lower	Upper	Length
(2&4)	X_{24}	0.9785	0.8543	1.6689	0.8146
	X_{25}	0.9932	0.8802	1.9117	1.0315
	X_{26}	1.1370	0.9437	2.1995	1.2558
	X_{27}	1.1631	0.9788	2.8209	1.8421
	X_{28}	1.3320	1.2236	3.8573	2.6337

Table 3. The 95% one-sample BPI for $X_{\Im}; \Im = 25, 26, 27, 28$.

Cases	$X_{\Im}$	True value	Lower	Upper	Length
(3&5)	X_{25}	0.9932	0.8946	1.8829	0.9883
	X_{26}	1.1370	0.9475	2.1356	1.1881
	X_{27}	1.1631	0.9810	2.7994	1.8184
	X_{28}	1.3320	1.2376	3.8297	2.5921

the future data from G_2 (0.9932, 1.1370, 1.1631, 1.3320) can be predicted using Equation (34).

Case (6):

Let $G_1 : T_{11} = 0.20,\ T_{12} = 0.30,\ k_1 = 11,\ r_1 = 12$ and $G_2 : T_{21} = 0.85,\ T_{22} = 0.95,\ k_2 = 12,\ r_2 = 13,$ where the test terminated at $x_{k_1}(k_1 = 11)$ and $x_{k_2}(k_2 = 12)$ for use and accelerated conditions respectively. So, the future data from G_2 (1.1370, 1.1631, 1.3320) can be predicted by solving Equation (35).

The Bayesian prediction intervals for the future failure times using MCMC method for Cases (1), (2&4), (3&5),(6) are summarizes in Tables 1–4 respectively.

Table 4. The 95% one-sample BPI for $X_{\Im}; \Im = 26, 27, 28$.

Case	$X_{\Im}$	True value	Lower	Upper	Length
(6)	X_{26}	1.1370	0.9653	2.0923	1.1270
	X_{27}	1.1631	0.9984	2.7269	1.7285
	X_{28}	1.3320	1.2668	3.7421	2.4753

6. Real data analysis

In this Section, the theoretical findings and simulation studies are supported with a real data example. We consider the data used by Lawless [27, p. 208 (4.19)], which represent the failure times of electrical insulation in a test. We have divided each data by 100 and then consider the failure times for type-A of electrical insulation as a use condition with size 11 (g_1) and the failure times for type-B as an accelerated condition with size 13 (g_2).

$\boldsymbol{g_1}$: 0.185, 0.217, 0.351, 0.405, 0.423, 0.794, 0.860, 1.219, 1.471, 1.502, 2.193.

$\boldsymbol{g_2}$: 0.123, 0.218, 0.244, 0p.286, 0.432, 0.469, 0.487, 0.707, 0.753, 0.953, 0.981, 1.386, 1.519.

First, we compute the KS distances and the *p*-values for groups 1 and 2. The KS (*p*-values) for G_1 ($Gompertz(\alpha, \gamma)$) and G_2 ($Gompertz(\alpha, \gamma\lambda)$) are 0.17298(0.8428) and 0.13639(0.9422), respectively. The probability–probability plots for the accelerated and use conditions are presented in Figure 2. Moreover, the empirical and fitted Gompertz distribution plots are displayed in Figure 3. Figures 2 and 3 also suggest, the considered model well fitted with the considered real data sets.

Now, we consider the UH censored samples and predict the censored sample for accelerated condition data. Therefore, we consider the following UHCSs.

Case (1):

$G_1 : T_{11} = 1.47,\ T_{12} = 2.15,\ k_1 = 4,\ r_1 = 6$ and $G_2 : T_{21} = 0.71,\ T_{22} = 0.95,\ k_2 = 4,\ r_2 = 6$. So, the future data from G_2 are: 0.753, 0.953, 0.981, 1.386 and 1.519.

Case (2 & 4):

$G_1 : T_{11} = 0.72,\ T_{12} = 2.15,\ k_1 = 5\ or\ 6,\ r_1 = 7$ and $G_2 : T_{21} = 0.60,\ T_{22} = 1.20,\ k_2 = 6\ or\ 8,\ r_2 = 9$. So, the

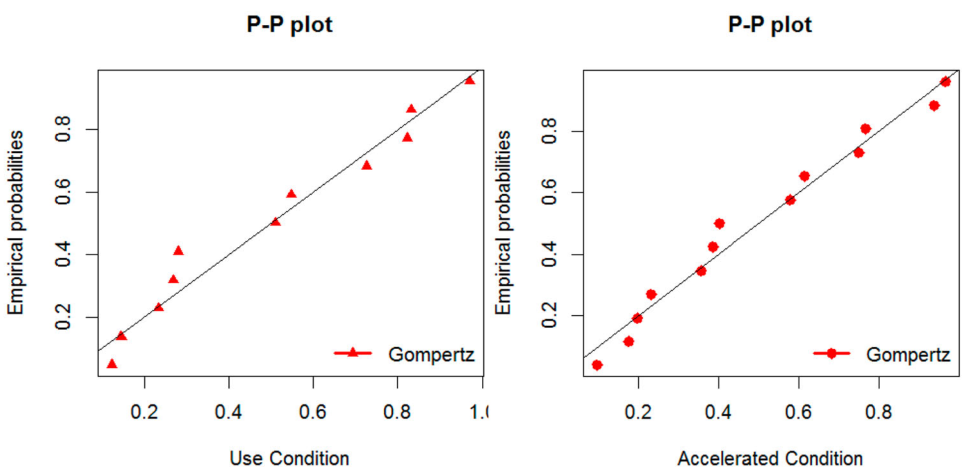

Figure 2. The probability–probability plots for use (left) and accelerated (right) conditions.

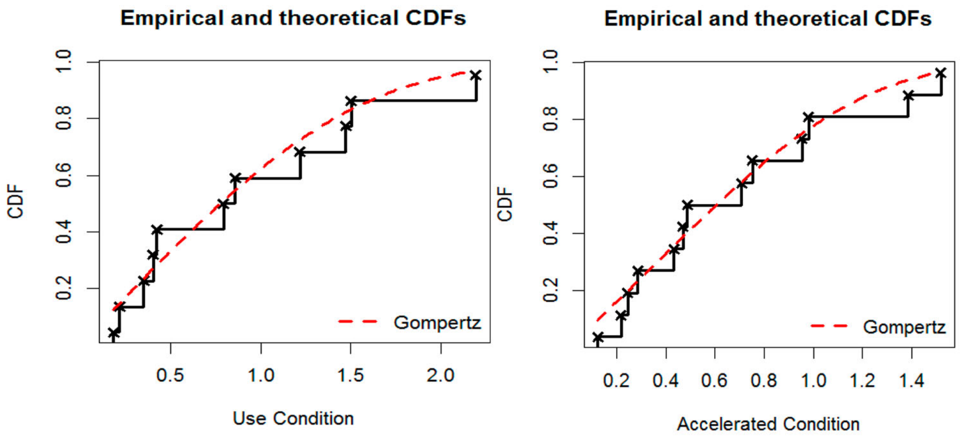

Figure 3. The empirical and fitted distribution plots for use (left) and accelerated (right) conditions.

Table 5. The 95% one-sample BPI for $X_{\Im}$; $\Im = 20, 21, 22, 23, 24$.

Case	$X_{\Im}$	True value	Lower	Upper	Length
(1)	X_{20}	0.753	0.7245	0.8378	0.1133
	X_{21}	0.953	0.9107	1.2169	0.3062
	X_{22}	0.981	0.9322	1.3679	0.4357
	X_{23}	1.386	1.3140	2.3465	1.0325
	X_{24}	1.519	1.4224	2.5819	1.1595

Table 6. The 95% one-sample BPI for $X_{\Im}$; $\Im = 21, 22, 23, 24$.

Cases	$X_{\Im}$	True value	Lower	Upper	Length
(2&4)	X_{21}	0.953	0.9165	1.1954	0.2789
	X_{22}	0.981	0.9437	1.3447	0.4010
	X_{23}	1.386	1.3246	2.3299	1.0053
	X_{24}	1.519	1.4296	2.5680	1.1384

Table 7. The 95% one-sample BPI for $X_{\Im}$; $\Im = 22, 23, 24$.

Cases	$X_{\Im}$	True value	Lower	Upper	Length
(3&5)	X_{22}	0.981	0.9499	1.3274	0.3775
	X_{23}	1.386	1.3318	2.3104	0.9786
	X_{24}	1.519	1.4420	2.5572	1.1152

Table 8. The 95% one-sample BPI for $X_{\Im}$; $\Im = 23, 24$.

Case	$X_{\Im}$	True value	Lower	Upper	Length
(6)	X_{23}	1.386	1.3521	2.2894	0.9373
	X_{24}	1.519	1.4317	2.5164	1.0847

future data from G_2 are: 0.953, 0.981, 1.386 and 1.519.

Case (3 & 5):

$G_1 : T_{11} = 0.72,\ T_{12} = 1.47,\ k_1 = 5 \text{ or } 7,\ r_1 = 10$ and $G_2 : T_{21} = 0.60,\ T_{22} = 0.96,\ k_2 = 6 \text{ or } 8,\ r = 11$. So, the future data from G_2 are: 0.981, 1.386 and 1.519.

Case (6):

$G_1 : T_{11} = 0.50,\ T_{12} = 0.80,\ k_1 = 9,\ r_1 = 10$ and $G_2 : T_{21} = 0.45,\ T_{22} = 0.75,\ k_2 = 11,\ r_2 = 12$ So, the future data from G_2 are: 1.386 and 1.519.

From the UHCS, 95% one-sample BPI for $X_{i\Im_i:n_i-D_i}$; $\Im_i = D_i + 1, D_i + 2, \ldots, n_i - D_i$, $i = 1, 2$ for Cases (1)–(6) are presented in Tables 5–8 respectively.

7. Conclusion

In this article, we have considered one-sample BPI for future samples from Gompertz distribution under UHCS based on CSPALT. The main reason for selecting this censoring plan is that it provides at least a specific number of failures. The MCMC method is applied for BPIs for future observations under UHCS. The theoretical results are carried out with the simulation and real data studies. We observed consistent and expected results. We observed that the BPIs contain the true values. Therefore, the proposed prediction intervals work well. Also, the results indicate that the BPIs affected by decreasing the number of censored data, where the length of prediction intervals decreases with increasing the number of observed data. So, case (6) is more preferable for predicting the unobserved data. Thus, reducing the censoring percentages causes smaller average interval lengths. It is an expected result because the number of observed failures increases parallel to smaller percentages of censoring. Moreover, a real-life data is analyzed for the purpose of illustration. It shows that the proposed Bayesian prediction method is practical.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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