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量子计算参数化电路搜索QuantumNAS的设计及实现

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[HPCA'22] QuantumNAS: Noise-adaptive search for robust quantum circuits

[DAC'22] QuantumNAT: Quantum Noise-Aware Training with Noise Injection, Quantization and Normalization

[DAC'22] QOC: Quantum On-Chip Training with Parameter Shift and Gradient Pruning

Outline

- Background
- QuantumNAS
- QuantumNAT
- TorchQuantum Library
- Conclusion

Quantum Bit

- Quantum Bit (Qubit)
 - Statevector: contains 2^n complex numbers for n qubit system
 - The square sum of magnitude of 2^n numbers are 1
 - 1 qubit:

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad a_0, a_1 \in \mathbb{C}$$
$$|a_0|^2 + |a_1|^2 = 1$$

- 2 qubits:
$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad a_0, a_1, a_2, a_3 \in \mathbb{C}$$
$$|a_0|^2 + |a_1|^2 + |a_2|^2 + |a_3|^2 = 1$$

Quantum Bit

- Classical bits represented in statevector

- Classical 0:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- Classical 1:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- An arbitrary quantum states:

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = a_0 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_1 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Quantum Gates

- Qubit gates: operations on one qubit or multiple qubits
- The qubit gates can be represented with matrix format with dimension $2^n \times 2^n$
- All gate matrices are unitary matrices: the conjugate transpose is the same as its inverse
- Single qubit gates:
 - Not (X) gate:

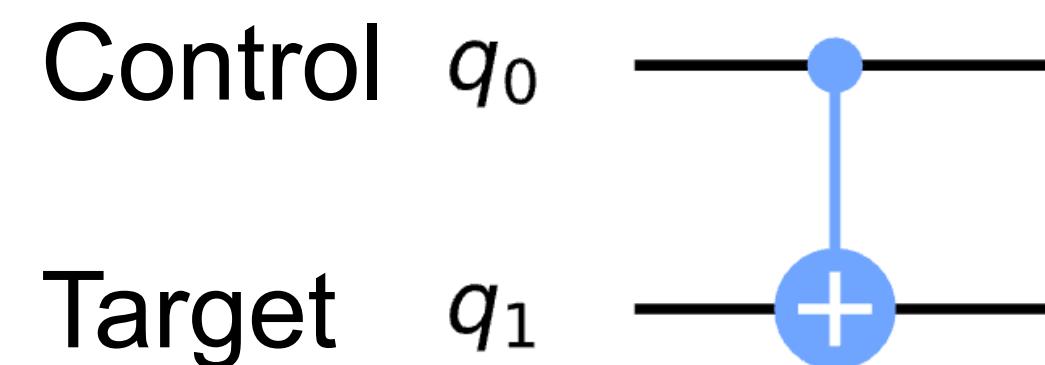
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- Parameterized gate: Rotation X (RX) with parameter theta

$$RX(\theta) = \begin{bmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}$$

Quantum Gates

- 2-qubit gates:
 - Controlled Not (CNOT) gate:



Input	Output			
	00	01	10	11
00	1	0	0	0
01	0	1	0	0
10	0	0	0	1
11	0	0	1	0

- Controlled Rotation X (CRX) gate

$$CRX(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ 0 & 0 & -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}$$

Quantum Gates

- Applying a gate to qubits is performing matrix-vector multiplication between the gate matrix and statevector
 - Apply an X gate to classical state 0, we get 1

$$X \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Apply an CNOT gate to state 10, we get 11

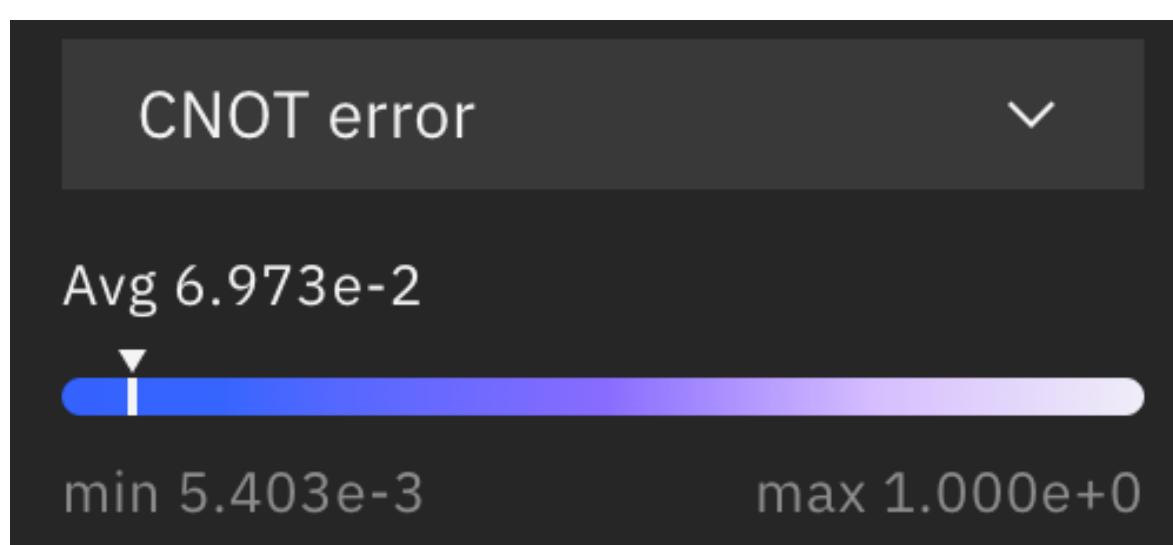
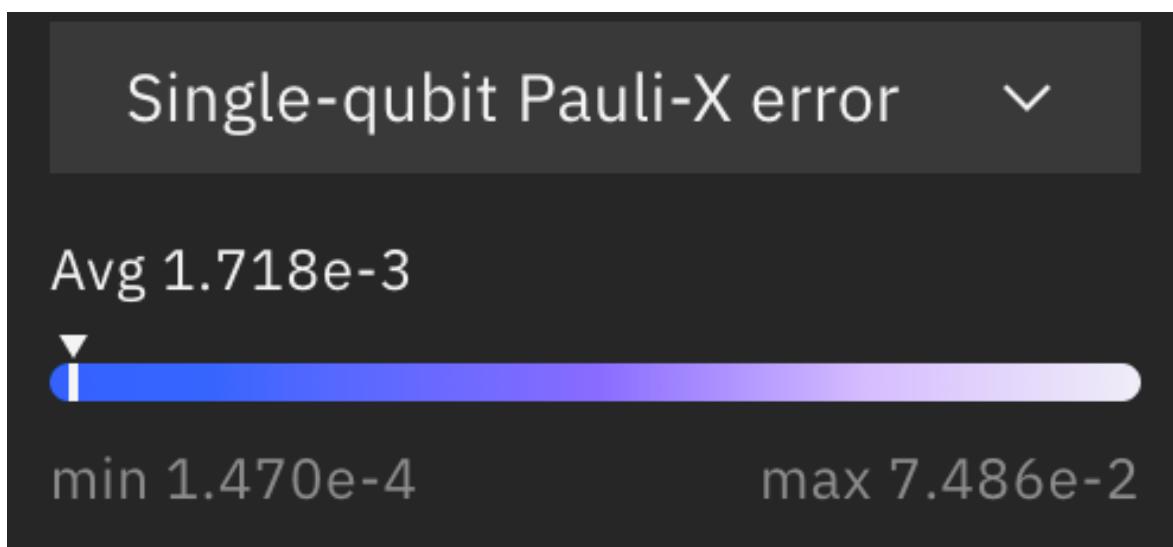
$$CNOT \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Source of Quantum Advantage

- One qubit carries more information than one classic bit
- The statevector length is exponentially to the number of qubits

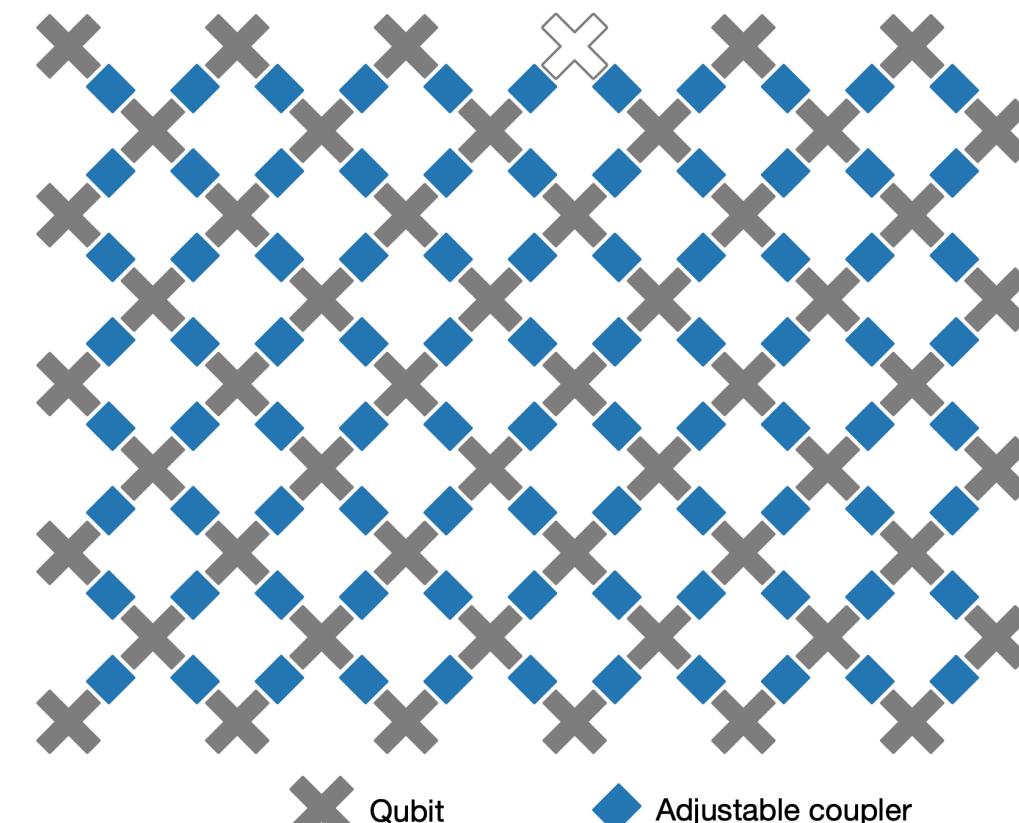
NISQ Era

- Noisy Intermediate-Scale Quantum (NISQ)
 - **Noisy**: qubits are sensitive to environment; quantum gates are unreliable
 - **Limited number** of qubits: tens to hundreds of qubits



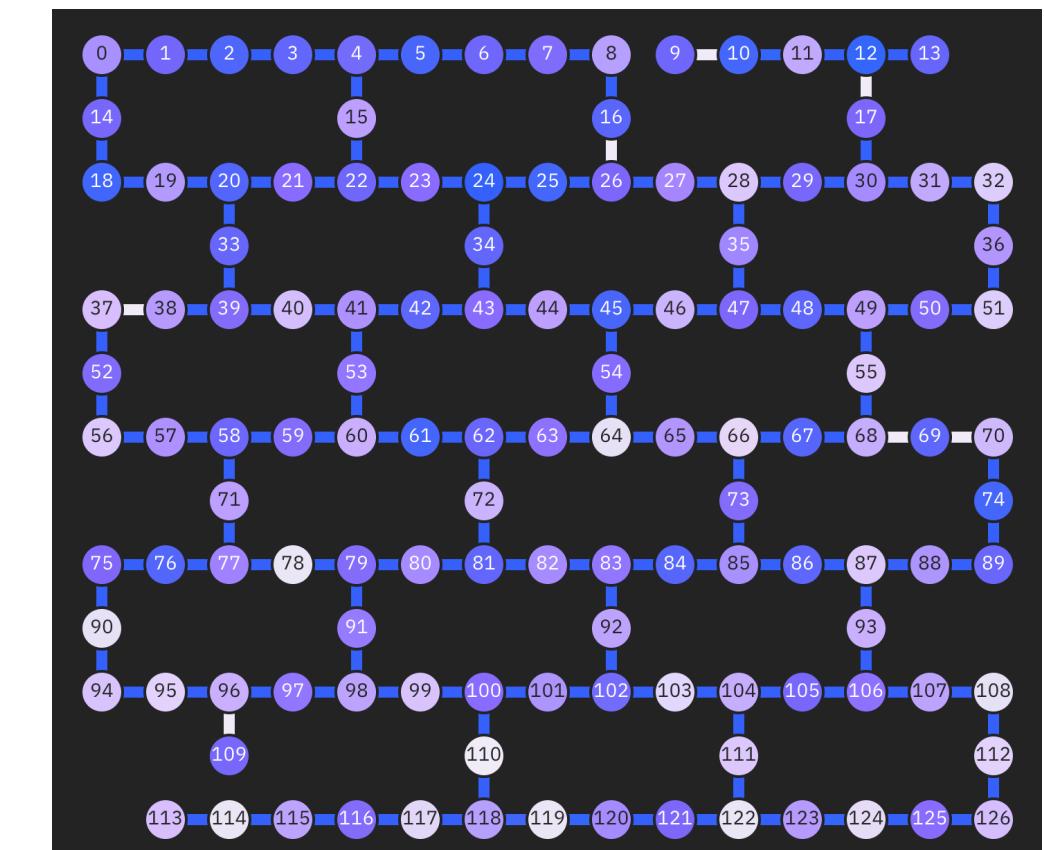
Gate Error Rate

<https://quantum-computing.ibm.com/>



Google Sycamore

<https://www.nature.com/articles/s41586-019-1666-5>

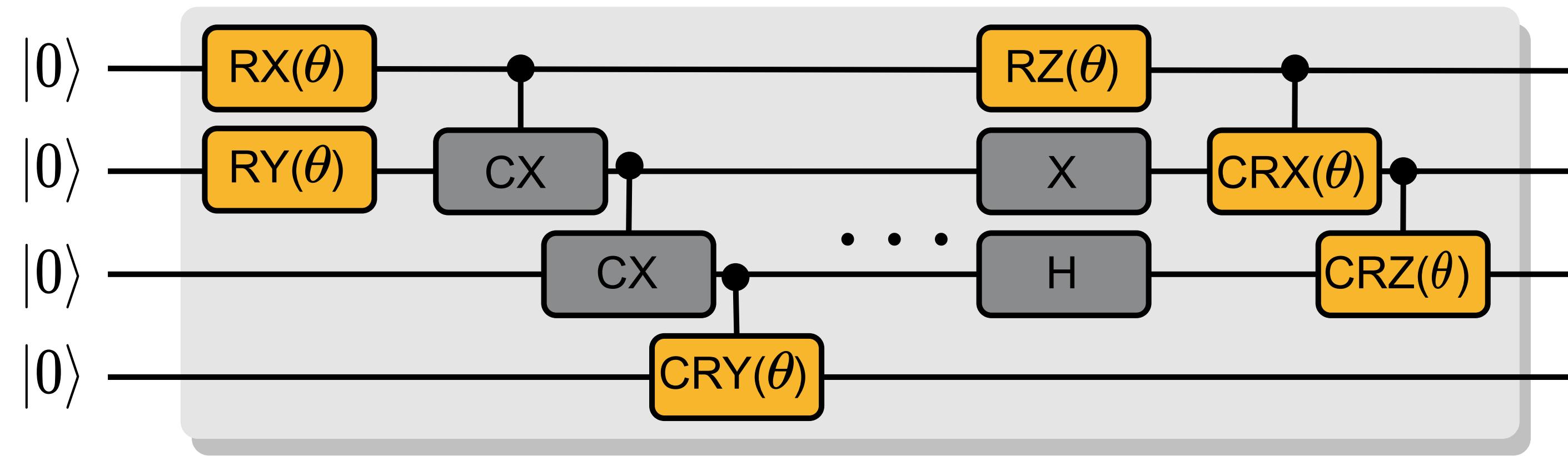


IBM Washington

<https://quantum-computing.ibm.com/>

Parameterized Quantum Circuits

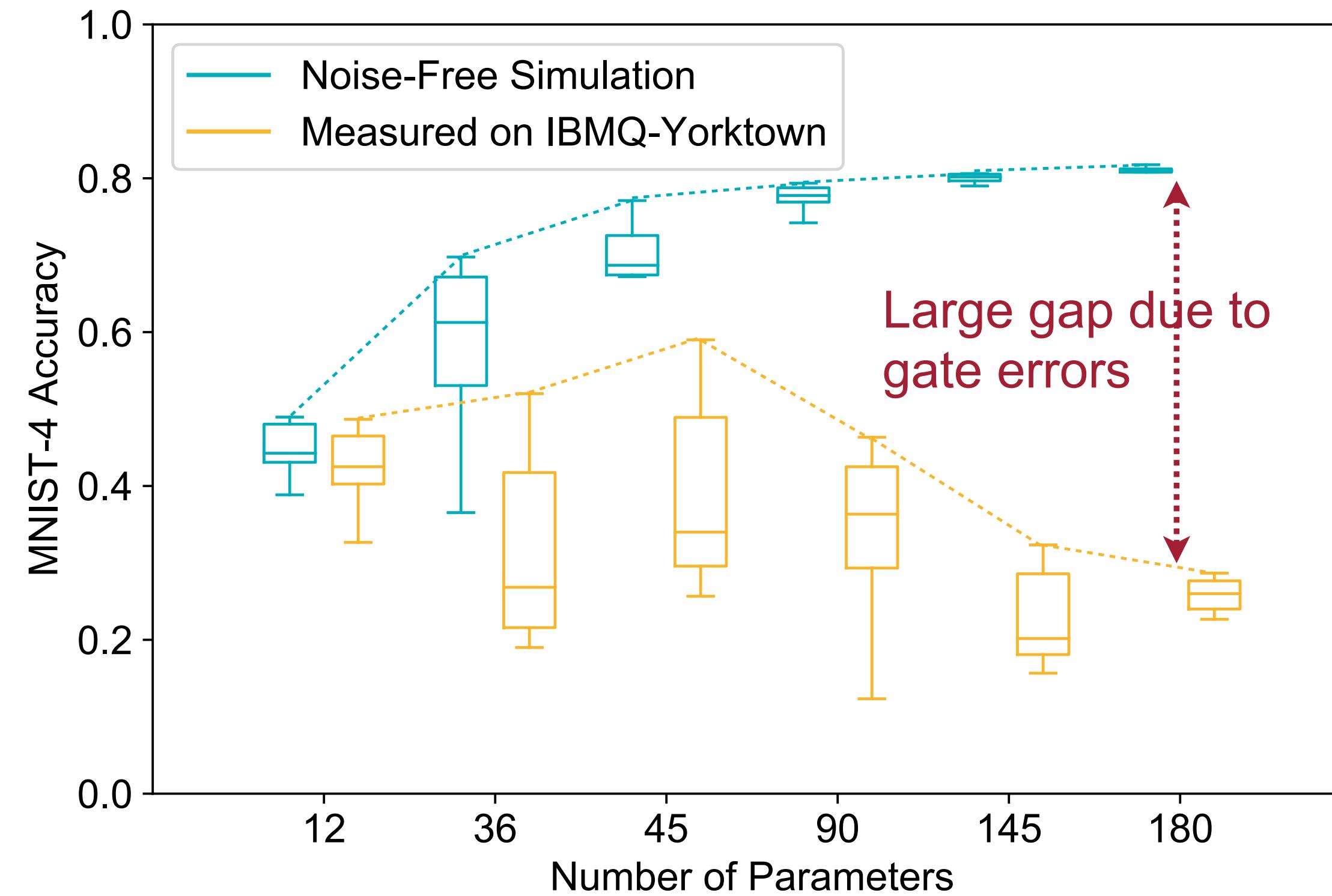
- Parameterized Quantum Circuits (PQC)
- Quantum circuit with fixed gates and **parameterized gates**



- PQCs are commonly used in **hybrid classical-quantum models** and show promises to achieve quantum advantage
 - Variational Quantum Eigensolver (VQE)
 - Quantum Neural Networks (QNN)
 - Quantum Approximate Optimization Algorithm (QAOA)

Challenges of PQC — Noise

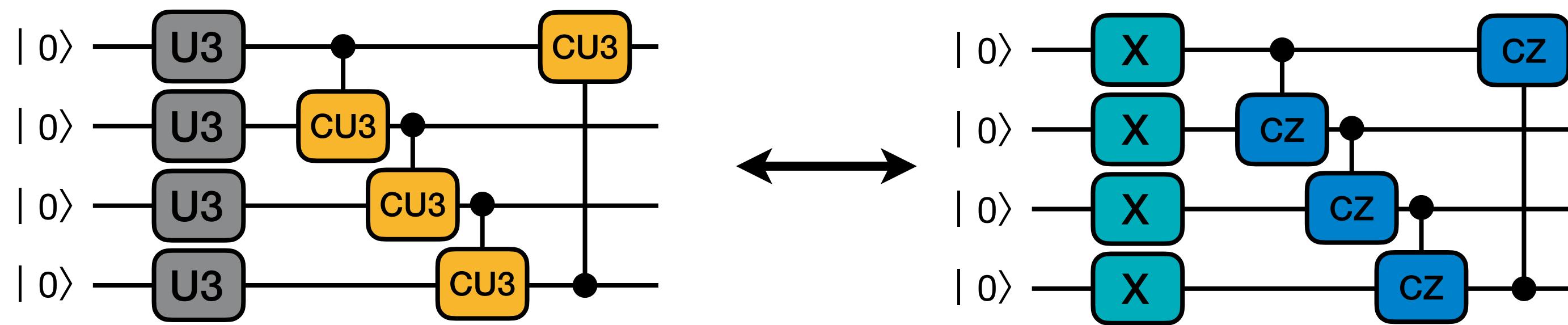
- Noise **degrades** the Parameterized Quantum Circuit (PQC) reliability
- More parameters increase the noise-free accuracy but degrade the measured accuracy
- Under same #parameters, measured accuracy of different circuit architecture (ansatz) varies a lot
- Therefore, circuit architecture is critical



Challenges of PQC – Large Design Space

- Large design space for circuit architecture

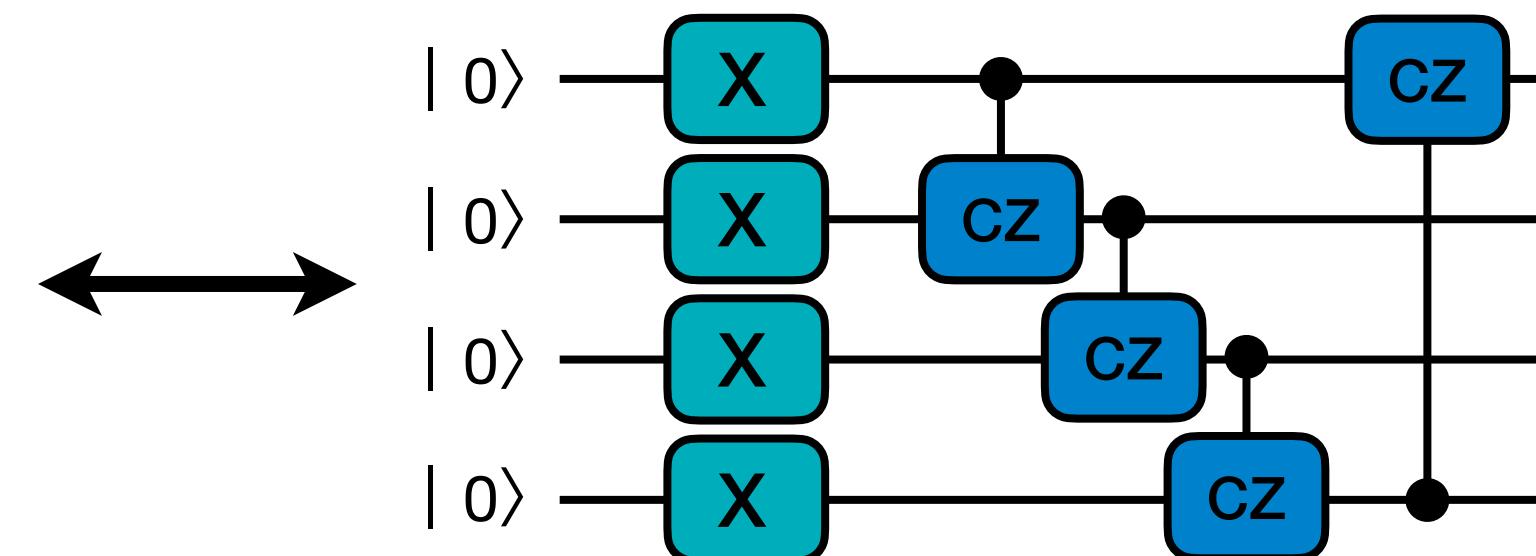
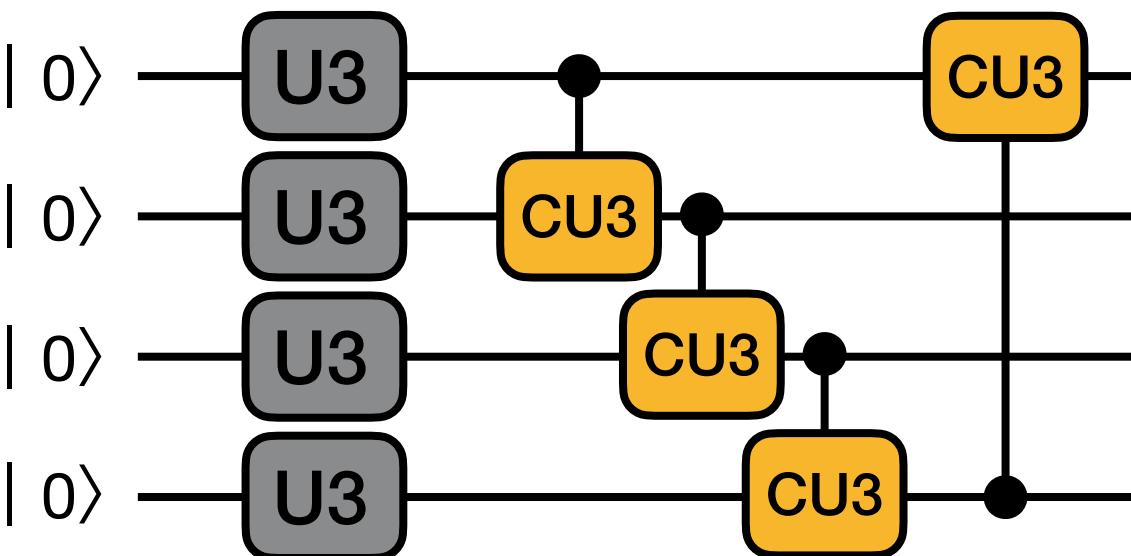
- Type of gates



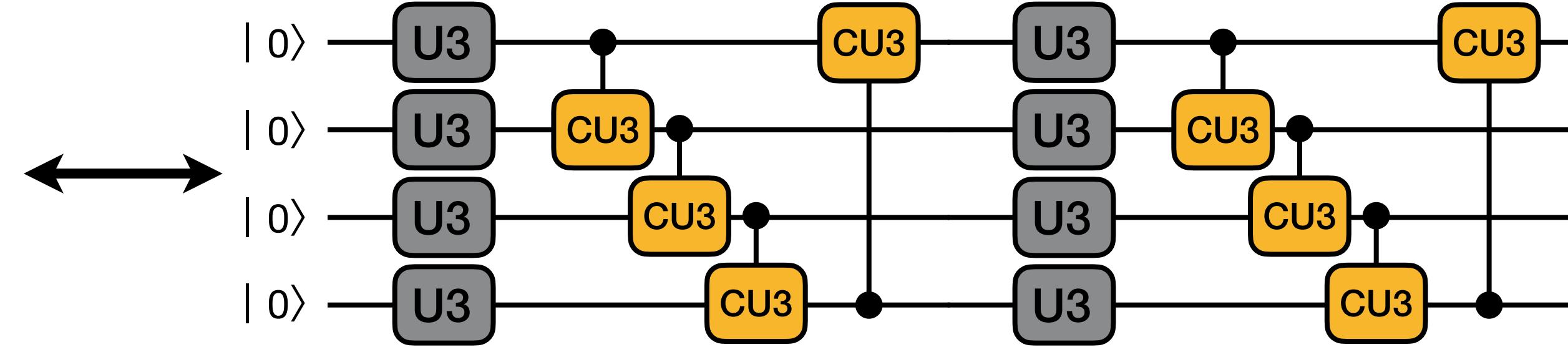
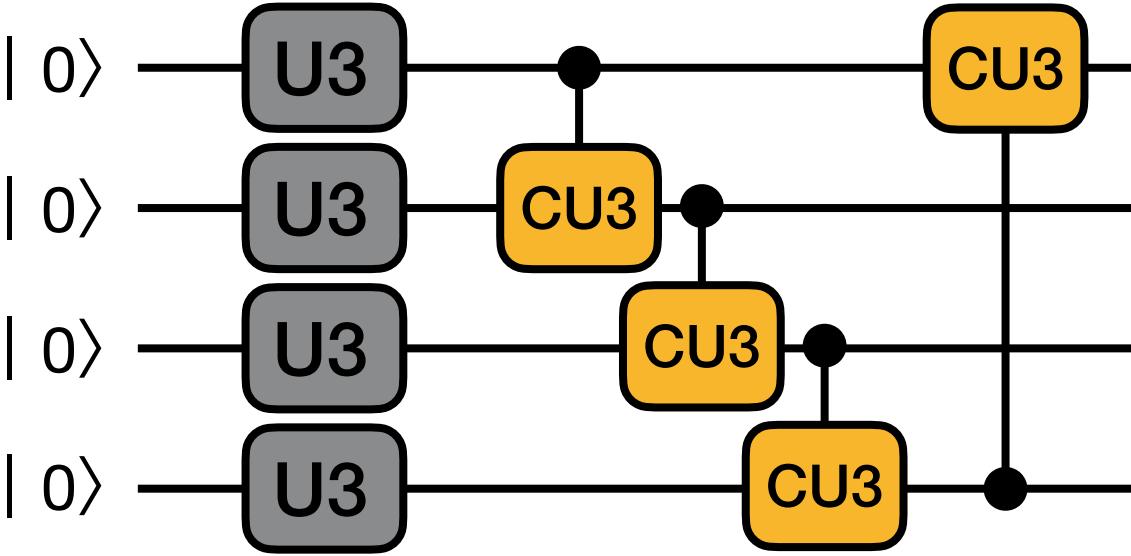
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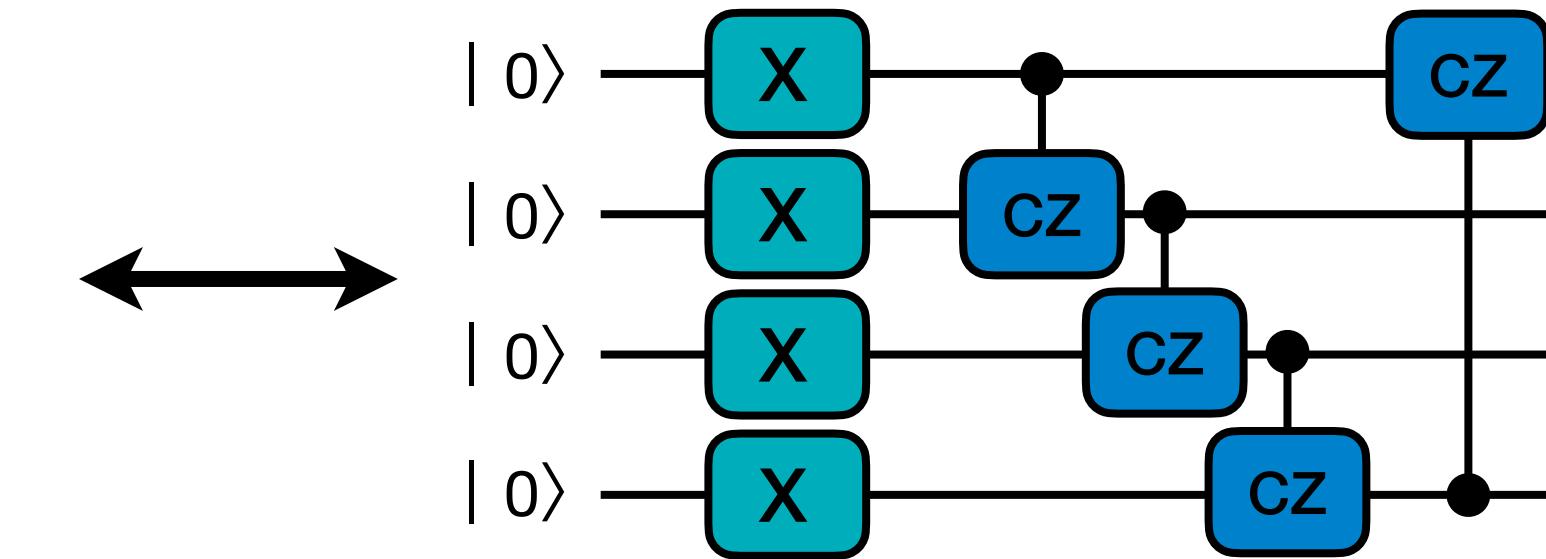
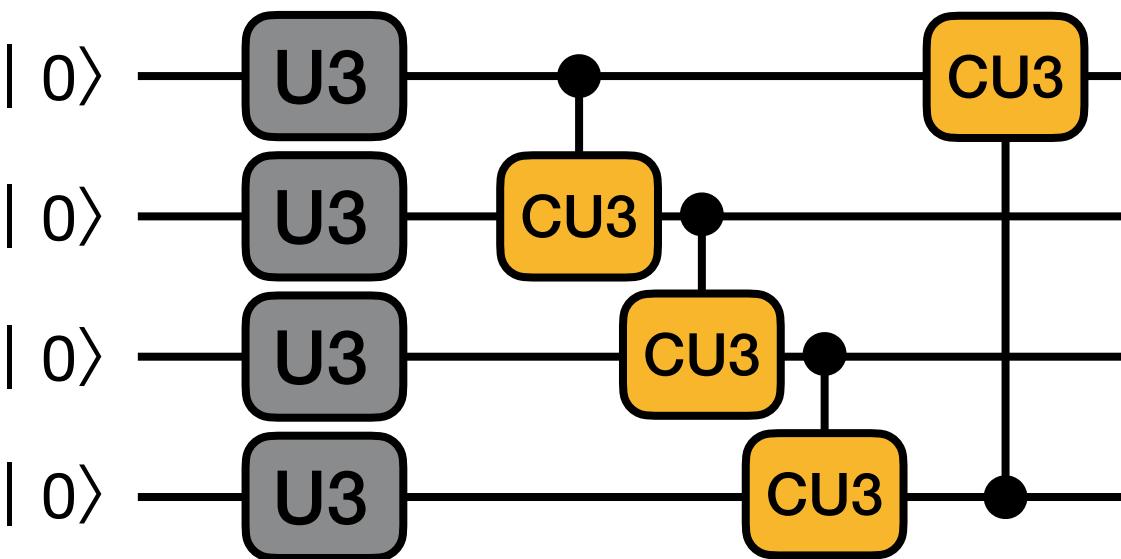
- Number of gates



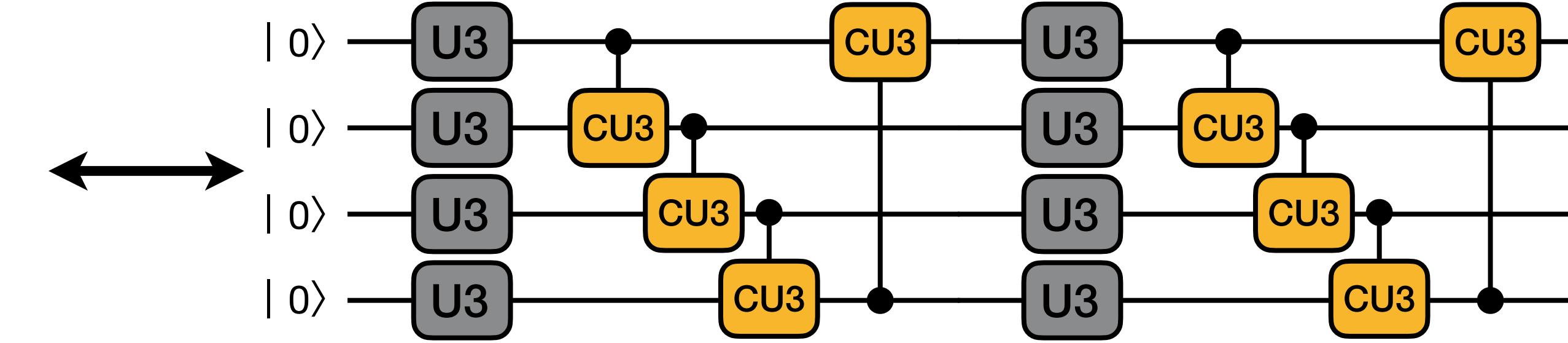
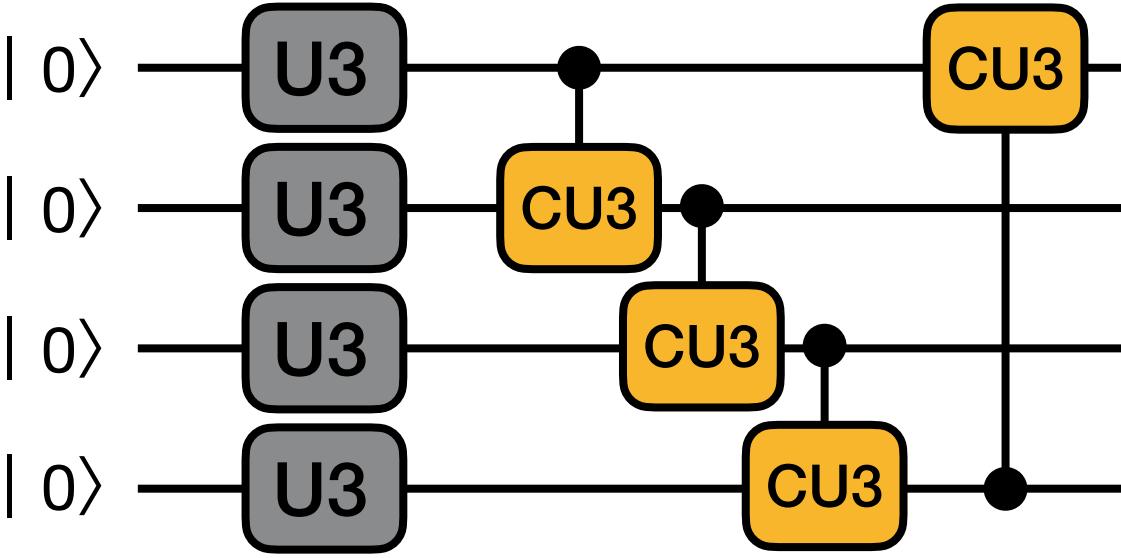
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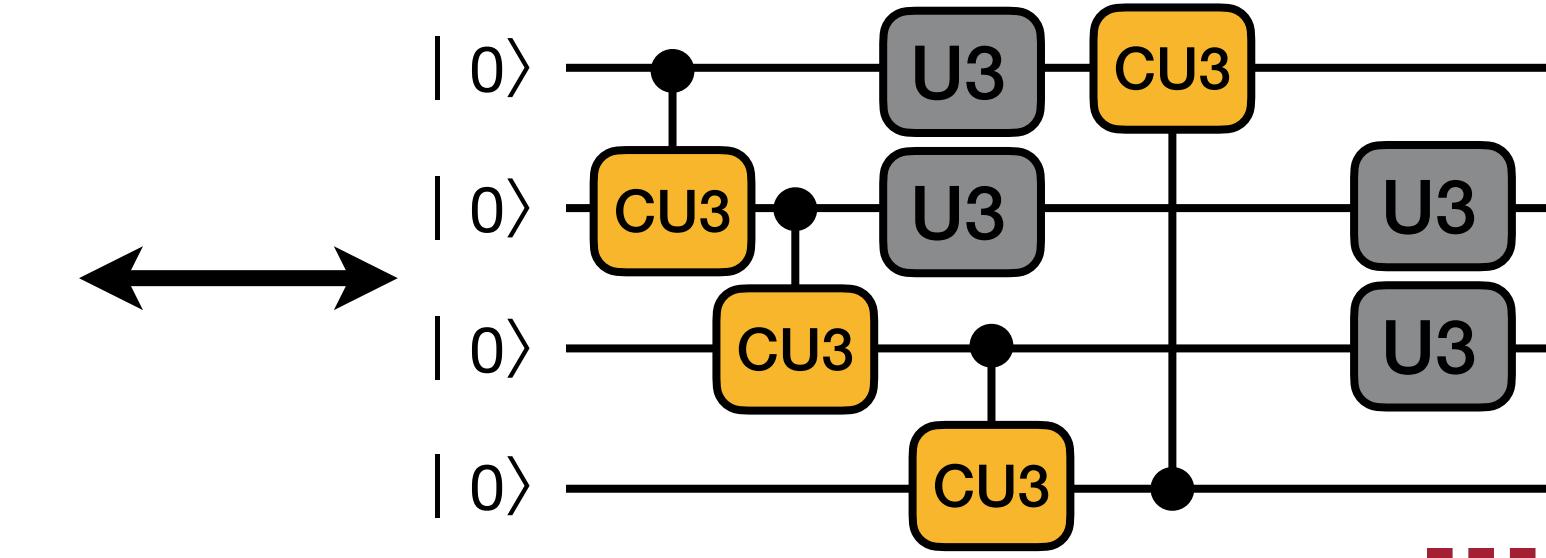
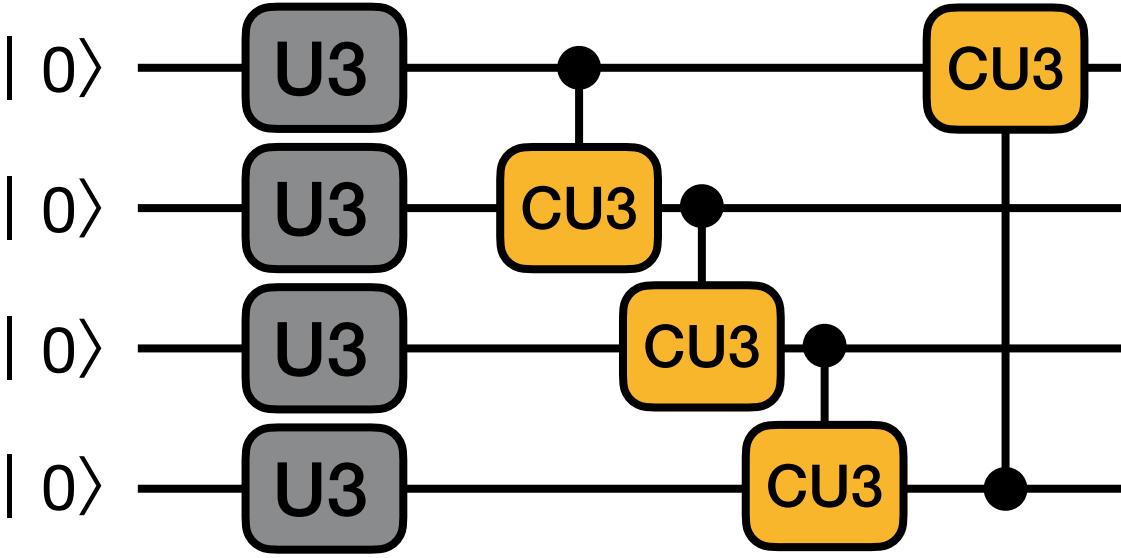
- Type of gates



- Number of gates



- Position of gates



Goal of QuantumNAS

Automatically & efficiently search for noise-robust quantum circuit

Train one “SuperCircuit”,
providing parameters to
many “SubCircuits”

Solve the challenge of large
design space

(1) Quantum noise feedback in
the search loop
(2) Co-search the circuit
architecture and qubit mapping

Solve the challenge of large
quantum noise

QuantumNAS: Decouple the Training and Search

Naive Search

For q_devices:

For search episodes: // meta controller

For circuit training iterations:

 update_parameters(); **Expensive**

If good_circuit: **break**;

QuantumNAS: Decouple the Training and Search

Naive Search

```
For q_devices:  
  For search episodes: // meta controller  
    For circuit training iterations:  
      update_parameters(); Expensive  
    If good_circuit: break;
```

=>

QuantumNAS

```
For SuperCircuit training iterations: Expensive  
  update_parameters(); training  
  .....  
  decouple  
  For q_devices:  
    For search episodes:  
      sample from SuperCircuit; Light-Weight  
      If good_circuit: break;  
      //no training
```

QuantumNAS

- SuperCircuit Construction and Training
- Noise-Adaptive Evolutionary Co-Search of SubCircuit and Qubit Mapping
- Train the Searched SubCircuit
- Iterative Quantum Gate Pruning

QuantumNAS

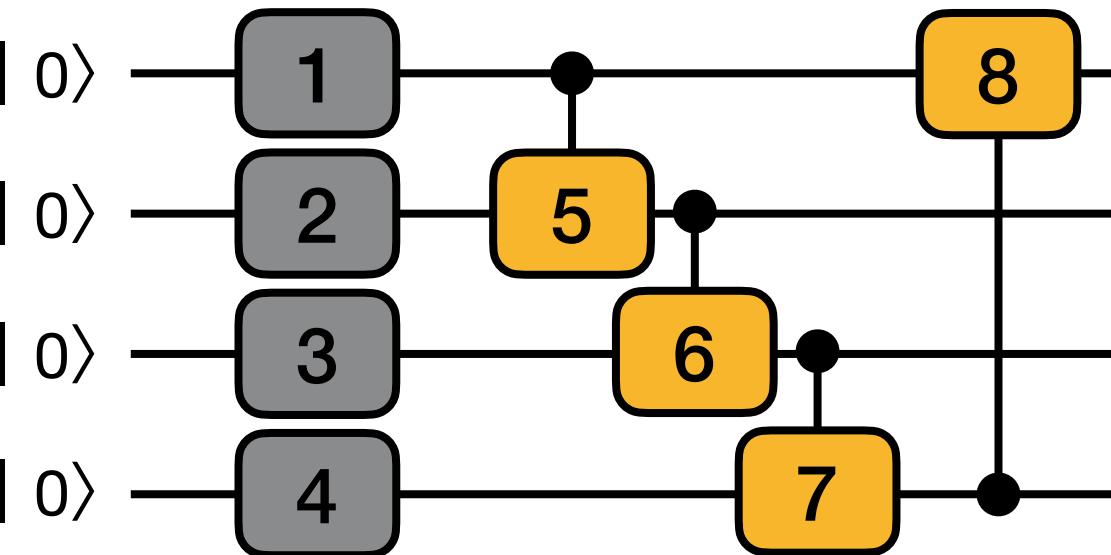
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SuperCircuit & SubCircuit

- Firstly construct a design space. For example, a design space of maximum 4 U3 in the first layer and 4 CU3 gates in the second layer
 - Contains $2^8 = 256$ candidates

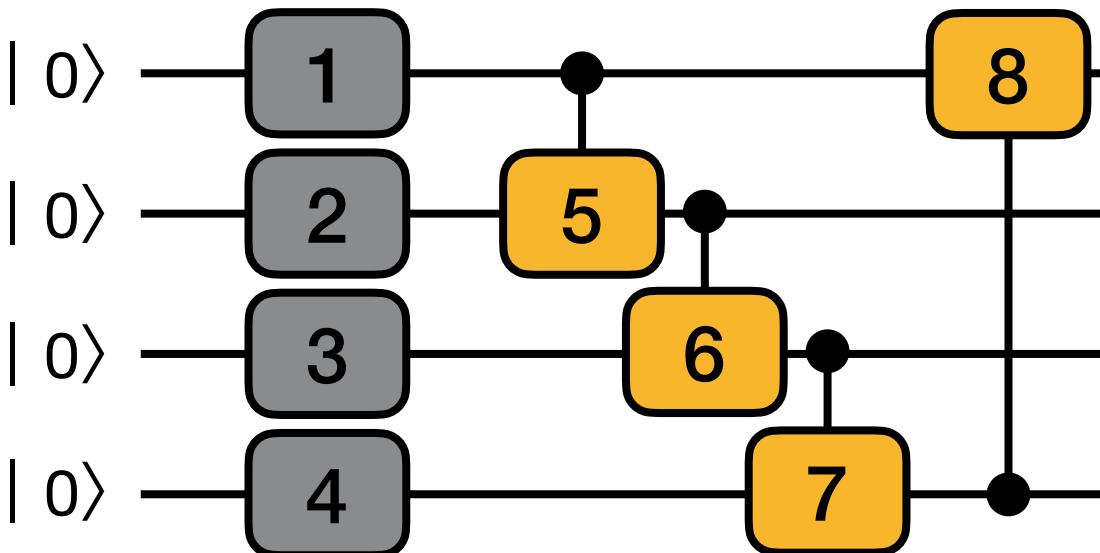
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 - SuperCircuit: the circuit with the **largest** number of gates in the design space
 - Example: SuperCircuit in U3+CU3 space

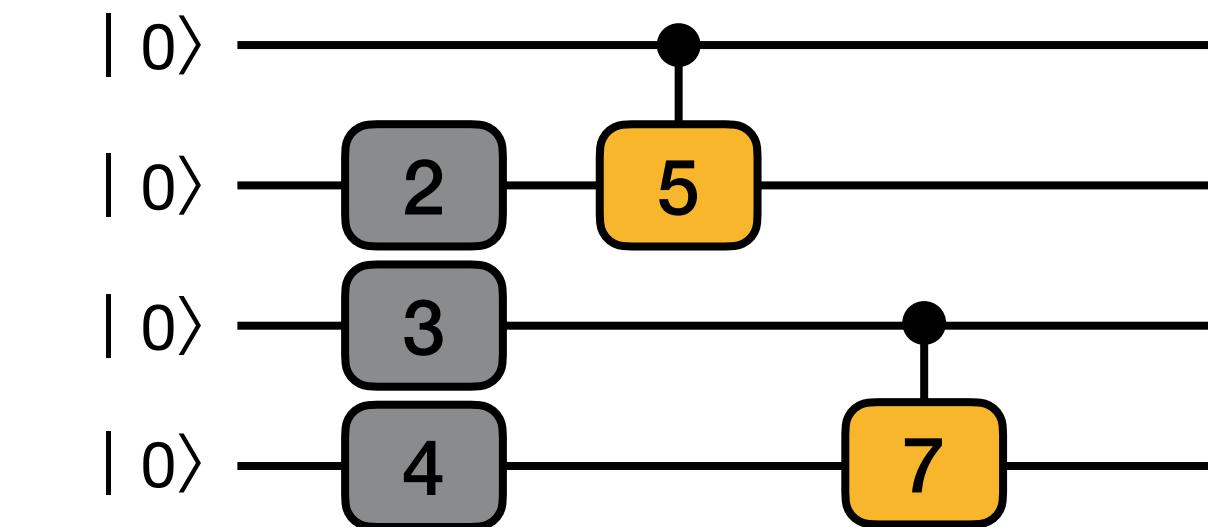
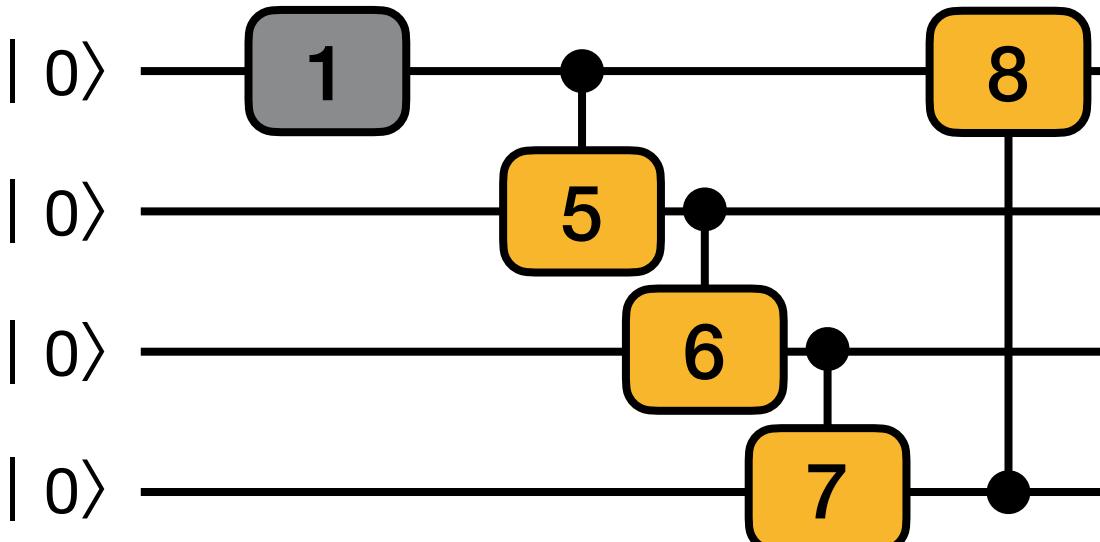
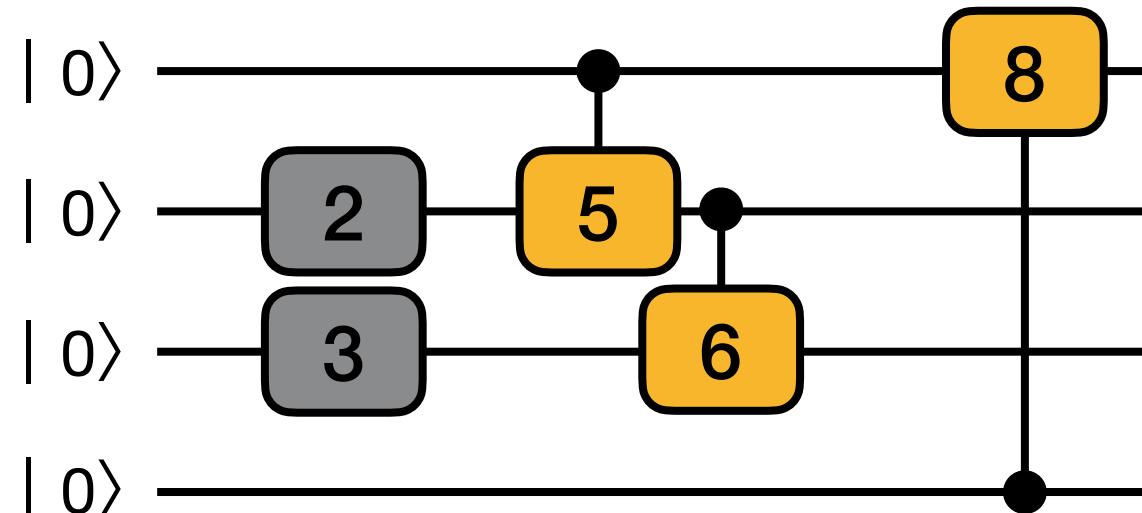


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 - Example: SuperCircuit in U3+CU3 space



- Each candidate circuit in the design space (called SubCircuit) is a **subset** of the SuperCircuit



SuperCircuit Construction

- Why use a SuperCircuit?
 - It enables **efficient** search of circuit architecture candidates with no need of training each of them individually
 - For one SubCircuit candidate, we can directly inherit parameters from SuperCircuit and consider that the SubCircuit can operate **as if it is trained individually from scratch**

SuperCircuit Construction

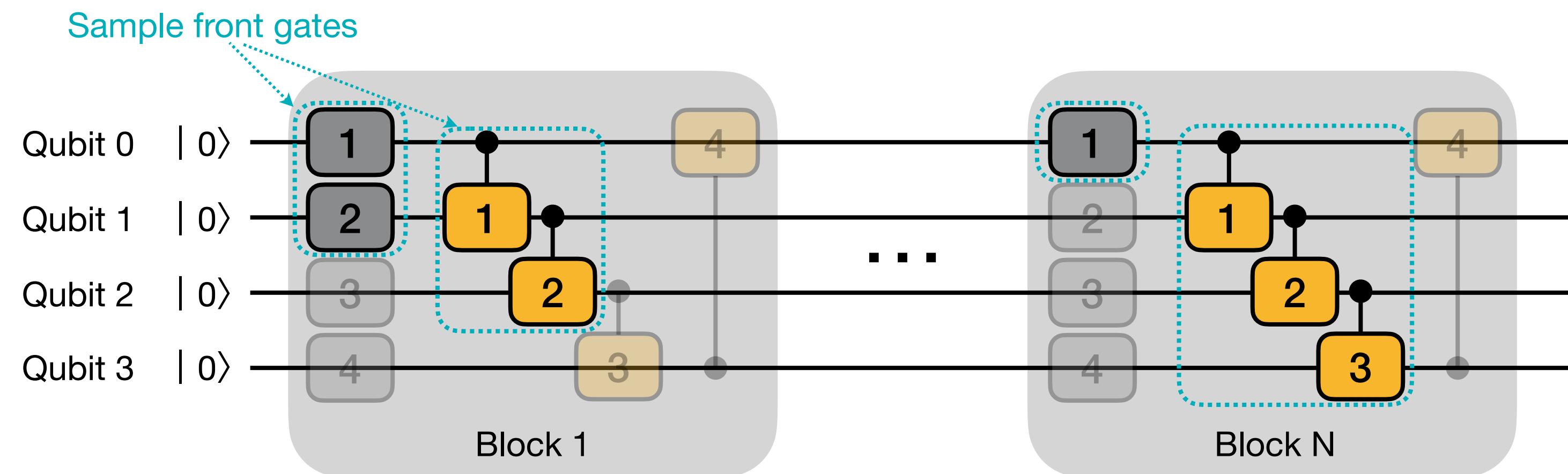
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 - It enables **efficient** search of circuit architecture candidates with no need of training each of them individually
 - For one SubCircuit candidate, we can directly inherit parameters from SuperCircuit and consider that the SubCircuit can operate **as if it is trained individually from scratch**
- Need to prevent interference of SubCircuits from each other

SuperCircuit Training

- In one SuperCircuit Training step:
 - Sample a gate subset of SuperCircuit (a SubCircuit)
 - Front Sampling and Restricted Sampling
 - Only use the subset to perform the task and updates the parameters in the subset
 - Parameter updates are cumulative across steps

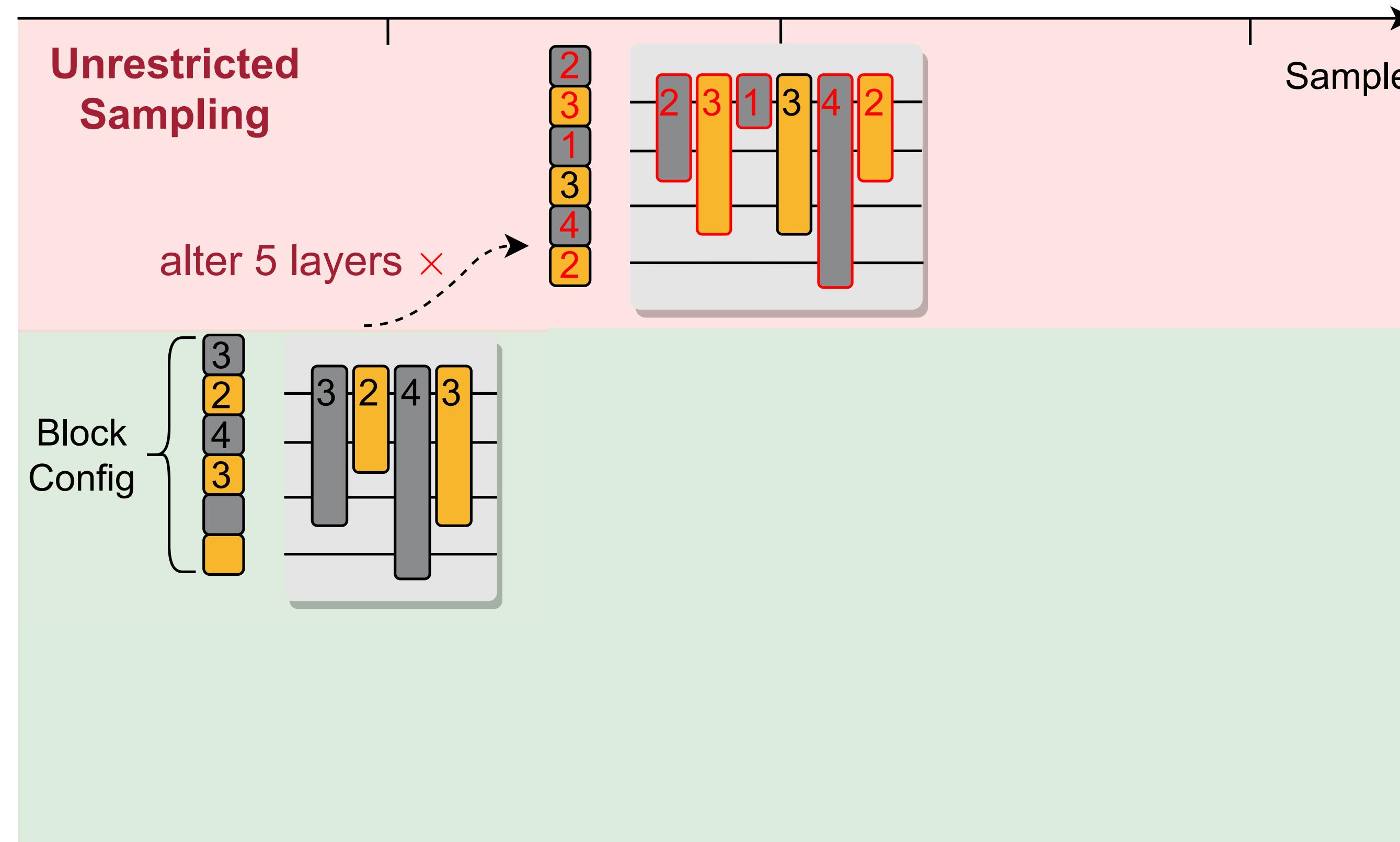
Front Sampling

- During sampling, we first sample total number of blocks, then sample gates within each block
 - Front sampling: Only the **front** several blocks and **front** several gates can be sampled to make SuperCircuit training more stable



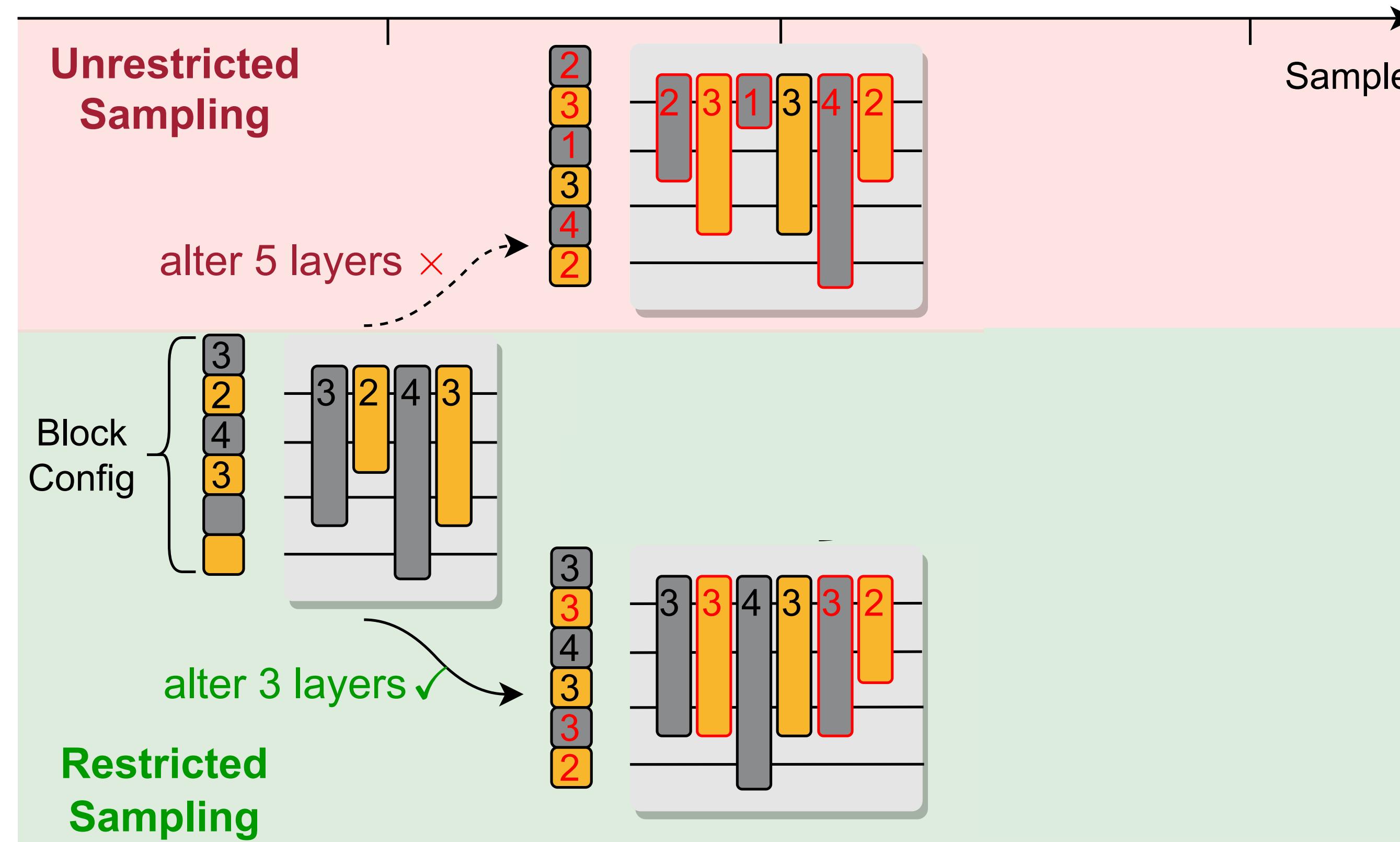
Restricted Sampling

- Restricted Sampling:
 - Restrict the difference between SubCircuits of two consecutive steps
 - For example: restrict to at most 4 different layers



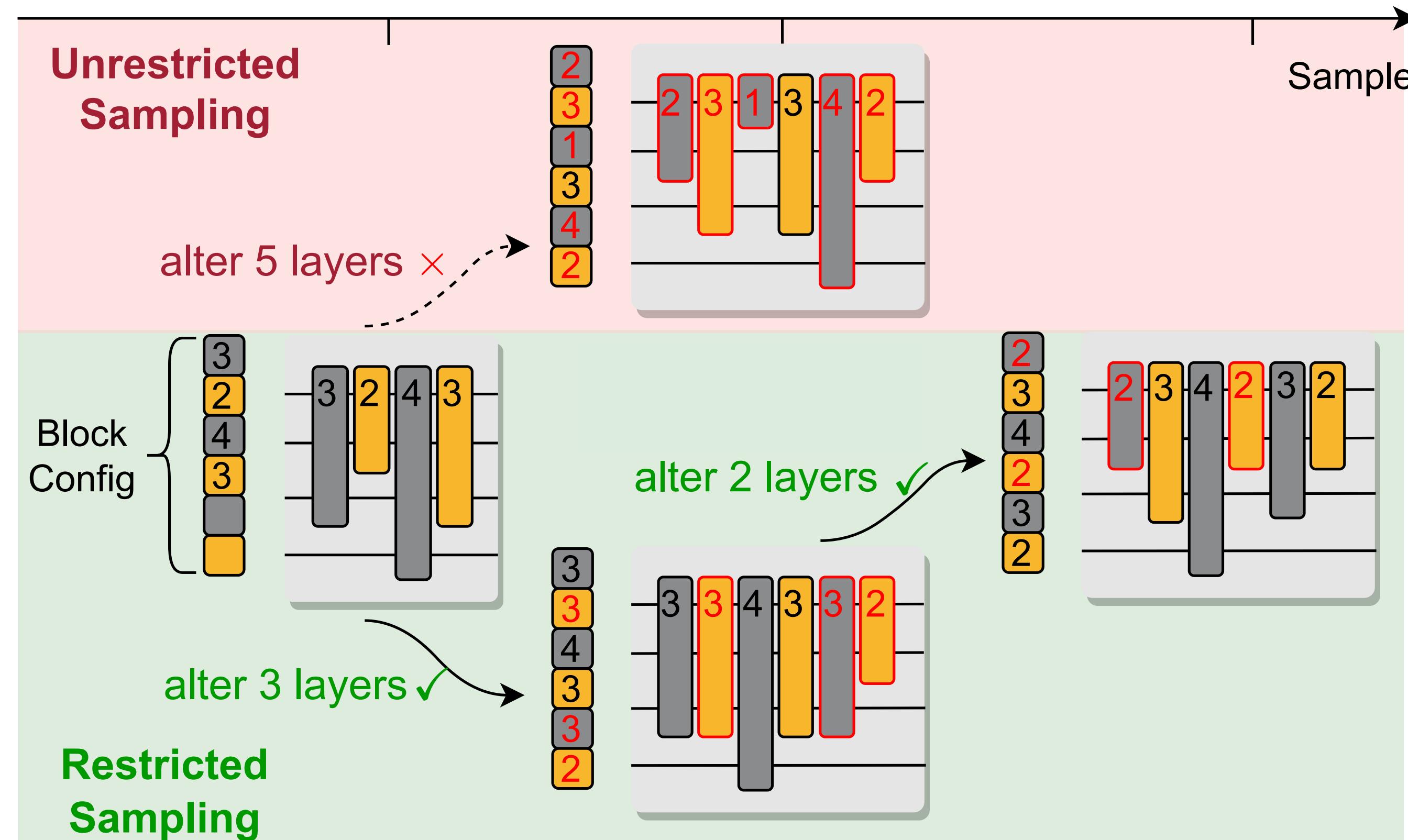
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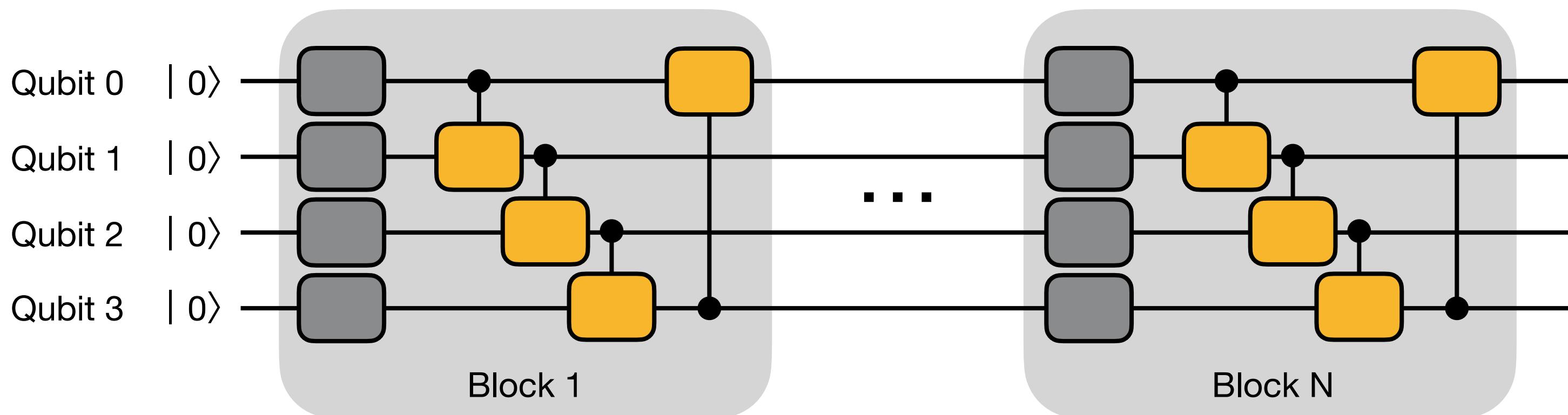
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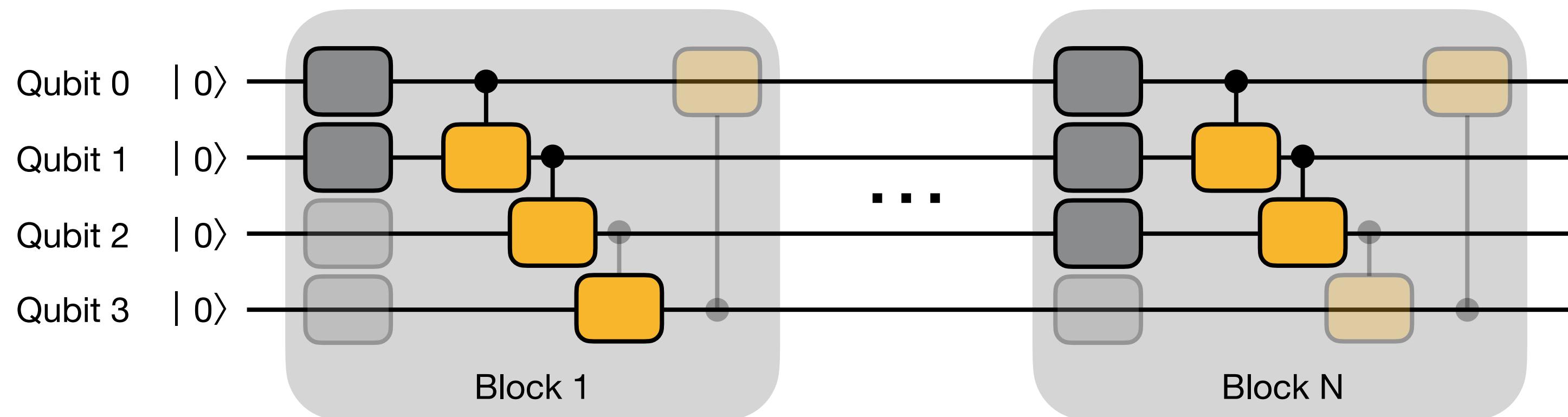
Train SuperCircuit for Multiple Steps

- In one SuperCircuit Training step: Sample and Train



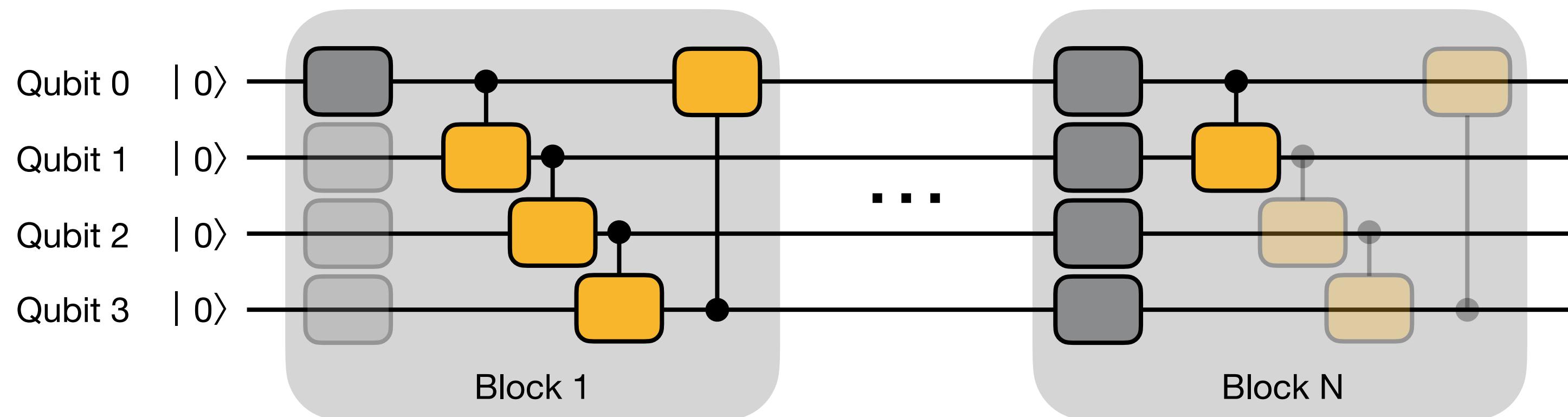
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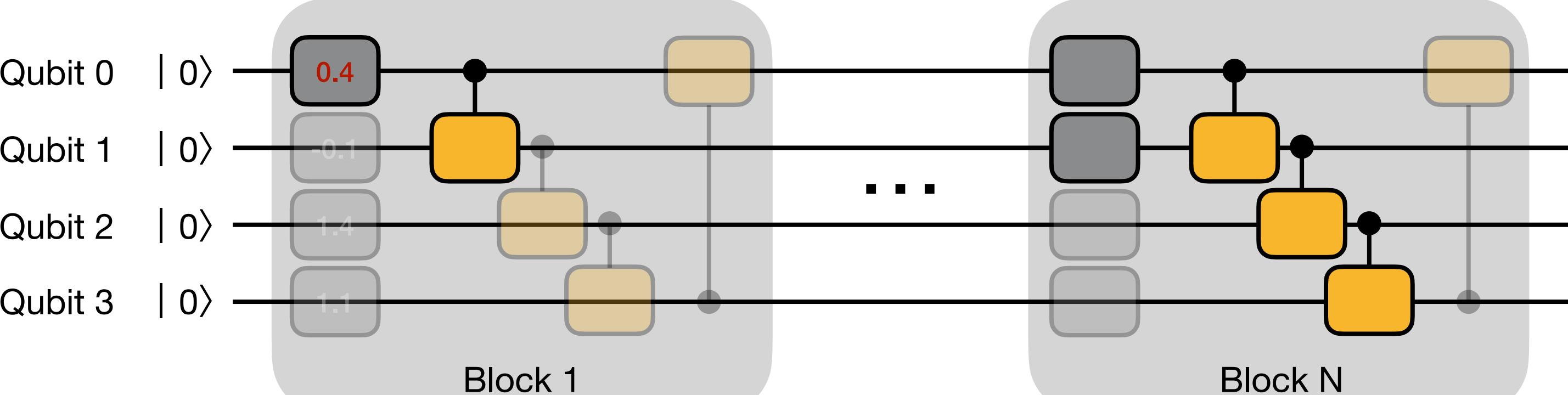
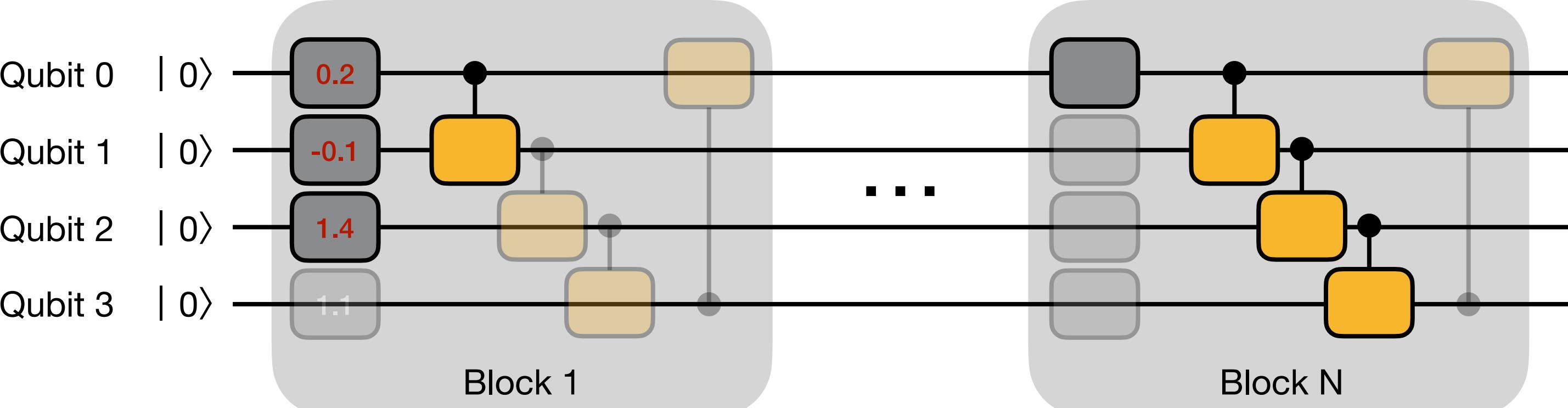
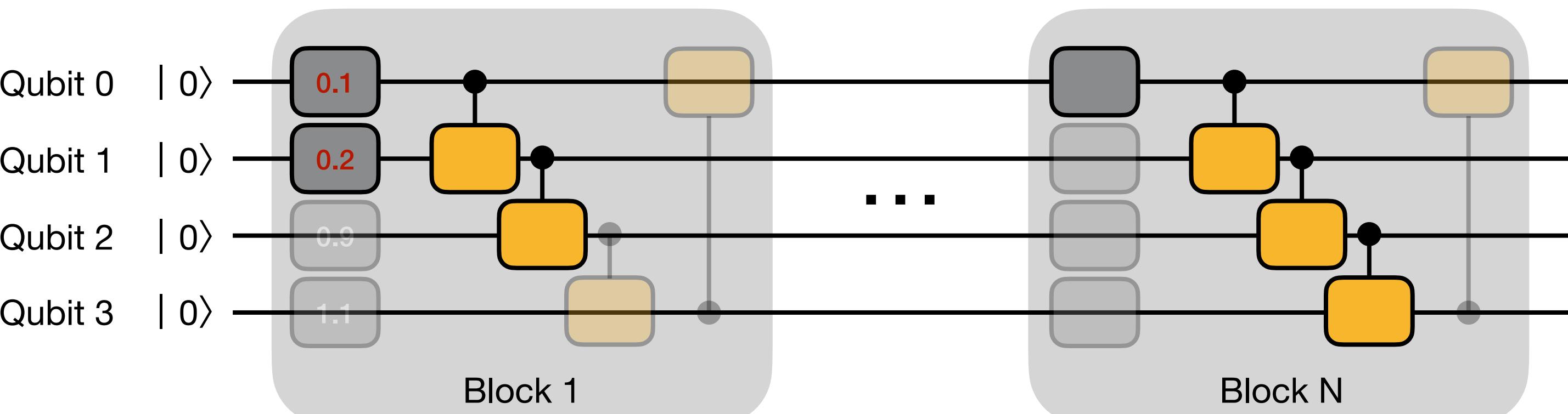


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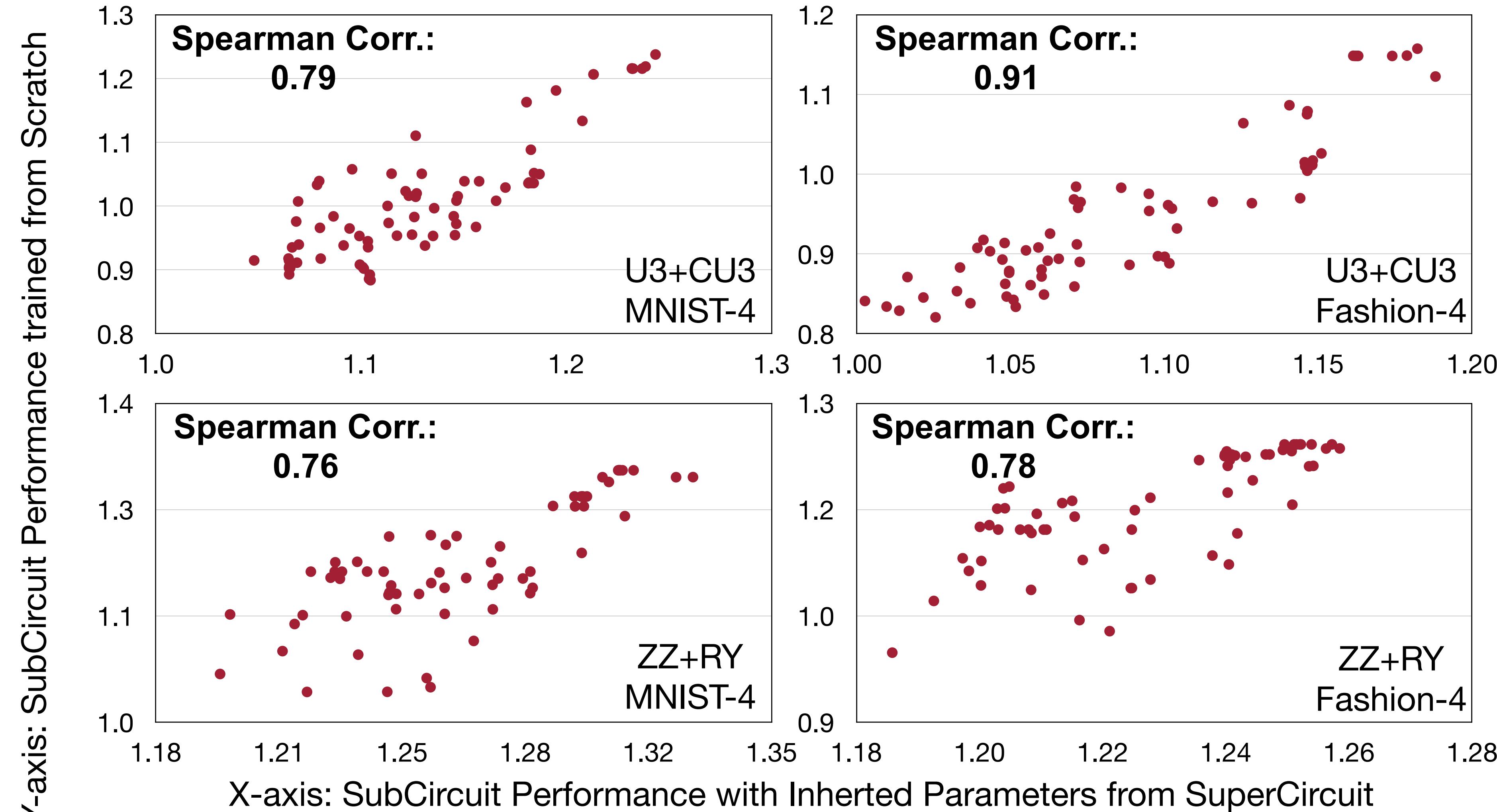


Train SuperCircuit for Multiple Steps



How Reliable is the SuperCircuit?

- Inherited parameters from SuperCircuit can provide accurate relative performance

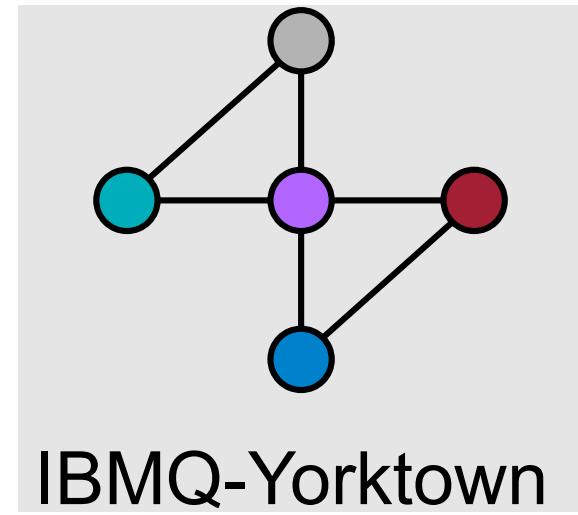


QuantumNAS

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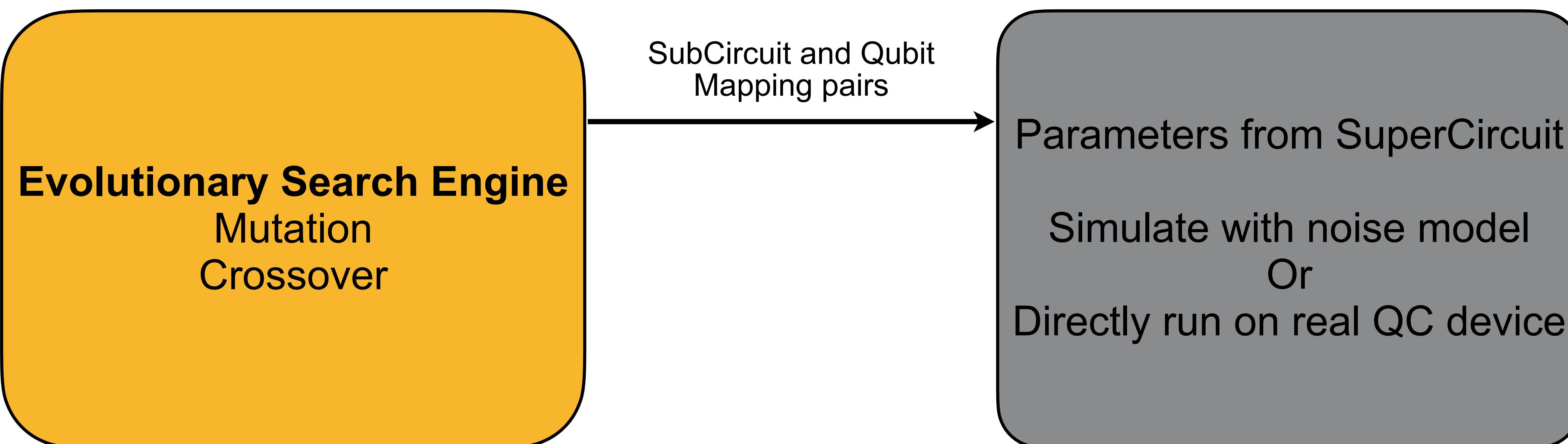
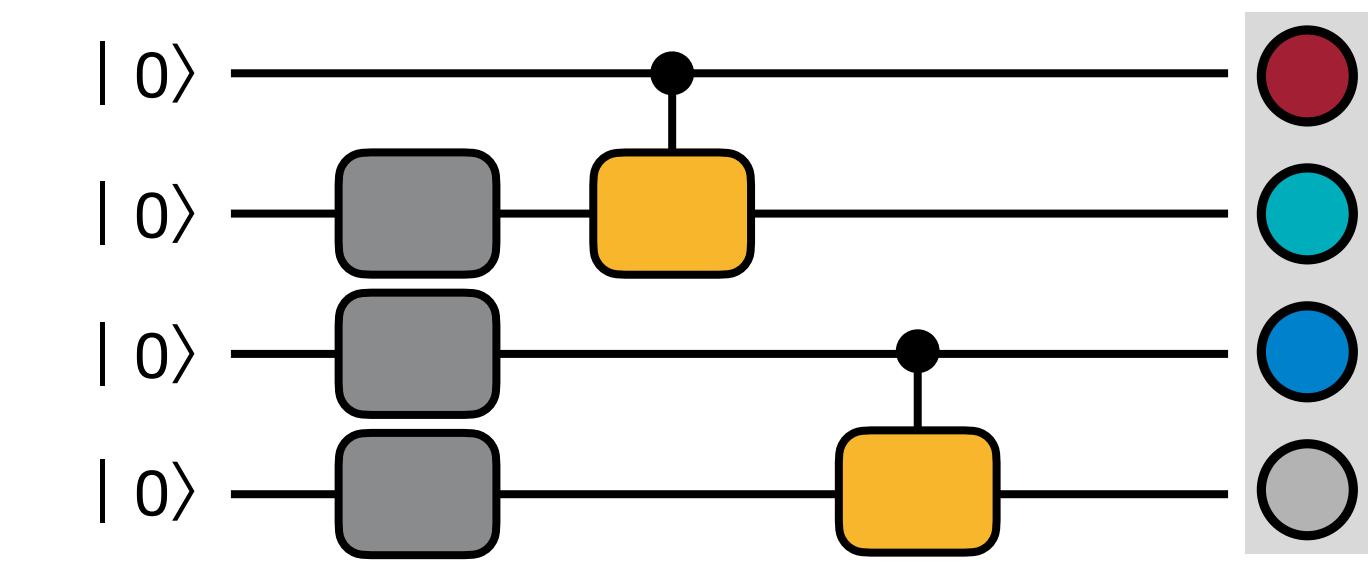
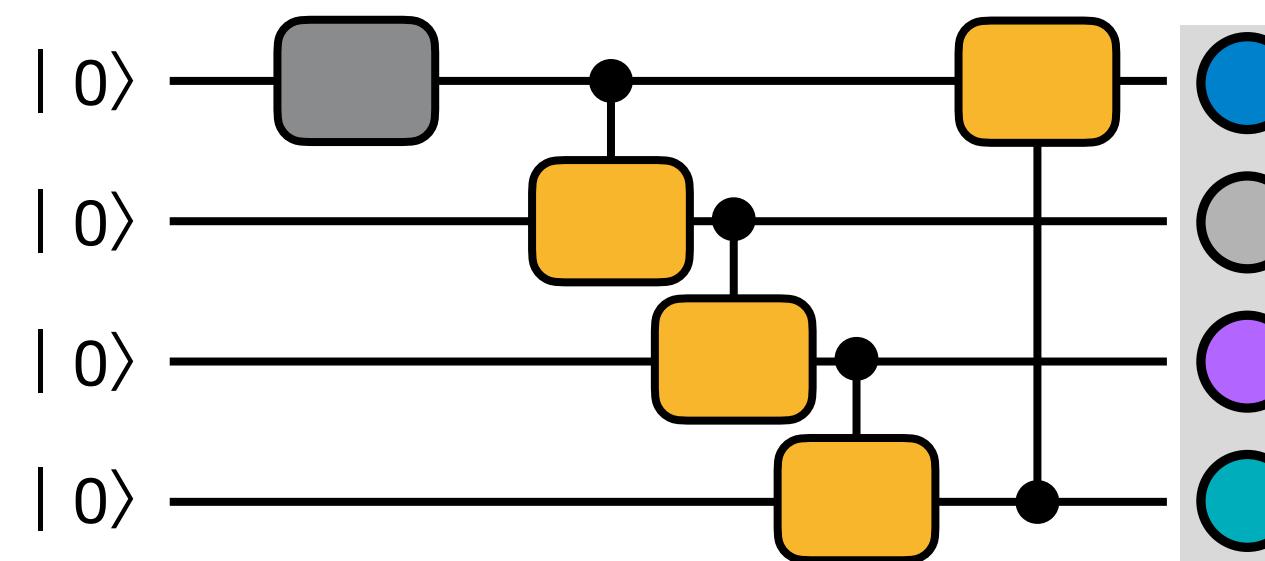
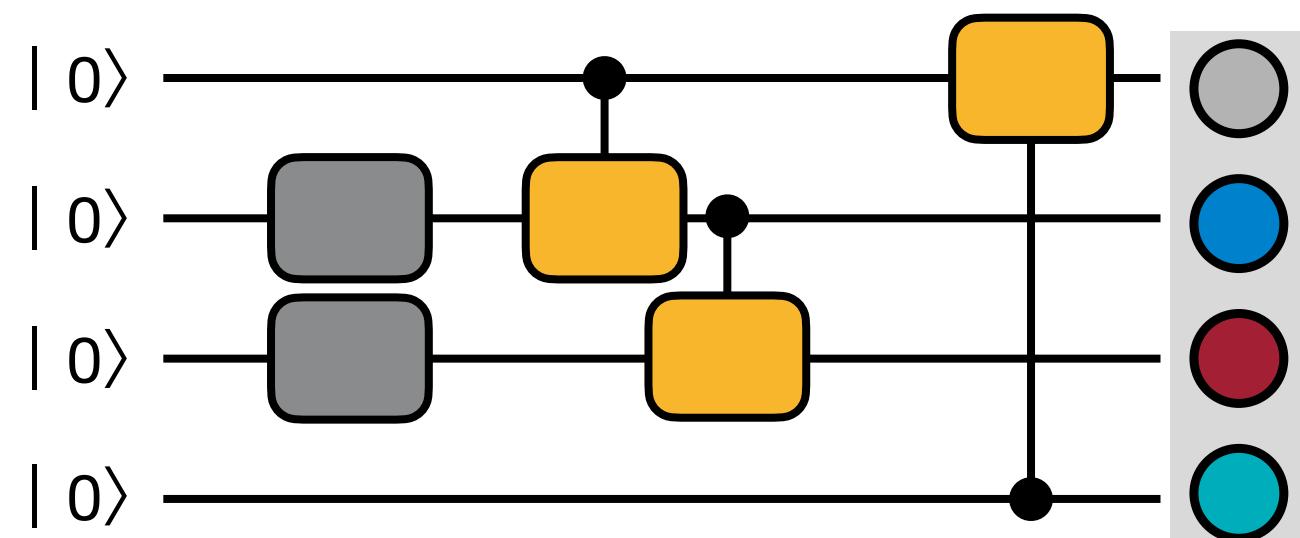
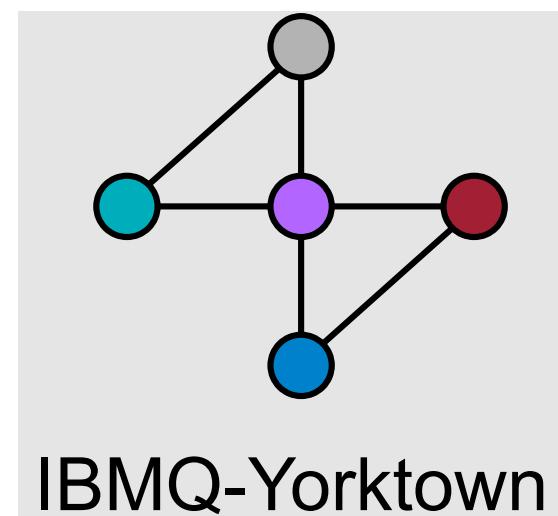
Noise-Adaptive Evolutionary Co-Search

- Search the best SubCircuit and its qubit mapping on target device



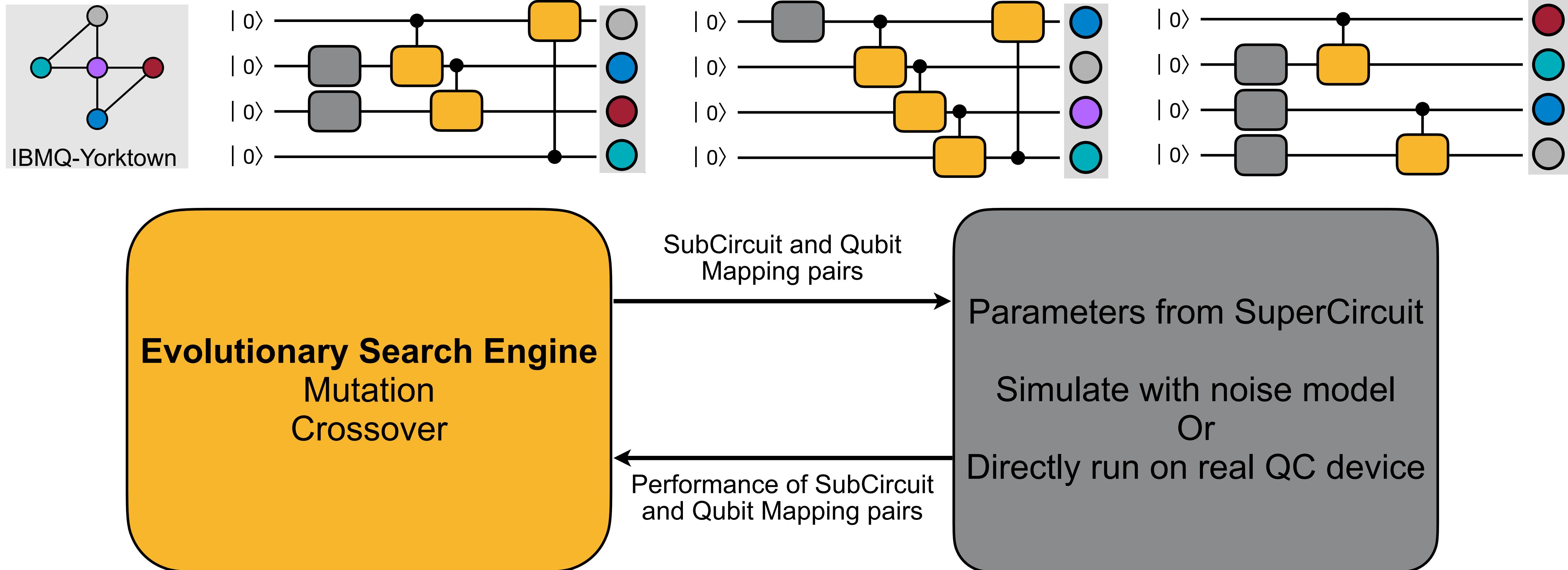
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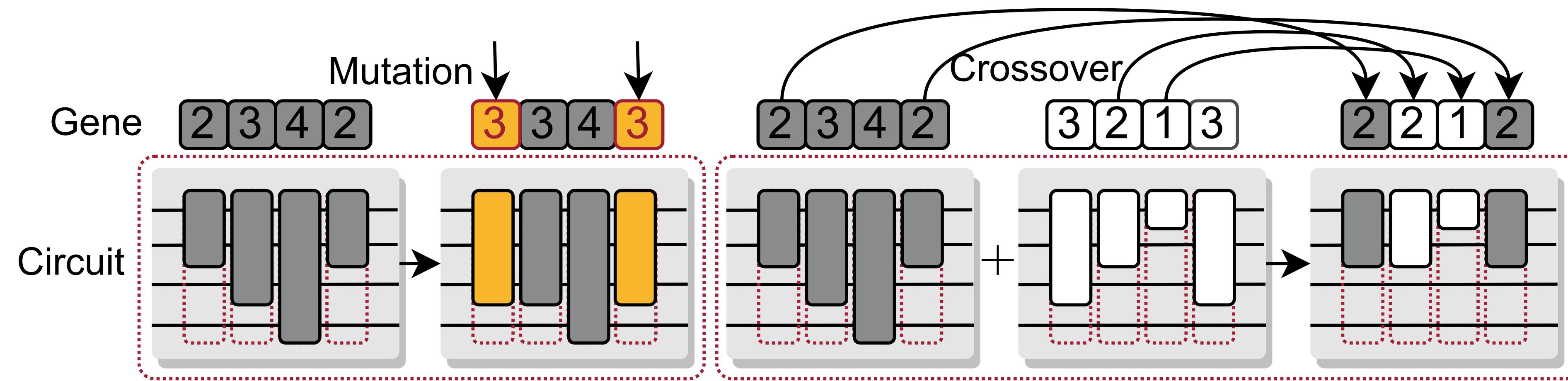
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Mutation and Crossover

- Mutation and crossover create new SubCircuit candidates



QuantumNAS

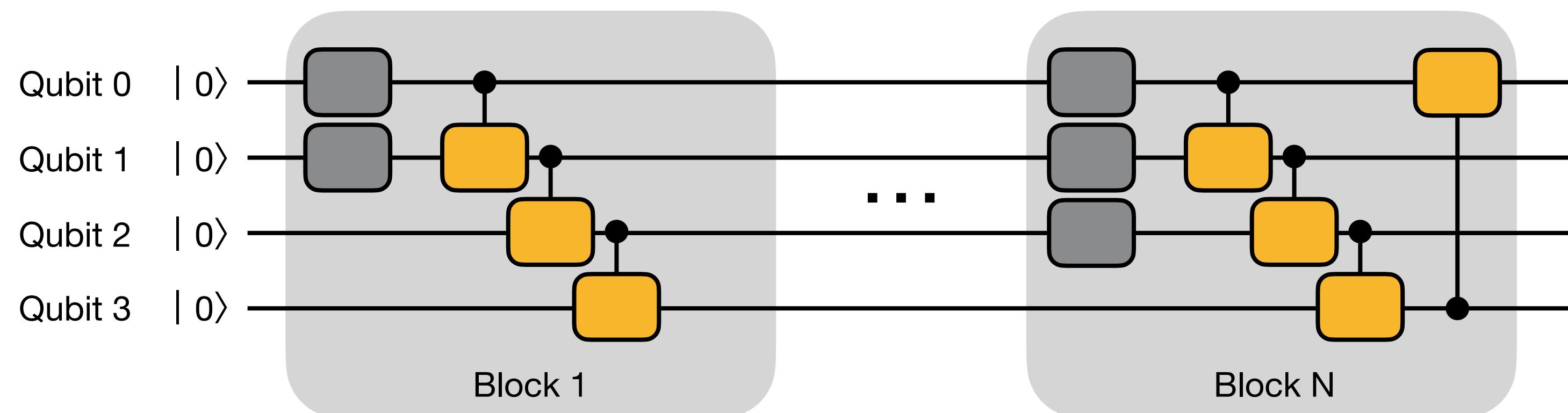
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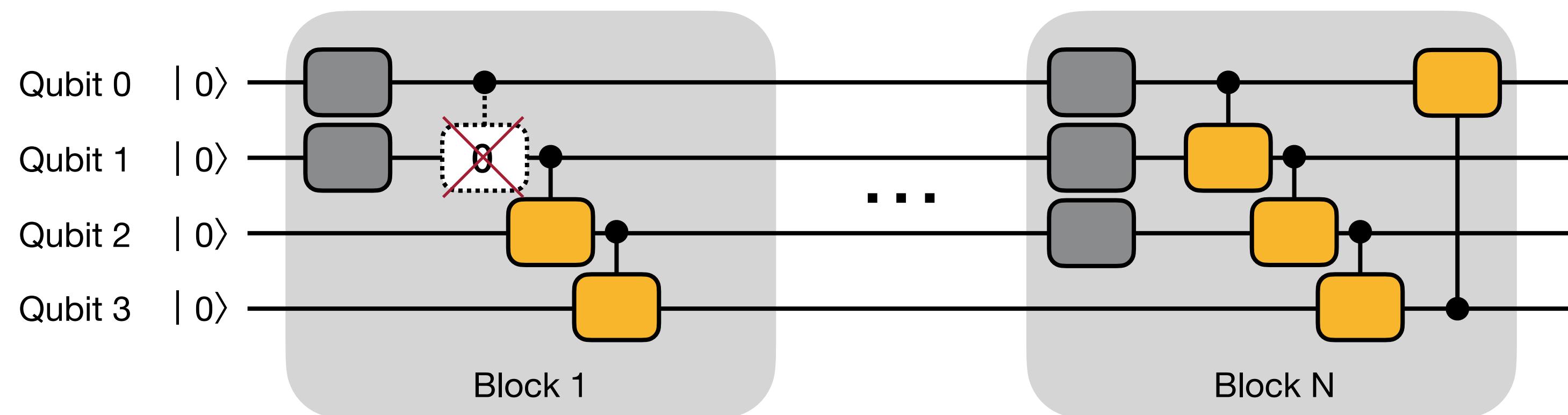
Iterative Pruning

- Some gates have parameters close to 0
 - Rotation gate with angle close to 0 has small impact on the results
 - Iteratively prune small-magnitude gates and fine-tune the remaining parameters



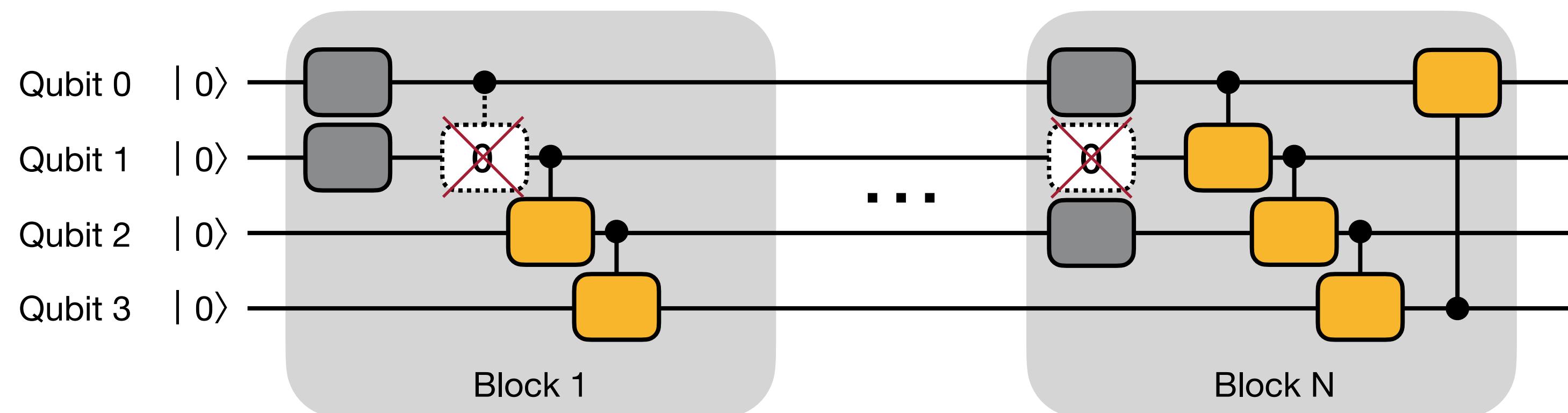
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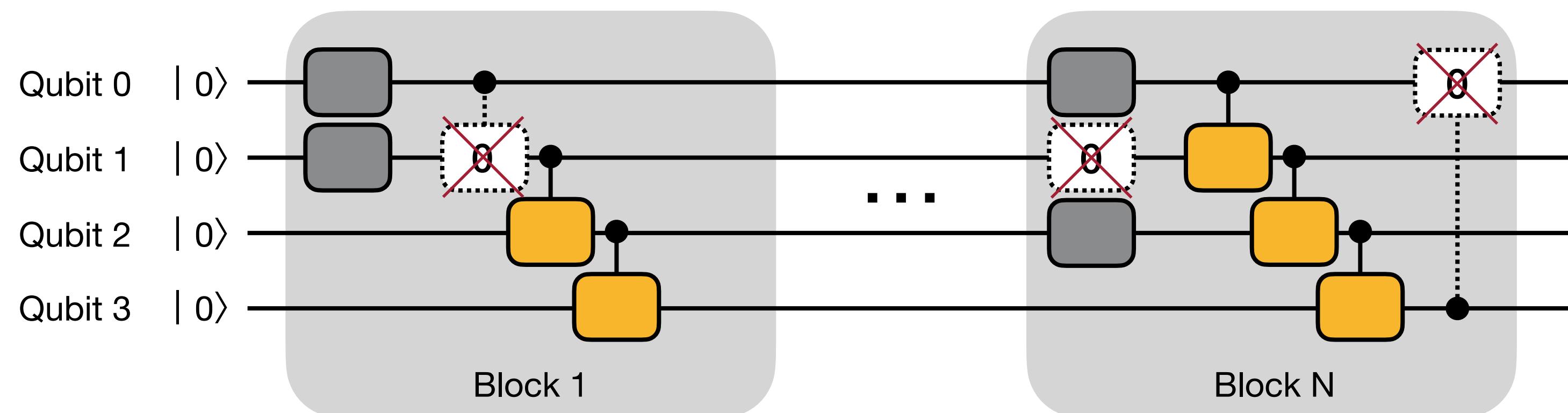
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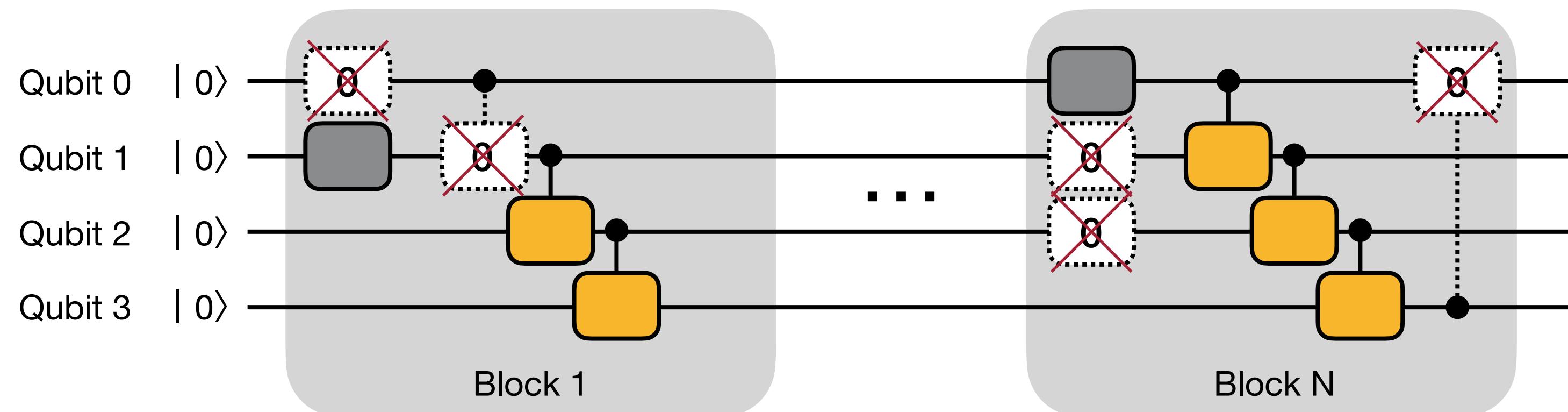
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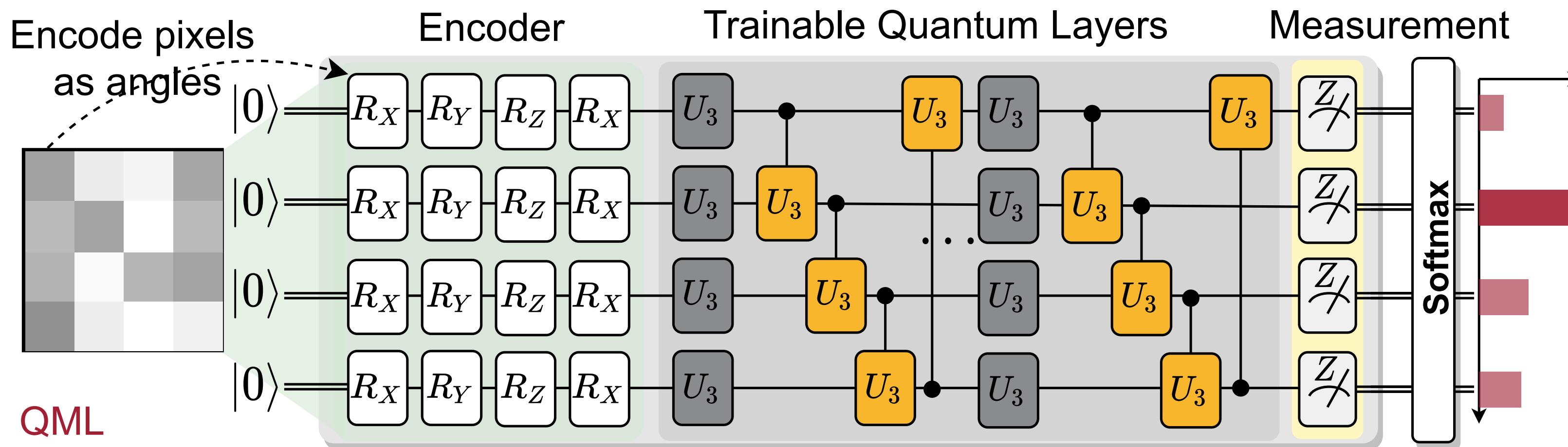


Evaluation Setups: Benchmarks and Devices

- Benchmarks
 - QML classification tasks: MNIST 10-class, 4-class, 2-class, Fashion 4-class, 2-class, Vowel 4-class
 - VQE task molecules: H₂, H₂O, LiH, CH₄, BeH₂
- Quantum Devices
 - IBMQ
 - #Qubits: 5 to 65
 - Quantum Volume: 8 to 128

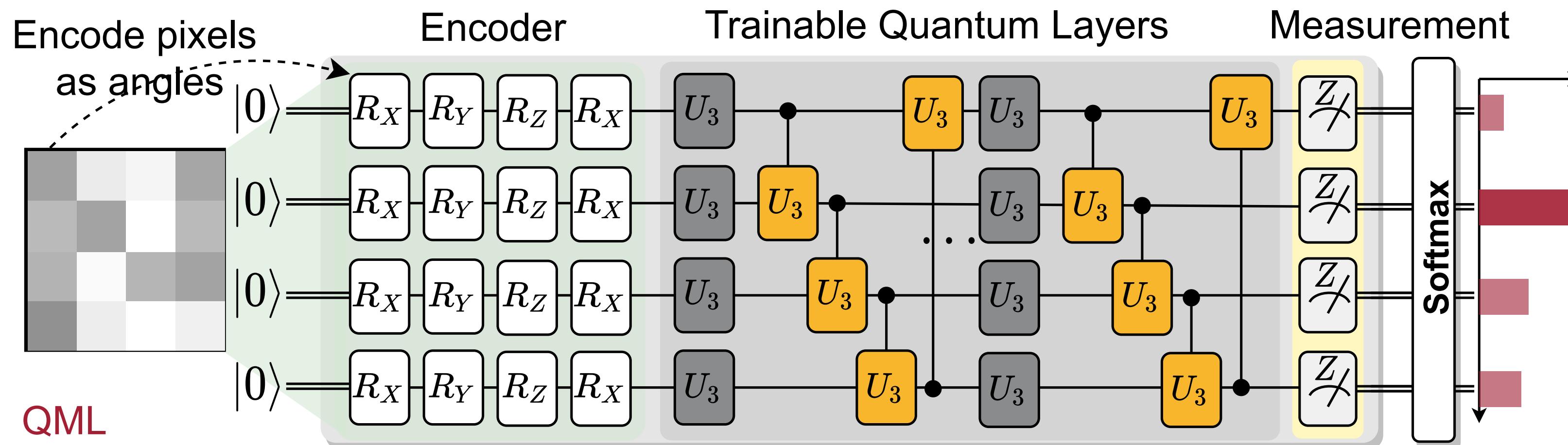
Benchmarks: QNN and VQE

- Quantum Neural Networks: classification

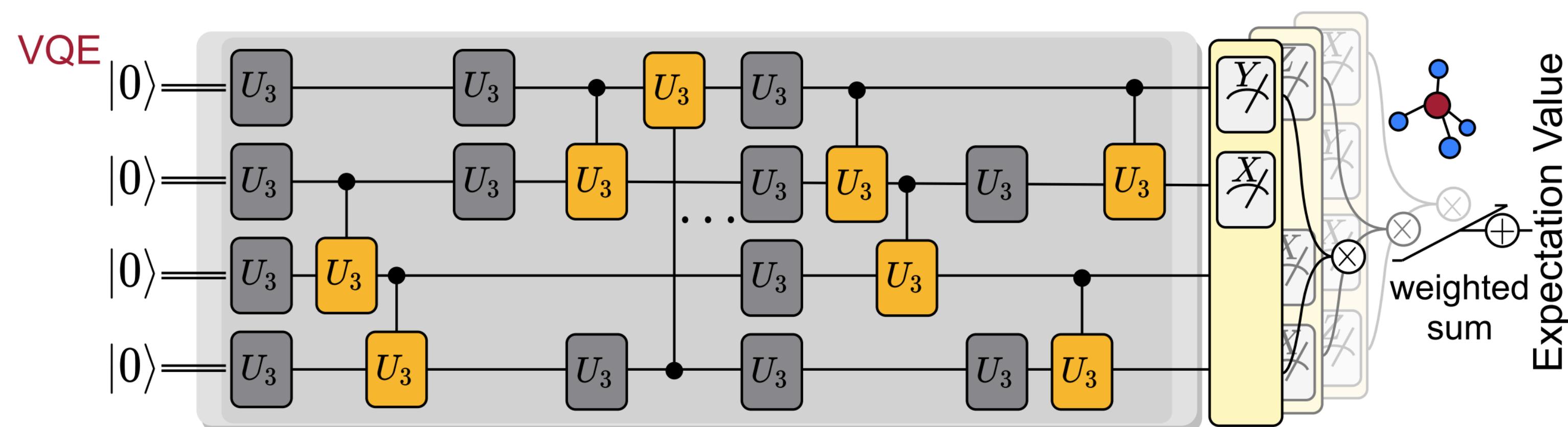


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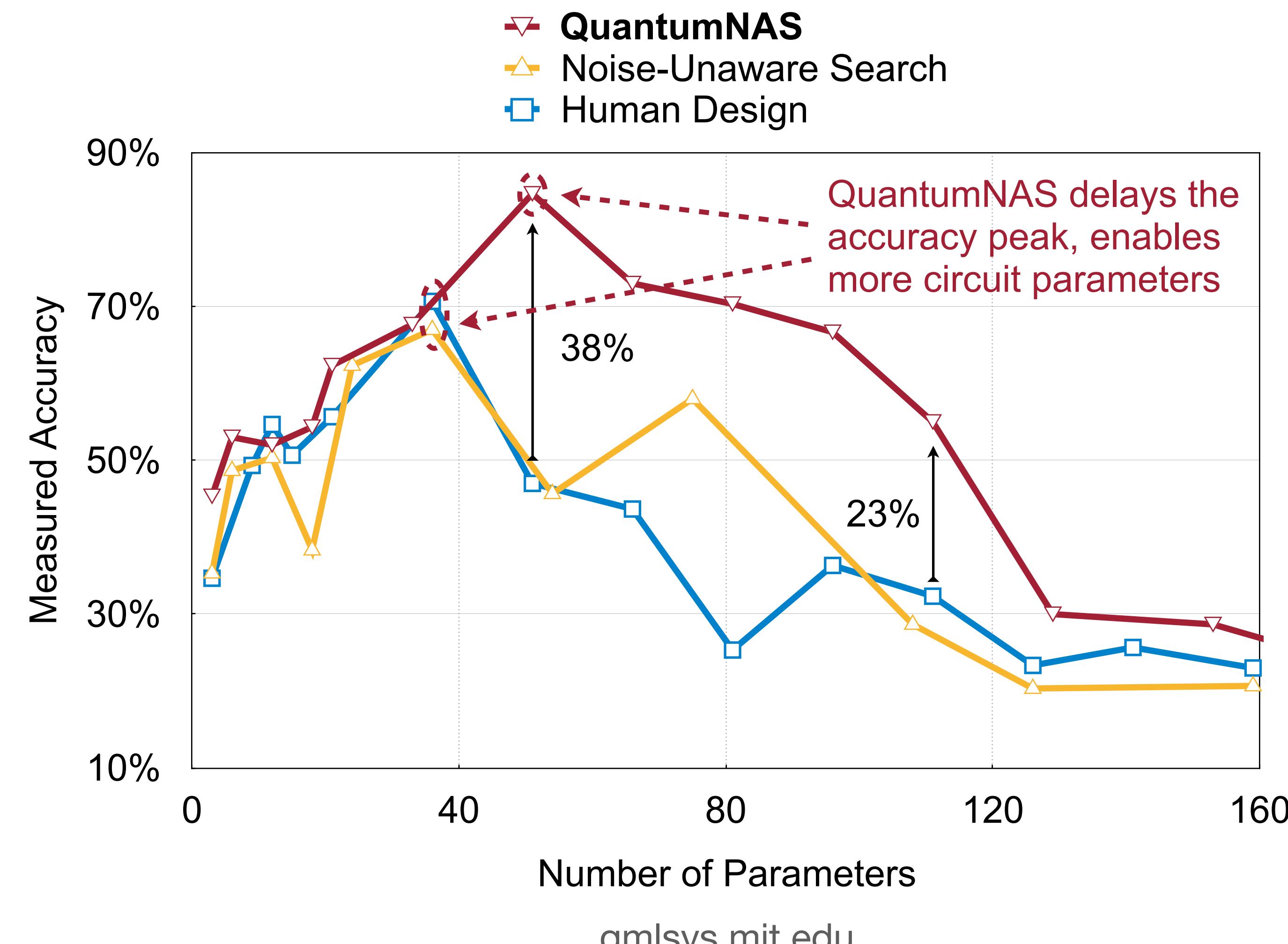


- Variational Quantum Eigensolver: finds the ground state energy of molecule Hamiltonian



QML Results

- 4-classification: MNIST-4 U3+CU3 on IBMQ-Yorktown



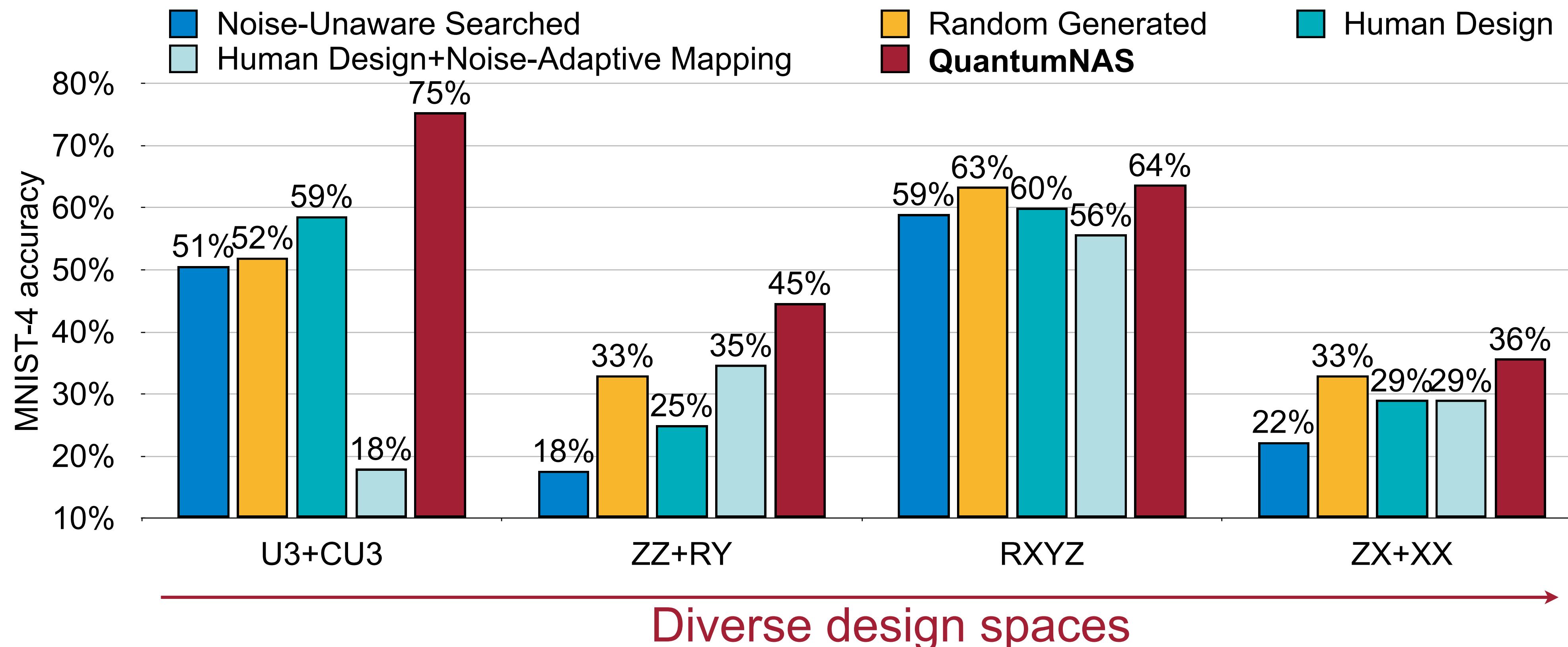
Analysis of Searched Circuit

- QuantumNAS searched circuit has fewer #gates and #params but higher accuracy

MNIST-2	Depth	#Gates	#Params	QuantumNAS
Human Design	64	135	36	88%
QuantumNAS	70	116	22	92%

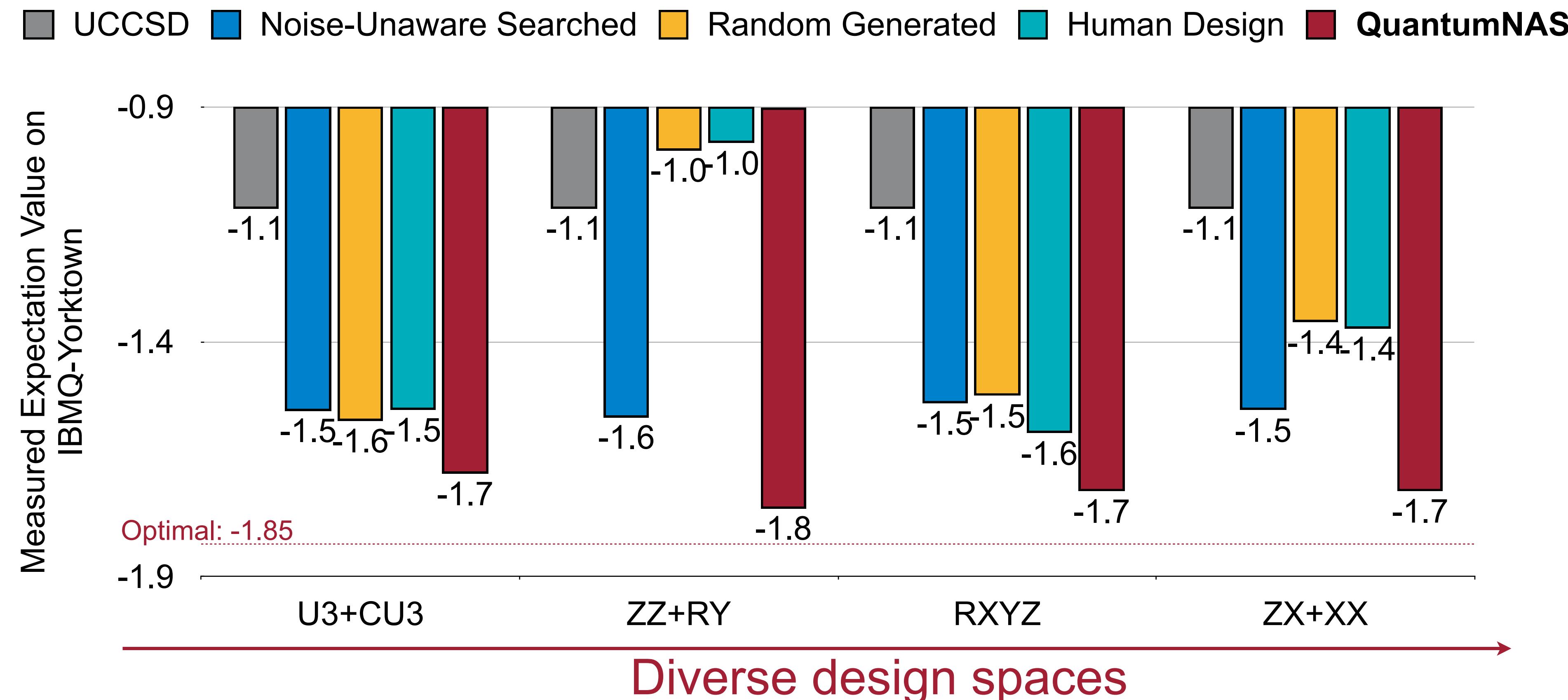
Consistent Improvements on Diverse Design Spaces

- ‘U3+CU3’, ‘ZZ+RY’, ‘RXYZ’, ‘ZX+XX’ spaces with different gates



Consistent Improvements on Diverse Design Spaces

- H2 in different design spaces on IBMQ-Yorktown



Consistent Improvements on Diverse Devices

- QuantumNAS is effective for different real quantum devices
- On different 5-Qubit devices
- MNIST-4, Fashion-4, Vowel-4, MNIST-2, Fashion-2 averaged accuracy

Diverse devices



Method	Noise-Unaware Searched	Random	Human	QuantumNAS
Belem (5Q, 16QV)	47%	50%	67%	76%
Quito (5Q, 16QV)	73%	68%	74%	79%
Athens (5Q, 32QV)	50%	63%	68%	77%
Santiago (5Q, 32QV)	74%	73%	75%	80%

Scalable to Large #Qubits

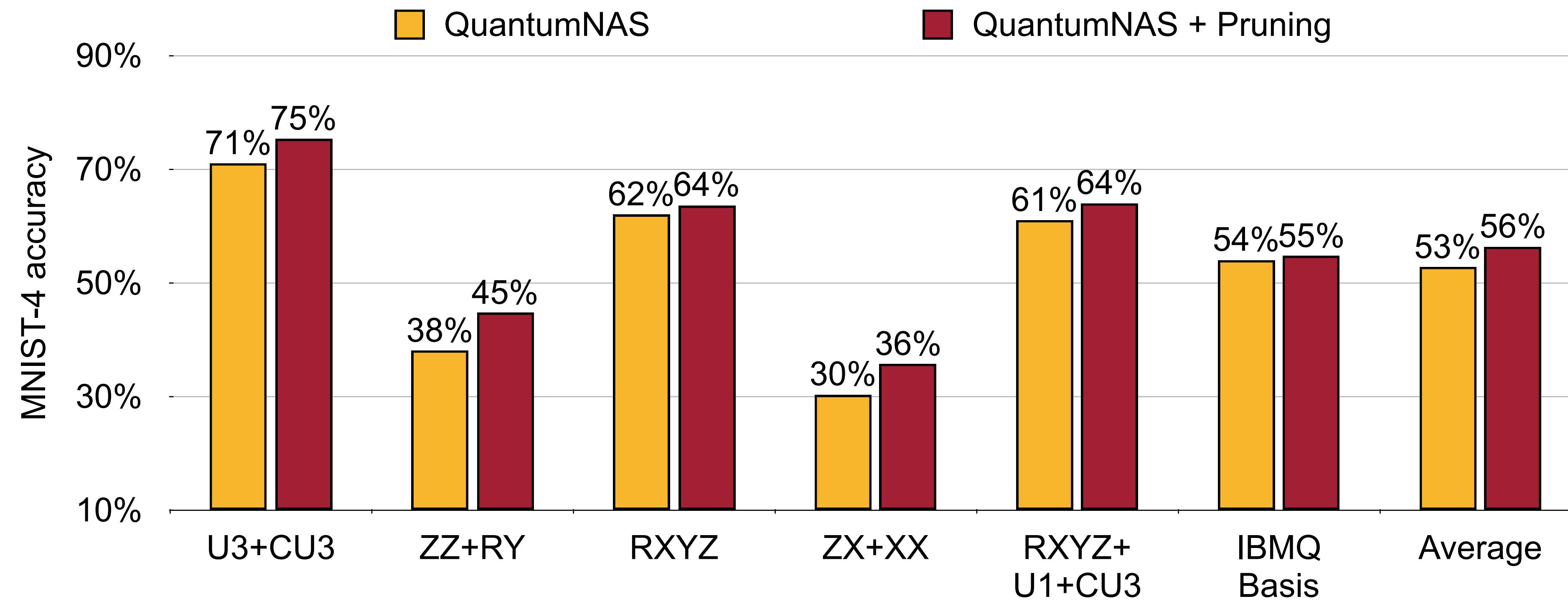
- On large devices
- MNIST-10 accuracy

More Qubits

Method	Noise-Unaware Searched	Random	Human	QuantumNAS
Melbourne (15Q, 8QV, use 15Q)	11%	10%	15%	32%
Guadalupe (16Q, 32QV, use 16Q)	14%	12%	10%	15%
Montreal (27Q, 128QV, use 21Q)	13%	7%	14%	16%
Manhattan (65Q, 32QV, use 21Q)	11%	11%	15%	18%

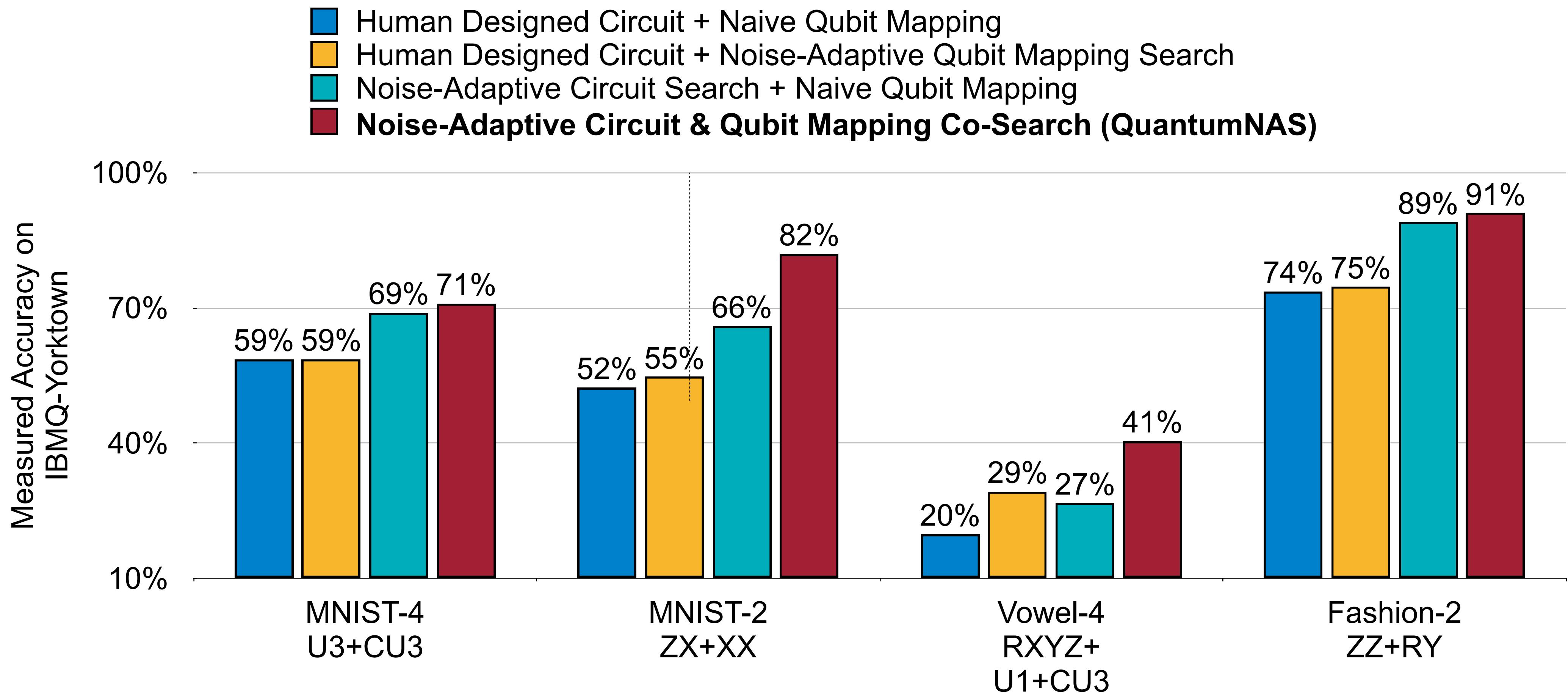
Effective of Quantum Gate Pruning

- For MNIST-4, Quantum gate pruning improves accuracy by 3% on average



Effect of Circuit & Qubit Mapping Co-Search

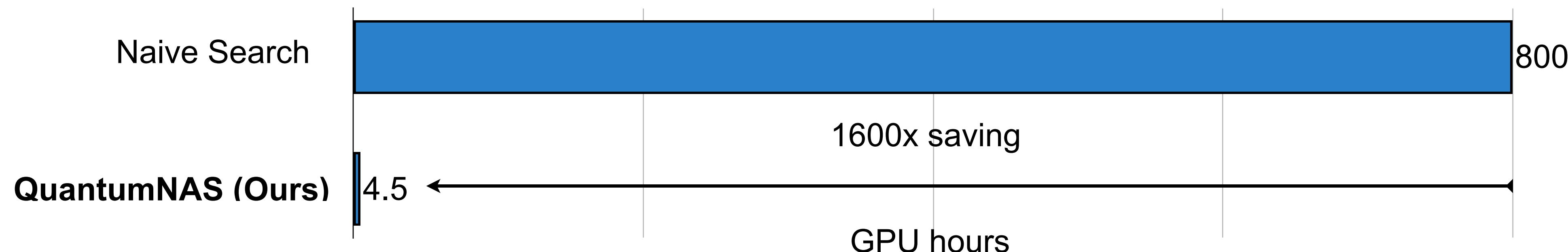
- Co-search is better than only search architecture/mapping



Time Cost

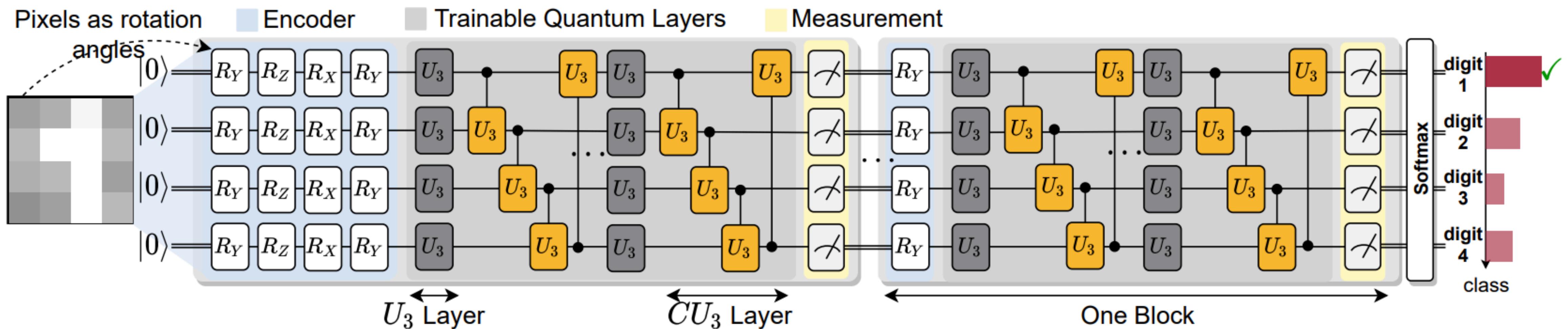
- On 1 Nvidia Titan RTX 2080 ti GPU

#qubits	Step	SuperCircuit Training	Noise-Adaptive Co-search	SubCircuit Training	Deployment on Real QC
4 Qubits		0.5h	3h	0.5h	0.5h
15 Qubits		5h	5h	5h	1h
21 Qubits		20h	10h	15h	1h



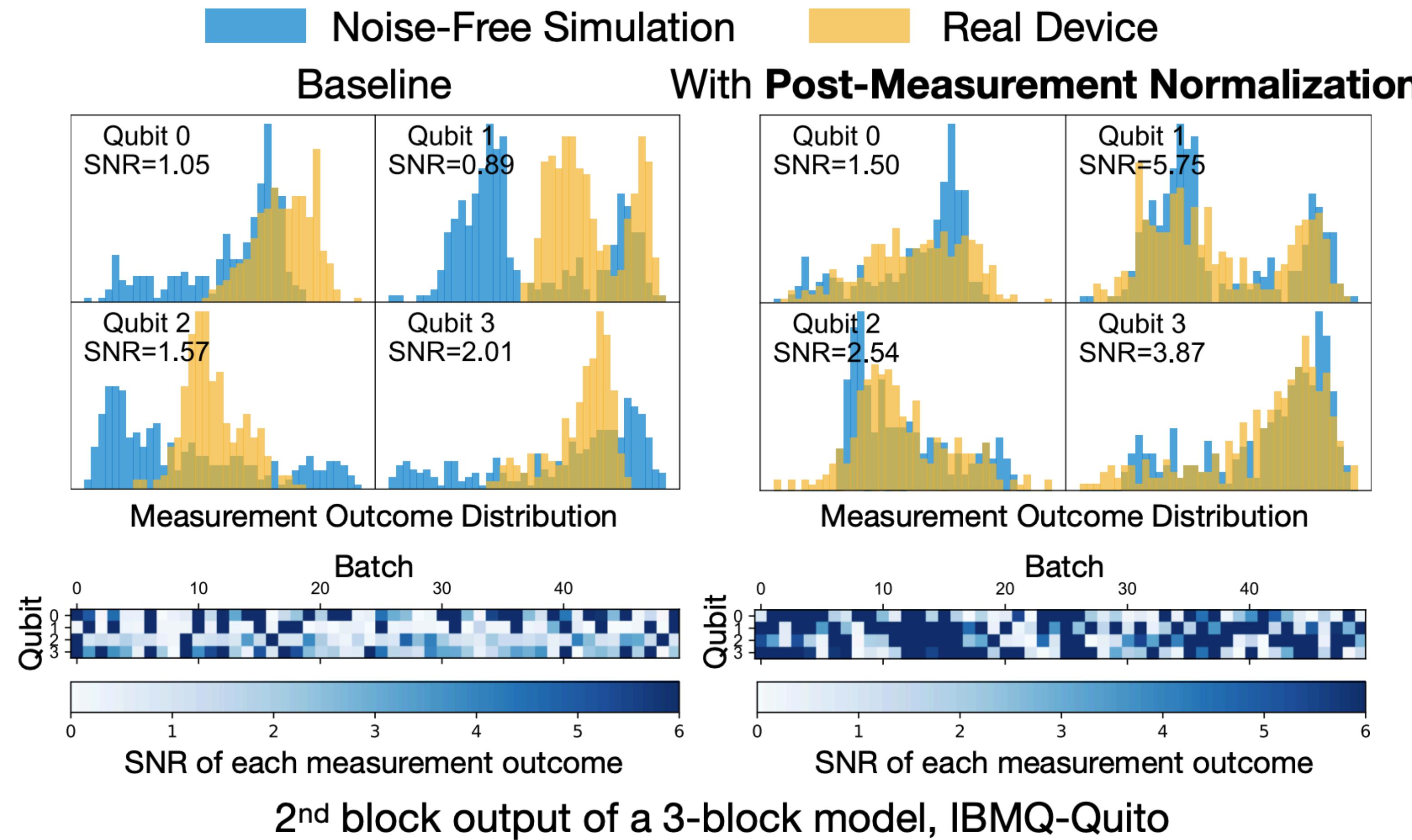
QuantumNAT

- QuantumNAS: find the **circuit architecture** robust to noise
- QuantumNAT: further make the **parameter** robust to noise



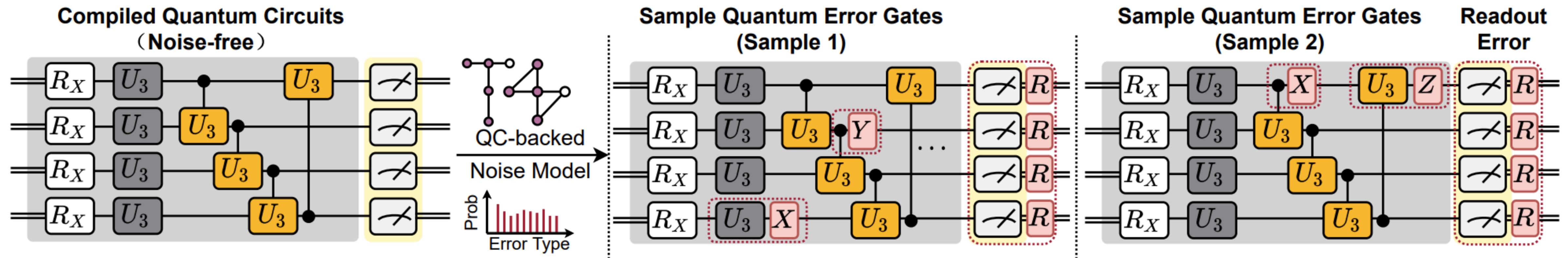
Post-Measurement Normalization

- Normalize the measurement outcomes on the batch dimension



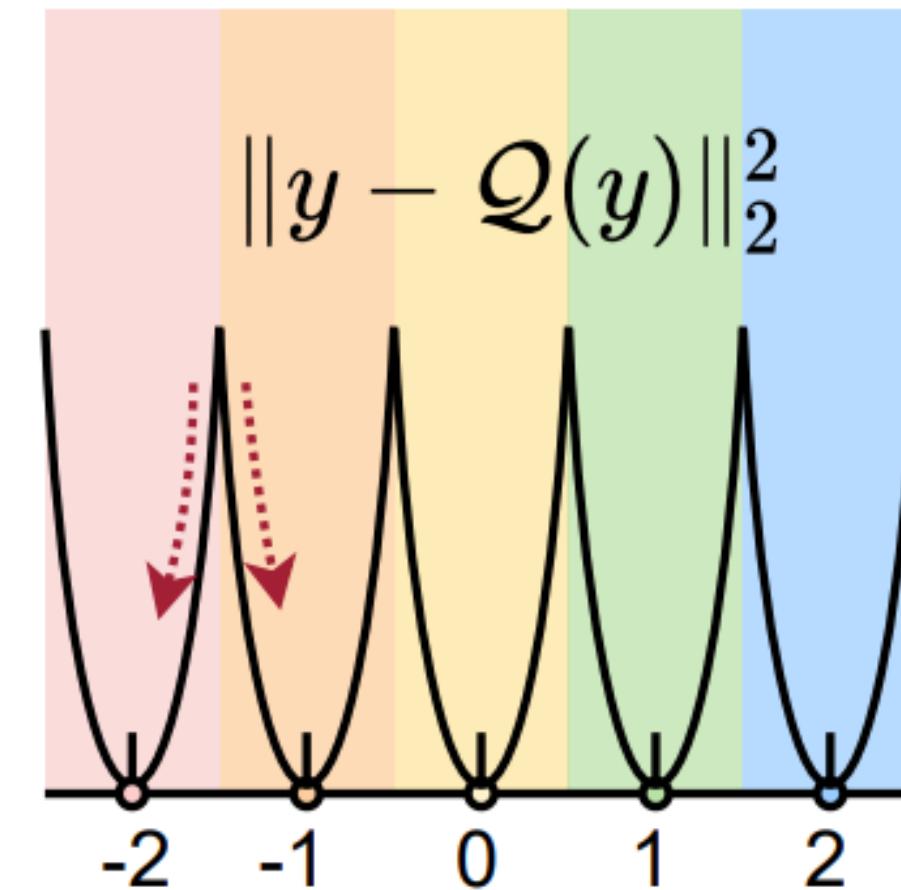
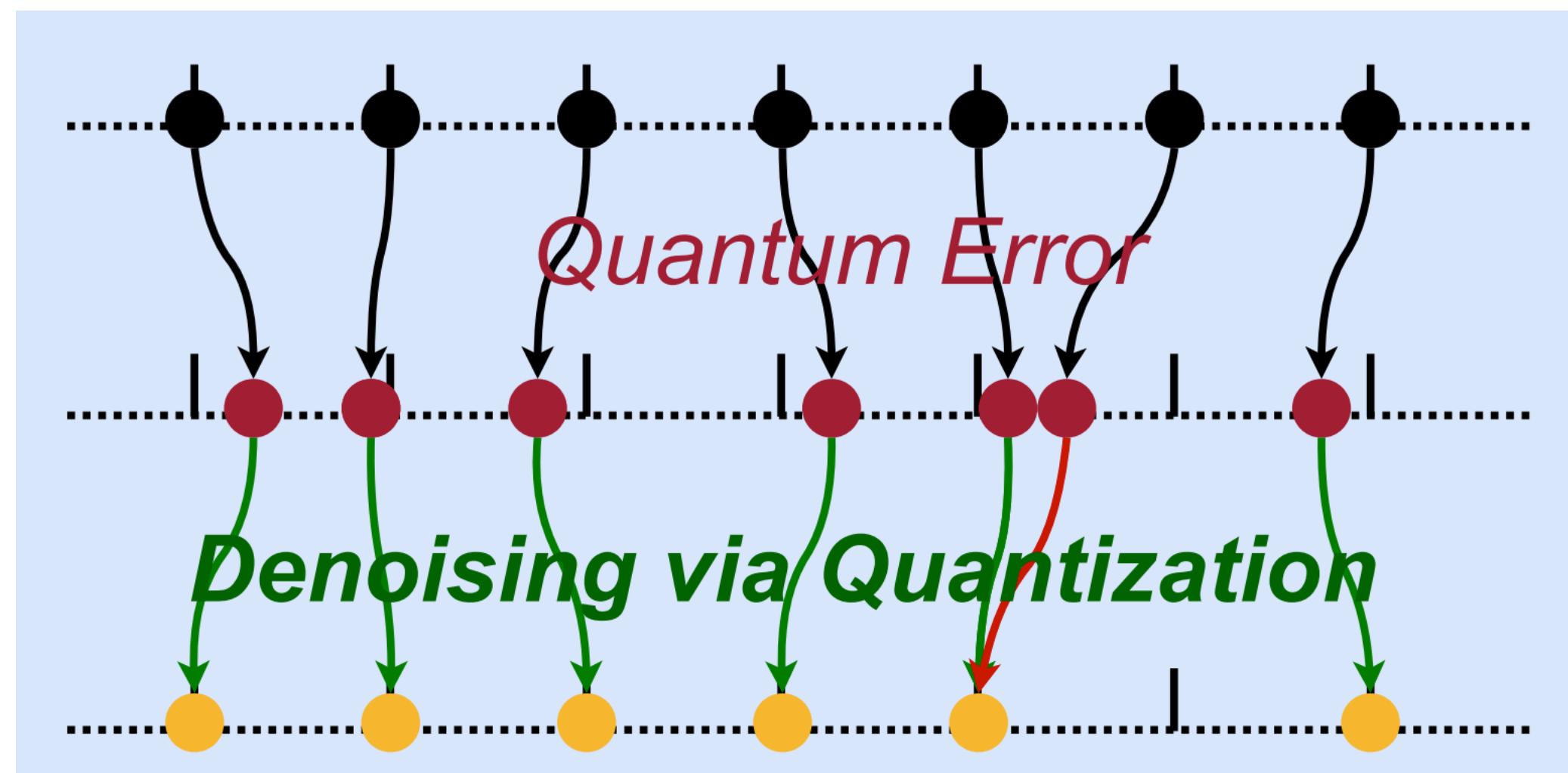
Noise Injection

- Insert noise gate during training, according to the noise model
- For each step, sample new positions for noise gates



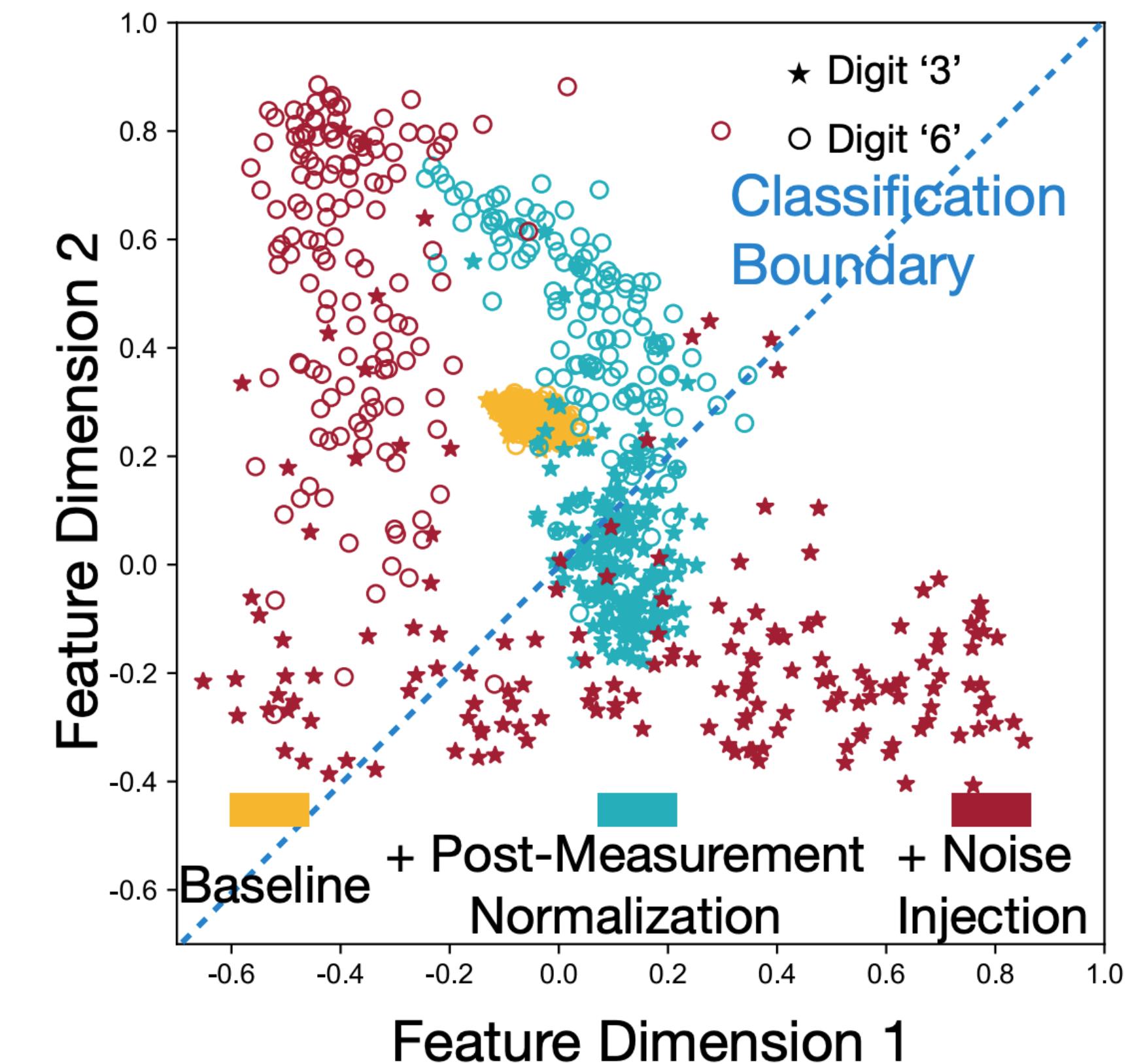
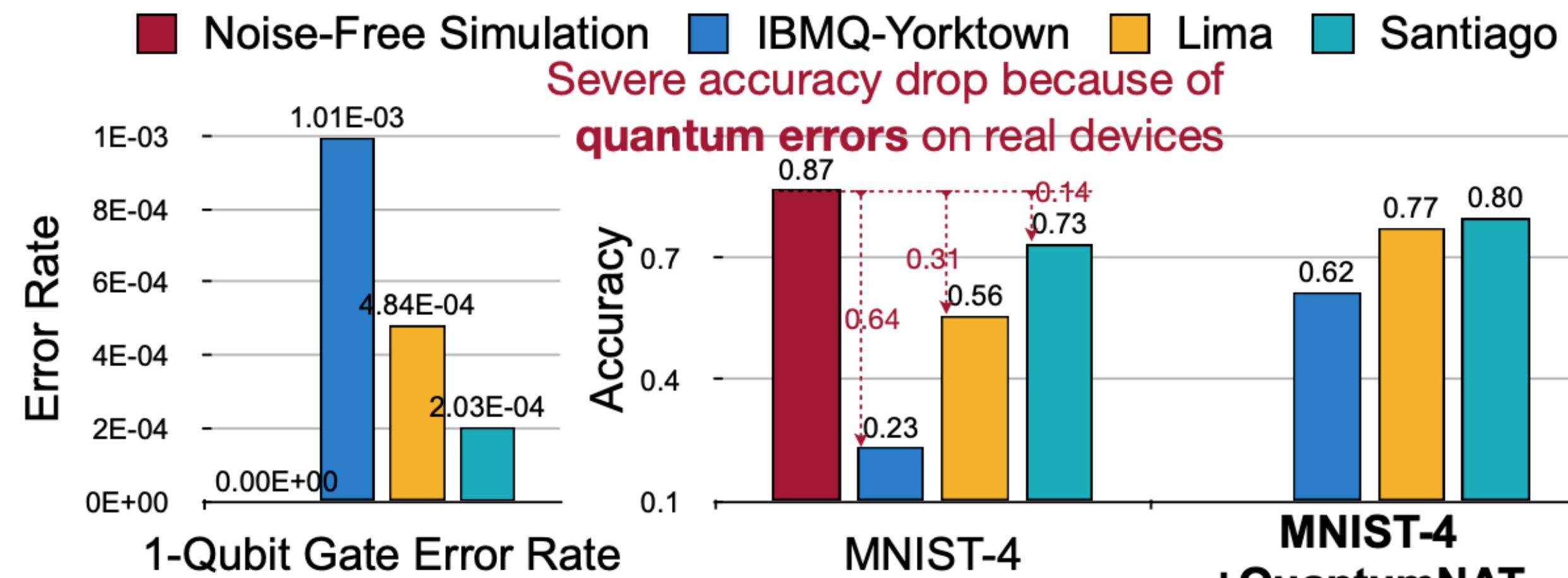
Post-Measurement Quantization

- Quantization provides denoising effects
- Quadratic penalty loss to encourage measurement outcomes close to quantization centroids



Evaluation

- On classification tasks with QNN

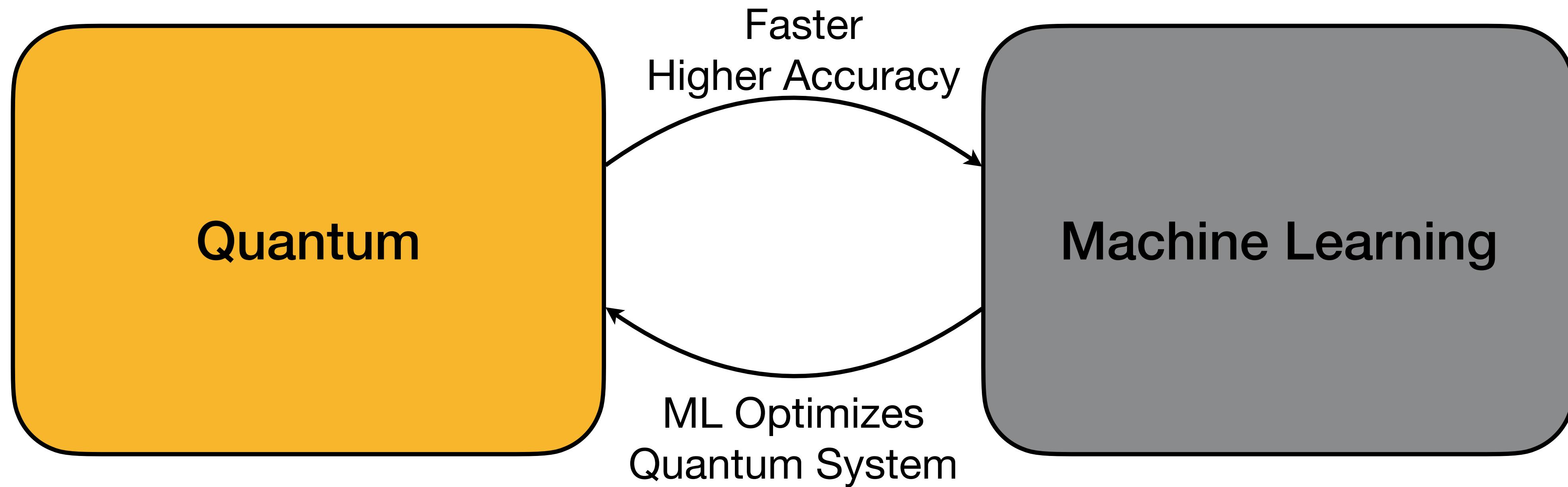




Torch
Quantum

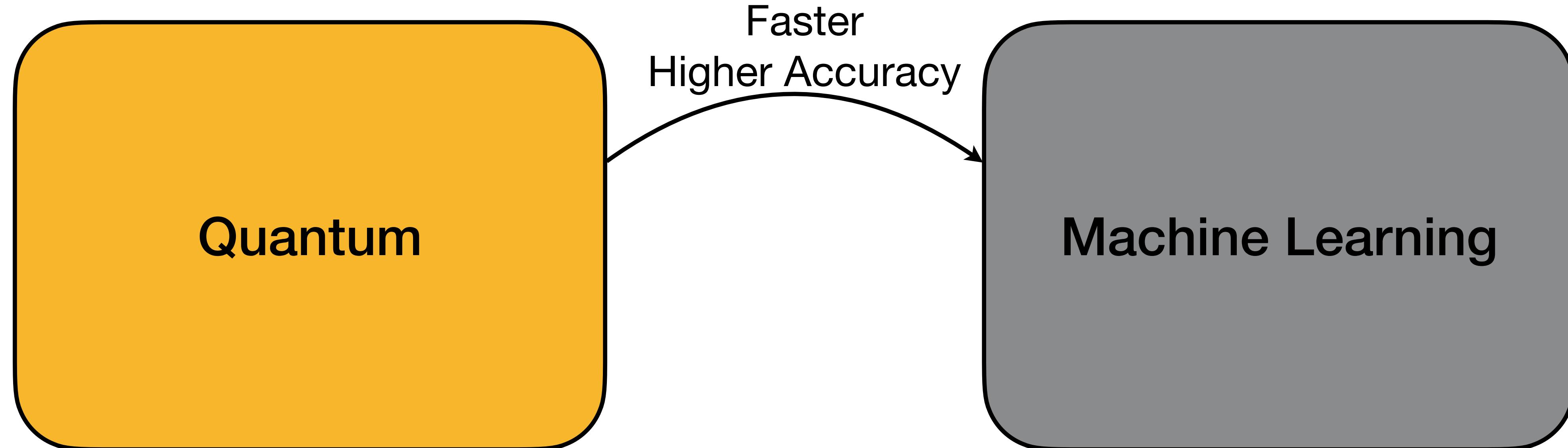
Open-source: TorchQuantum

- TorchQuantum — An open-source library for interdisciplinary research of quantum computing and machine learning
- <https://github.com/mit-han-lab/torchquantum>



Open-source: TorchQuantum

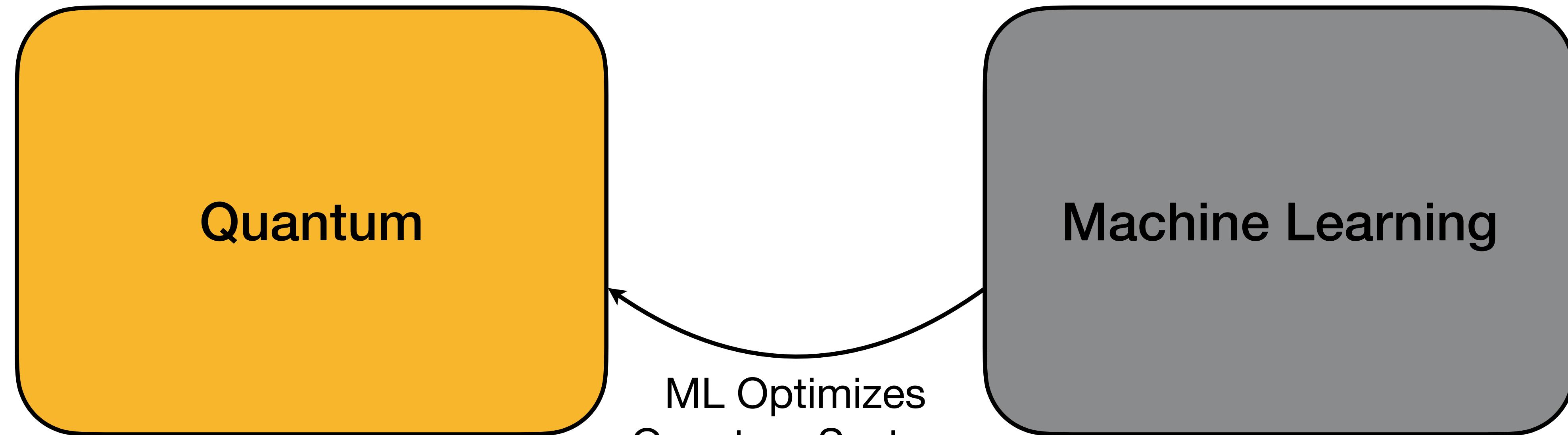
- TorchQuantum — An open-source library for interdisciplinary research of quantum computing and machine learning
- <https://github.com/mit-han-lab/torchquantum>



- Quantum for Machine learning
 - Quantum neural networks
 - Quantum kernel methods

Open-source: TorchQuantum

- TorchQuantum — An open-source library for interdisciplinary research of quantum computing and machine learning
- <https://github.com/mit-han-lab/torchquantum>



- Machine Learning for Quantum
 - ML for quantum compilation (qubit mapping, unitary synthesis)
 - ...

TorchQuantum

- Features
 - Easy construction of **parameterized quantum circuits** such as Quantum Neural Networks in PyTorch
 - Support **batch mode inference and training** on GPU/CPU, supports highly-parallelized training
 - Support **easy deployment** on real quantum devices such as IBMQ
 - Provide tutorials, videos and example projects of QML and using ML to optimize quantum computer system problems

PyTorch Implementations

- Statevector

```
_state = torch.zeros(2 ** self.n_wires, dtype=C_DTYPE)
_state[0] = 1 + 0j
```

- Quantum Gates

```
'cnot': torch.tensor([[1, 0, 0, 0],
                      [0, 1, 0, 0],
                      [0, 0, 0, 1],
                      [0, 0, 1, 0]], dtype=C_DTYPE),
```

PyTorch Implementations

- Quantum Gates

```
def crx_matrix(params):
    theta = params.type(C_DTYPE)
    co = torch.cos(theta / 2)
    jsi = 1j * torch.sin(-theta / 2)

    matrix = torch.tensor([[1, 0, 0, 0],
                          [0, 1, 0, 0],
                          [0, 0, 0, 0],
                          [0, 0, 0, 0]], dtype=C_DTYPE, device=params.device
                         ).unsqueeze(0).repeat(co.shape[0], 1, 1)

    matrix[:, 2, 2] = co[:, 0]
    matrix[:, 2, 3] = jsi[:, 0]
    matrix[:, 3, 2] = jsi[:, 0]
    matrix[:, 3, 3] = co[:, 0]

    return matrix.squeeze(0)
```

- Matrix-vector multiplication:
torch.einsum and torch.bmm

Examples and tutorials

- Tutorial Colab and videos



TorchQuantum Tutorials Opening

Hanrui Wang
MIT HAN Lab



TorchQuantum Tutorials Quanvolutional Neural Network

Zirui Li, Hanrui Wang
MIT HAN Lab



MIT HAN LAB



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Thank you for listening!

- **QuantumNAS** exploits **SuperCircuit**-based co-search for most **noise-robust** circuit architecture and qubit mapping
- Iterative **quantum gate pruning** to further remove redundant gates
- **QuantumNAT** exploits noise injection, post-measurement Normalization, quantization to improve robustness of parameters
- Improves MNIST 2-class accuracy from 88% to **95%**, 10-class from 15% to **34%**
- Save search cost by over **1,000 times**
- Open-sourced **TorchQuantum** library for Quantum + ML research



Torch
Quantum

<https://github.com/mit-han-lab/torchquantum>

qmlsys.mit.edu



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