

Sample Solution for Problem Set 7.5

Data Structures and Algorithms, Fall 2021

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1 Problem 1

The actual cost of each operation:

- **OrderedpPush(x)**: Suppose there are k elements at the top of the stack that is smaller than x . Then, cost k to pop k elements and cost 1 to push.

$$C_{push} = k + 1$$

- **Pop(x)**:

$$C_{pop} = 1$$

Use potential function $\Phi(S) =$ the number of elements in the stack. Obviously, we have $\Phi(S_i) \geq \Phi(S_0)$ holds for all $i \geq 0$. The amortized cost of each operation:

- **OrderedpPush(x)**:

$$\hat{C}_{push} = C_{push} + \Delta\Phi = (k + 1) + (-k + 1) \leq 2$$

- **Pop(x)**:

$$\hat{C}_{pop} = C_{pop} + \Delta\Phi = 1 + (-1) \leq 0$$

The amortized cost of each operation is $O(1)$.

2 Problem 2

Let $\Phi(x) = 20 \times x$.

- **Push**: $T_{\text{amortized}} = T_{\text{actual}} + \Phi(x + 1) - \Phi(x) \leq 21$.
- **Pop**: $T_{\text{amortized}} = T_{\text{actual}} + \Phi(x - 1) - \Phi(x) \leq 0$.
- **Decimate**: $T_{\text{amortized}} = T_{\text{actual}} + \Phi(\lceil \frac{9x}{10} \rceil) - \Phi(x) \leq 20$.