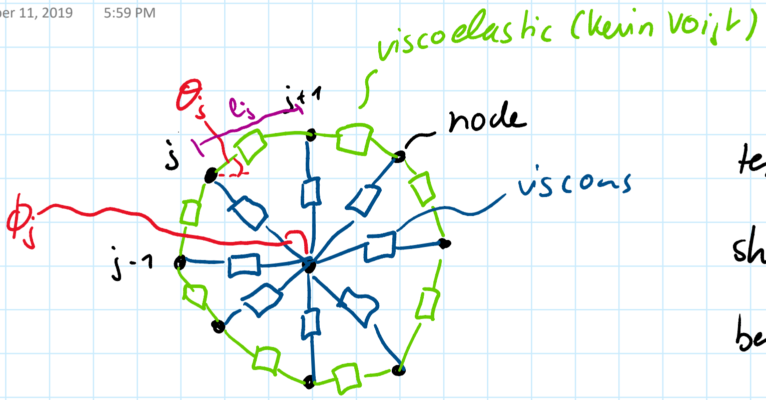


tension force $t_j(s)$

shear force $q_j(s)$

bending momentum $m_j(s)$

normal force $f_j(s)$

tangential force $g_j(s)$

average tension force : $\bar{t}_j$

average tension T_j

assumption:

3d: interior incompressable => constant volume ~> membrane almost constant surface area

↳ 2d: perimeter & area are considered constrained

↳ external: elastic control of element length

internal: interior pressure $p_{int} = k_p \left(1 - \frac{A}{A_{ref}}\right)$

<u>external</u>

→ elastic & viscous component parallel to each other

↳ viscoelastic responce

→ elastic nodes

↳ resistance to bending => bending momentum

internal

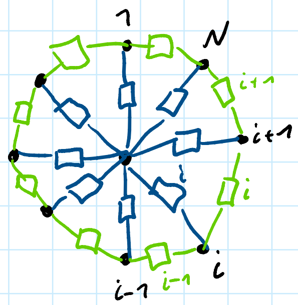
viscous elements

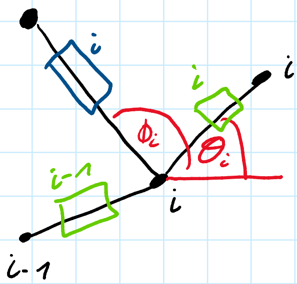
↳ viscous resistance to tank treading

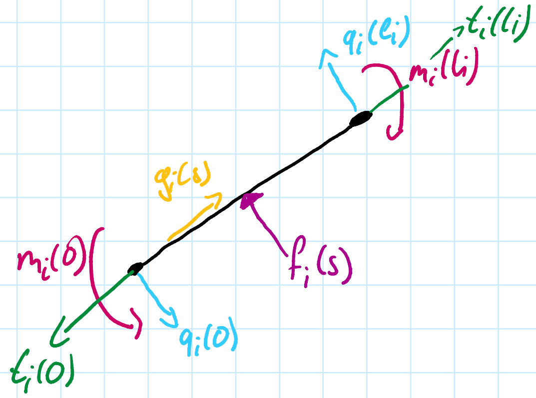
shear viscosity of membrane → viscous resistance

viscosity of the fluid cell interior → tank treading

put together to avoid computation of interior flow field







$$\frac{d}{ds} t_i = -g_i \qquad \frac{d}{ds} q_i = -f_i \qquad (*)\ \frac{d}{ds} m_i = q_i$$

now define mean tension $\bar{t}_i = \frac{1}{l_i}\int_0^{l_i} t_i(s)\, ds = k_t\left(\frac{l_i}{l_0} - 1\right) + \mu_m \frac{1}{l_i}\frac{dl_i}{dt}$

and mean shear $\bar{q}_i = \frac{1}{l_i}\int_0^{l_i} q_i(s)\, ds$

(**)

k_t: elastic modulus; l_0: reference length; μ_m: viscosity

bending moments $m_i(0) = -k_b \frac{\alpha_i}{l_0}$ $\qquad \alpha_i = \theta_i - \theta_{i-1}$

$m_i(l_i) = -k_b \frac{\alpha_{i+1}}{l_0}$ $\qquad \alpha_{i+1} = \theta_{i+1} - \theta_i$

integrate (*) $\rightarrow$ $\bar{q}_i = k_b \frac{(\alpha_i - \alpha_{i+1})}{l_0\, l_i}$

integration by parts for $**$

$$t_i(0) = \bar{t}_i + \frac{1}{l_i}\int_0^{l_i} g_i(s)\,(l_i - s)\,ds \qquad q_i(0) = \bar{q}_i + \int_0^{l_i} f_i(s)\,(l_i - s)\,ds$$

$$t_i(l_i) = \bar{t}_i + \frac{1}{l_i}\int_0^{l_i} g_i(s)\, s\, ds \qquad q_i(l_i) = \bar{q}_i + \int_0^{l_i} f_i(s)\, s\, ds$$

internal viscosity

Internal element

$$T_i = \mu_m' \frac{1}{l_i}\frac{dl_i}{dt}$$

length of internal element

$\Longrightarrow$ equilibrium of forces at node i

$$\text{external}\begin{cases} 0 = t_i(0)\cos\theta_i - t_{i-1}(l_{i-1})\cos\theta_{i-1} - q_i(0)\sin\theta_i + q_{i-1}(l_{i-1})\sin\theta_{i-1} + T_i\cos\varphi_i \\ 0 = t_i(0)\sin\theta_i - t_{i-1}(l_{i-1})\sin\theta_{i-1} + q_i(0)\cos\theta_i - q_{i-1}(l_{i-1})\cos\theta_{i-1} + T_i\sin\varphi_i \end{cases}$$

$$\text{internal}\{\, 0 = \sum_{i=1}^{n} T_i\cos\varphi_i = \sum_{i=1}^{n} T_i\sin\varphi_i$$

Fluid flow

↳ pressure field $p(x,y)$

velocity field $\bar{u}(x,y) = \begin{pmatrix} u \\ v \end{pmatrix}$

↳ stress

$$\sigma = \mu \begin{pmatrix} 2\frac{\partial u}{\partial x} - p & \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & 2\frac{\partial v}{\partial y} - p \end{pmatrix}$$

↳ conservation of momentum

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}$$

↳ divergence of the velocity field

$$e = 0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

↳ $f_i = -p_{int} - \sigma_{xx} \sin^2\theta_i + 2\sigma_{xy} \sin\theta_i \cos\theta_i - \sigma_{yy} \cos^2\theta_i$

$$g_i = (\sigma_{xx} - \sigma_{yy}) \sin\theta_i \cos\theta_i - \sigma_{xy} (\cos^2\theta_i - \sin^2\theta_i)$$