第十二次作业.

2.(1) 设第i个人的索赔金额为 X_i

则由题意得 $E(X_i)=280$. $D(X_i)=800^2>0$.

令算术平均 $\bar{X}=\frac{1}{10000}\sum_{i=1}^{10000}X_i$

由中心极限定理 $\frac{\bar{X}-280}{800\sqrt{10000}}\sim N(0,1)$.

$\Rightarrow \bar{X}\sim N(280,\frac{800^2}{10000})$

$\Rightarrow \bar{X}\sim N(280,64)$.

设索赔总金额超过2700000美元为事件A

$P(A)=P(\sum_{i=1}^{10000}X_i>2700000)$

$=P(10000\bar{X}>2700000)$

$=P(\bar{X}>270)\xRightarrow{\bar{X}\sim N(280,64)}$

$\approx\Phi(\frac{5}{4})$

查表得到结果 $P(A)=0.8925$.

(2). $E(X_i)=\overset{=\mu}{5}$, $D(X_i)=6=\sigma^2$.

设50张保险单索赔的合计金额大于300为事件B.

$\bar{X}=\frac{1}{50}\sum_{i=1}^{50}x_i\sim N(5,\frac{6}{50})$.

$P(B)=P(\sum_{i=1}^{50}x_i>300)$

$=P(50\bar{X}>300)$ $\qquad \frac{50}{\sqrt{300}}=\frac{5}{\sqrt{3}}$

$=P(\bar{X}>6)$

$\approx1-\Phi(\frac{6-5}{\sqrt{\frac{6}{50}}})=1-\Phi(\frac{5}{3}\sqrt{3})$

$=1-0.9981$

$=0.0019$.

7. (1). 设第i个蛋糕卖出的价格是 X_i

$E(X_i)=1\times0.3+1.2\times0.2+1.5\times0.5=0.3+0.24+0.75=0.54+0.75=1.29$

$E(X_i^2)=1\times0.3+\overset{1.44}{2.24}\times0.2+2.25\times0.5=0.3+\overset{0.588}{0.288}+1.125=\quad 1.713$

$D(X_i)=E(X_i^2)-E(X_i)^2$

$=1.713-1.29^2$

$=0.0489.$

设 $\bar{X}=\frac{1}{300}\sum_{i=1}^{300}X_i \Rightarrow \bar{X}\sim N(1.29,\frac{0.0489}{300})$.

$P(\sum_{i=1}^{300}X_i\geq400)=P(300\bar{X}\geq400)=P(\bar{X}\geq\frac{4}{3})$

$=1-\Phi(\frac{\frac{4}{3}-1.29}{\sqrt{\frac{0.0489}{300}}})$

$=1-\Phi(3.394)$

$=0.0003.$

(2). 设第i个蛋糕为1.2元个数为 Y_i.

Y_i	0	1
P	0.8	0.2

$E(Y_i)=0.2$

$D(Y_i)=E(Y_i^2)-E(Y_i)^2=0.2-0.04=0.16.$

设 $\bar{Y}=\frac{1}{300}\sum_{i=1}^{300}Y_i$

$\bar{Y}\sim N(0.2,\frac{0.16}{300})$.

$P(\sum_{i=1}^{300}Y_i>60)=P(300\bar{Y}>60)=P(\bar{Y}>\frac{1}{5})=1-P(\bar{Y}\leq\frac{1}{5})$.

$=1-\Phi(\frac{\frac{1}{5}-\frac{1}{5}}{\quad})$

$=1-\Phi(0)=0.5.$

9. (1) $\bar{X}=\frac{1}{52}\sum_{i=1}^{52}X_i$

$\bar{X}\sim N(2.2,\frac{1.4^2}{52})$.

(2) $P(\sum_{i=1}^{52}X_i<100)=P(52\bar{X}<100)$

$=P(\bar{X}<\frac{25}{13})$

$=\Phi(\frac{\frac{25}{13}-\frac{11}{5}}{\sqrt{\frac{1.4^2}{52}}})$

$=1-0.923$

$=0.077.$

$\begin{array}{r}1.2\\ \times1.2\\ \hline 24\\ 12\\ \hline\end{array}$ $\begin{array}{r}2.25\\ \times.25\\ \hline 11.25\end{array}$ $\begin{array}{r}1.29\\ 1.29\\ \hline 1.61\end{array}$

11. (1).

$\bar{X} \sim N(5, \frac{0.3}{80})$.

$P(4.9 < \bar{X} < 5.1)$

$= P(\bar{X} \leq 5.1) - P(\bar{X} \leq 4.9)$.

$= \Phi(\frac{5.1-5}{\sqrt{\frac{0.3}{80}}}) - \Phi(\frac{4.9-5}{\sqrt{\frac{0.3}{80}}}) \approx 0.8968$.

(2). $E(\bar{X}-\bar{Y}) = E(\bar{X}) - E(\bar{Y}) = 0$.

$D(\bar{X}-\bar{Y}) = D(\bar{X}) + D(\bar{Y}) = 0.6$.

$\bar{X}-\bar{Y} \sim N(0, \frac{3}{400})$

$P(-0.1 < \bar{X}-\bar{Y} < 0.1)$

$= P(\bar{X}-\bar{Y} < 0.1) - P(\bar{X}-\bar{Y} \leq -0.1)$.

$= \Phi(\frac{0.1-0}{\sqrt{\frac{3}{400}}}) - \Phi(\frac{-0.1-0}{\sqrt{\frac{3}{400}}}) \approx 0.75$.

12. $E(X)=0\times0.1+1\times0.6+2\times0.3=0.6+0.6=1.2$.

$E(X^2)=0\times0.1+1\times0.6+2^2\times0.3=0.6+1.2=1.8$.

$D(X)=E(X^2)-E(X)^2=1.8-1.44=0.36$.

$\bar{X}=\frac{1}{200}\sum_{i=1}^{200}X_i\sim N(1.2,\frac{0.36}{200})$.

设需要k个车位

$P(\sum_{i=1}^{200}X_i\le k)=P(200\bar{X}\le k)=P(\bar{X}\le\frac{k}{200})$.

$=\Phi(\frac{\frac{k}{200}-1.2}{0.6})$

$=\Phi(\frac{k-240}{120})\ge0.95$

$$\begin{array}{r}1.645\\ \times\ \ 12\\ \hline 3290\\ 1645\ \\ \hline 19.740\end{array}\qquad\begin{array}{r}197.4\\ 240\\ \hline 437\end{array}$$

$\frac{k-240}{120}\ge1.645$

$k\ge1.645\times120+240$

$k\ge437.4$.

$\Rightarrow$至少需438个车位.

13. $\bar{X}\sim N(u,\frac{400}{n})$. $\Rightarrow\frac{\bar{X}-u}{\frac{20}{\sqrt{n}}}\sim N(1,0)$.

$P(|X-u|<1)$.

$=P(|\frac{\bar{X}-u}{\frac{20}{\sqrt{n}}}|<\frac{\sqrt{n}}{20})=\Phi(\frac{\sqrt{n}}{20})-\Phi(-\frac{\sqrt{n}}{20})$

$=2\Phi(\frac{\sqrt{n}}{20})-1$

≥0.95.

$\Phi(\frac{\sqrt{n}}{20})\ge0.975$.

$n\ge1536.6$.

14. (1) 设治愈的人数为X

则 $X \sim N(80, 16)$.

设接受这一断言为事件A

$P(A) = P(X > 75)$

$= 1 - P\left(\frac{X-80}{4} \le \frac{75-80}{4}\right)$

$\approx 1 - \Phi\left(-\frac{5}{4}\right)$

$= 0.8944$.

(2) $X \sim N(70, 21)$.

$P(A) = P(X > 75)$

$= 1 - P(X \le 75)$

$= 1 - P\left(\frac{X-70}{\sqrt{21}} \le \frac{75-70}{\sqrt{21}}\right)$

$\approx 1 - \Phi\left(\frac{5\sqrt{21}}{21}\right)$

$= 0.1379$.