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# Noise Robust Quantum Circuits Search and Training with QuantumNAS and QuantumNAT

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MIT HAN Lab

[HPCA'22] QuantumNAS: Noise-adaptive search for robust quantum circuits

[DAC'22] QuantumNAT: Quantum Noise-Aware Training with Noise Injection, Quantization and Normalization

[DAC'22] QOC: Quantum On-Chip Training with Parameter Shift and Gradient Pruning

# Outline

- Background
- QuantumNAS
- QuantumNAT
- TorchQuantum Library
- Conclusion

# Quantum Bit

- Quantum Bit (Qubit)
  - Statevector: contains  $2^n$  complex numbers for n qubit system
  - The square sum of magnitude of  $2^n$  numbers are 1
  - 1 qubit:

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad a_0, a_1 \in \mathbb{C}$$
$$|a_0|^2 + |a_1|^2 = 1$$

- 2 qubits:
$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad a_0, a_1, a_2, a_3 \in \mathbb{C}$$
$$|a_0|^2 + |a_1|^2 + |a_2|^2 + |a_3|^2 = 1$$

# Quantum Bit

- Classical bits represented in statevector

- Classical 0:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- Classical 1:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- An arbitrary quantum states:

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = a_0 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_1 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

# Quantum Gates

- Qubit gates: operations on one qubit or multiple qubits
- The qubit gates can be represented with matrix format with dimension  $2^n \times 2^n$
- All gate matrices are unitary matrices: the conjugate transpose is the same as its inverse
- Single qubit gates:
  - Not (X) gate:

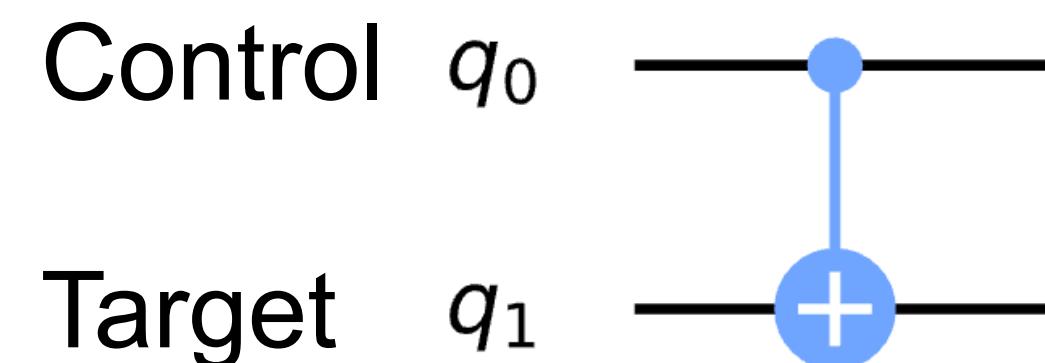
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- Parameterized gate: Rotation X (RX) with parameter theta

$$RX(\theta) = \begin{bmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}$$

# Quantum Gates

- 2-qubit gates:
  - Controlled Not (CNOT) gate:



Input	Output			
	00	01	10	11
00	1	0	0	0
01	0	1	0	0
10	0	0	0	1
11	0	0	1	0

- Controlled Rotation X (CRX) gate

$$CRX(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ 0 & 0 & -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}$$

# Quantum Gates

- Applying a gate to qubits is performing matrix-vector multiplication between the gate matrix and statevector
  - Apply an X gate to classical state 0, we get 1

$$X \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Apply an CNOT gate to state 10, we get 11

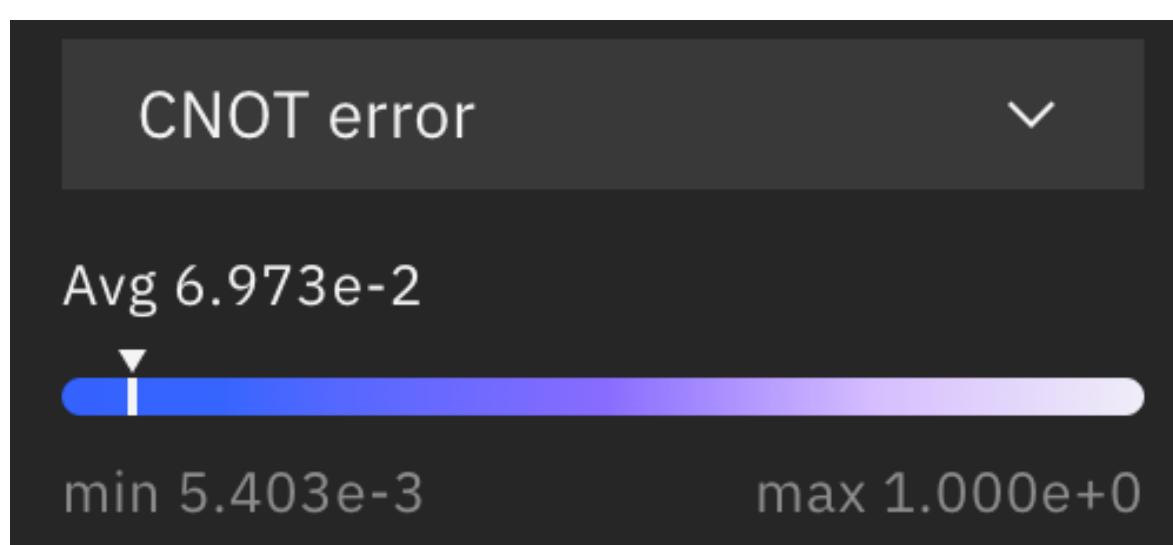
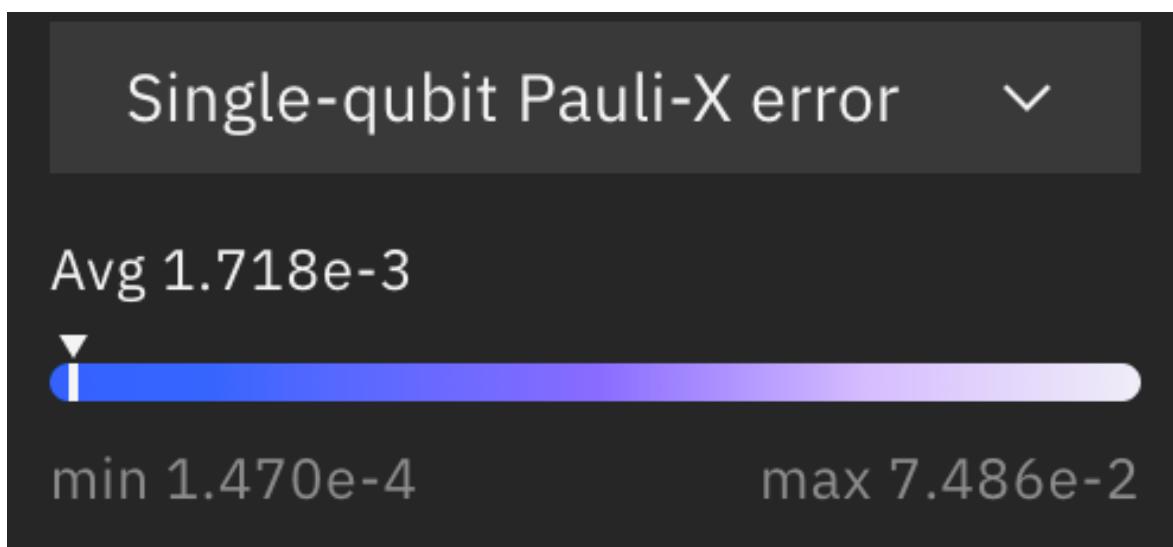
$$CNOT \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

# Source of Quantum Advantage

- One qubit carries more information than one classic bit
- The statevector length is exponentially to the number of qubits

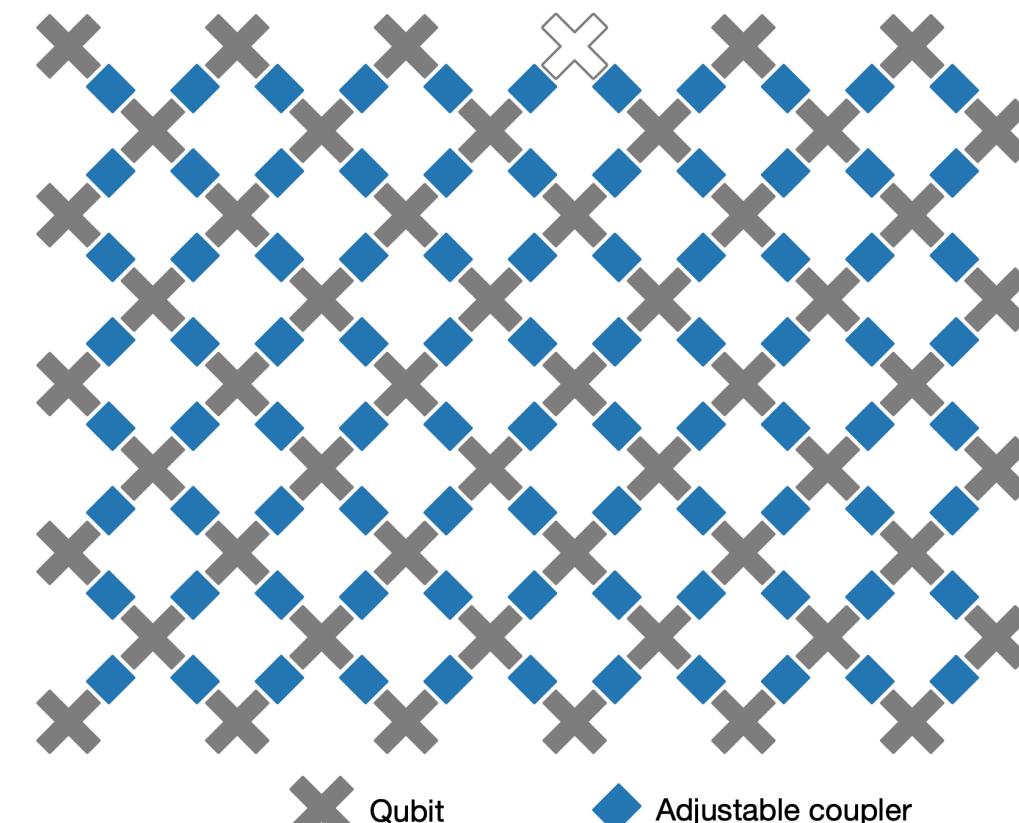
# NISQ Era

- Noisy Intermediate-Scale Quantum (NISQ)
  - **Noisy**: qubits are sensitive to environment; quantum gates are unreliable
  - **Limited number** of qubits: tens to hundreds of qubits



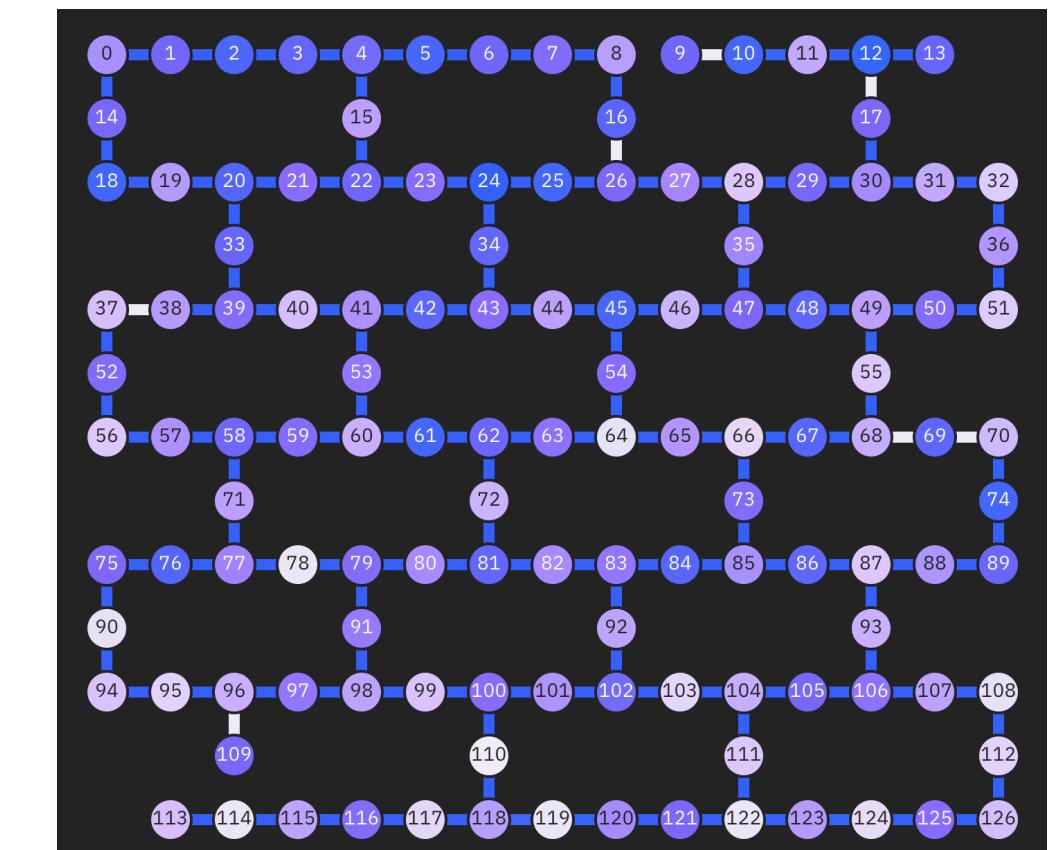
Gate Error Rate

<https://quantum-computing.ibm.com/>



Google Sycamore

<https://www.nature.com/articles/s41586-019-1666-5>

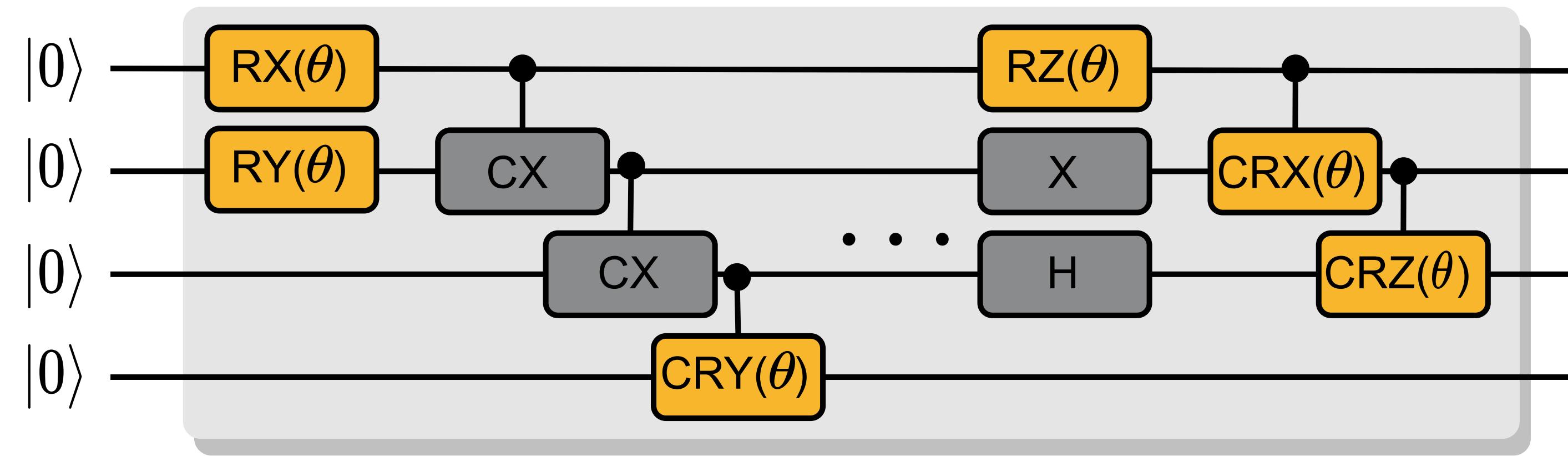


IBM Washington

<https://quantum-computing.ibm.com/>

# Parameterized Quantum Circuits

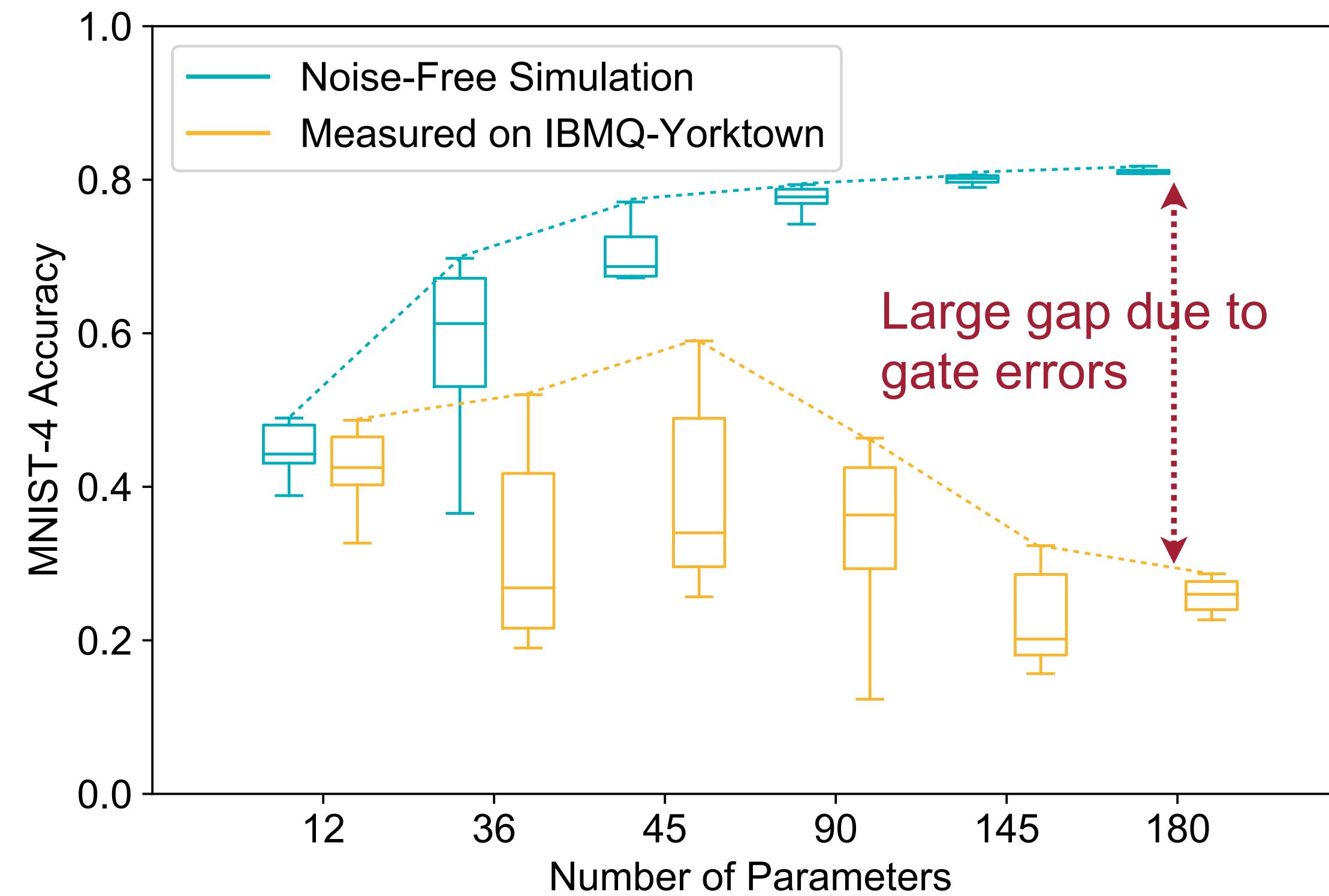
- Parameterized Quantum Circuits (PQC)
- Quantum circuit with fixed gates and parameterized gates



- PQCs are commonly used in **hybrid classical-quantum models** and show promises to achieve quantum advantage
  - Variational Quantum Eigensolver (VQE)
  - Quantum Neural Networks (QNN)
  - Quantum Approximate Optimization Algorithm (QAOA)

# Challenges of PQC — Noise

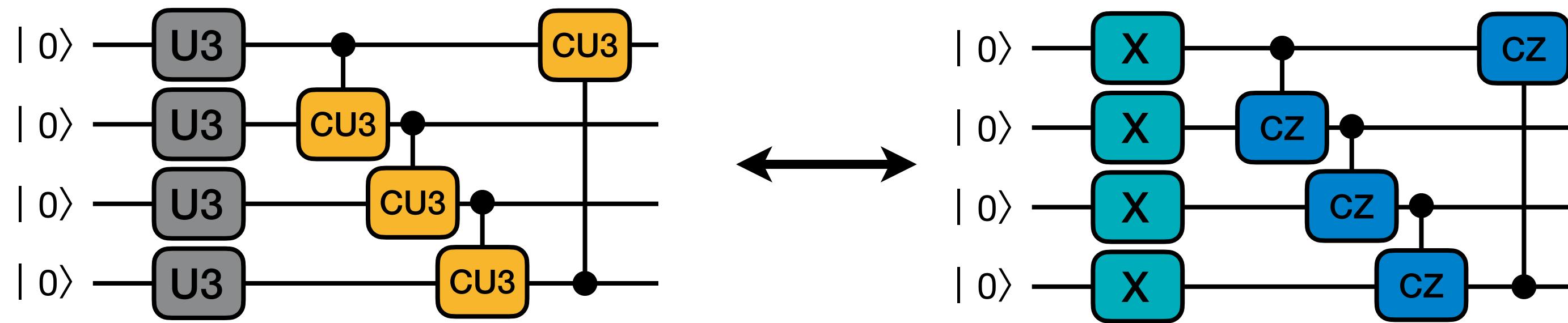
- Noise **degrades** the Parameterized Quantum Circuit (PQC) reliability
- More parameters increase the noise-free accuracy but degrade the measured accuracy
- Under same #parameters, measured accuracy of different circuit architecture (ansatz) varies a lot
- Therefore, circuit architecture is critical



# Challenges of PQC – Large Design Space

- Large design space for circuit architecture

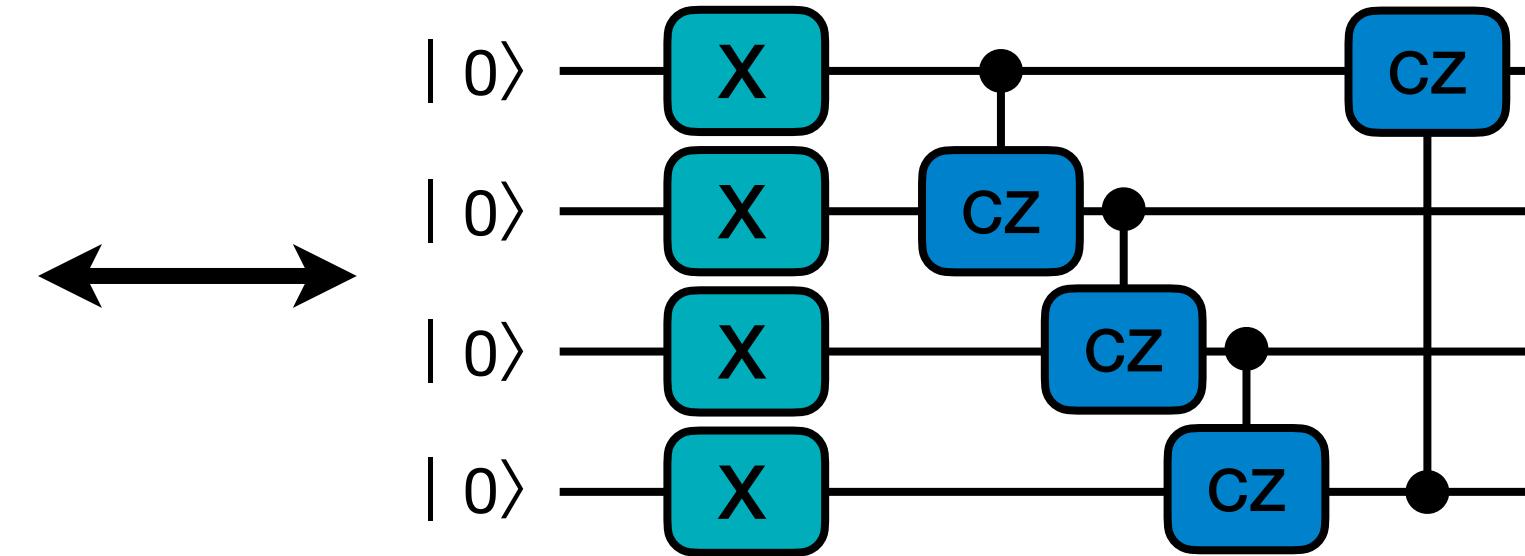
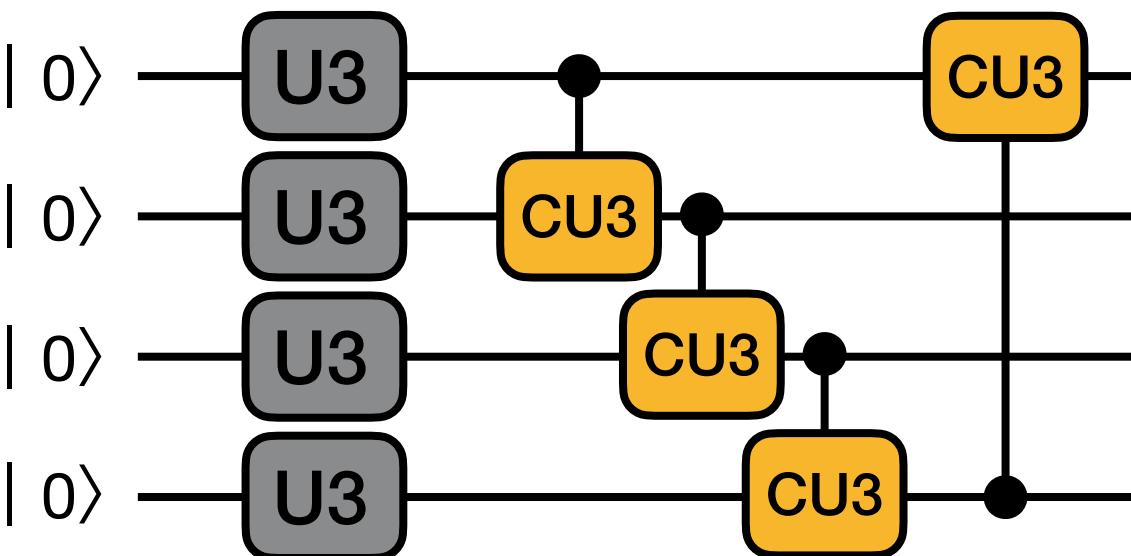
- Type of gates



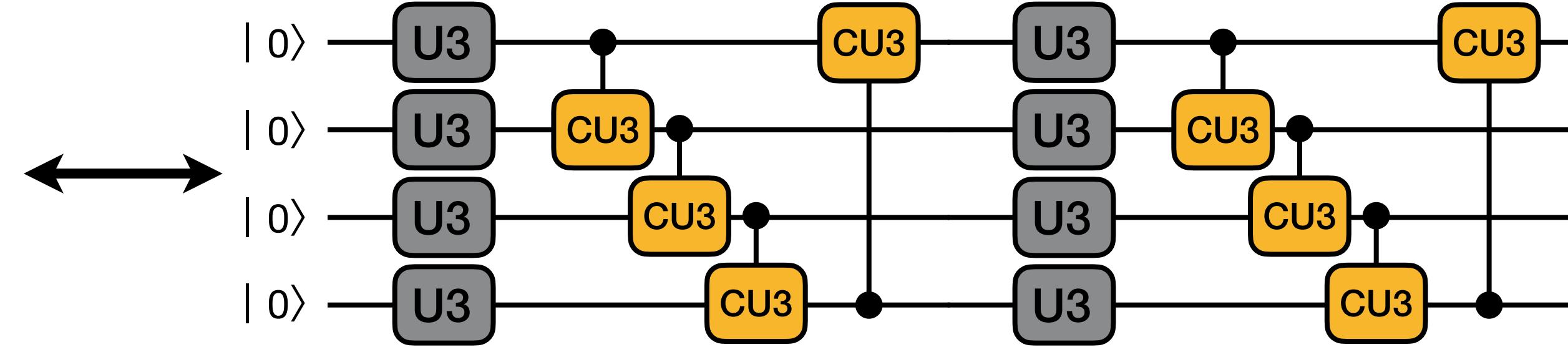
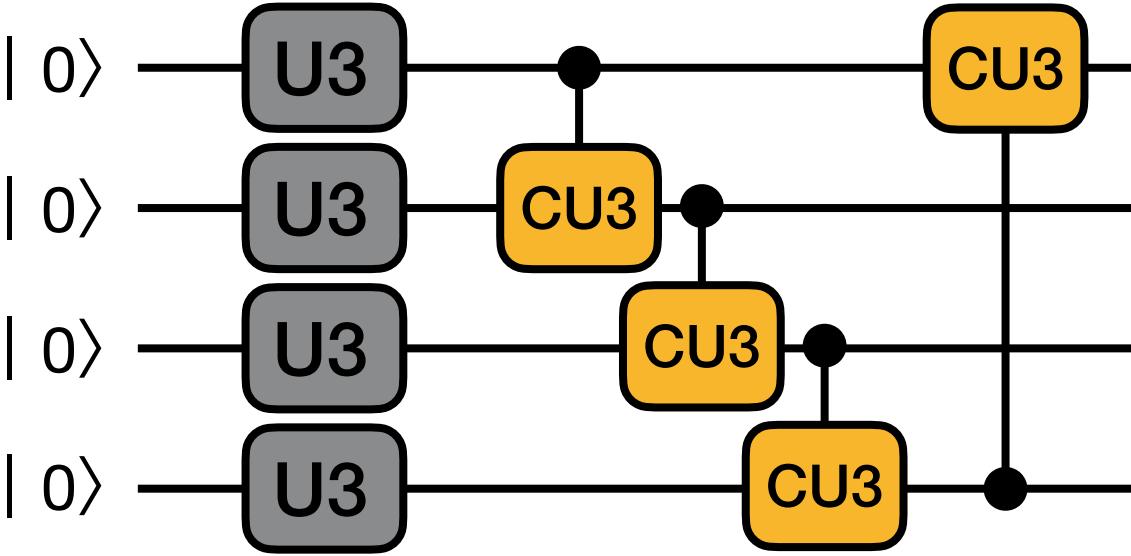
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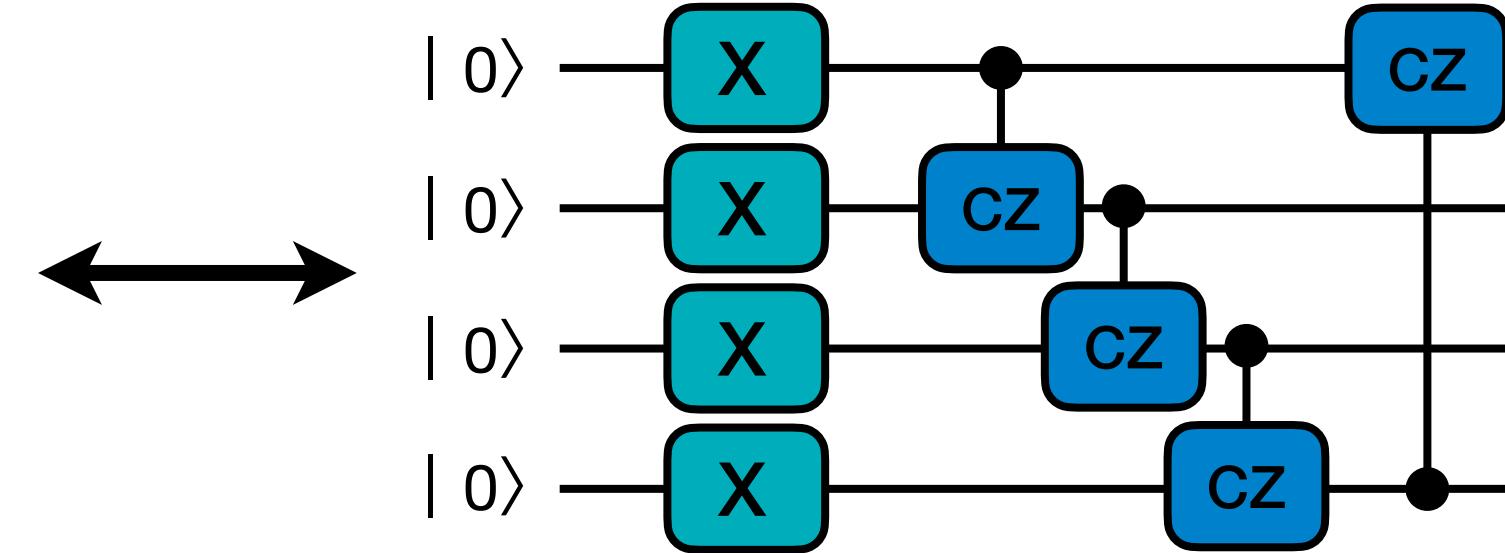
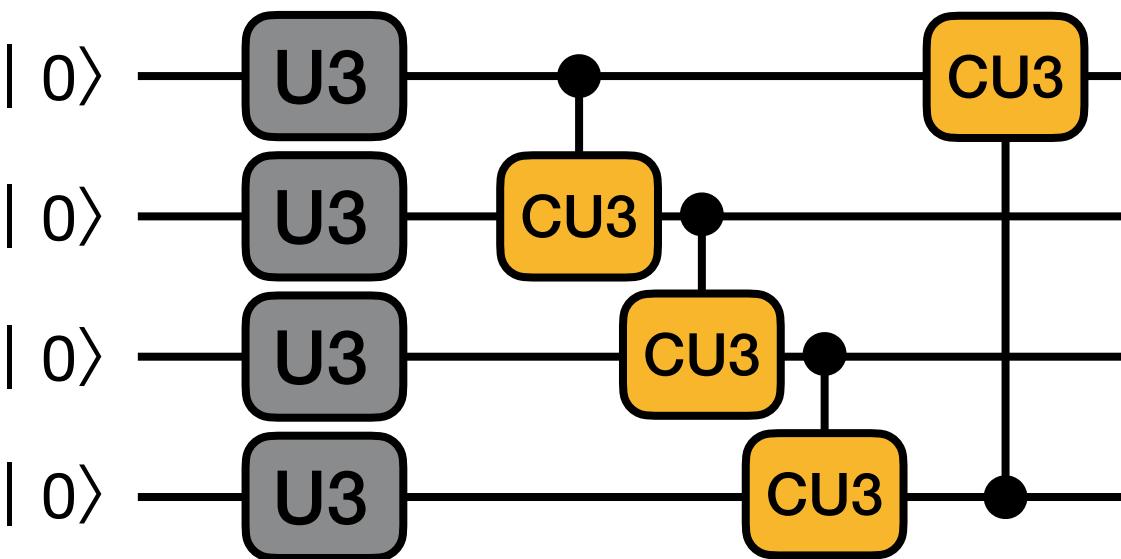
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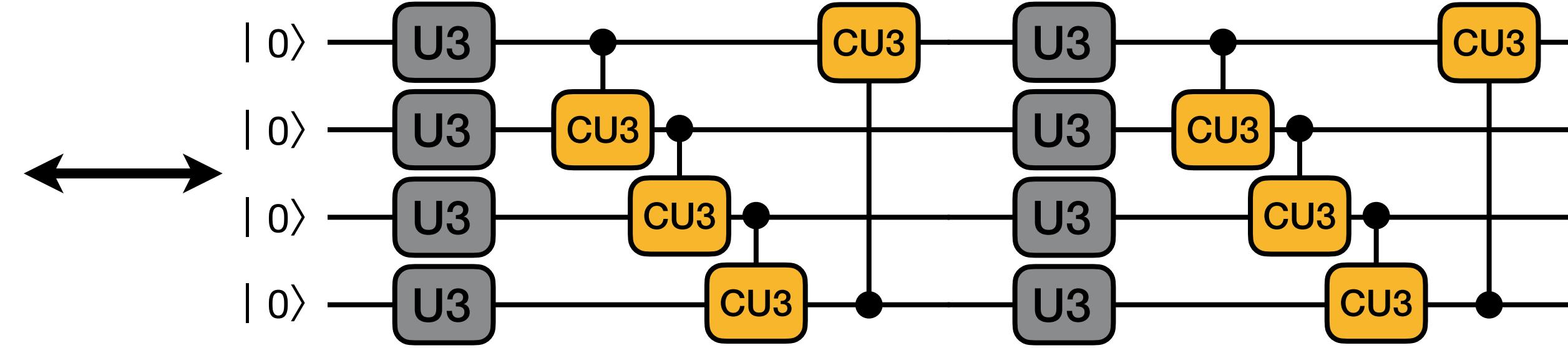
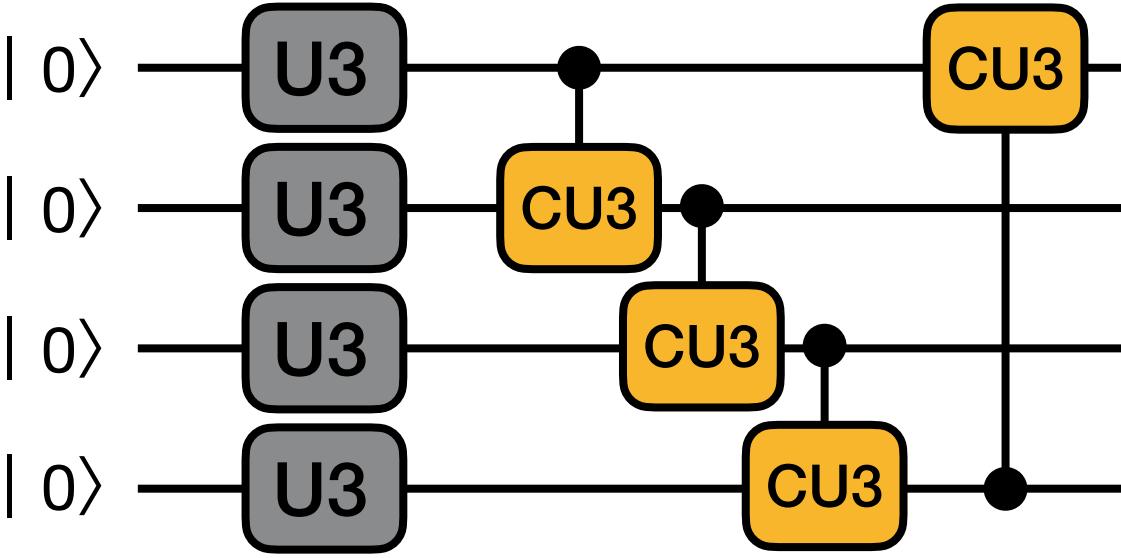
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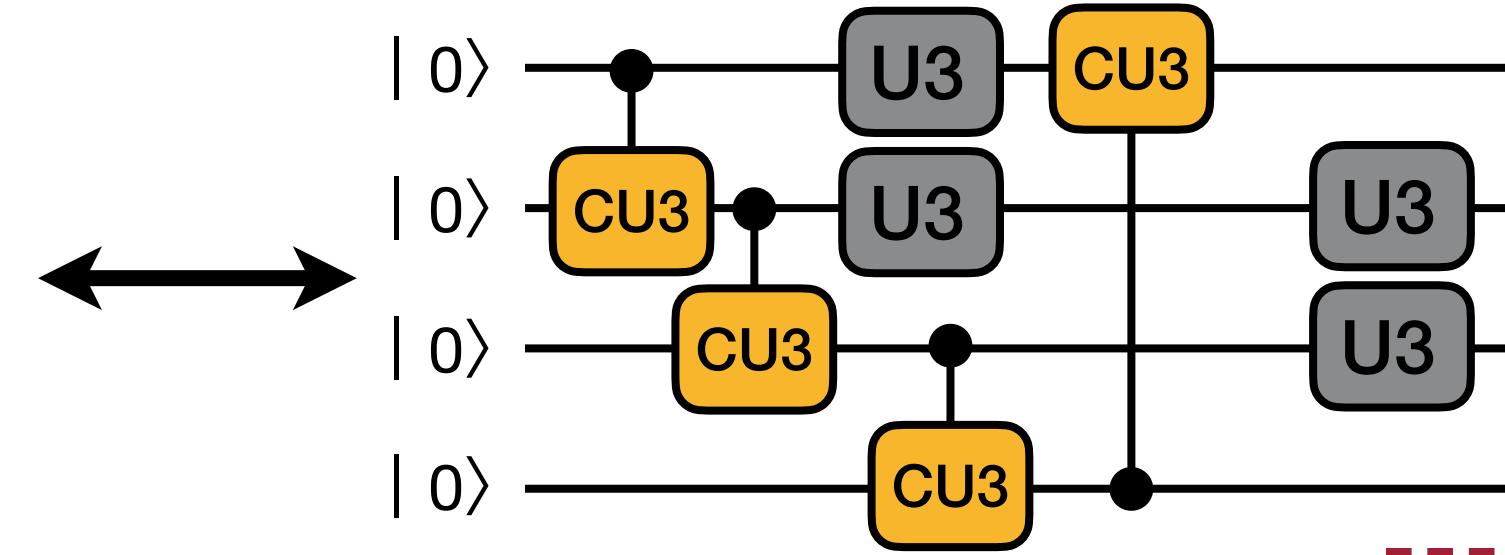
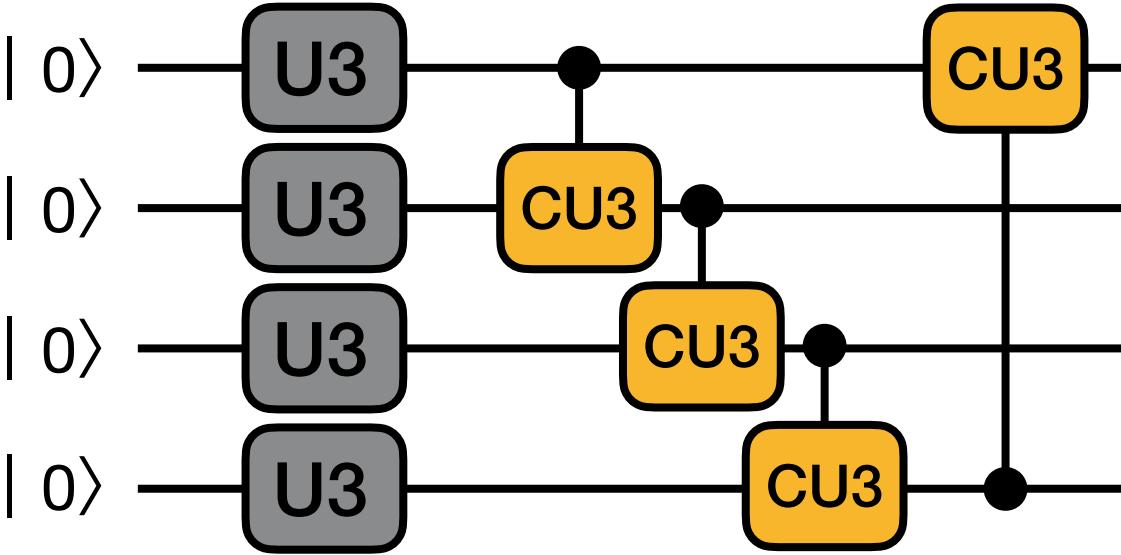
- Type of gates



- Number of gates



- Position of gates



# Goal of QuantumNAS

Automatically & efficiently search for noise-robust quantum circuit

Train one “SuperCircuit”,  
providing parameters to  
many “SubCircuits”

Solve the challenge of large  
design space

(1) Quantum noise feedback in  
the search loop  
(2) Co-search the circuit  
architecture and qubit mapping

Solve the challenge of large  
quantum noise

# QuantumNAS: Decouple the Training and Search

## Naive Search

**For** q\_devices:

**For** search episodes: // meta controller

**For** circuit training iterations:

            update\_parameters(); **Expensive**

**If** good\_circuit: **break**;

# QuantumNAS: Decouple the Training and Search

Naive Search

```
For q_devices:  
  For search episodes: // meta controller  
    For circuit training iterations:  
      update_parameters(); Expensive  
    If good_circuit: break;
```

=>

QuantumNAS

```
For SuperCircuit training iterations: Expensive  
  update_parameters(); training  
  .....  
  decouple  
  For q_devices:  
    For search episodes:  
      sample from SuperCircuit; Light-Weight  
      If good_circuit: break;  
      //no training
```

# QuantumNAS

- SuperCircuit Construction and Training
- Noise-Adaptive Evolutionary Co-Search of SubCircuit and Qubit Mapping
- Train the Searched SubCircuit
- Iterative Quantum Gate Pruning

# QuantumNAS

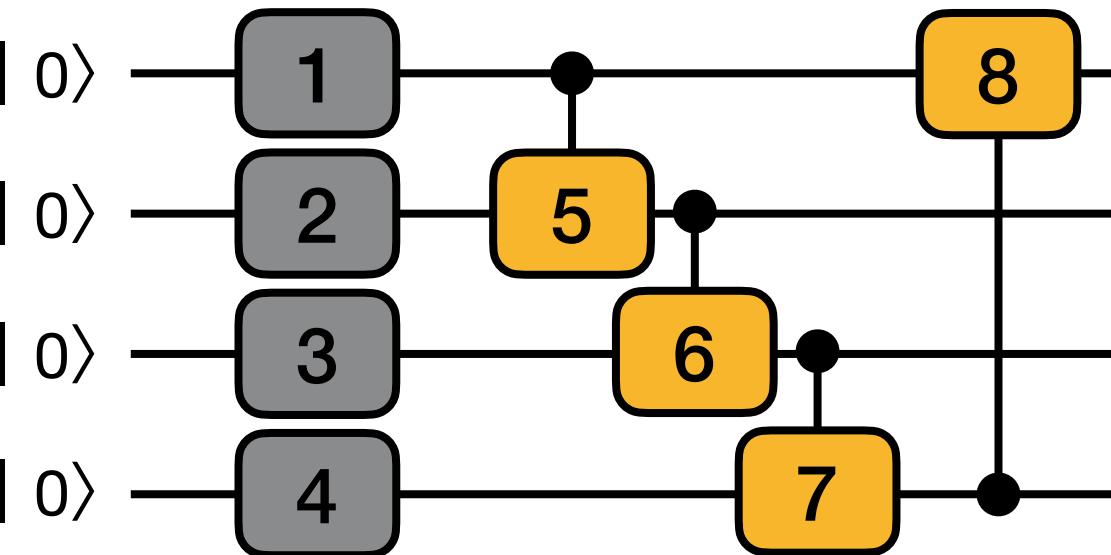
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# SuperCircuit & SubCircuit

- Firstly construct a design space. For example, a design space of maximum 4 U3 in the first layer and 4 CU3 gates in the second layer
  - Contains  $2^8 = 256$  candidates

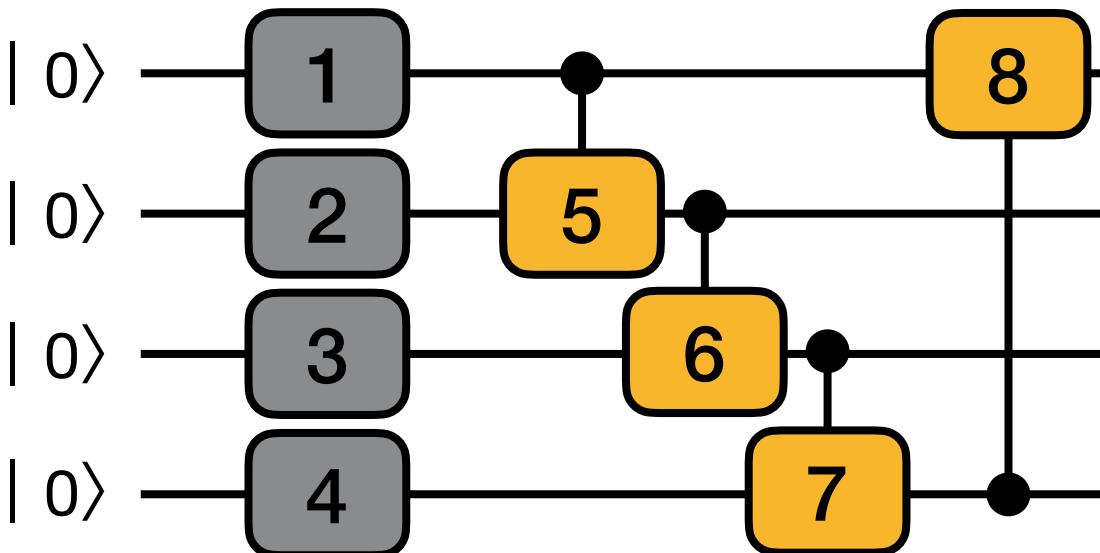
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  - SuperCircuit: the circuit with the **largest** number of gates in the design space
  - Example: SuperCircuit in U3+CU3 space

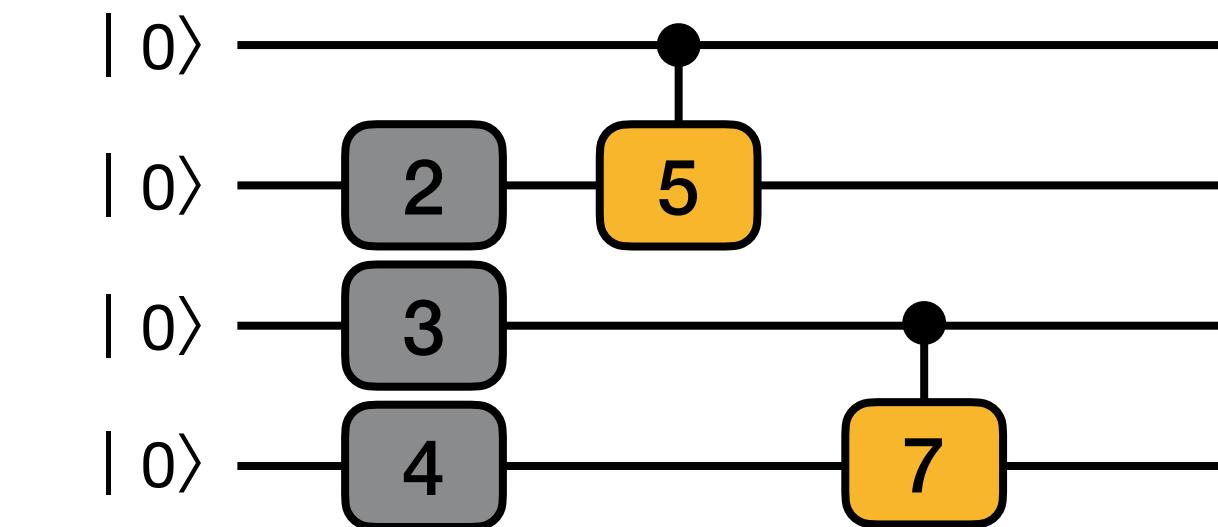
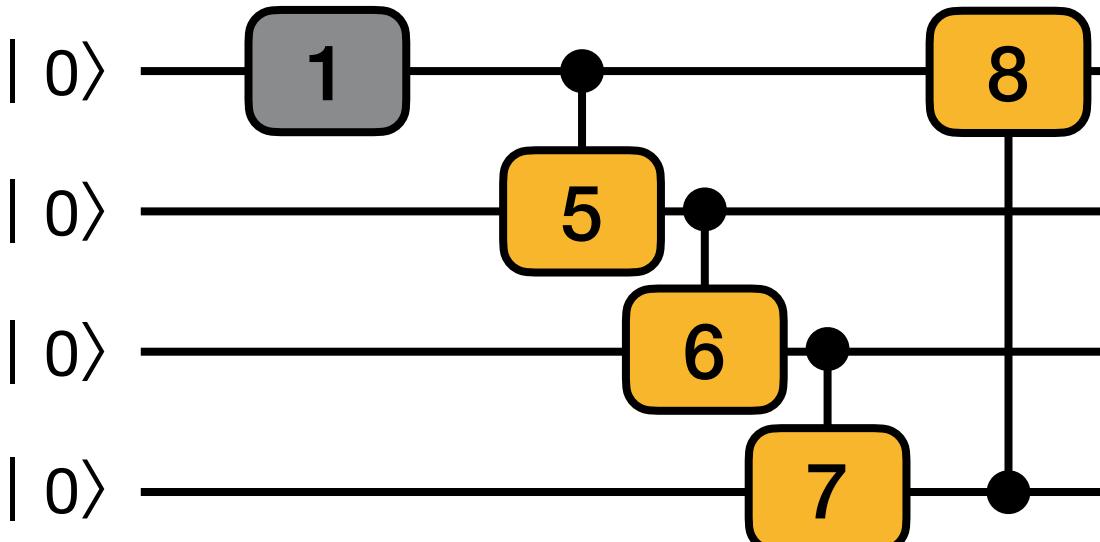
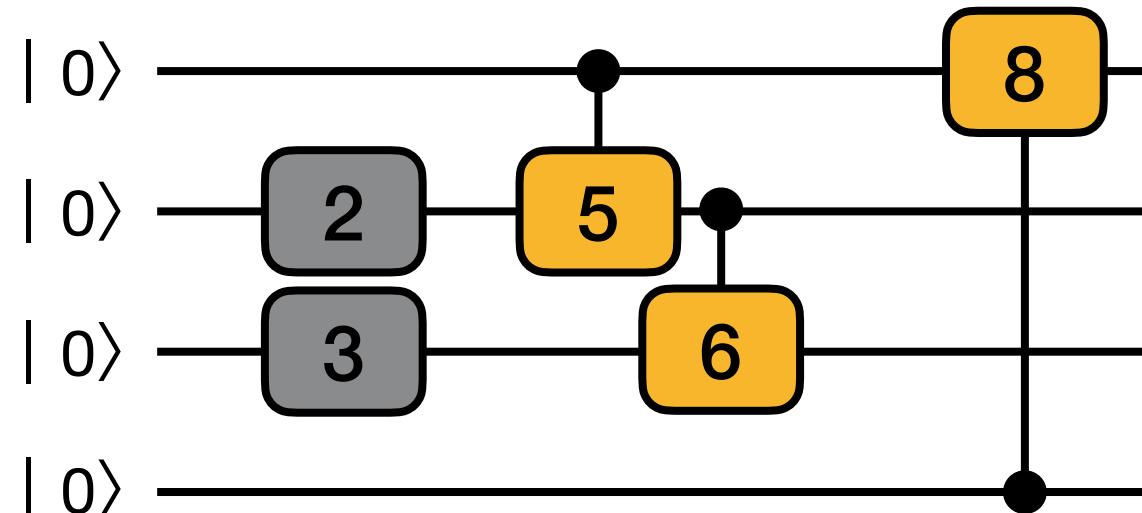


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  - Example: SuperCircuit in U3+CU3 space



- Each candidate circuit in the design space (called SubCircuit) is a **subset** of the SuperCircuit



# SuperCircuit Construction

- Why use a SuperCircuit?
  - It enables **efficient** search of circuit architecture candidates with no need of training each of them individually
  - For one SubCircuit candidate, we can directly inherit parameters from SuperCircuit and consider that the SubCircuit can operate **as if it is trained individually from scratch**

# SuperCircuit Construction

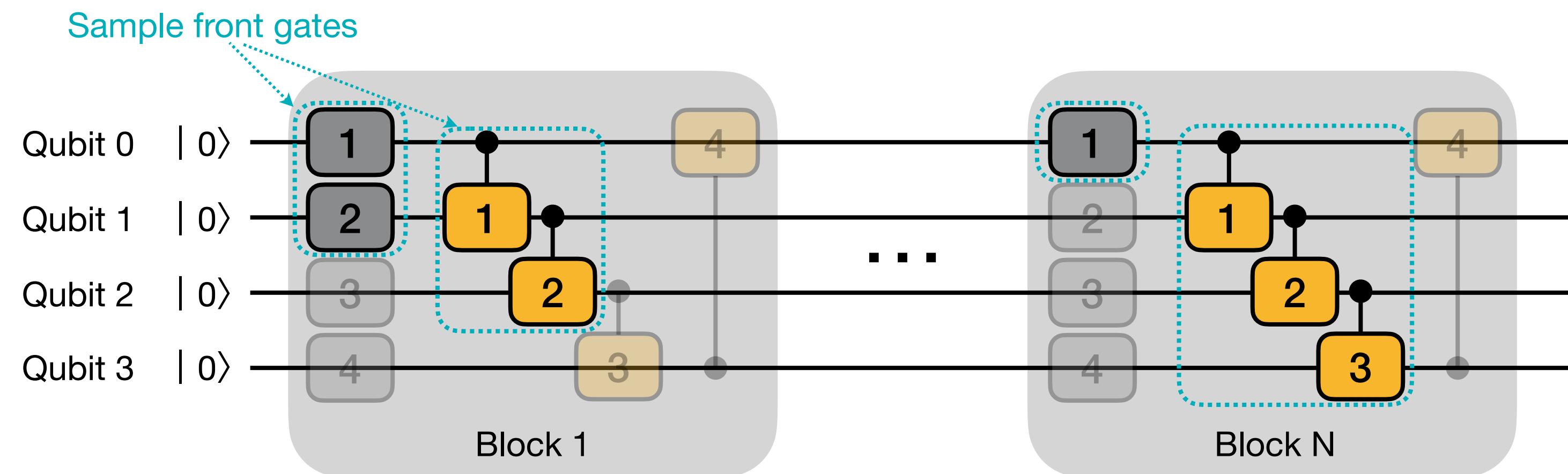
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  - It enables **efficient** search of circuit architecture candidates with no need of training each of them individually
  - For one SubCircuit candidate, we can directly inherit parameters from SuperCircuit and consider that the SubCircuit can operate **as if it is trained individually from scratch**
- Need to prevent interference of SubCircuits from each other

# SuperCircuit Training

- In one SuperCircuit Training step:
  - Sample a gate subset of SuperCircuit (a SubCircuit)
    - Front Sampling and Restricted Sampling
  - Only use the subset to perform the task and updates the parameters in the subset
  - Parameter updates are cumulative across steps

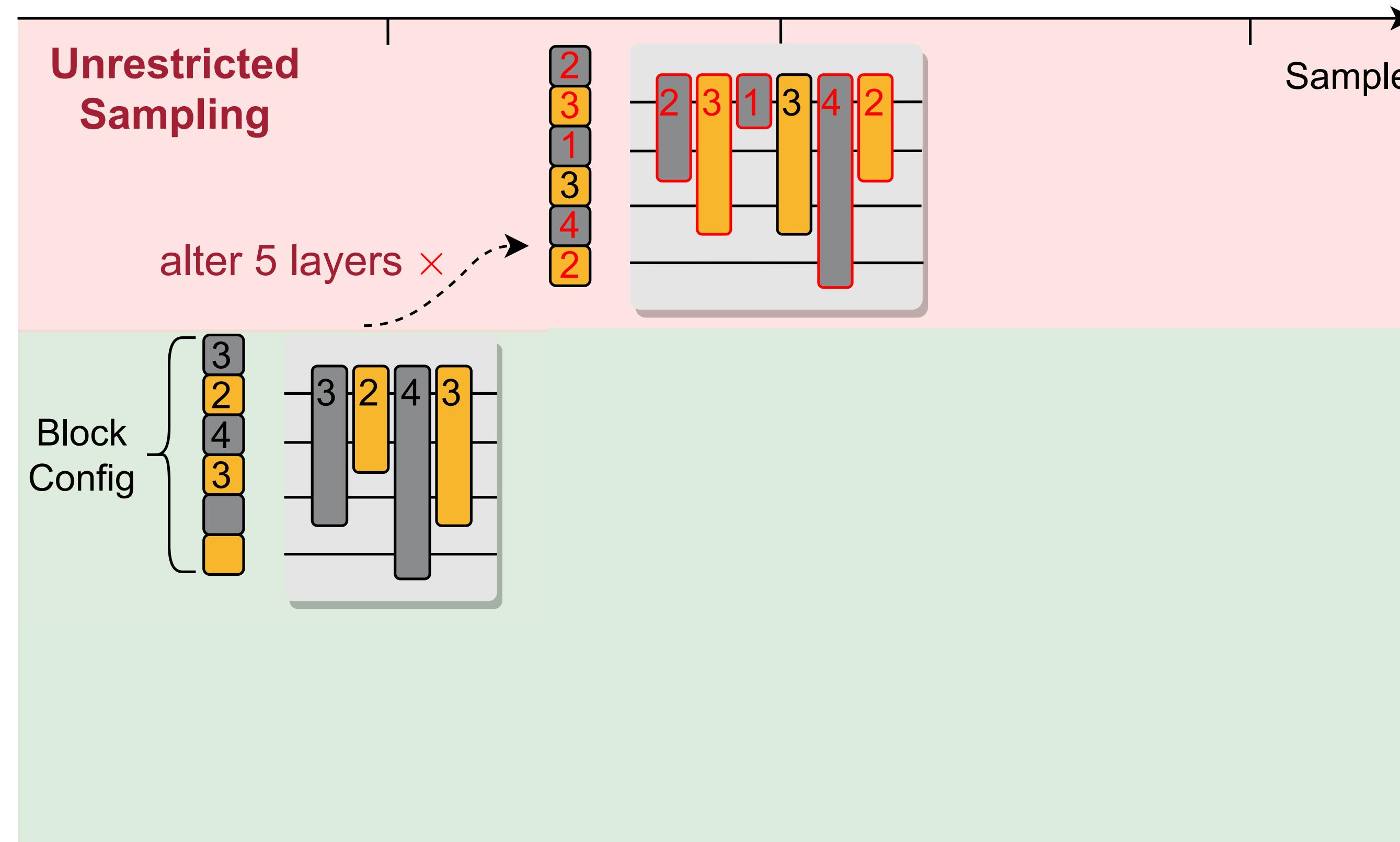
# Front Sampling

- During sampling, we first sample total number of blocks, then sample gates within each block
  - Front sampling: Only the **front** several blocks and **front** several gates can be sampled to make SuperCircuit training more stable



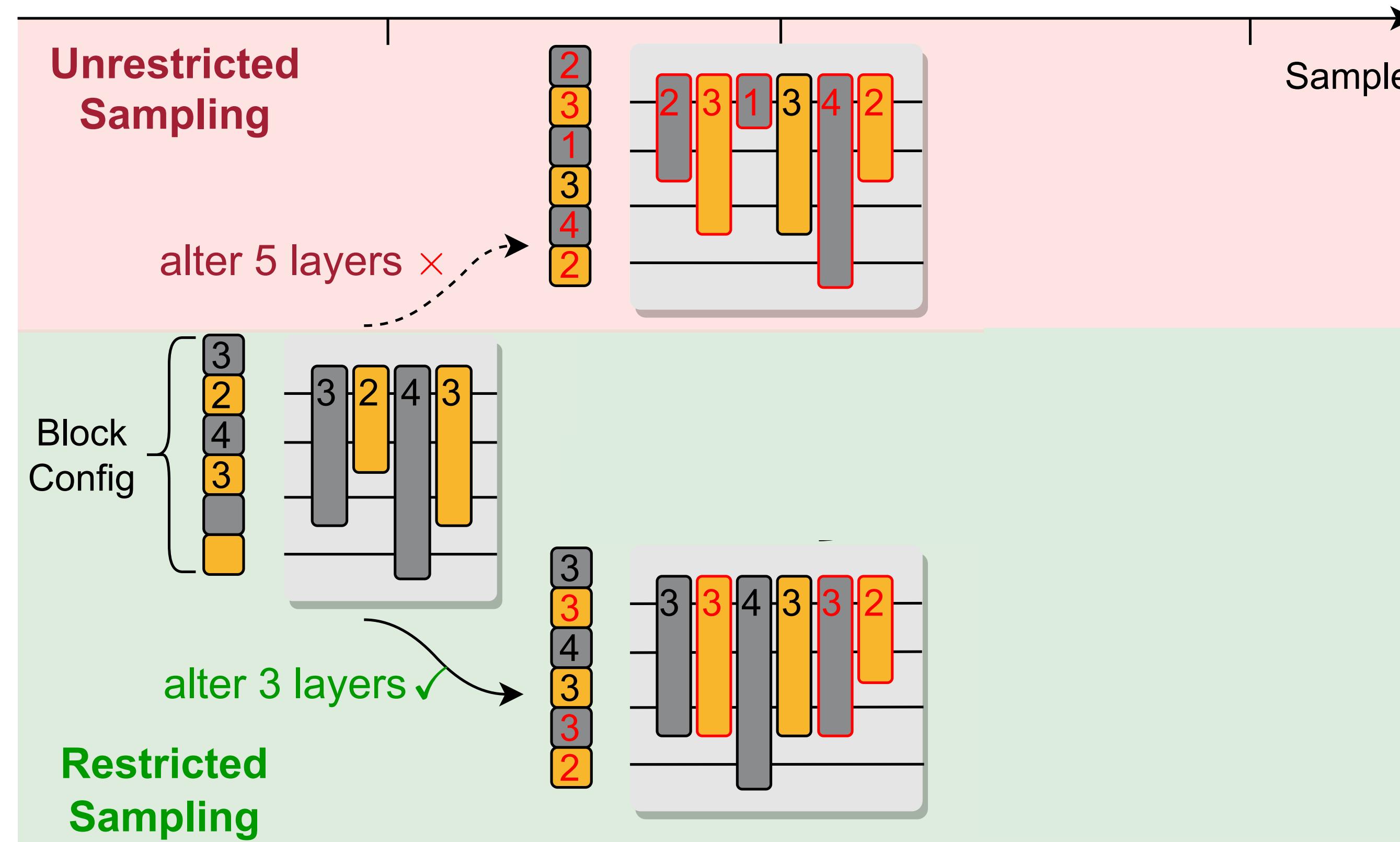
# Restricted Sampling

- Restricted Sampling:
  - Restrict the difference between SubCircuits of two consecutive steps
  - For example: restrict to at most 4 different layers



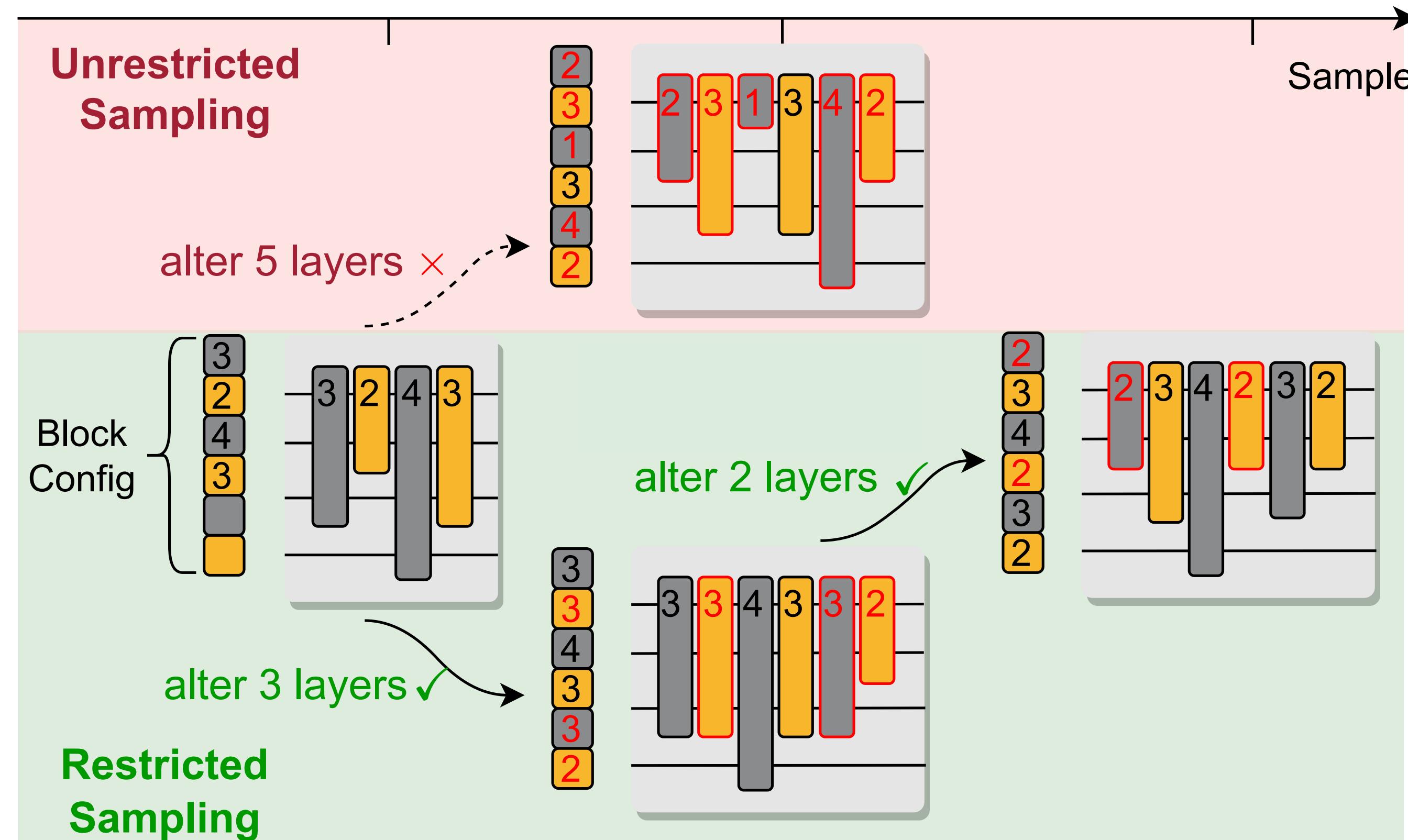
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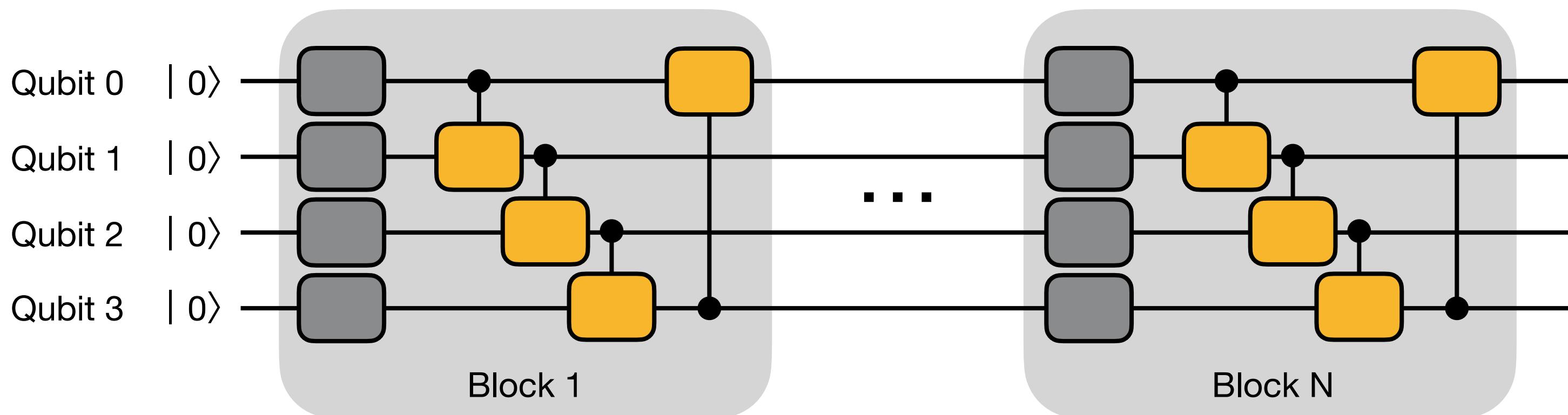
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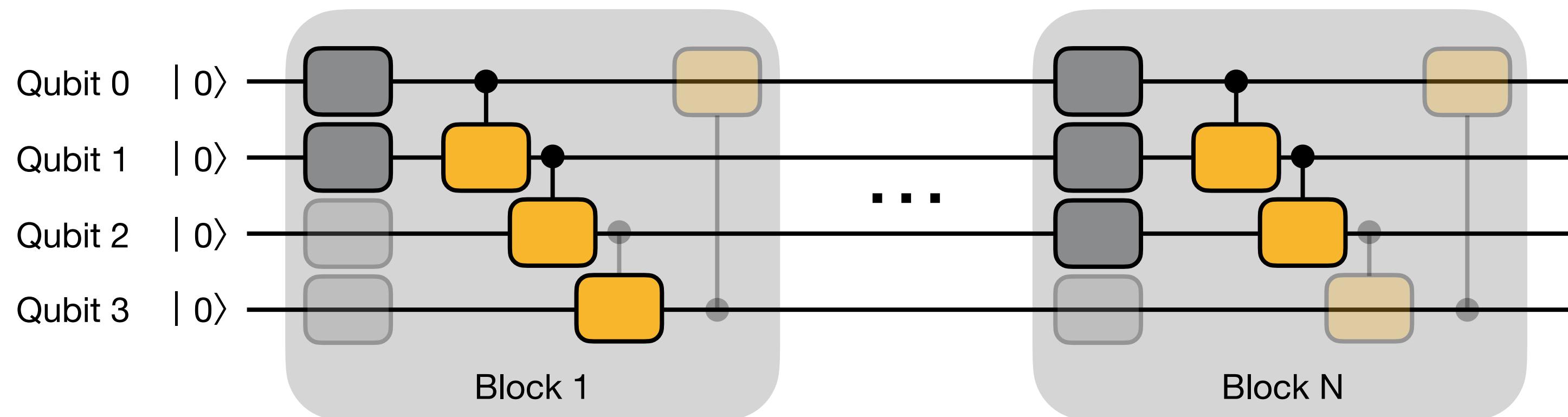
# Train SuperCircuit for Multiple Steps

- In one SuperCircuit Training step: Sample and Train



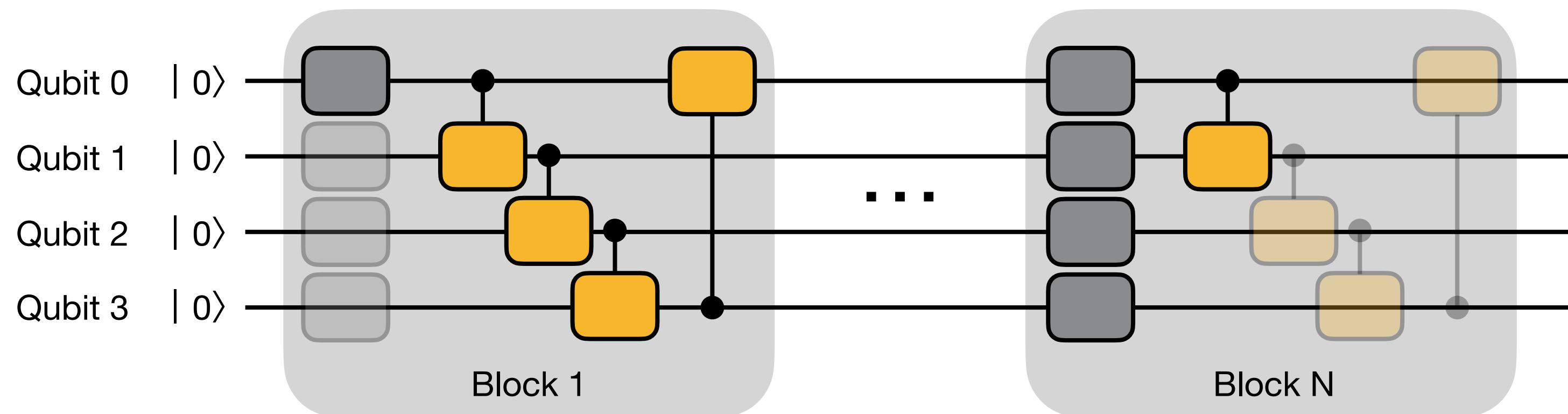
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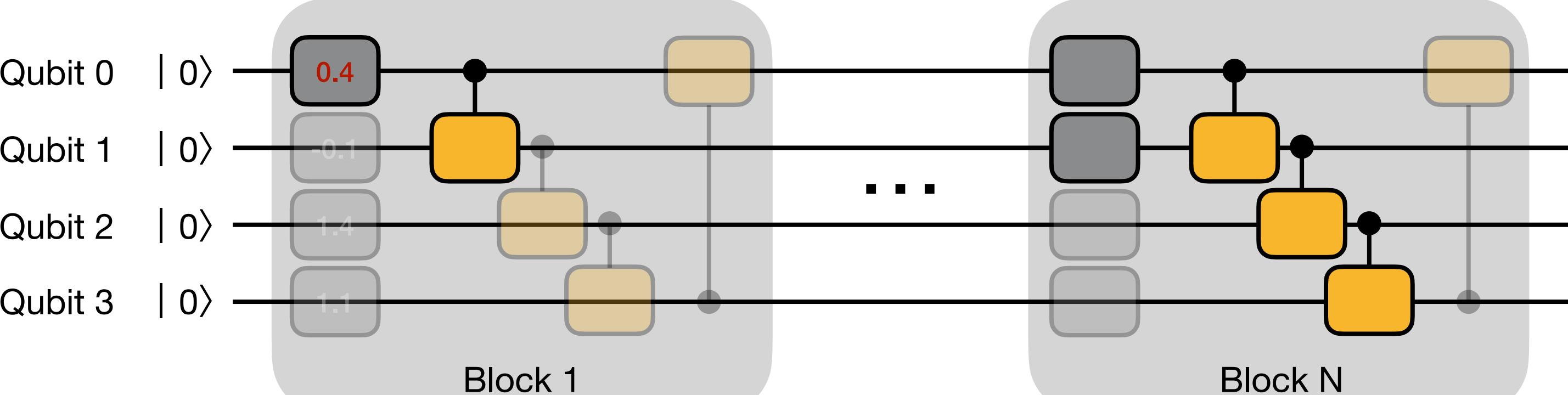
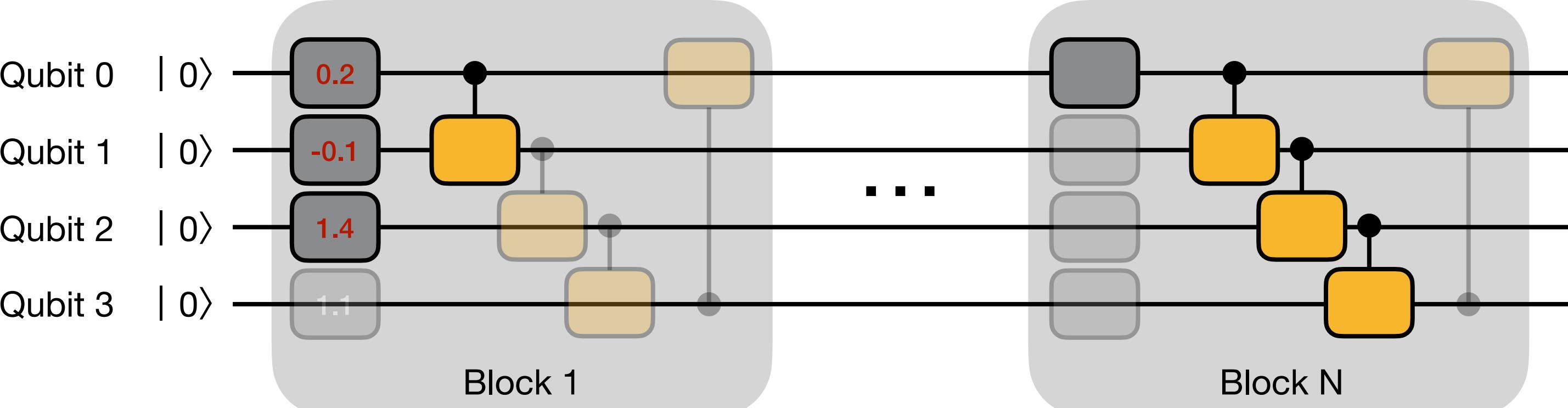
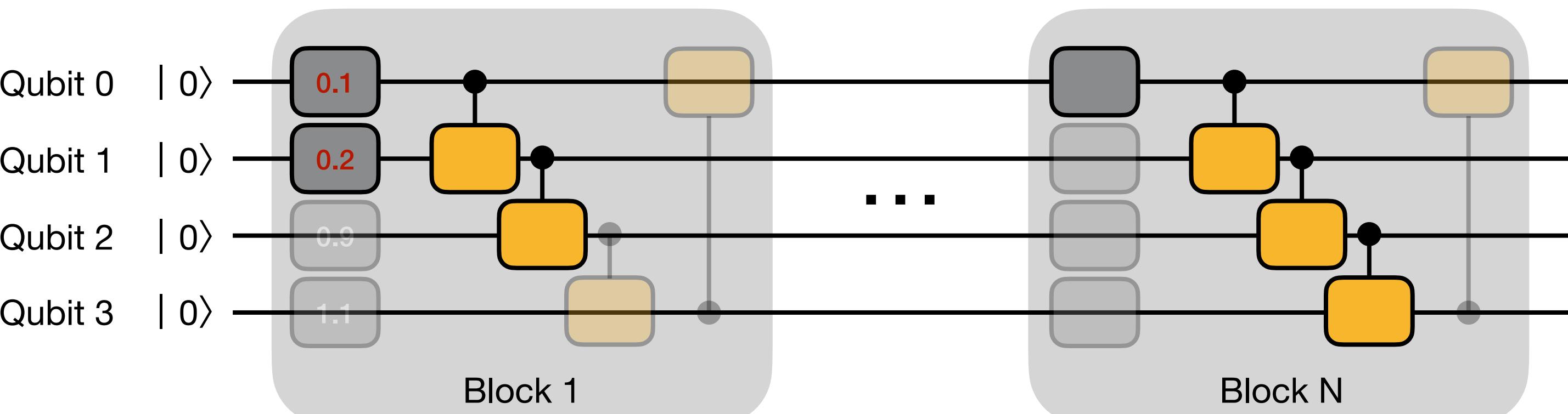


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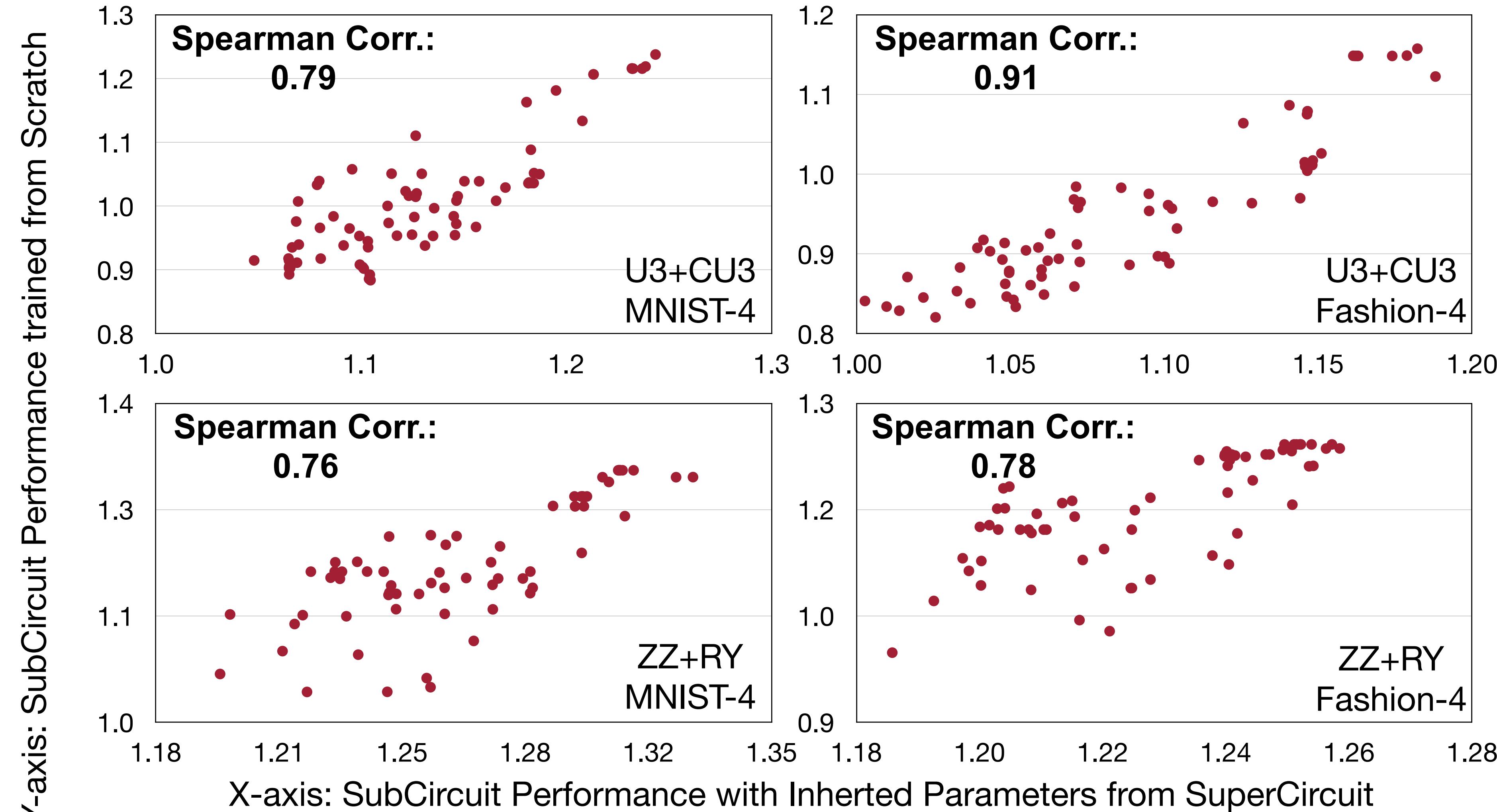


# Train SuperCircuit for Multiple Steps



# How Reliable is the SuperCircuit?

- Inherited parameters from SuperCircuit can provide accurate relative performance

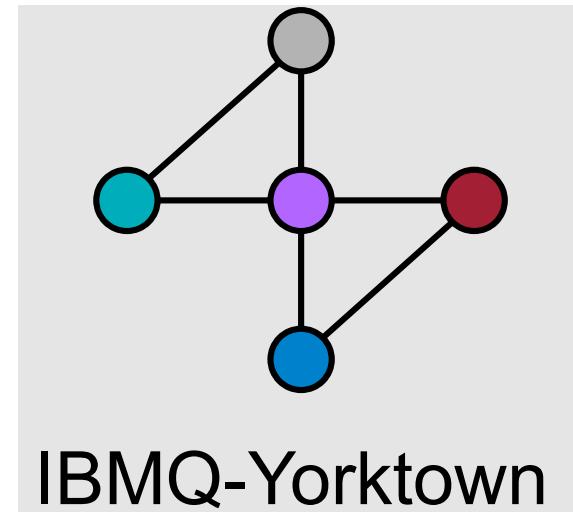


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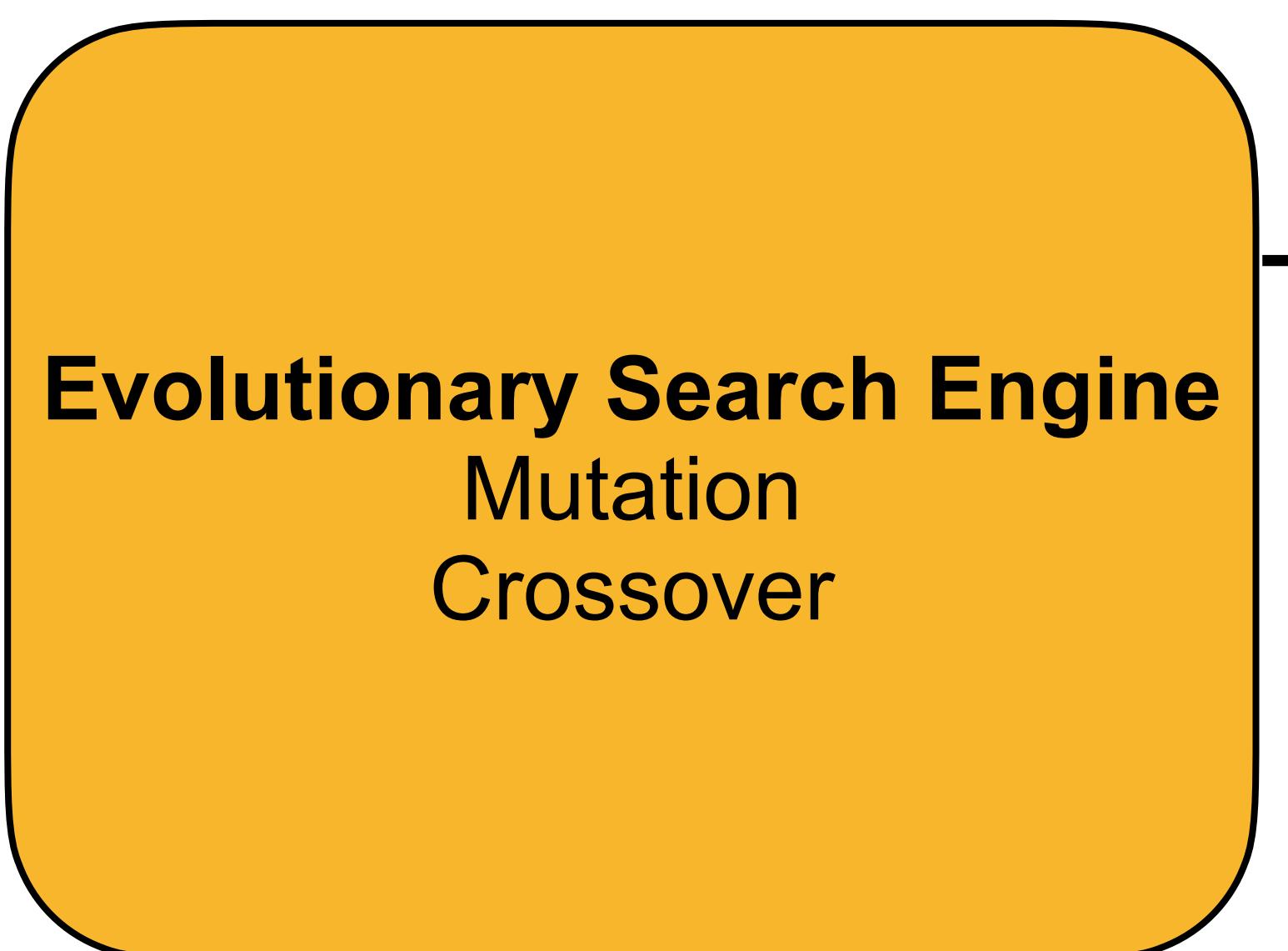
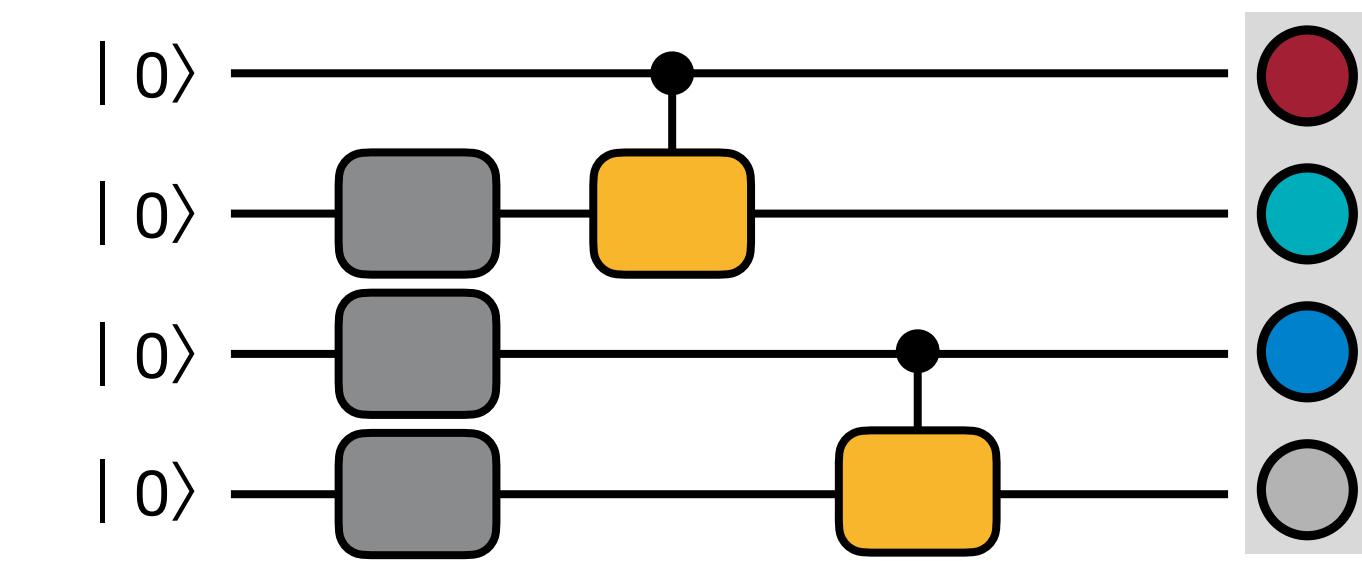
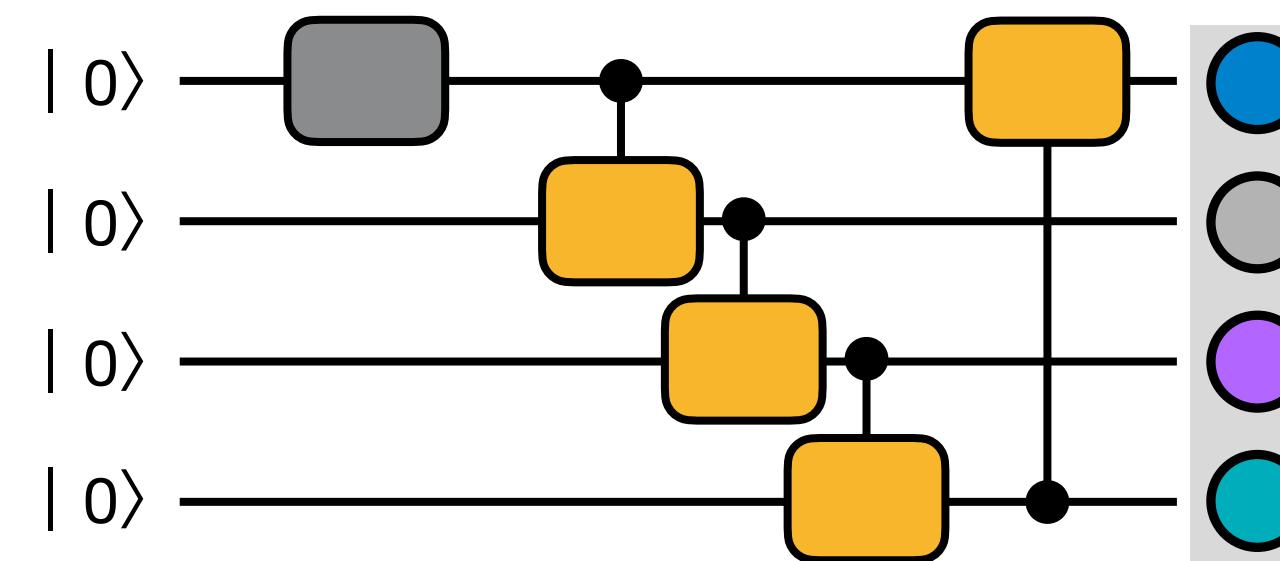
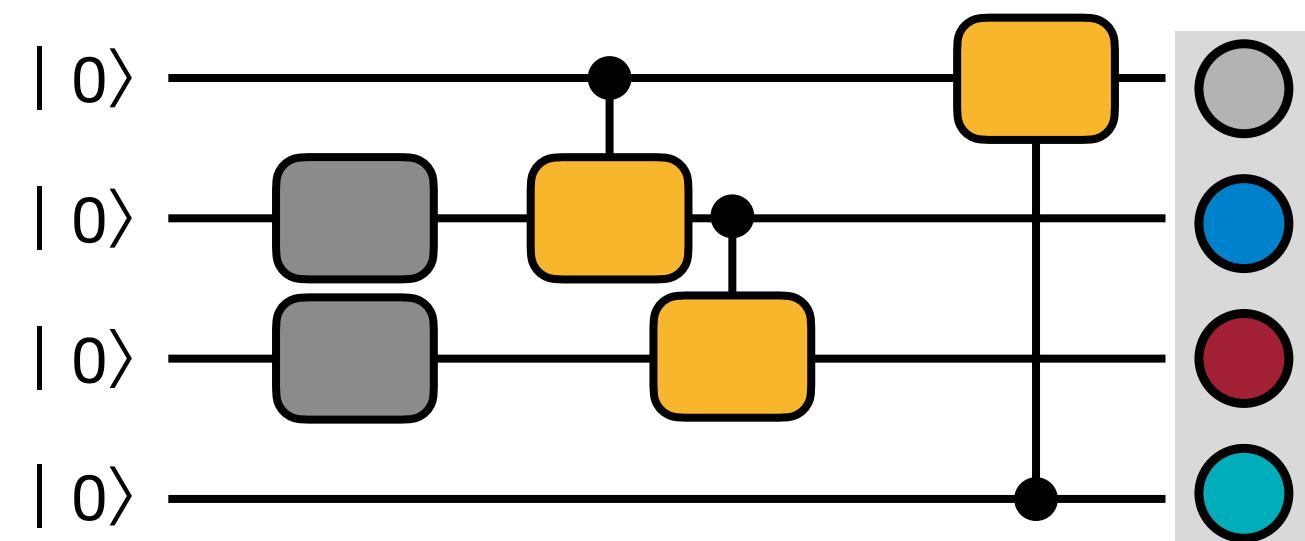
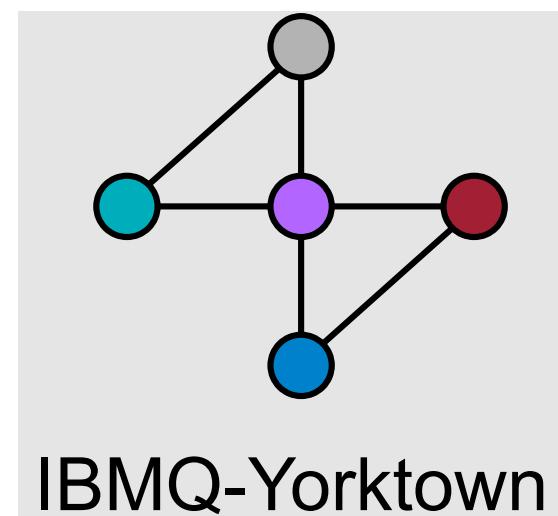
# Noise-Adaptive Evolutionary Co-Search

- Search the best SubCircuit and its qubit mapping on target device

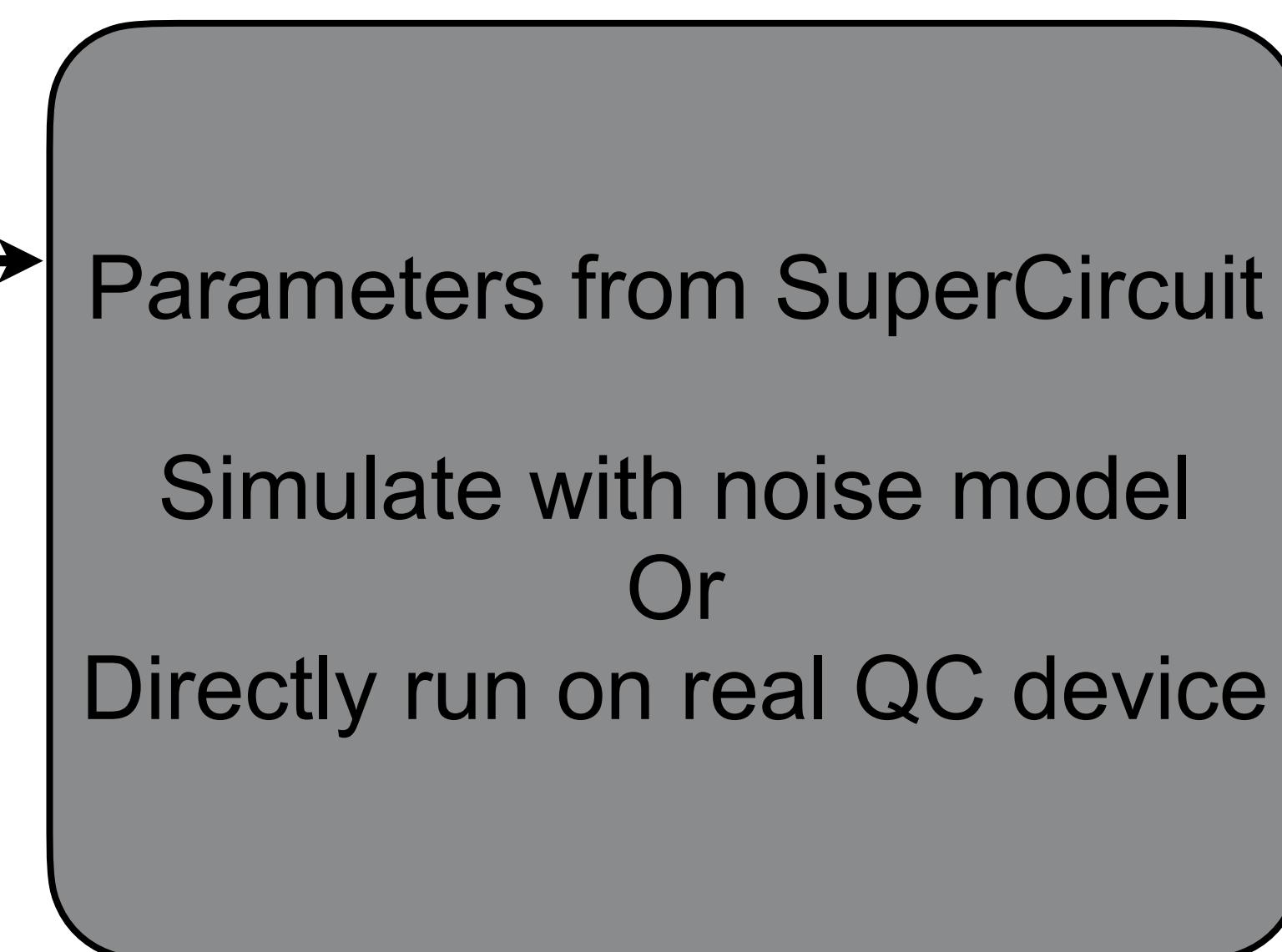


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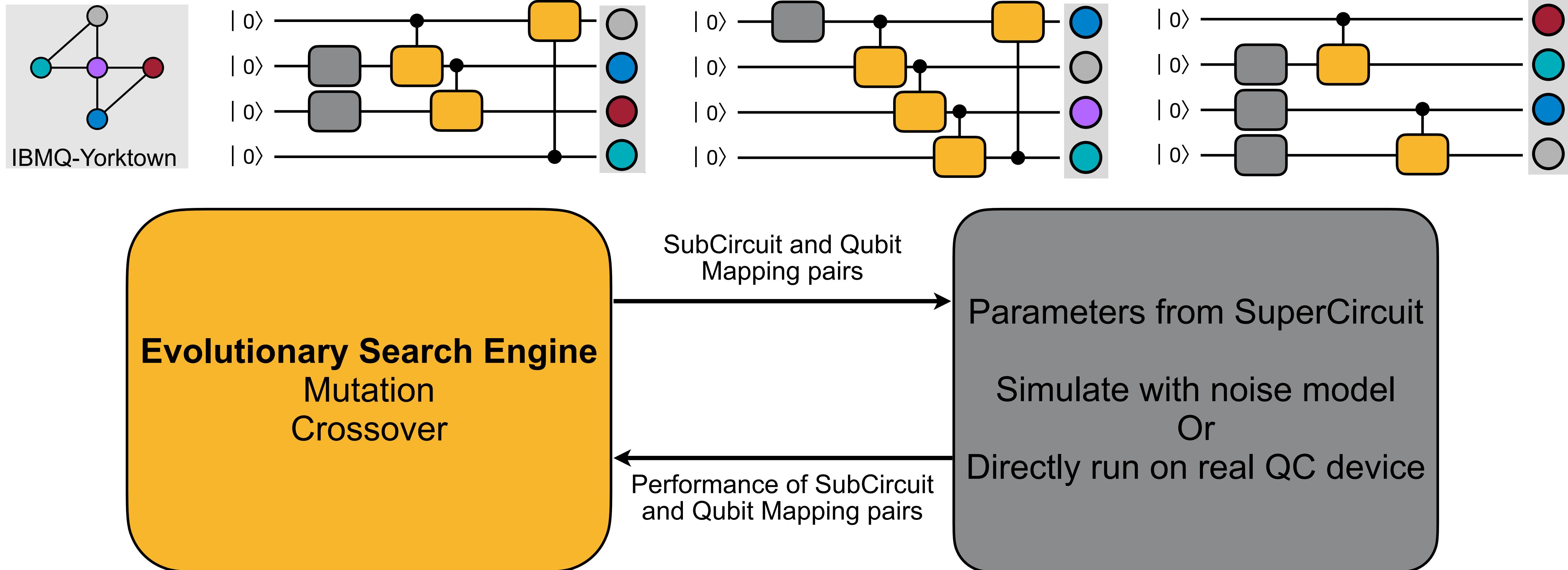


SubCircuit and Qubit  
Mapping pairs



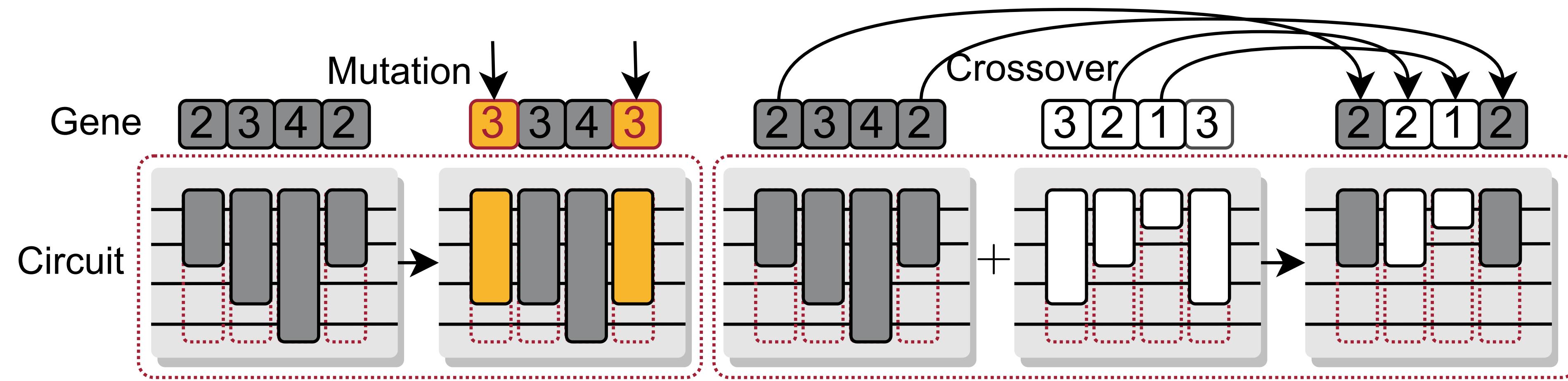
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# Mutation and Crossover

- Mutation and crossover create new SubCircuit candidates



# QuantumNAS

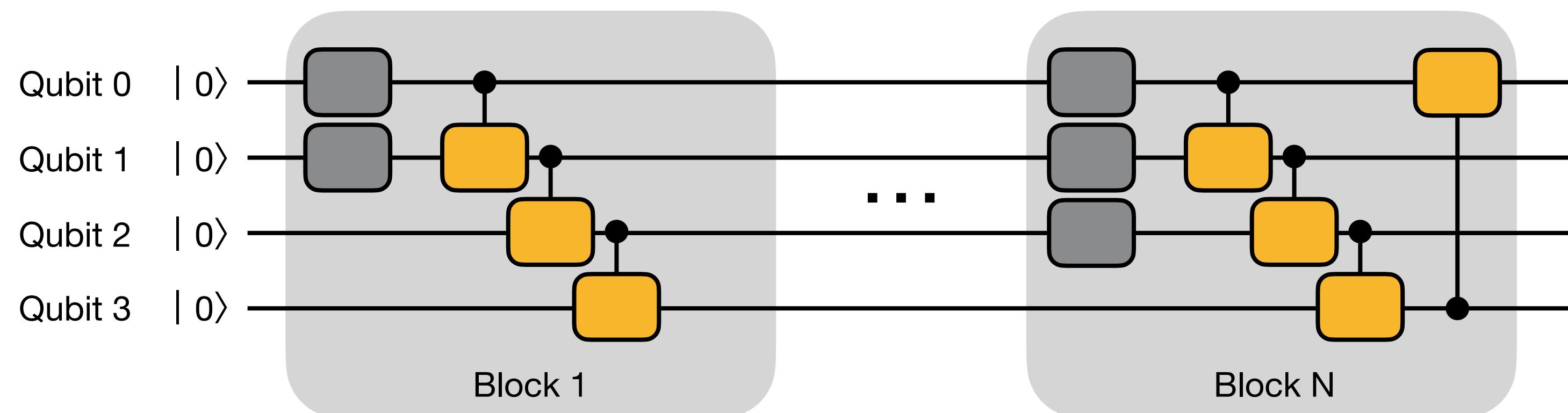
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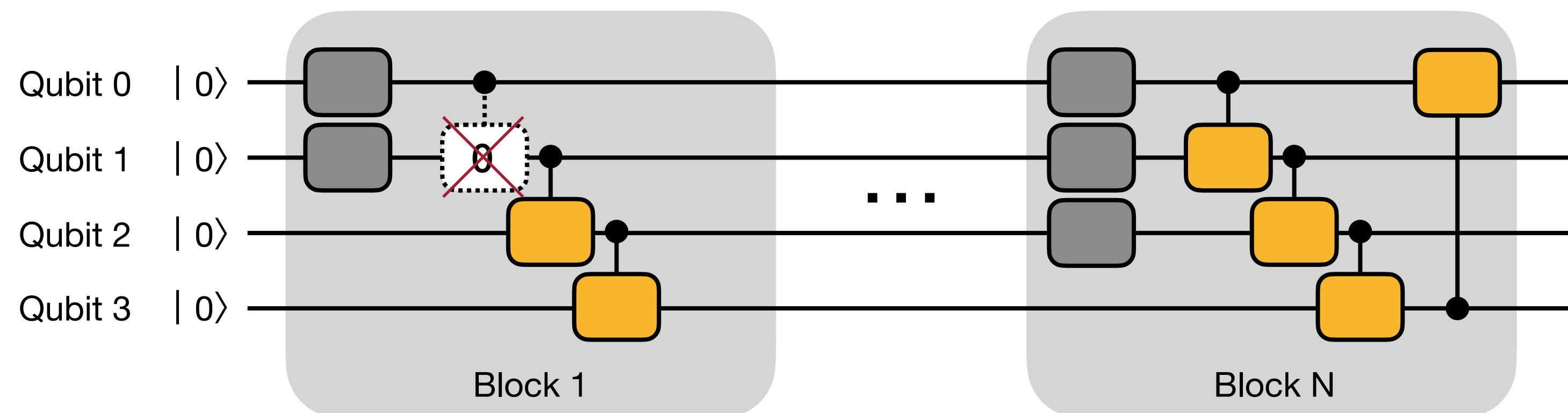
# Iterative Pruning

- Some gates have parameters close to 0
  - Rotation gate with angle close to 0 has small impact on the results
  - Iteratively prune small-magnitude gates and fine-tune the remaining parameters



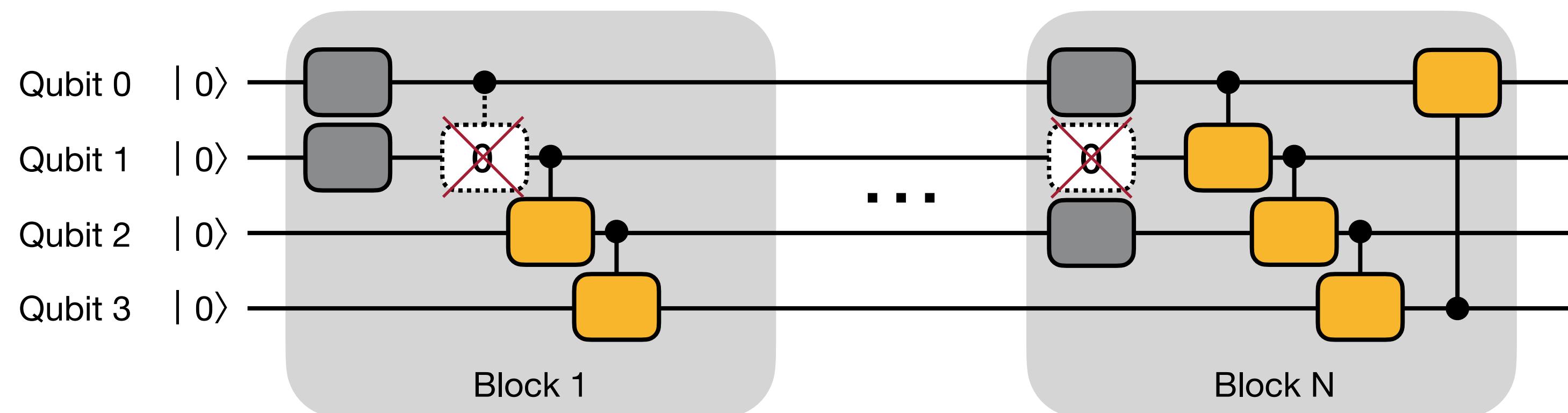
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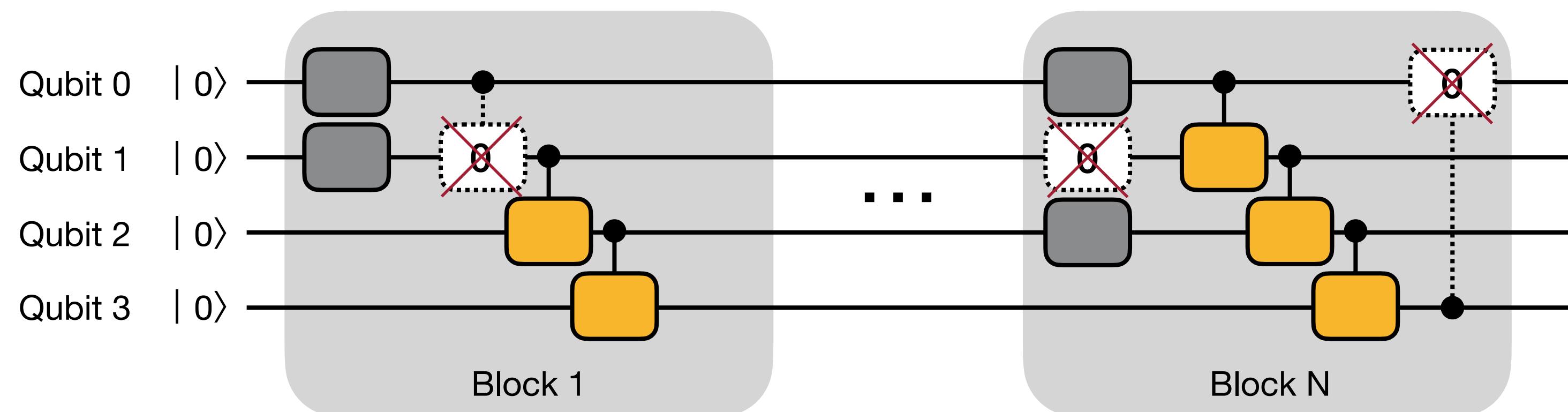
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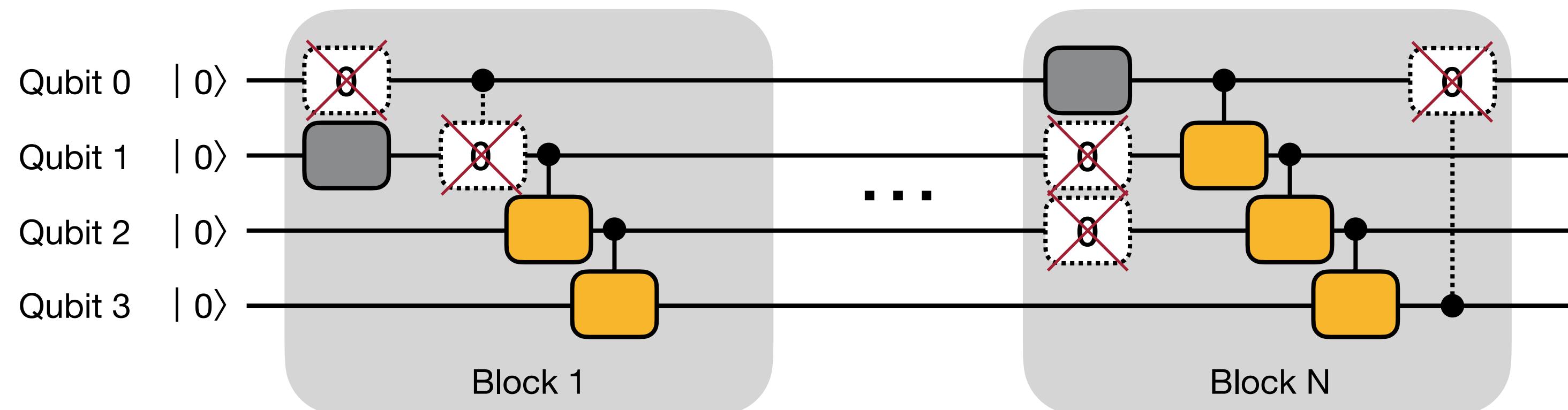
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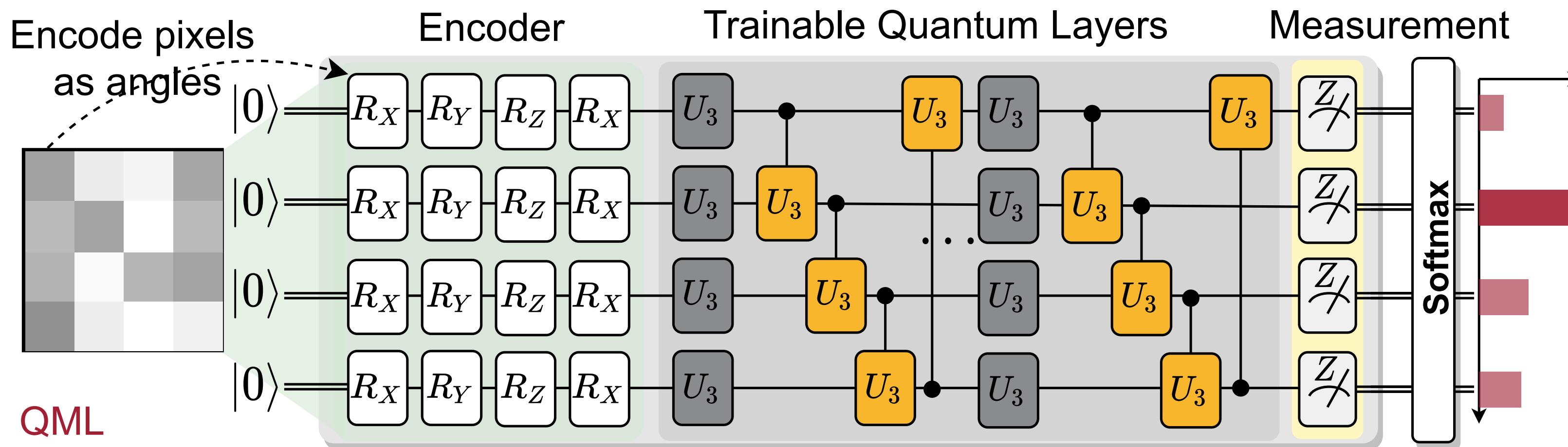


# Evaluation Setups: Benchmarks and Devices

- Benchmarks
  - QML classification tasks: MNIST 10-class, 4-class, 2-class, Fashion 4-class, 2-class, Vowel 4-class
  - VQE task molecules: H<sub>2</sub>, H<sub>2</sub>O, LiH, CH<sub>4</sub>, BeH<sub>2</sub>
- Quantum Devices
  - IBMQ
  - #Qubits: 5 to 65
  - Quantum Volume: 8 to 128

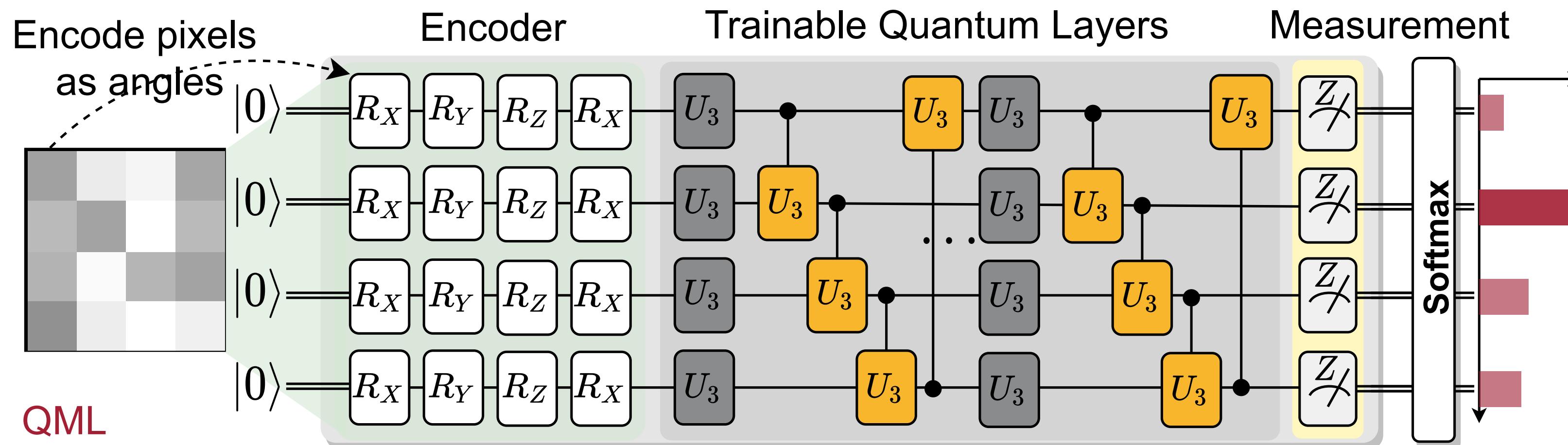
# Benchmarks: QNN and VQE

- Quantum Neural Networks: classification

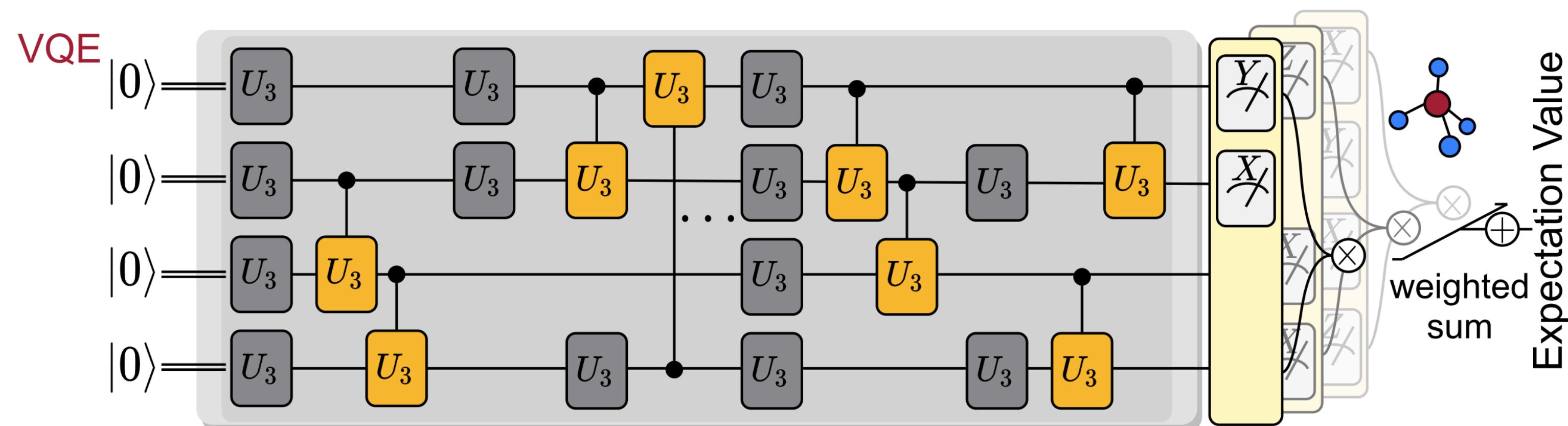


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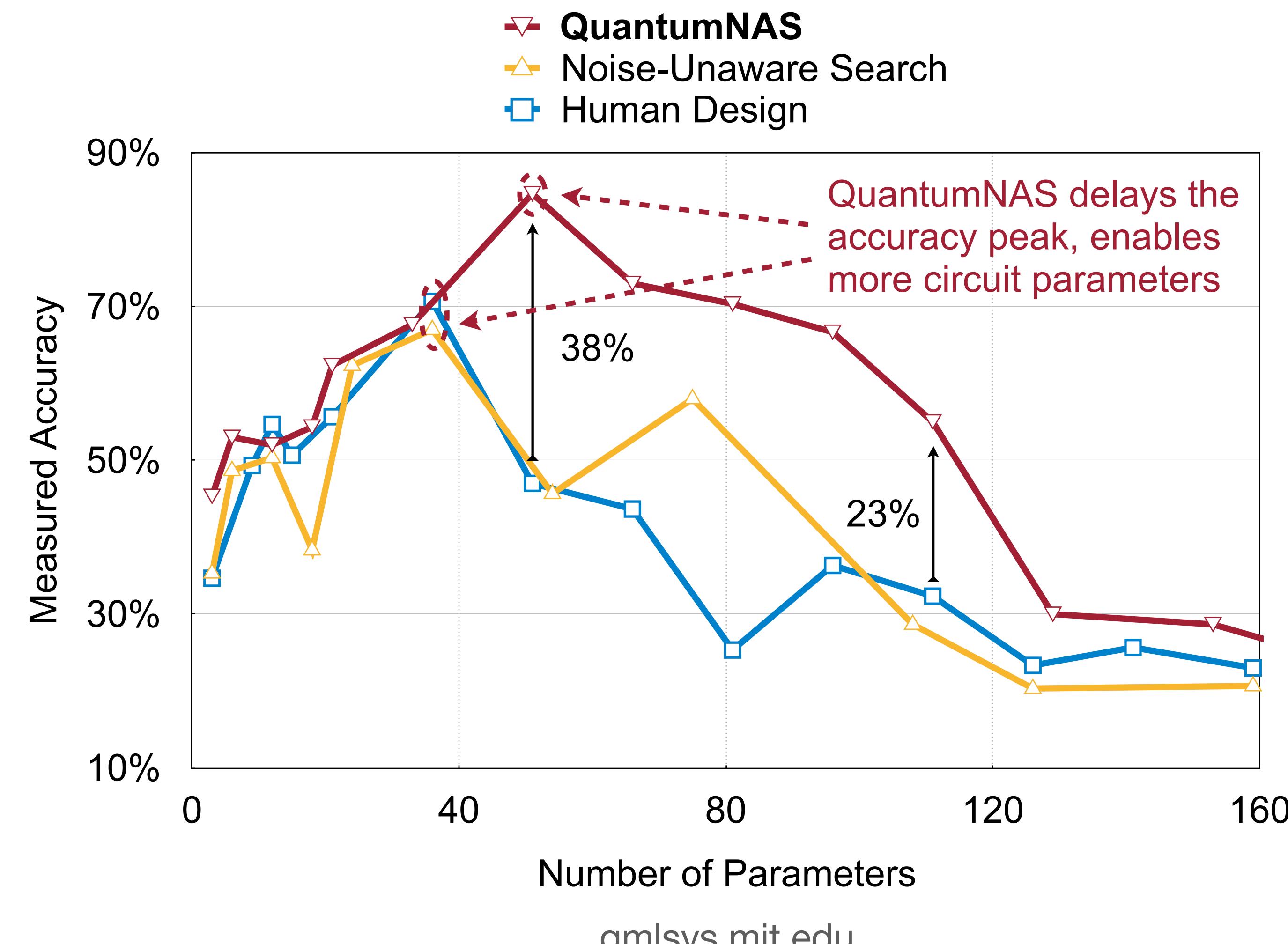


- Variational Quantum Eigensolver: finds the ground state energy of molecule Hamiltonian



# QML Results

- 4-classification: MNIST-4 U3+CU3 on IBMQ-Yorktown



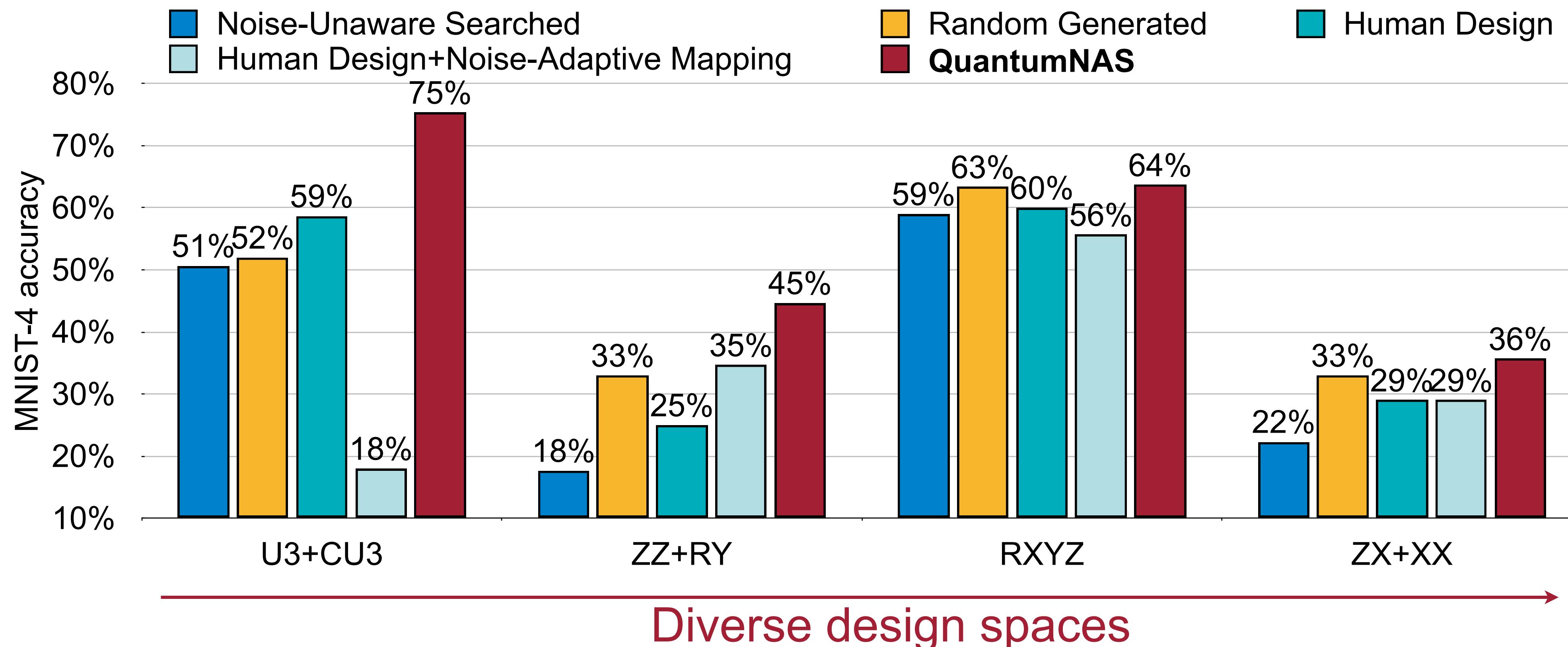
# Analysis of Searched Circuit

- QuantumNAS searched circuit has fewer #gates and #params but higher accuracy

MNIST-2	Depth	#Gates	#Params	QuantumNAS
Human Design	64	135	36	<b>88%</b>
QuantumNAS	70	116	22	<b>92%</b>

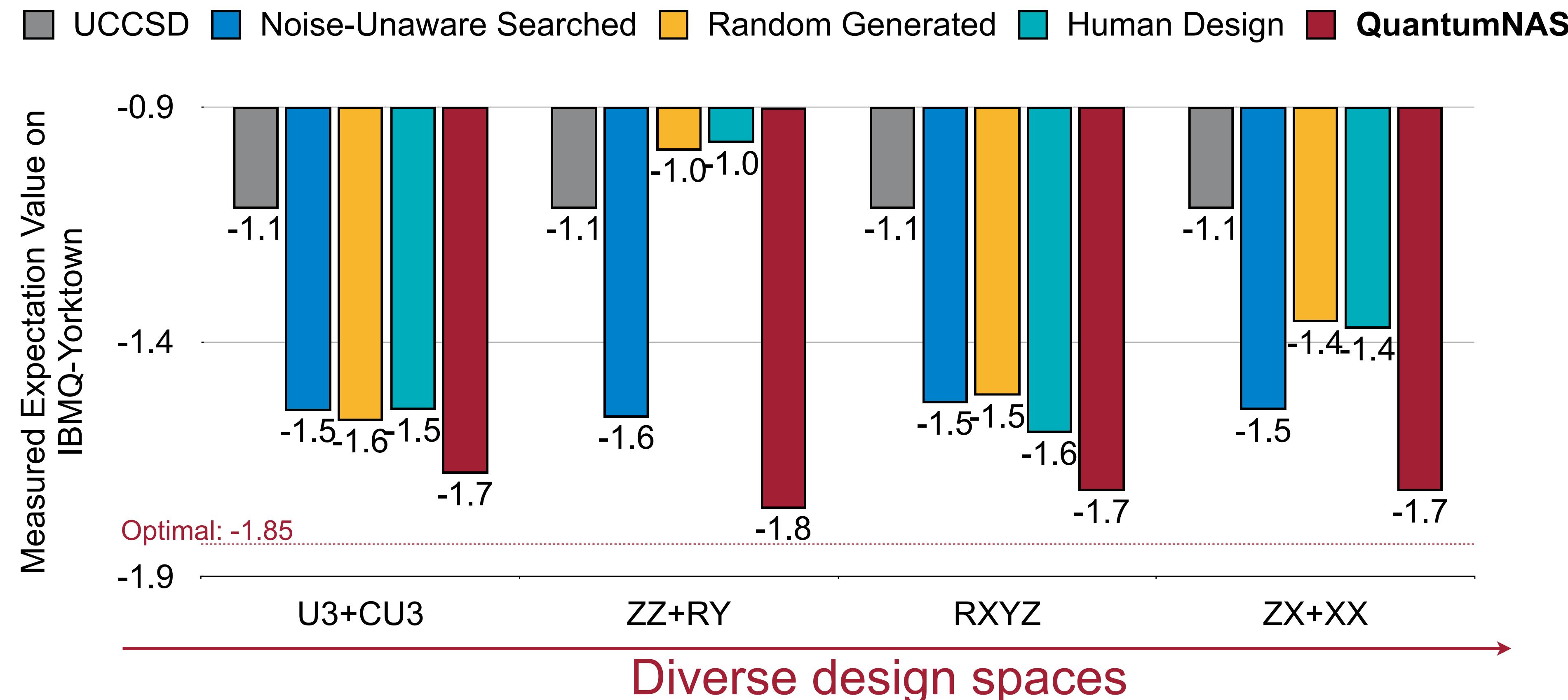
# Consistent Improvements on Diverse Design Spaces

- ‘U3+CU3’, ‘ZZ+RY’, ‘RXYZ’, ‘ZX+XX’ spaces with different gates



# Consistent Improvements on Diverse Design Spaces

- H2 in different design spaces on IBMQ-Yorktown



# Consistent Improvements on Diverse Devices

- QuantumNAS is effective for different real quantum devices
- On different 5-Qubit devices
- MNIST-4, Fashion-4, Vowel-4, MNIST-2, Fashion-2 averaged accuracy

Diverse devices



Method	Noise-Unaware Searched	Random	Human	QuantumNAS
Belem (5Q, 16QV)	47%	50%	67%	<b>76%</b>
Quito (5Q, 16QV)	73%	68%	74%	<b>79%</b>
Athens (5Q, 32QV)	50%	63%	68%	<b>77%</b>
Santiago (5Q, 32QV)	74%	73%	75%	<b>80%</b>

# Scalable to Large #Qubits

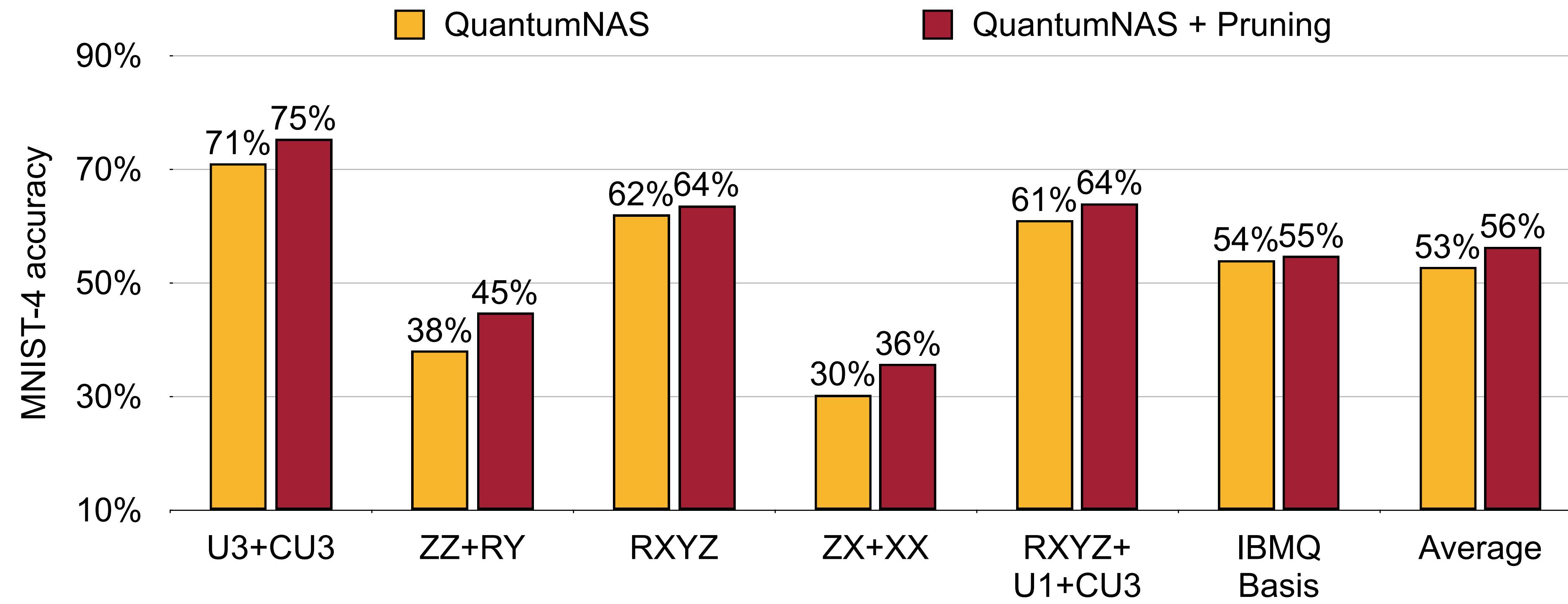
- On large devices
- MNIST-10 accuracy

More Qubits

Method	Noise-Unaware Searched	Random	Human	QuantumNAS
Melbourne (15Q, 8QV, use 15Q)	11%	10%	15%	<b>32%</b>
Guadalupe (16Q, 32QV, use 16Q)	14%	12%	10%	<b>15%</b>
Montreal (27Q, 128QV, use 21Q)	13%	7%	14%	<b>16%</b>
Manhattan (65Q, 32QV, use 21Q)	11%	11%	15%	<b>18%</b>

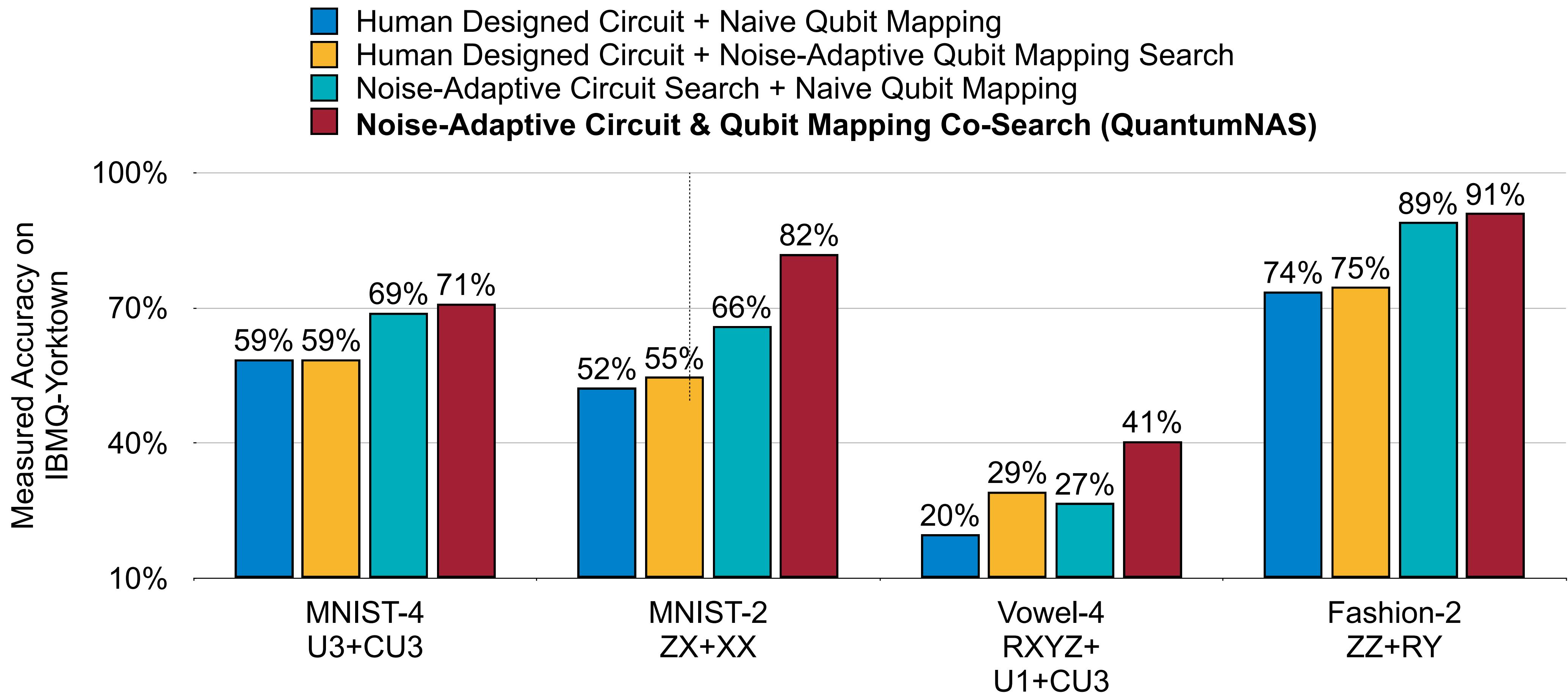
# Effective of Quantum Gate Pruning

- For MNIST-4, Quantum gate pruning improves accuracy by 3% on average



# Effect of Circuit & Qubit Mapping Co-Search

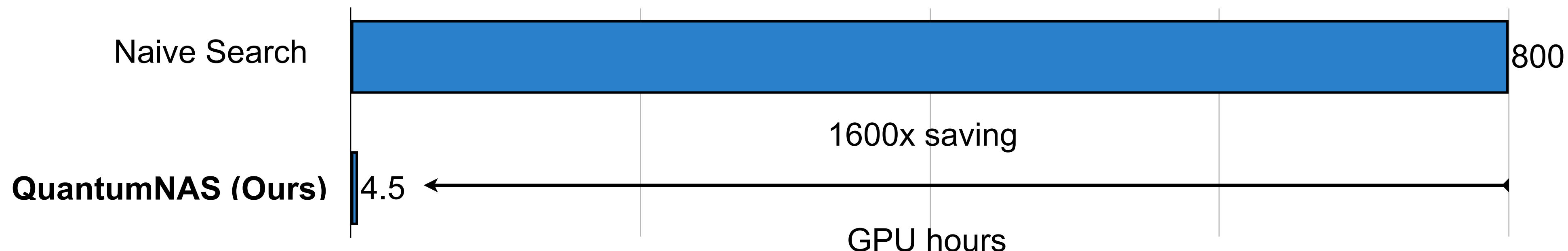
- Co-search is better than only search architecture/mapping



# Time Cost

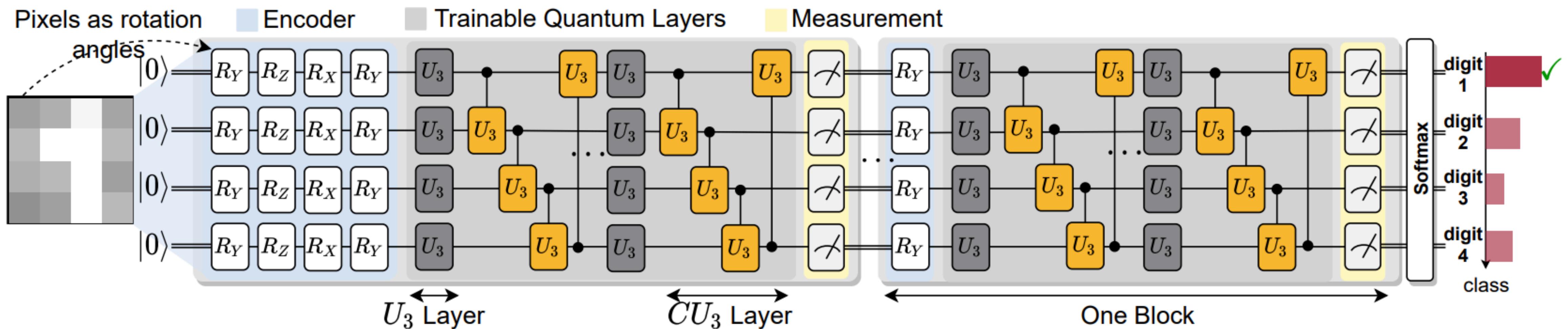
- On 1 Nvidia Titan RTX 2080 ti GPU

#qubits	Step	SuperCircuit Training	Noise-Adaptive Co-search	SubCircuit Training	Deployment on Real QC
4 Qubits		0.5h	3h	0.5h	0.5h
15 Qubits		5h	5h	5h	1h
21 Qubits		20h	10h	15h	1h



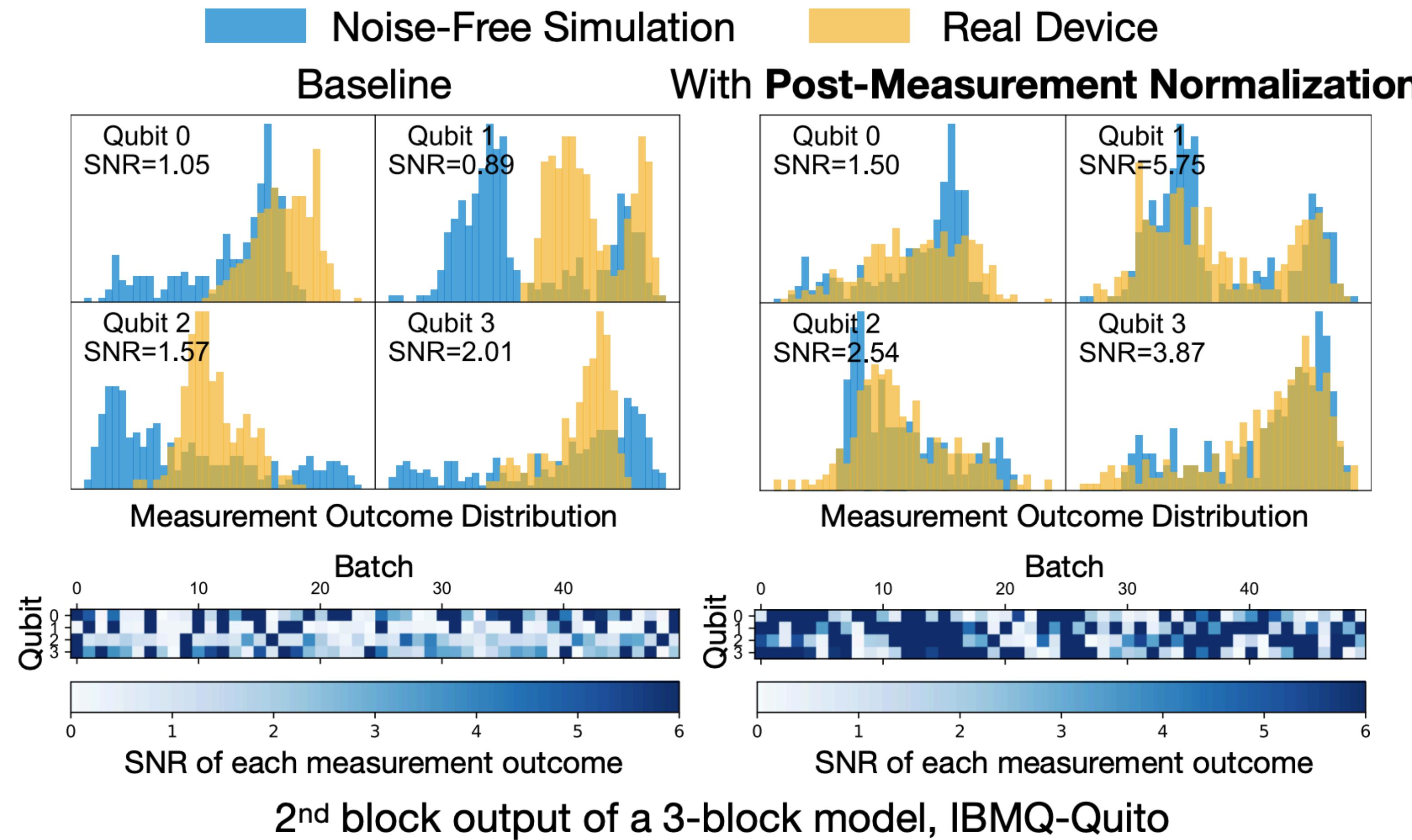
# QuantumNAT

- QuantumNAS: find the **circuit architecture** robust to noise
- QuantumNAT: further make the **parameter** robust to noise



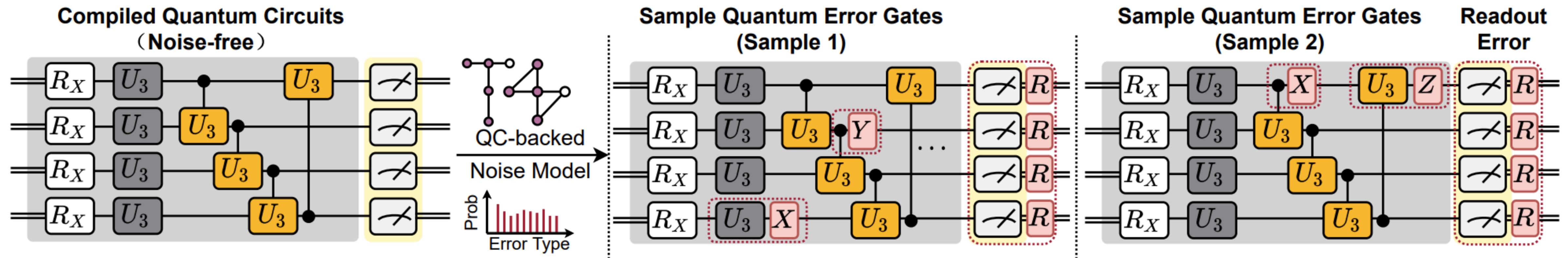
# Post-Measurement Normalization

- Normalize the measurement outcomes on the batch dimension



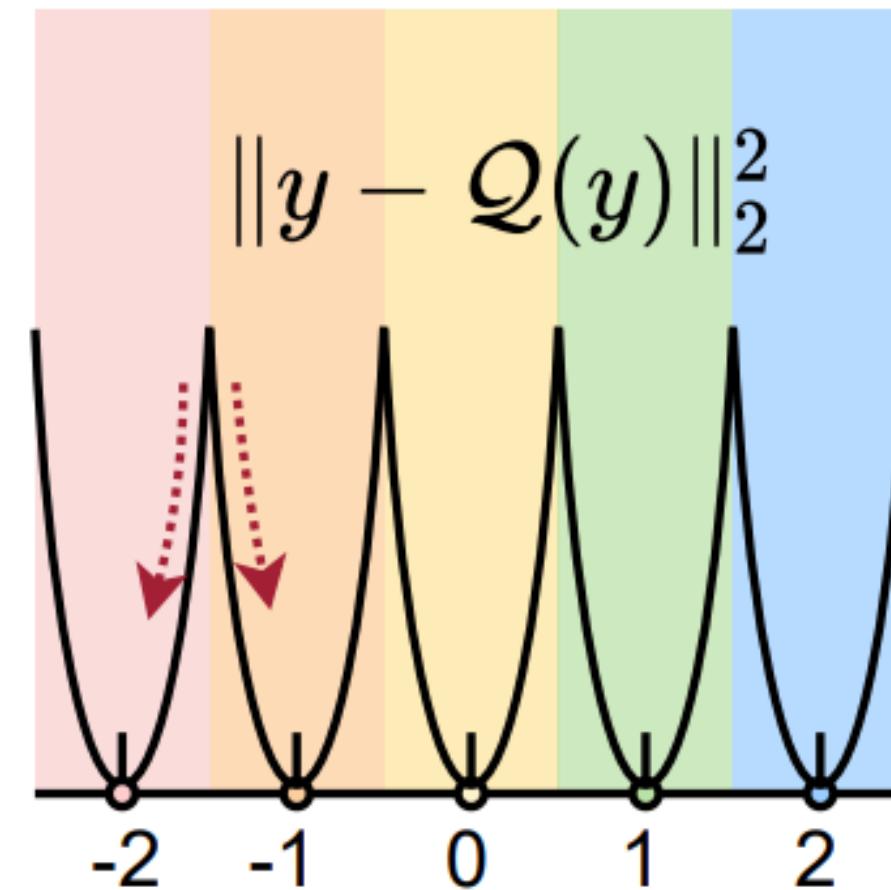
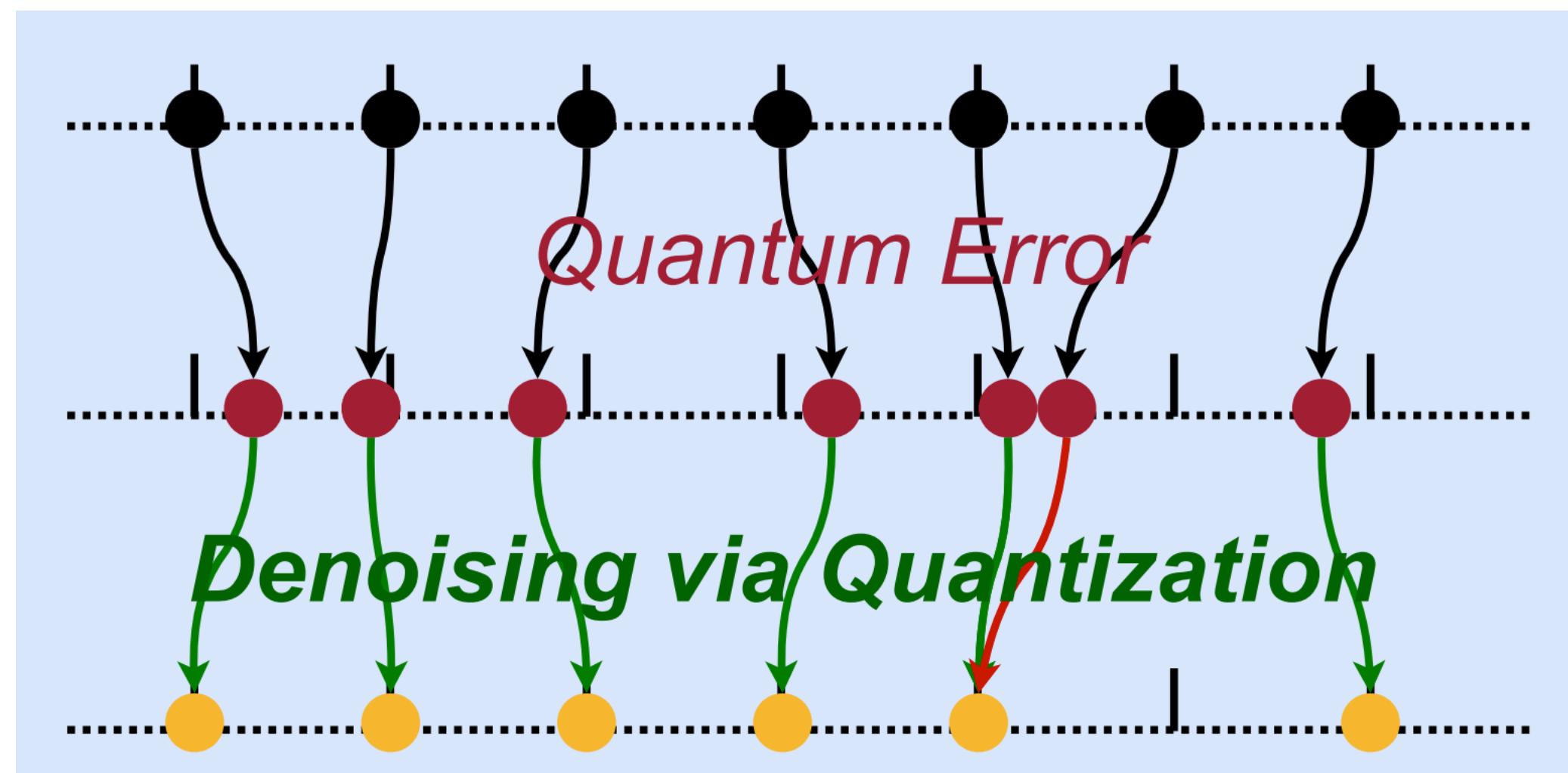
# Noise Injection

- Insert noise gate during training, according to the noise model
- For each step, sample new positions for noise gates



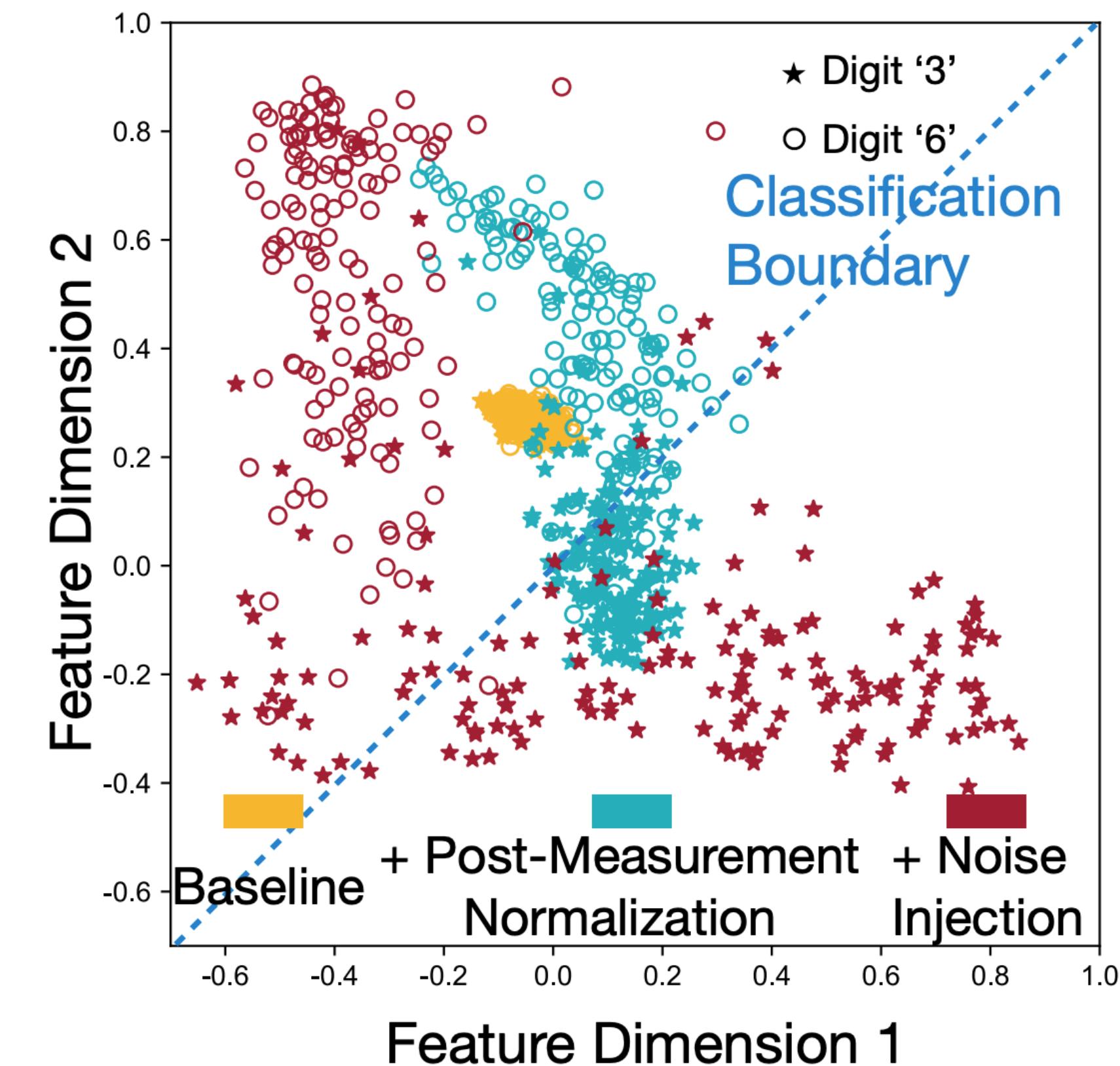
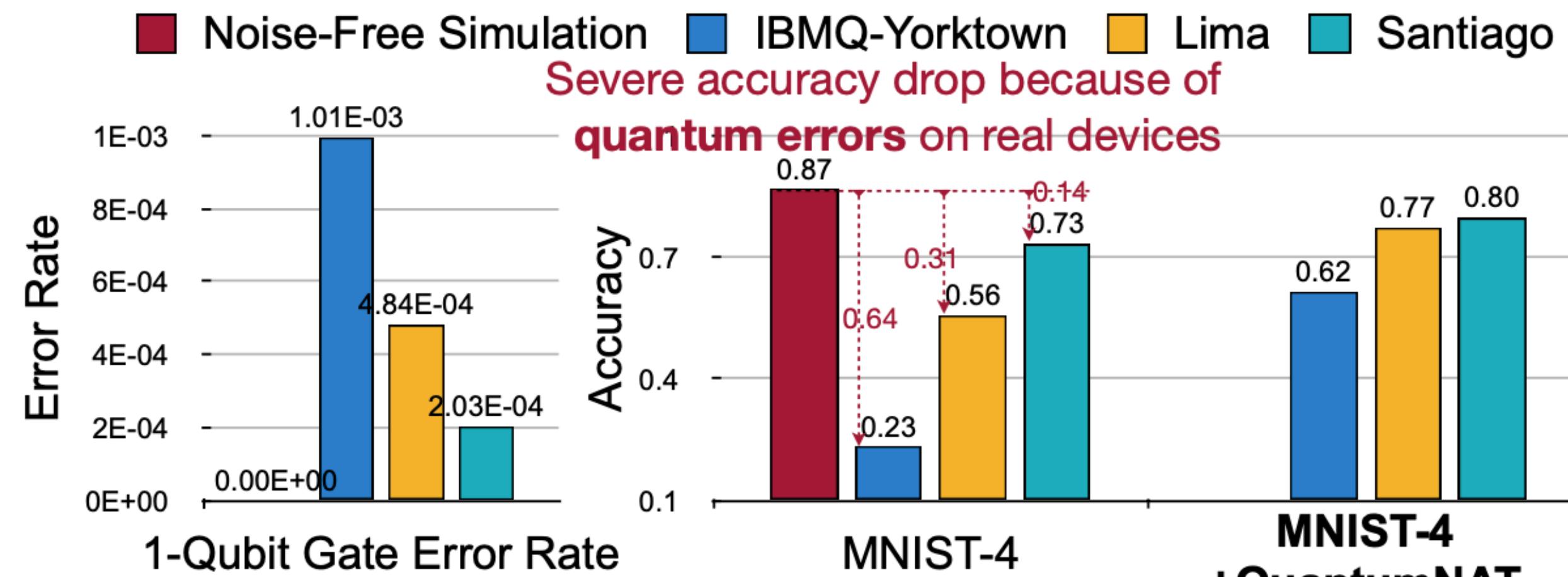
# Post-Measurement Quantization

- Quantization provides denoising effects
- Quadratic penalty loss to encourage measurement outcomes close to quantization centroids



# Evaluation

- On classification tasks with QNN

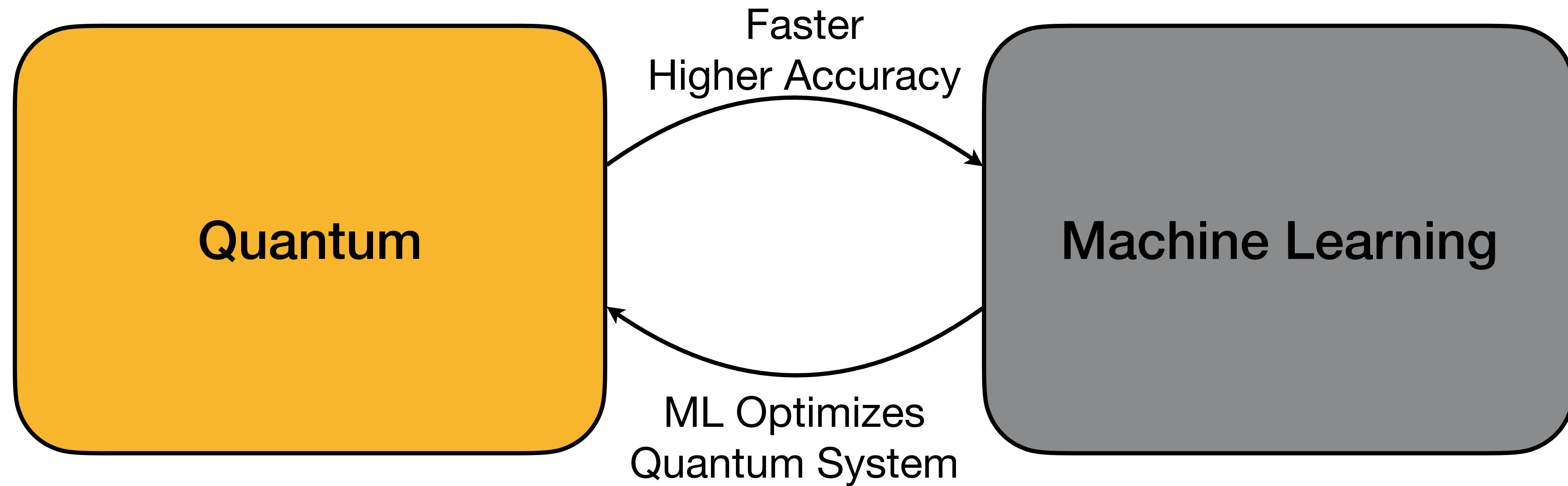




Torch  
Quantum

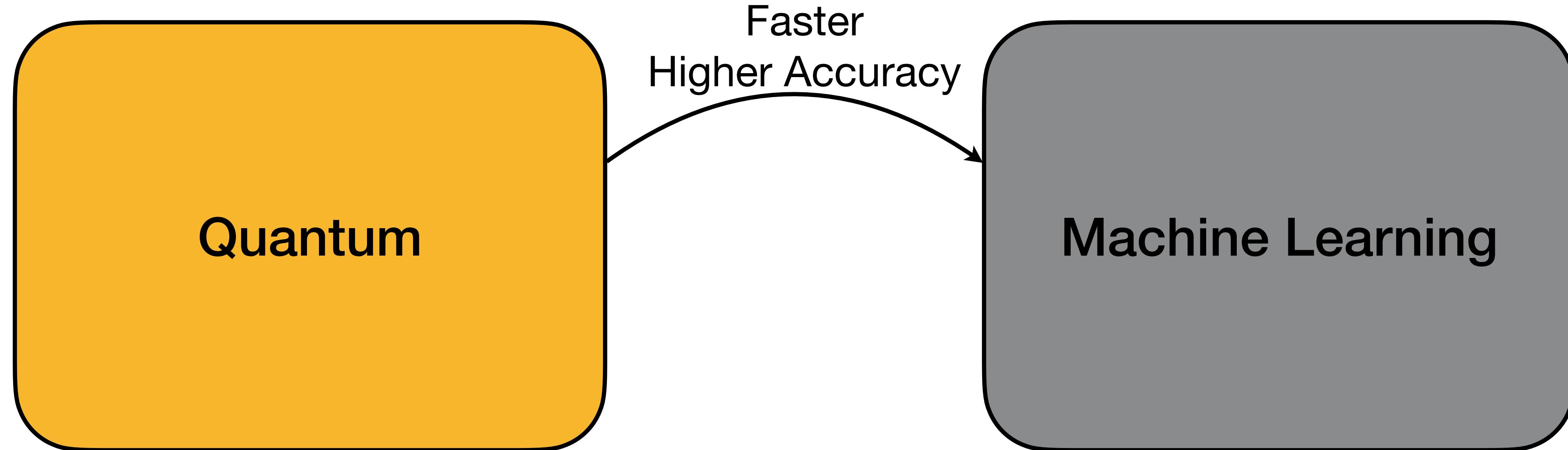
# Open-source: TorchQuantum

- TorchQuantum — An open-source library for interdisciplinary research of quantum computing and machine learning
- <https://github.com/mit-han-lab/torchquantum>



# Open-source: TorchQuantum

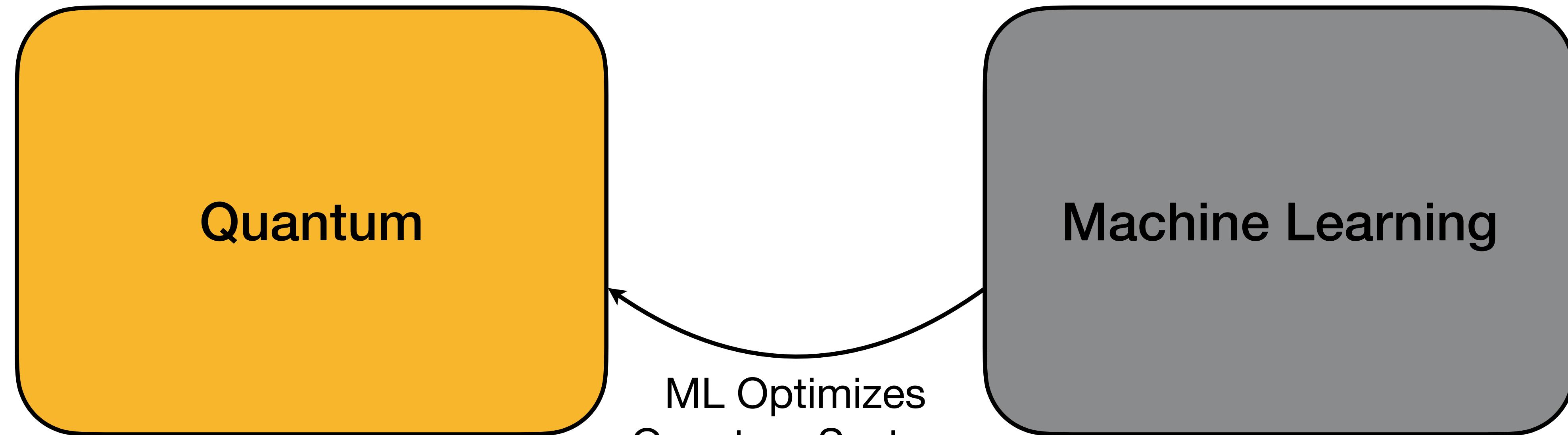
- TorchQuantum — An open-source library for interdisciplinary research of quantum computing and machine learning
- <https://github.com/mit-han-lab/torchquantum>



- Quantum for Machine learning
  - Quantum neural networks
  - Quantum kernel methods

# Open-source: TorchQuantum

- TorchQuantum — An open-source library for interdisciplinary research of quantum computing and machine learning
- <https://github.com/mit-han-lab/torchquantum>



- Machine Learning for Quantum
  - ML for quantum compilation (qubit mapping, unitary synthesis)
  - ...

# TorchQuantum

- Features
  - Easy construction of **parameterized quantum circuits** such as Quantum Neural Networks in PyTorch
  - Support **batch mode inference and training** on GPU/CPU, supports highly-parallelized training
  - Support **easy deployment** on real quantum devices such as IBMQ
  - Provide tutorials, videos and example projects of QML and using ML to optimize quantum computer system problems

# PyTorch Implementations

- Statevector

```
_state = torch.zeros(2 ** self.n_wires, dtype=C_DTYPE)
_state[0] = 1 + 0j
```

- Quantum Gates

```
'cnot': torch.tensor([[1, 0, 0, 0],
                      [0, 1, 0, 0],
                      [0, 0, 0, 1],
                      [0, 0, 1, 0]], dtype=C_DTYPE),
```

# PyTorch Implementations

- Quantum Gates

```
def crx_matrix(params):
    theta = params.type(C_DTYPE)
    co = torch.cos(theta / 2)
    jsi = 1j * torch.sin(-theta / 2)

    matrix = torch.tensor([[1, 0, 0, 0],
                          [0, 1, 0, 0],
                          [0, 0, 0, 0],
                          [0, 0, 0, 0]], dtype=C_DTYPE, device=params.device
                         ).unsqueeze(0).repeat(co.shape[0], 1, 1)

    matrix[:, 2, 2] = co[:, 0]
    matrix[:, 2, 3] = jsi[:, 0]
    matrix[:, 3, 2] = jsi[:, 0]
    matrix[:, 3, 3] = co[:, 0]

    return matrix.squeeze(0)
```

- Matrix-vector multiplication:  
torch.einsum and torch.bmm

# Examples and tutorials

- Tutorial Colab and videos



## TorchQuantum Tutorials Opening

Hanrui Wang  
MIT HAN Lab



## TorchQuantum Tutorials Quanvolutional Neural Network

Zirui Li, Hanrui Wang  
MIT HAN Lab



MIT HAN LAB



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# Thank you for listening!

- **QuantumNAS** exploits **SuperCircuit**-based co-search for most **noise-robust** circuit architecture and qubit mapping
- Iterative **quantum gate pruning** to further remove redundant gates
- **QuantumNAT** exploits noise injection, post-measurement Normalization, quantization to improve robustness of parameters
- Improves MNIST 2-class accuracy from 88% to **95%**, 10-class from 15% to **34%**
- Save search cost by over **1,000 times**
- Open-sourced **TorchQuantum** library for Quantum + ML research



Torch  
Quantum

<https://github.com/mit-han-lab/torchquantum>

[qmlsys.mit.edu](http://qmlsys.mit.edu)



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