TNM087 – Image Processing and Analysis
Lab 3 – Filtering in the frequency domain

TASK 1 Preparation

The preparation task consists of a number of problems that should be solved using Matlab. Your answers (in Swedish or English) should be written in the document *Lab_3.1_Preparation_Answers.docx*, where you also insert the required images. To save the images you can use the MATLAB functions `imwrite` or `imsave`. Make sure to save the images in an uncompressed format, such as **.tif** or **.png**.
Don't scale the images when inserting them in the word document. Before submitting the answer document on Lisam, first save the document as **.pdf**!

For the preparation tasks you do not need to submit your m-file. However, it is strongly recommended that you save your experiments in an m-file, in case you need to go back and correct anything later. Sometimes, you can also re-use your code in later tasks.

There are a number of MATLAB functions that are useful regarding Fourier transform of an image, for example, $fft2$, $ifft2$, $fftshift$, $ifftshift$, abs and $angle$. See the lecture notes for Chapter 4 (**FÖ 4 and 5 in the course**) on **page 74** to learn how they work.

1) 2D Fourier spectrum

In this section of the task, you will examine the Fourier spectrum of several test images. Our main objective in this section is to establish connections between specific components of an image and its Fourier transform. As discussed in the lecture notes for Chapter 4 **FÖ 4 and 5 in the course**) on **page 73**, the DC term of the Fourier transform typically dominates the values in the spectrum. To enhance the details, a **log transformation** of the spectrum is applied. We recommend writing a simple MATLAB code to compute and display the log transformation of the spectrum of an image in order to conduct the following experiments (you do not need to submit this m-file). On page 74 of the lecture notes for Chapter 4 (**FÖ 4 and 5 in the course**), you will find an example of how such a MATLAB function should be structured.

Read the image $characterTestPattern.tif$ into MATLAB and scale it to the range $[0, 1]$. We will refer to this image as cTP.
Display the test image cTP, paying particular attention to the various horizontal, vertical, and diagonal lines (bars) present in the image.

Problem 1) Compute the log transformation of the Fourier spectrum of cTP and refer to the result as $Spec1$. Insert your result into the answer document.

Pay special attention to how the vertical, horizontal, and diagonal bars in cTP influence its spectrum, $Spec1$.

Next, apply the following MATLAB command to circularly shift the test image cTP by 100 pixels in the x-direction and -200 pixels in the y-direction:

$$cTP\_shift = circshift(cTP, [100, -200]);$$

Display the shifted image cTP_shift to observe how this circular shift has affected the test image.

Problem 2) Compute the log transformation of the Fourier spectrum of cTP_shift and refer to the result as $Spec2$. Insert your result into the answer document.

Problem 3) Analyze the differences between $Spec2$ and $Spec1$. How does the shift applied to the test image affect the spectrum of its Fourier transform?

Now, rotate the test image using the following MATLAB command, which rotates cTP by 15° counterclockwise:

$$cTP\_rot = imrotate(cTP, 15, 'bicubic');$$

Display the rotated image cTP_rot to observe how this rotation has affected the test image.

Problem 4) Compute the log transformation of the Fourier spectrum of cTP_rot and refer to the result as $Spec3$. Insert your result into the answer document.

Problem 5) Compare $Spec3$ with $Spec1$. What differences do you observe? How does rotation in the spatial domain affect the Fourier spectrum? (Please ignore any distortions caused by the black area added around the image after rotation.)

Read the image $characterTestPattern\_2.tif$ into MATLAB and scale it to the range $[0, 1]$. We will refer to this image as $cTP2$. Display the test image $cTP2$, noting that it is almost identical to cTP, except that the vertical bars under the large "a" have been removed.

Problem 6) Compute the log transformation of the Fourier spectrum of $cTP2$ and refer to the result as $Spec4$. Insert your result into the answer document.

Problem 7) Compare $Spec4$ with $Spec1$ and explain how the elimination of the vertical bars affected the spectrum. **HINT**: Pay special attention to the horizontal axis of the spectrum.

Problem 8) Discuss what would happen to the spectrum if the horizontal bars were eliminated from cTP.

Next, read the image $characterTestPattern\_3.tif$ into MATLAB and scale it to the range [0, 1]. We will refer to this image as $cTP3$. Display the test image $cTP3$, noting that it is almost identical to cTP, except that the diagonal bars in the bottom-left corner have been removed.

Problem 9) Compute the log transformation of the Fourier spectrum of $cTP3$ and refer to the result as $Spec5$. Insert your result into the answer document.

Problem 10) Compare $Spec5$ with $Spec1$ and explain how the elimination of the diagonal bars affected the spectrum. **HINT**: Pay special attention to the diagonal axes of the spectrum.

2) Period and Frequency

In this section of the task, you will explore the relationship between the period in the spatial domain and the frequency in the Fourier domain.
First, read the image $verticalbars\_2.tif$ into MATLAB and name it $v2$. This image is binary and of logical type, meaning it contains only values of 0 or 1. Therefore, do not divide it by 255 in MATLAB. The index "2" in the file name indicates the period of the vertical bars, which is $P = 2$. Display the image $v2$ and observe that each black-and-white stripe (bar) is 1 pixel wide. One-pixel-wide stripes (resulting in a period of $P = 2$) represent the narrowest possible bars in a digital image. Consequently, this results in the smallest possible period, implying the highest possible frequency. This means that the peaks in the Fourier spectrum will appear at the furthest points on each side of the center along the horizontal axis, corresponding to a frequency of $f = \frac{1}{P} = \frac{1}{2} = 0.5$ cycles/pixel.
Compute the **log transformation** of the Fourier spectrum of $v2$ and refer to the result as $Spec6$. You do not need to insert your result into the answer document. Carefully examine $Spec6$; you should observe three dominant peaks in the spectrum. One of these peaks represents the DC term located at the center of the spectrum. The other two peaks should appear at the furthest points on each side of the horizontal axis. **Since this image has an even size, you will only see one of these two peaks on the left side.**
HINT: Refer to assignment 6 in "Lektion 3" for additional context.

Problem 11) Where would the three dominant peaks appear if the image $v2$ is transposed, meaning the vertical bars become horizontal?

Next, read the image $verticalbars\_4.tif$ into MATLAB and name it $v4$. This image is binary and of logical type. The index "4" in the file name indicates the period of the vertical bars, which is $P = 4$.

Problem 12) What is the frequency of the stripes in $v4$? Where would the three dominant peaks in the spectrum for this image appear?

As you may have noticed, these images are 300×300 pixels. If we increase the period to $P = 300$, the image will consist of only one white bar and one black bar, each 150 pixels wide.

Problem 13) What is the frequency of these stripes? Where would the three most dominant peaks in the spectrum for this image appear?

3) The importance of the spectrum and the phase angle

In this section of the task, you will investigate the significance of the spectrum and phase angle by exchanging the spectra and phase angles of two different images.
To obtain the spectrum and phase angle of a Fourier transform image F, you can use the MATLAB functions $abs(F)$ and $angle(F)$ respectively.

If you want to combine the spectrum $Spec$ and the phase angle Ang to reconstruct the image in the spatial domain, you can use the following command:

$$real(ifft2(Spec.* \exp(i * Ang)))$$

Make sure you understand how this command works.

Read the two Einstein images $Einstein1.jpg$ and $Einstein2.jpg$, into MATLAB. Scale them to the range [0, 1] and assign them to the variables $E1$ and $E2$, respectively.

Problem 14) Calculate the Fourier spectrum of $E1$ and the Fourier phase angle of $E2$. Combine these two components to reconstruct an image in the spatial domain. Name your result $E1\_E2$ and insert it into the answer document.

Problem 15) Now, compute the Fourier spectrum of $E2$ and the Fourier phase angle of $E1$. Combine these two components to create a new image in the spatial domain. Name this result $E2\_E1$ and insert it into the answer document.

Problem 16) Based on your visual analysis of the results from Problems 14 and 15, which has a greater effect on the structure of an image: the spectrum or the phase angle?