4.1

T18.

$\because X \sim U(0,a)$

$\therefore$ 分布函数 $F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{a}, & 0 < x < a \\ 1, & x \geq a. \end{cases}$

T19. (1) $P(X \leq 3) = F_X(3)$ 由分布函数定义

$= 1 - e^{1.2}$

(2) $P(X \geq 4) = 1 - P(X < 4)$

$= 1 - \cancel{P}(\cancel{F_X(4) = 1}\lim_{x \to 4^-} F_X(x))$

$= 1 - F_X(4)$

$= (1 - e^{-1.6}) = e^{-1.6}$

(3) $P(3 \leq X \leq 4) = F_X(4) - F_X(3)$

$\cancel{= e^{-1.6} - e^{-1.2}} = 1 - e^{-1.6} - 1 + e^{-1.2} = e^{-1.2} - e^{-1.6}$

(4) $P(X \leq 3 \cup X \geq 4) = P(X \leq 3) + P(X \geq 4)$

$= F_X(3) + 1 - F_X(4)$

$= 1 - e^{-1.2} + 1 - (1 - e^{-1.6})$

$= 1 - e^{-1.2} + e^{-1.6}$

(5) $P(X = 2.5) = P(X \leq 2.5) - P(X < 2.5)$

$= F_X(2.5) - \lim_{x \to 2.5^-} F_X(x) = F_X(2.5) - F_X(2.5)$

$= 0.$

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T20. (1) $P(X<2)=F_X(2)=\ln 2$

$P(0<X\leq 3)=P(X\leq 3)-P(X\leq 0)$

$=F_X(3)-F_X(0)$

$=1-0=1$

$P(2<X<\frac{5}{2})=P(X<\frac{5}{2})-P(X\leq 2)$

$=\lim\limits_{x\to\frac{5}{2}^-}F_X(x)-F_X(2)$

$=F_X(\frac{5}{2})-F_X(2)$

$=\ln\frac{5}{2}-\ln 2$

$=\ln\frac{5}{4}$

(2) $f_X(x)=\begin{cases}\frac{1}{x}, & 1\leq x<e\\ 0, & 其他.\end{cases}$

T21. (1) $F(x)=\begin{cases}0, & x<1\\ 2(x+\frac{1}{x}-2), & 1\leq x\leq 2\\ 1, & x>2.\end{cases}$

(2) $0\leq x<1$时

$F(x)=\int_0^x x\,dx=\frac{1}{2}x^2\Big|_0^x=\frac{1}{2}x^2$

$1\leq x<2$时

$F(x)=\int_1^x(2-x)\,dx=(2x-\frac{1}{2}x^2)\Big|_1^x$

$=2x-\frac{1}{2}x^2-2+\frac{1}{2}$

$=2x-\frac{1}{2}x^2-\frac{3}{2}$

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(2) $0\le x<1$时

$F(x)=\int_0^x x\,dx$

$=\frac{1}{2}x^2\Big|_0^x=\frac{1}{2}x^2$

$1\le x<2$时

$F(x)=2x-\frac{1}{2}x^2-1$

$x<0$时

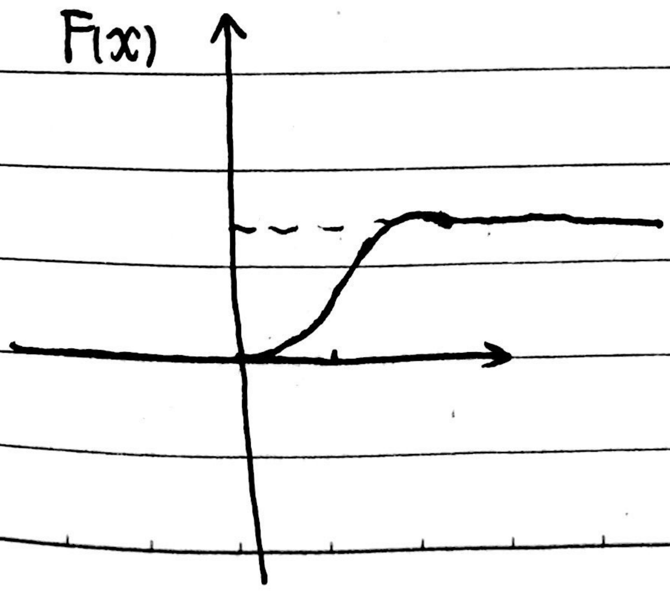
$F(x)=0$

$x\ge 2$时

$F(x)=1$

$$F(x)=\begin{cases}0, & x<0\\ \frac{1}{2}x^2, & 0\le x<1\\ 2x-\frac{1}{2}x^2-1, & 1\le x<2\\ 1, & x\ge 2\end{cases}$$

图形：



CJP

T23. $P(X>1500)=1-P(X\leq 1500)$

$=1-F(1500)$

$F(x)=\int_{1000}^{x}\frac{1000}{t^2}dt=-\frac{1000}{t}\Big|_{1000}^{x}$

$=-\frac{1000}{x}+1$

$\Rightarrow F(1500)=-\frac{2}{3}+1=\frac{1}{3}$

$\Rightarrow P(X>1500)=1-\frac{1}{3}=\frac{2}{3}.$

设其中有Y只寿命大于1500，$Y\sim b(5,\frac{2}{3})$

$P(Y\geq 2)=1-P(Y=0)-P(Y=1)$

$=1-\binom{5}{5}(\frac{1}{3})^5-\binom{5}{1}(\frac{1}{3})^4(\frac{2}{3})$

$=1-\frac{1}{243}-\frac{10}{243}$

$=\frac{232}{243}$

T24. $P(X\geq 10)=1-P(X<10)$

$=1-F(10)$

$F(x)=\int_0^x\frac{1}{5}e^{-\frac{x}{5}}=-e^{-\frac{x}{5}}\Big|_0^x=-e^{-\frac{x}{5}}+1$

$F(10)=-e^{-2}+1$

$\Rightarrow P(X\geq 10)=1+e^{-2}-1$

$=e^{-2}=\frac{1}{e^2}$

$P(Y\geq 1)=1-P(Y=0)$

$=1-\binom{5}{5}(1-\frac{1}{e^2})^5$

$=1-(1-\frac{1}{e^2})^5$

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T15. 方程 $4x^2+4kx+k+2=0$

有实根。

$\Rightarrow \Delta=16k^2-4(4k+8)$

$=16k^2-16k-32$

$=16(k^2-k-2)$

$=16(k-2)(k+1)\geqslant 0$

$\Rightarrow k\geqslant 2$ 或 $k\leqslant -1$

$k\sim U(0,5)$

$$\Rightarrow F(k)=\begin{cases}0, & \cancel{x}\ k\leqslant 0\\ \frac{x}{5}, & 0<\cancel{x}\ k<5\\ 1, & \cancel{x}\ k\geqslant 5\end{cases}$$

$P(k\geqslant 2\cup k\leqslant -1)$

$=P(k\geqslant 2)+P(k\leqslant -1)$

$=1-P(k<2)+P(k\leqslant -1)$

$=1-F(2)+F(-1)$

$=1-\frac{2}{5}+0=\frac{3}{5}$

P115 T18.

$E(x)=\int_{-\infty}^{+\infty}xf(x)dx$

$=\int_0^{+\infty}x\times\frac{x}{\sigma^2}e^{-x^2/(2\sigma^2)}dx$

$=\int_0^{+\infty}\frac{x^2}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}dx=\int_0^{+\infty}e^{-\frac{x^2}{2\sigma^2}}dx$

$=\int_0^{+\infty}\sqrt{2\pi\sigma^2}\times\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}dx$

$=\sqrt{2\pi\sigma^2}\times\frac{1}{2}$

$=\sqrt{\frac{\pi}{2}}\sigma.$

CJP

$$E(X^2)=\int_{-\infty}^{+\infty}x^2 f(x)dx$$
$$=\int_0^{+\infty}\frac{x^3}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}dx$$
$$=2\int_0^{+\infty}xe^{-\frac{x^2}{2\sigma^2}}dx$$
$$=2\sigma^2\int_0^{+\infty}\frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}dx=2\sigma^2$$

故 $D(X)=E(X^2)-(E(X))^2$

$=2\sigma^2-\frac{\pi}{2}\sigma^2$

4.2 设长方形的宽为 X，$X\sim U(0,2)$

$\Rightarrow E(X)=\frac{0+2}{2}=1$，$Var(X)=\frac{2^2}{12}=\frac{4}{12}=\frac{1}{3}$

长方形的面积为10

$\Rightarrow$ 长为 $\frac{10}{X}$ $\Rightarrow$ 周长为 $X+\frac{10}{X}$

$\Rightarrow E(X+\frac{10}{X})=E(X)+E(\frac{10}{X})$

$E(\frac{10}{X})$ 不存在

$\Rightarrow$ 期望不存在

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4.3.

$$E(e^{-2X}) = \int_{-\infty}^{+\infty} e^{-2t} \times Ae^{-t}\,dt$$
$$= \int_{0}^{+\infty} e^{-2t} \times Ae^{-t}\,dt$$
$$= \int_{0}^{+\infty} Ae^{-3t}\,dt$$
$$= -\frac{A}{3} \times [e^{-3t}]_0^{+\infty}$$
$$= -\frac{A}{3}(0-1)$$
$$= \frac{A}{3}.$$

4.4. 证明：$\int_{-\infty}^{+\infty} e^{-\frac{(t-u)^2}{2\sigma^2}}\,dt = \sqrt{2\pi}\sigma$

令 $t-u=m$ 即 $t=m+u$

即要证 $\int_{-\infty}^{+\infty} e^{-\frac{m^2}{2\sigma^2}}\,d(m+u) = \int_{-\infty}^{+\infty} e^{-\frac{m^2}{2\sigma^2}}\,dm$

$$\left(\int_{-\infty}^{+\infty} e^{-\frac{m^2}{2\sigma^2}}\,dm\right)^2$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{m^2+n^2}{2\sigma^2}}\,dm\,dn$$

令 $\begin{cases} m = r\cos\theta \\ n = r\sin\theta \end{cases}$

故原式 $= \int_0^{2\pi} d\theta \int_0^{+\infty} e^{-\frac{r^2}{2\sigma^2}}\,r\,dr$
$$= \int_0^{2\pi} d\theta \int_0^{+\infty} e^{-\frac{r^2}{2\sigma^2}}\,d\frac{r^2}{2}$$
$$= 2\pi \times \sigma^2$$
$$= 2\pi\sigma^2$$

$$\Rightarrow \int_{-\infty}^{+\infty} e^{-\frac{(t-u)^2}{2\sigma^2}}\,dt = \sqrt{2\pi\sigma^2} = \sqrt{2\pi}\sigma.$$