COS2661

ASSIGNMENT 02

100% Marks

SUBMISSION: Electronically through myUnisa (as one .pdf file)

It will be to your own advantage to check whether the assignment has been registered on the system after a few days.

This assignment is submitted electronically and if myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.

Due date	24 June 2022
Extension	No extension
Tutorial matter	Textbook Chapters: Chapter 7: 7.1 – 7.4 Chapter 8: 8.1 - 8.4 Chapter 9: 9.1 – 9.6 Chapter 10: 10.1 – 10.4 Tutorial Letter 102 Chapters 7 to 10
Weight of contribution to semester mark	40%
Unique number	184931

QUESTION 1 **[14]**

For the following sentence determine whether the indicated pairs of sentences are tautologically equivalent. Use truth tables to show your answer.

1.1 $A \leftrightarrow B$ and $(A \wedge B) \vee (\neg A \wedge \neg B)$ (3)

1.2 $(A \wedge B) \rightarrow C$ and $A \rightarrow (B \rightarrow C)$ (4)

1.3 $A \leftrightarrow (B \leftrightarrow (C \leftrightarrow D))$ and $((A \leftrightarrow B) \leftrightarrow C) \leftrightarrow D$ (7)

QUESTION 2 **[10]**

Study the table below and, translate the following English sentences into FOL.

	English	FOL
Names	Max, Claire, Jenny, Nancy	Same
Predicates	x marry y x is y x is in love with y x is intelligent.	Marry (x, y) x = y InLove (x, y) Intelligent (x)
Functions	the oldest daughter of x the youngest daughter of x	o-daughter (x) y-daughter (x)

2.1 Max can marry either Nancy's oldest daughter or her youngest daughter (in a monogamous society).

2.2 Jenny is Nancy's youngest daughter and Claire is her oldest daughter.

2.3 Neither Claire nor Jenny is in love with Max.

2.4 Jenny will not marry Max unless he is intelligent and in love with her.

2.5 Max is not both intelligent and in love with Jenny.

QUESTION 3 **[10]**

Translate the following formal language of Tarski's World (FOL) sentences into English sentences.

3.1 (FrontOf(a, c) $\wedge$ FrontOf(b, c)) $\wedge$ (Smaller(a, c) $\wedge$ Smaller(b, c))

3.2 $\neg$Between (c, a, b) $\wedge$ $\neg$ (FrontOf(c, a) $\vee$ FrontOf(c, b))

3.3 $\neg$(FrontOf(d, b) $\vee$ FrontOf(d, e)) $\wedge$ $\neg$(FrontOf(c, b) $\vee$ FrontOf(c, e))

3.4 (Between (a, c, d) $\vee$ Between (b, c, d)) $\wedge$ $\neg$ (Between (a, c, d) $\wedge$ Between (b, c, d))

3.5 [(Small(a) $\wedge$ Cube(a) $\wedge$ Large(b) $\wedge$ Dodec(b) $\wedge$ FrontOf(a, b)] $\wedge$ [LeftOf(a, b) $\vee$ (Tet(c) $\wedge$ Tet(d) $\wedge$ Between(a , c, d))]

QUESTION 4 **[10]**

The following arguments are all valid. Give the informal proofs of these arguments.

4.1 b is small unless it's a cube.
If c is small, then either d or e is too.
If d is small, then c is not.
If b is a cube, then e is not small.

If c is small, then so is b.

4.2

The unicorn, if horned, is elusive and dangerous.
If elusive or mythical, the unicorn is rare.
If a mammal, the unicorn is not rare.
∴ The unicorn, if horned, is not a mammal.

QUESTION 4 **[10]**

Below are a number of expressions. State which are terms, some are atomic wffs (well-formed formulae) and some are neither. If it is neither a term nor a wff, state the reason.

a) Small(y, Large(x, a, y))
b) sister_of(thembi)
c) **$**aP(a, a)
d) Happy(x; y; 4:00)
e) ¬ y ∧ A(y)
f) Cube(a)
g) Shorter(mother(z), z)

QUESTION 5 **[46]**

In this question you have to construct formal proofs using the natural deduction rules. The Fitch system makes use of these rules.

A summary of the rules of natural deduction is given on pages 573 to 578 of your textbook. Consult this when you do question 5. Remember that De Morgan's laws and other tautologies are not permissible natural deduction rules. You are also not allowed to use Taut Con, Ana Con or FO Con. It is important to number your statements, to indicate subproofs and at each step to give the rule that you are using.

Hint: If you have access to a computer, take advantage of the fact and use Fitch.

Question 5.1 **(8)**

Using the natural deduction rules, give a formal proof that the following three sentences are inconsistent:

A ∨ B
¬ A
¬ B

Question 5.2 **(12)**

Using the natural deduction rules, give a formal proof of:

$Same(b, c) \rightarrow (Large(c) \vee Small(c))$

from the premises:

1. $Small(a) \rightarrow Small(b)$
2. $Small(b) \rightarrow (SameSize(b, c) \rightarrow Small(c))$
3. $\neg Small(a) \rightarrow (Large(a) \wedge Large(c)$

Question 5.3 **(12)**

Using the natural deduction rules, give a formal proof of:

$(A \rightarrow [B \rightarrow C]) \leftrightarrow ([A \wedge B] \rightarrow C)$ from no premises.

Question 5.4 **(14)**

Using the natural deduction rules, give a formal proof of:

$A \leftrightarrow \neg B$ from premises

1. $A \vee B \vee C$
2. $B \rightarrow (A \rightarrow \neg C)$
3. $A \leftrightarrow C$

End of Assignment 2: