Hybrid Quantum and Classic Computing to Solve Optimization and Machine Learning Problems for Future Networking

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Outline

- Motivation and Quantum Computing Basics
- Quantum Annealing
  - Adiabatic Quantum Computing
  - Quadratic Unconstrained Binary Optimization (QUBO)
- Hybrid Quantum Benders' Decomposition
  - Benders Decomposition Basics
  - Quantum and Class Computing Bender Decomposition
- Applications
  - Federated Learning in Mobile Edge Computing
  - Feature Selection for Machine Learning
  - Cable-Routing Problem in Solar Power Plants
- Conclusions

Quantum Computing

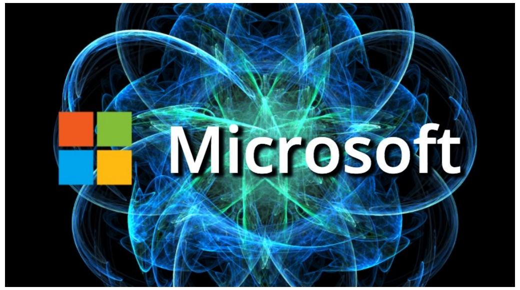
Started in the 1980s

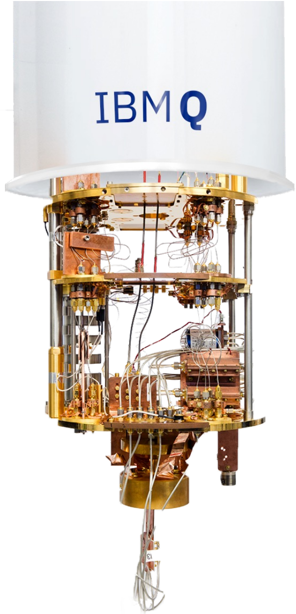
Maybe we need to use quantum mechanics in our computers.

Feynman

Many years later …

Quantum Computer





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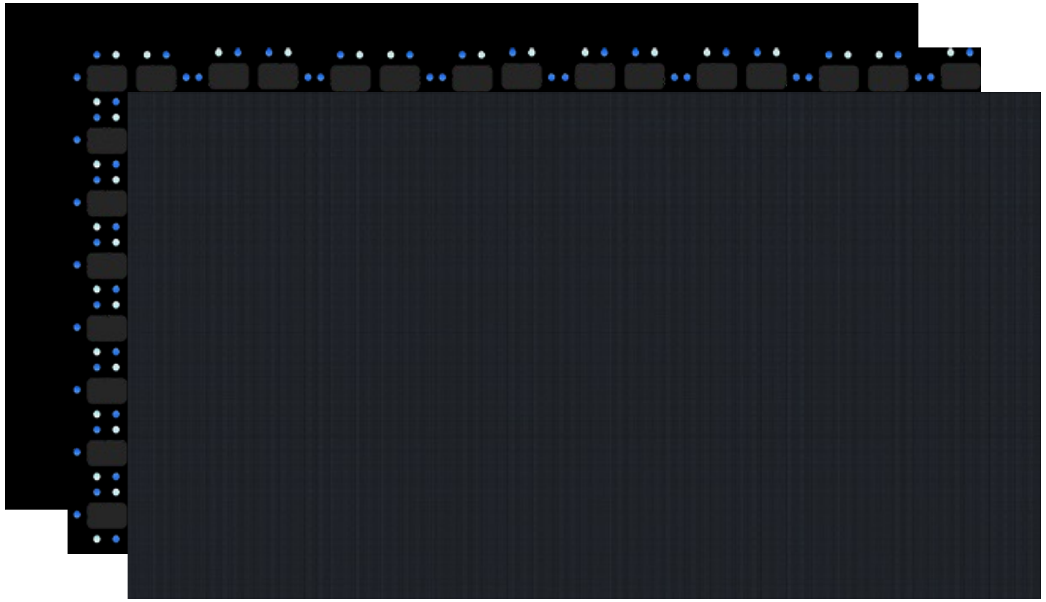
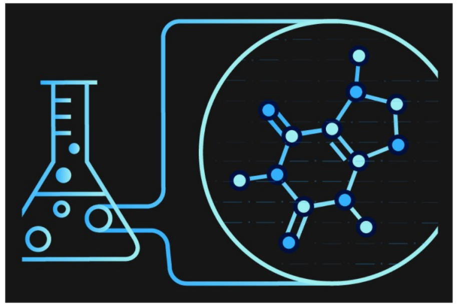
What is wrong in classic computing?

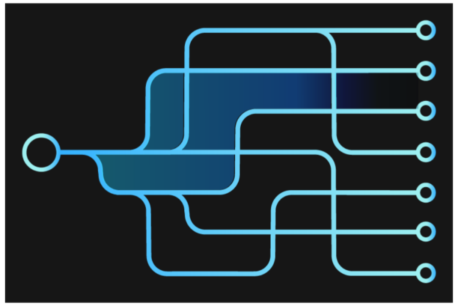


Total number of combinations exponentially grows

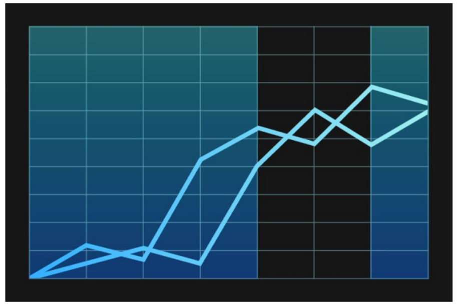
Many Practical NP hard Problems



Life science



Manufacturing & Logistics



Financial Services

Quantum Computing

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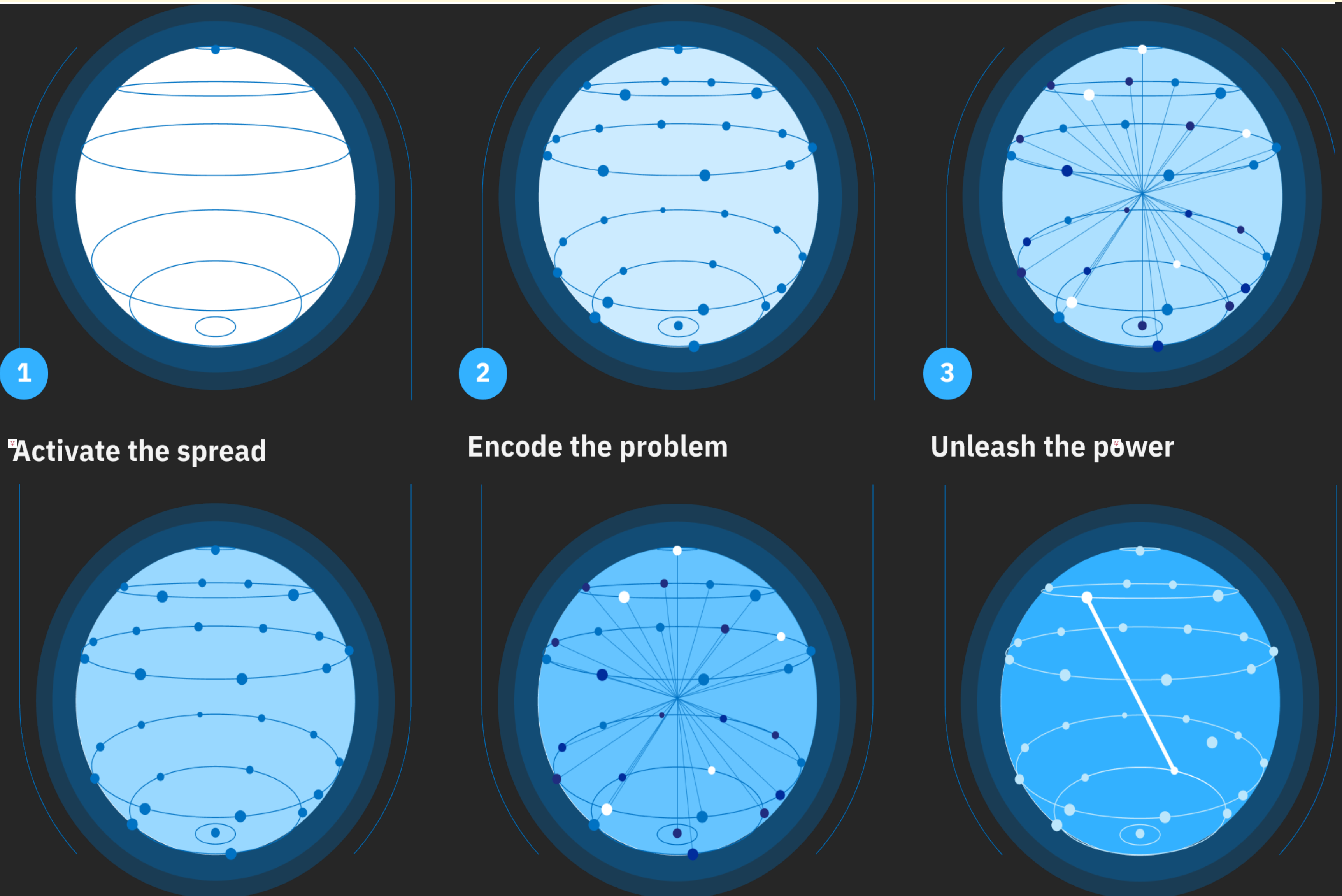

Quantum computers
Can

create vast multidimensional spaces
to deal with large problems,

and translate them back into
what we can use,

while
classical computers
may have
difficulties
to do the same.

Known Types of Quantum Computing and Their Applications and Generality.

Application:	Optimization	**Application:**	Chemistry Sampling Quantum Dynamics	**Application:**	Cryptography Searching Secure Computing
Generality :	Restrictive	**Generality :**	Partial	**Generality :**	High

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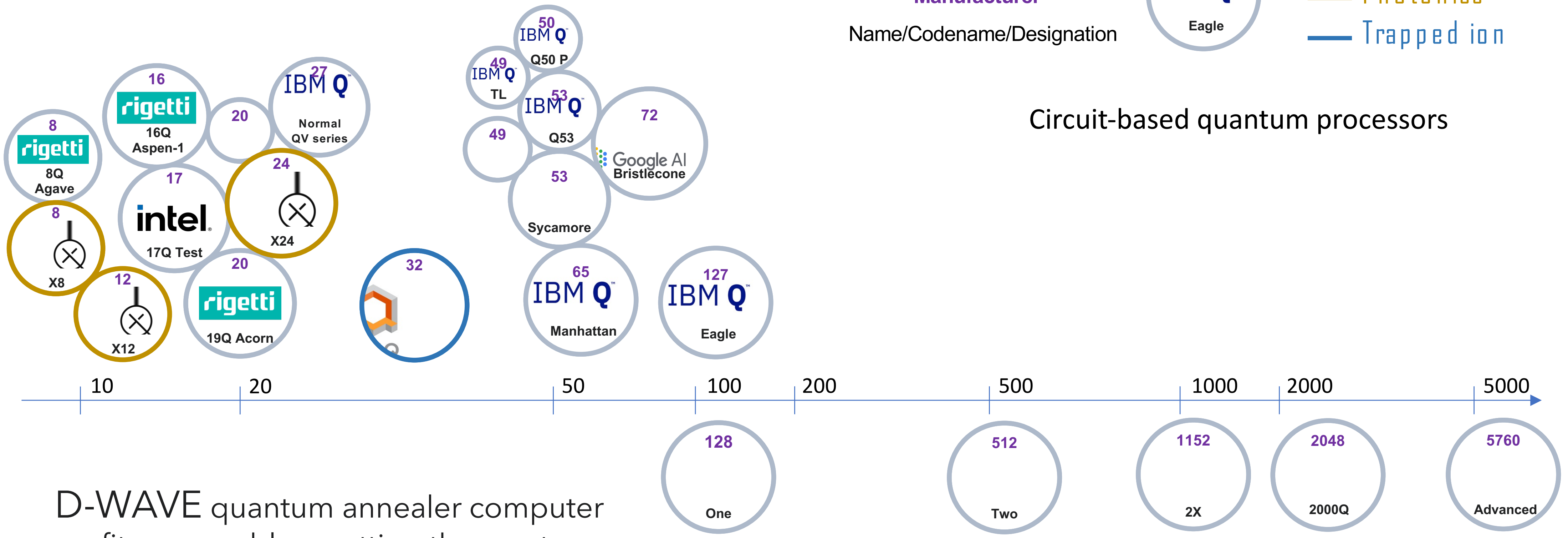
Rank of quantum processors



D-WAVE quantum annealer computer fits our problem setting the most.

Annealing quantum processors

Quantum Computing

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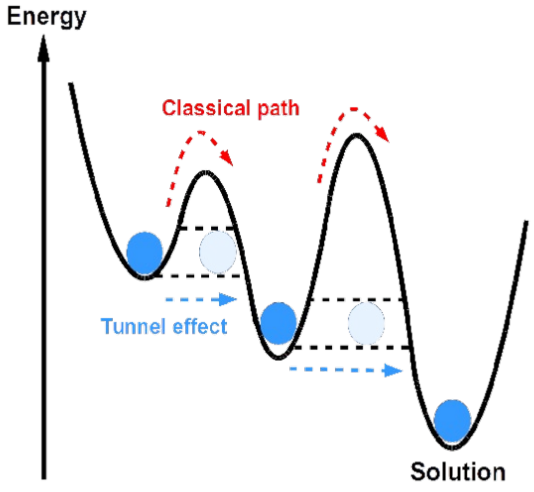
Quantum Computer

- Gate Model
- Analog Quantum Model
- Quantum Annealing

Why Quantum Annealing?



Quantum Tunnelling

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Quantum Annealing

Annealing a Metal

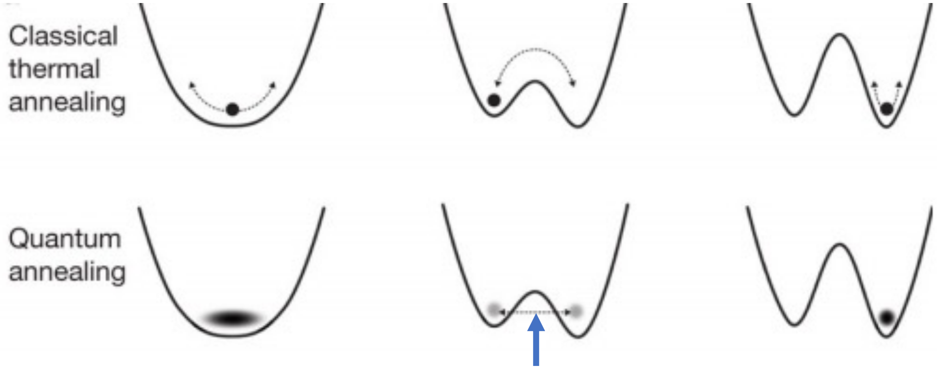
- Heat the metal to a temperature;
- Lower the temperature;

Simulated Annealing

- Heuristic algorithm;
- Random search method;
- **The temperature variable;**

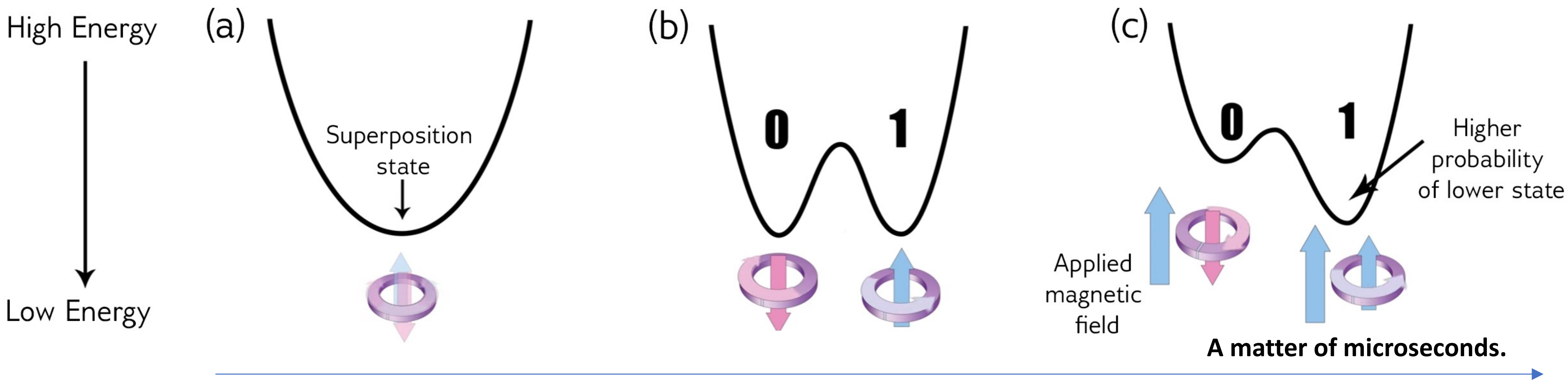
Quantum Annealing

- **Enables jumping from one classical state to another**
  – Decreases likelihood of getting stuck in a local minimum
- **Width of energy barrier is important, but height is not**



Quantum Annealing

Energy diagram in Quantum Annealing



- Initial Qubits
- Superposition at $|0\rangle$s and $|1\rangle$s.
- Not yet coupled.

- Qubits are entangled.
- At state of many possible answers.
- Couplers & biases applied

- Inputs' energy are set.
- Lowest energy is at or closes to the optima.
- Energy possibility

Qubits and Quantum Operation

- Superposition
  - A quantum bit $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$
  - $||\alpha||^2$: the probability in state $|0\rangle$
  - $||\beta||^2$: the probability in state $|1\rangle$
  - $||\alpha||^2 + ||\beta||^2 = 1$

A general n-qubit system:

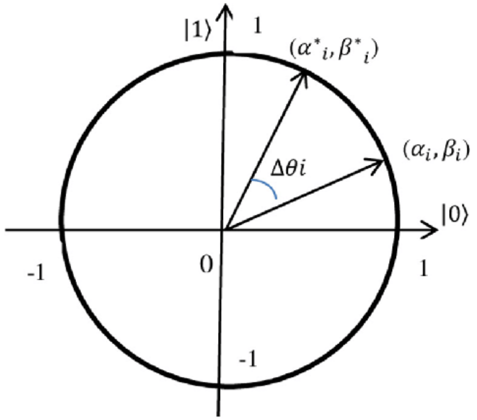
$$|\Phi\rangle = \sum_{i=0}^{N-1} \alpha_i |i\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \cdots \\ \alpha_{N-2} \\ \alpha_{N-1} \end{pmatrix}, N = 2^n.$$

$|i\rangle$ is the i^{th} computational basis of the space, and α_i is the amplitude of the i^{th} computational basis.

- Tensor Product

$|\psi\rangle \otimes |\phi\rangle$ represents the overall state of two quantum bits.

$$|\psi\rangle \otimes |\phi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle,$$
$$||\alpha_{00}||^2 + ||\alpha_{01}||^2 + ||\alpha_{10}||^2 + ||\alpha_{11}||^2 = 1.$$



Schrödinger Equation

- Evolution from $t = 0$ to $t = T$
- $i\frac{\partial|\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle$
- $|\psi(t)\rangle$: The actual state of the system
- $H(t)$: a time-dependent Hamiltonian (i.e., Kinetic Energy + Potential Energy)
- $H(t)|\phi_j(t)\rangle = E_j(t)|\phi_j(t)\rangle$
- $|\phi_j(t)\rangle$: the $j-$th instantaneous eigenstate
- $E_j(t)$: the $j-$th instantaneous eigenvalue

Before studying quantum

after studying quantum

Adiabatic Quantum Computing & Algorithm

Overview

- A computing approach utilizing the quantum mechanics (e.g., superposition, entanglement).
- Prepare the system in an initial state and transform it to the final state.
- Has the potential to speed up the computing process.
- Polynomial equivalent to circuit model
- Applications: PageRank algorithm, Quadratic Unconstrained Binary Optimization, Machine Learning.

Algorithm

- 1: Encoding the solution
  - 1) Encoding target solution in final state $|\phi_j(T)\rangle, j = 0$.
  - 2) Encoding target function in the final eigenvalue $E_j(T)$.
  - 3) Find the Hamiltonian $H(T) = H_{fin}$ based the encoding rules.
- 2: Prepare the initial Hamiltonian $H(0) = H_{ini}$ and its eigenstates $|\phi_j(0)\rangle, j = 0$.
- 3: Prepare the time dependent Hamiltonian $H(t) = (1 - f(t))H_{ini} + f(t)H_{fin}$, $f(0) = 0, f(1) = 1, 0 \leq f(t) \leq 1, \mathit{for}\ t \in [0, T]$. function $f(t)$ is at least twice differentiable.
- 4: Evolve the system from $t = 0$ to $t = T$, then observe the final state to obtain the solution.

Link Schrodinger with QUBO Model

Core idea: Encoding the objective function as the eigenvalue of the final ground state of Schrodinger equation, based on Adiabatic Quantum Computing Model

Ising Model

$$H(s) = \sum_i h_i s_i + \sum_i \sum_{i<j} J_{i,j} s_i \, s_j$$

Neighboring spins interact with each other

Spins interact with applied field

QUBO (Quadratic Unconstrained Binary Optimization)

$$f(x) = \sum_i \boldsymbol{Q}_{i,i} x_i + \sum_i \sum_{i<j} \boldsymbol{Q}_{i,j} x_i \, x_j$$

$\boldsymbol{Q}$: upper-diagonal matrix

These important optimization problems can be transformed into QUBO model:

- Knapsack Problems
- Assignment Problems
- Task Allocation Problems
- Capital Budgeting Problems
- … (NP-hard problem)

Quantum Computing: provide an alternative method to solve some NP-hard problems

Dwave Hamiltonian

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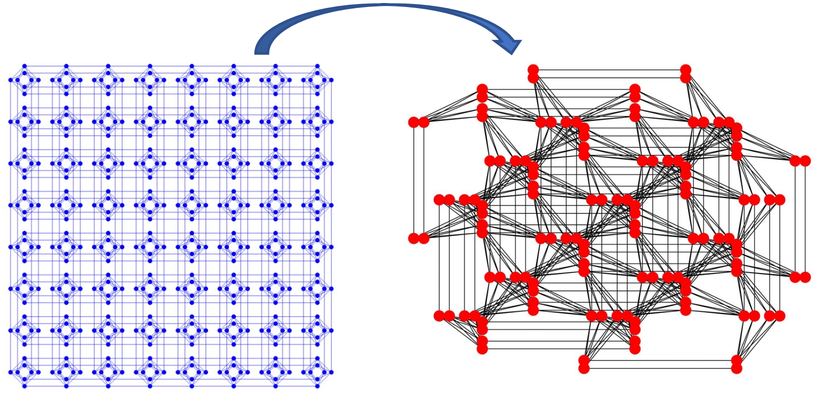
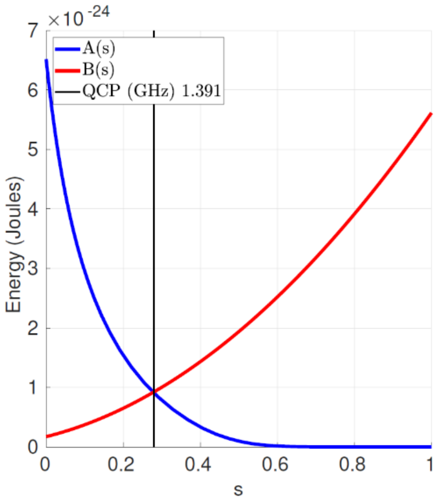
- **Problem of Hamiltonian:**

$$H_{ising} = \underbrace{-\frac{A(s)}{2}\left(\sum_i \hat{\sigma}_x^{(i)}\right)}_{\text{Initial Hamiltonian}} + \underbrace{\frac{B(s)}{2}\left(\sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}\right)}_{\text{Final Hamiltonian}}$$



- **Goal (what the hardware does)**
  - Minimize $\sigma_i \in \{-1,+1\}$ subject to provided $J_{i,j} \in \mathrm{R}$ and $h_i \in \mathrm{R}$ coefficients
  - In other words, a quantum optimization program is merely a list of $J_{i,j}$ and h_i
- **Classical**
  - Much easier to reason than a quantum Hamiltonian
  - Quantum effects are used internally to work towards the goal
- **Sparsely connected**
  - Possible to map fully connected problems onto the D-Wave's Chimera graph



Dwave Hamiltonian

- **Considering only the external field:**

$$\mathcal{H}_P = \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} J_{i,j}\sigma_i^z\sigma_j^z + \sum_{i=0}^{N-1} h_i\sigma_i^z$$

- **We arbitrarily call** $\sigma^z = +1$ **"TRUE" and** $\sigma^z = -1$ **"FALSE"**
- **Here are the optimal values of** σ^z **for different values of h**

Negative (say, $h_i = -5$)

σ_i^z	$h_i\sigma_i^z$
–1	+5
+1	–5

Zero

σ_i^z	$h_i\sigma_i^z$
–1	0
+1	0

Positive (say, $h_i = +5$)

σ_i^z	$h_i\sigma_i^z$
–1	–5
+1	+5

- **Observations**
  - A negative h_i means, "I want σ^z to be TRUE"
  - A zero h_i means, "I don't care if σ^z is TRUE or FALSE"
  - A positive h_i means, "I want σ^z to be FALSE"

Dwave Hamiltonian

- **Consider only the coupler strengths:**

$$\mathcal{H}_P = \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} J_{i,j}\sigma_i^z\sigma_j^z + \sum_{i=0}^{N-1} h_i\sigma_i^z$$

- **Here are the optimal values of σ^z and σ^z for different values of $J_{i,j}$:**

Negative ($J_{i,j} = -5$)

σ_i^z	σ_j^z	$J_{i,j}\sigma_i^z\sigma_j^z$
–1	–1	–5
–1	+1	+5
+1	–1	+5
+1	+1	–5

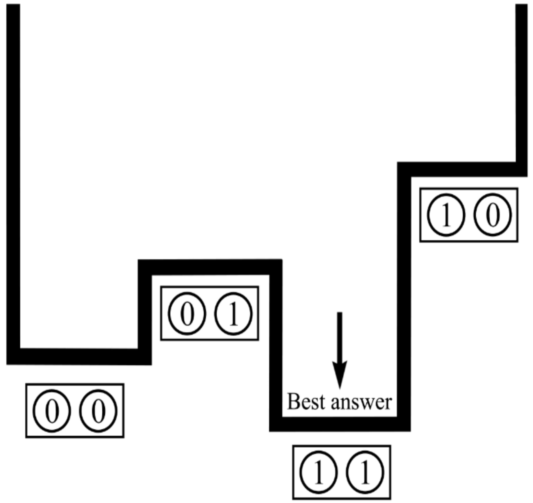
Zero

σ_i^z	σ_j^z	$J_{i,j}\sigma_i^z\sigma_j^z$
–1	–1	0
–1	+1	0
+1	–1	0
+1	+1	0

Positive ($J_{i,j} = +5$)

σ_i^z	σ_j^z	$J_{i,j}\sigma_i^z\sigma_j^z$
–1	–1	+5
–1	+1	–5
+1	–1	–5
+1	+1	+5

- **Observations**
  - A negative $J_{i,j}$ means, "I want σ^z_i and σ^z_j to be equal"
  - A zero $J_{i,j}$ means, "I don't care how σ^z_i and σ^z_j are related"
  - A positive $J_{i,j}$ means, "I want σ^z_i and σ^z_j to be different"



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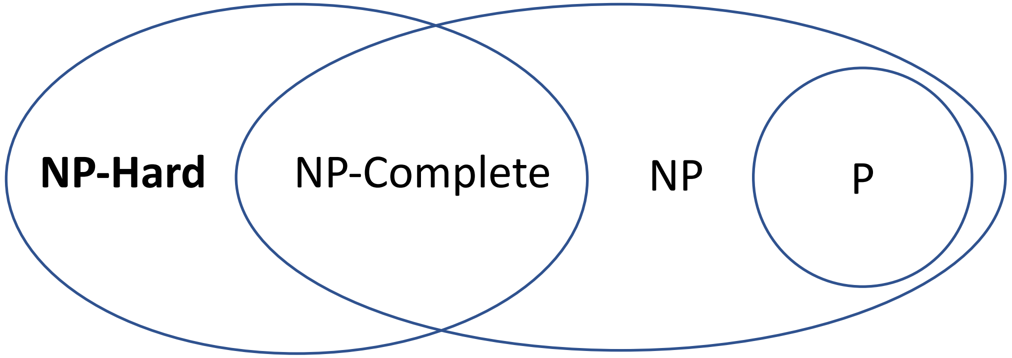
Mixed-integer Linear Programming is of:

$$\begin{aligned} \max_{\mathbf{x},\mathbf{y}} \quad & \mathbf{c}^\top \mathbf{x} + \mathbf{h}^\top \mathbf{y} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} + \mathbf{G}\mathbf{y} \le \mathbf{b} \\ & \mathbf{x} \in \mathbf{X}, \mathbf{x} \in \{0,1\}^n \\ & \mathbf{y} \in \mathbb{R}^p \end{aligned}$$

- Mixed-Integer linear Programming (MILP) is **NP-Hard**.
- It can't be solved in polynomial time unless P=NP.



Problem type	Example Problem
NP-Hard	Matrix Permanent Turing Halting Problem **MILP**
NP-Complete	Steiner Tree Graph 3-coloring Maximum Clique
NP	Factoring Graph Isomorphism
P	Linear Programming Graph Connectivity

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Consider a Mixed-integer Linear Programming

$$\begin{aligned} \max_{\mathbf{x},\mathbf{y}} \quad & \mathbf{c}^\intercal \mathbf{x} + \mathbf{h}^\intercal \mathbf{y} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} + \mathbf{G}\mathbf{y} \leq \mathbf{b} \\ & \mathbf{x} \in \mathbf{X}, \mathbf{x} \in \{0,1\}^n \\ & \mathbf{y} \in \mathbb{R}_+^p \end{aligned} \xrightarrow{\text{z}_{\text{LP}} \text{ Replacement}} \begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{c}^\intercal \mathbf{x} + z_{LP}(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in \mathbf{X},\ \mathbf{x} \in \{0,1\}^n \end{aligned}$$

We denote the value of the best choice for $\mathbf{y}$ by $\text{z}_{\text{LP}}(\mathbf{x})$

$$\begin{aligned} z_{LP}(\mathbf{x}) = \max_{\mathbf{y}} \quad & \mathbf{h}^\intercal \mathbf{y} \\ \text{s.t.} \quad & \mathbf{G}\mathbf{y} \leq \mathbf{b} - \mathbf{A}\mathbf{x} \\ & \mathbf{y} \in \mathbb{R}_+^p \end{aligned} \xrightarrow{\text{LP Duality}} \begin{aligned} z_{LP}(\mathbf{x}) = \min_{\mathbf{u}} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\intercal \mathbf{u} \\ \text{s.t.} \quad & \mathbf{G}^\intercal \mathbf{u} \geq \mathbf{h} \\ & \mathbf{u} \in \mathbb{R}_+^m \end{aligned}$$

Feasible Region Q

Continue:

$$z_{LP}(\mathbf{x}) = \min_{\mathbf{u}} \quad (\mathbf{b}-\mathbf{A}\mathbf{x})^\intercal \mathbf{u}$$
$$\text{s.t.} \quad \mathbf{G}^\intercal \mathbf{u} \geq \mathbf{h}, \quad \mathbf{u} \in \mathbb{R}^m_+$$

Feasible Region Q

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{c}^\intercal \mathbf{x} + z_{LP}(\mathbf{x}) \\ \text{s.t.} \quad & (\mathbf{b}-\mathbf{A}\mathbf{x})^\intercal u^k \geq z_{LP}(\mathbf{x}) \quad \text{for } k \in K \\ & (\mathbf{b}-\mathbf{A}\mathbf{x})^\intercal r^j \geq 0 \quad \text{for } j \in J \\ & z_{LP}(\mathbf{x}) \in \mathbb{R}, \ \mathbf{x} \in \mathbf{X}, \ \mathbf{x} \in \{0,1\}^n \end{aligned}$$

The feasible region Q does not depend on x.

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{c}^\intercal \mathbf{x} + z_{LP}(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in \mathbf{X}, \ \mathbf{x} \in \{0,1\}^n \end{aligned}$$

The feasible region Q does not exist $\longrightarrow$ The original problem is not feasible

extreme points: $u^k, \quad k \in K$ $\xrightarrow{z_{LP} \text{ bounded}}$

extreme rays: $r^j, \quad j \in J$ $\xrightarrow{z_{LP} = -\infty}$

$$\begin{aligned} \text{s.t.} \quad & (\mathbf{b}-\mathbf{A}\mathbf{x})^\intercal u^k \geq z_{LP}(\mathbf{x}) \\ & (\mathbf{b}-\mathbf{A}\mathbf{x})^\intercal r^j \geq 0 \\ & z_{LP}(\mathbf{x}) \in \mathbb{R} \end{aligned}$$

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Continue:

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{c}^\top \mathbf{x} + z_{LP}(\mathbf{x}) \\ \text{s.t.} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top u^k \geq z_{LP}(\mathbf{x}) \quad \text{for } k \in K \\ & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top r^j \geq 0 \quad \text{for } j \in J \\ & z_{LP}(\mathbf{x}) \in \mathbb{R}, \ \mathbf{x} \in \mathbf{X}, \ \mathbf{x} \in \{0,1\}^n \end{aligned}$$

$$\begin{aligned} z_{LP}(\mathbf{x}) = \min_{\mathbf{u}} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top \mathbf{u} \\ \text{s.t.} \quad & \mathbf{G}^\top \mathbf{u} \geq \mathbf{h} \\ & \mathbf{u} \in \mathbb{R}_+^m \end{aligned}$$

Feasible Region Q

Replace $z_{LP}(\mathbf{x})$ with symbol t

$$\begin{aligned} \max_{\mathbf{x},\, t} \quad & \mathbf{c}^\top \mathbf{x} + t \\ \text{s.t.} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top u^k \geq t \quad \text{for } k \in K \\ & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top r^j \geq 0 \quad \text{for } j \in J \\ & t \in \mathbb{R}, \ \mathbf{x} \in \mathbf{X}, \ \mathbf{x} \in \{0,1\}^n \end{aligned}$$

Master Problem

solution $\mathbf{x}$ →

← Feasible region Q
Either extreme ray or point

$$\begin{aligned} \min_{\mathbf{u}} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top \mathbf{u} \\ \text{s.t.} \quad & \mathbf{G}^\top \mathbf{u} \geq \mathbf{h} \\ & \mathbf{u} \in \mathbb{R}_+^m \end{aligned}$$

Subproblem

1 Benders' Decomposition

Algorithm:

1) Determine (possibly empty) initial sets $\hat{K}$ of extreme points and $\hat{J}$ of extreme rays of Q.
2) Solve problem (6), the relaxation of the Benders reformulation. Obtain solution $\bar{\boldsymbol{x}}$ and corresponding $\bar{t}$.
3) Determine $\mathrm{z_{LP}}(\bar{\boldsymbol{x}})$ by solving the dual (3) of the subproblem.
4) If $\bar{z}_{LP} = -\infty$, an extreme ray of Q has been found. Add the extreme ray to $\hat{J}$ and return to Step 2. (Feasibility Cuts).
5) If $\mathrm{z_{LP}}(\bar{\boldsymbol{x}}) < \bar{t}$ and finite, Add the extreme point of Q to $\hat{K}$ and return to Step 2. (Optimality Cuts)
6) If $\mathrm{z_{LP}}(\bar{\boldsymbol{x}}) = \bar{t}$ then $\bar{\boldsymbol{x}}$ solves the original mixed integer program (1), with optimal y equal to the solution to the primal subproblem (2) with $\boldsymbol{x} = \bar{\boldsymbol{x}}$.

Master Problem

$$\begin{aligned}
\max_{\mathbf{x},\, t} \quad & \mathbf{c}^\top \mathbf{x} + t \\
\text{s.t.} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top u^k \geq t \quad \text{for } k \in K \\
& (\mathbf{b} - \mathbf{A}\mathbf{x})^\top r^j \geq 0 \quad \text{for } j \in J \\
& t \in \mathbb{R},\ \mathbf{x} \in \mathbf{X},\ \mathbf{x} \in \{0,1\}^n
\end{aligned}$$

Feasible region Q
Either extreme ray or point

solution $\mathbf{x}$

$$\begin{aligned}
\min_{\mathbf{u}} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top \mathbf{u} \\
\text{s.t.} \quad & \mathbf{G}^\top \mathbf{u} \geq \mathbf{h} \\
& \mathbf{u} \in \mathbb{R}^m_+
\end{aligned}$$

Subproblem

Classical Benders' Decomposition (CBD)

$$\begin{aligned} \max_{\mathbf{x},\, t} \quad & \mathbf{c}^\top \mathbf{x} + t \\ \text{s.t.} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top u^k \geq t \quad \text{for } k \in K \\ & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top r^j \geq 0 \quad \text{for } j \in J \\ & t \in \mathbb{R},\ \mathbf{x} \in \mathbf{X},\ \mathbf{x} \in \{0,1\}^n \end{aligned}$$

Master Problem

solution $\mathbf{x}$ →

← Feasible region Q
Either an extreme ray
or a point

$$\begin{aligned} \min_{\mathbf{u}} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top \mathbf{u} \\ \text{s.t.} \quad & \mathbf{G}^\top \mathbf{u} \geq \mathbf{h} \\ & \mathbf{u} \in \mathbb{R}^m_+ \end{aligned}$$

Subproblem

QUBO (Quadratic Unconstrained Binary Optimization)

$$\mathbf{Q}_{\text{obj}} = \sum_i x_i \mathbf{Q}_{i,i} x_i + \sum_i \sum_{i<j} \mathbf{Q}_{i,j} x_i x_j$$

$\mathbf{Q}_{\text{obj}}$: Upper triangular matrix $\quad$ $\boldsymbol{x}_i$: Binary variable

Master problem of the **CBD** is **one** step **away** from pure **ILP** (Integer-linear programming).

The **last** barrier is the scalar t.

Classical Benders' Decomposition Master Problem

$$\begin{aligned}
\max_{\mathbf{x},\, t} \quad & \mathbf{c}^\top \mathbf{x} + t \\
\text{s.t.} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\top u^k \geq t \quad \text{for } k \in K \\
& (\mathbf{b} - \mathbf{A}\mathbf{x})^\top r^j \geq 0 \quad \text{for } j \in J \\
& t \in \mathbb{R},\ \mathbf{x} \in \mathbf{X}
\end{aligned}$$

In order to reformulate the master problem into the QUBO formulation,

We use a **binary** vector $\mathbf{w}$ with length of $M = \bar{m}_+ + \underline{m} + \bar{m}_- + 1$ bits to replace the continuous variable t.

$$\begin{aligned}
t &= \sum_{i=-\underline{m}}^{\bar{m}_+} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\bar{m}_-} 2^j w_{j+(1+\underline{m}+\bar{m}_+)} \\
&= \bar{t}(\mathbf{w})
\end{aligned}$$

$\bar{m}_+$: # of bits of $\mathbb{N}$ part.

$\underline{m}$: # of bits of the decimal part.

$\bar{m}_- + 1$: # of bits of $\mathbb{Z}_-$ part.

Classical Benders' Decomposition Master Problem

$$\begin{aligned} \max_{\mathbf{x},\, t} \quad & \mathbf{c}^\intercal \mathbf{x} + t \\ \text{s.t.} \quad & (\mathbf{b} - \mathbf{A}\mathbf{x})^\intercal u^k \geq t \quad \text{for } k \in K \\ & (\mathbf{b} - \mathbf{A}\mathbf{x})^\intercal r^j \geq 0 \quad \text{for } j \in J \\ & t \in \mathbb{R},\ \mathbf{x} \in \mathbf{X} \end{aligned}$$

MILP

$\longrightarrow$ t reformulation

Alternative Benders' Decomposition Subproblem

$$\begin{aligned} \max_{\mathbf{x},\mathbf{w}} \quad & c^\intercal \mathbf{x} + \sum_{i=-\underline{m}}^{\bar{m}_+} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\bar{m}_-} 2^j w_{j+(1+\underline{m}+\bar{m}_+)} \\ \text{s.t.} \quad & (b - A\mathbf{x})^\intercal u^k \geq \bar{t}(\mathbf{w}), \quad \text{for } k \in \hat{K}, \\ & (b - A\mathbf{x})^\intercal r^j \geq 0, \quad \text{for } j \in \hat{J}, \\ & \mathbf{x} \in X, \quad \mathbf{x} \in \{0,1\}^n, \\ & \mathbf{w} \in W, \quad \mathbf{w} \in \{0,1\}^M. \end{aligned}$$

Pure ILP

QUBO can be applied now.

Hybrid Quantum Benders' Decomposition Master Problem

Alternative Benders' Decomposition Master Problem

$$
\begin{aligned}
\max_{\mathbf{x},\mathbf{w}} \quad & c^\intercal \mathbf{x} + \sum_{i=-\underline{m}}^{\bar{m}_+} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\bar{m}_-} 2^j w_{j+(1+\underline{m}+\bar{m}_+)} \\
\text{s.t.} \quad & (b - A\mathbf{x})^\intercal u^k \geq \bar{t}\,(\mathbf{w}), \quad \text{for } k \in \hat{K}, \\
& (b - A\mathbf{x})^\intercal r^j \geq 0, \quad \text{for } j \in \hat{J}\,, \\
& \mathbf{x} \in X, \quad \mathbf{x} \in \{0,1\}^n, \\
& \mathbf{w} \in W, \quad \mathbf{w} \in \{0,1\}^M .
\end{aligned}
$$

TABLE I
TABLE OF COMMON CONSTRAINT-PENALTY PAIRS

Constraint	Equivalent Penalty
$x_1 + x_2 = 1$	$P(x_1 + x_2 - 1)^2$
$x_1 + x_2 \geq 1$	$P(1 - x_1 - x_2 + x_1 x_2)^2$
$x_1 + x_2 \leq 1$	$P(x_1 x_2)$
$x_1 + x_2 + x_3 \leq 1$	$P(x_1 x_2 + x_1 x_3 + x_2 x_3)$

1) Objective Function:

$$
c^\intercal \mathbf{x} + \sum_{i=-\underline{m}}^{\bar{m}_+} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\bar{m}_-} 2^j w_{j+(1+\underline{m}+\bar{m}_+)} \Rightarrow \mathbf{Q}_{obj} = \mathbf{x}^\intercal \mathrm{diag}(c)\mathbf{x} + \sum_{i=-\underline{m}}^{\bar{m}_+} w_{i+\underline{m}} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\bar{m}_-} w_{j+(1+\underline{m}+\bar{m}_+)} 2^j w_{j+(1+\underline{m}+\bar{m}_+)} .
$$

Hybrid Quantum Benders' Decomposition

Master Problem

$$\begin{aligned}
\max_{\mathbf{x},\mathbf{w}} \quad & c^\top \mathbf{x} + \sum_{i=-\underline{m}}^{\bar{m}_+} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\bar{m}_-} 2^j w_{j+(1+\underline{m}+\bar{m}_+)} \\
\text{s.t.} \quad & (b - A\mathbf{x})^\top u^k \ge \bar{t}(\mathbf{w}), \quad \text{for } k \in \hat{K}, \\
& (b - A\mathbf{x})^\top r^j \ge 0, \quad \text{for } j \in \hat{J}, \\
& \mathbf{x} \in X, \quad \mathbf{x} \in \{0,1\}^n, \\
& \mathbf{w} \in W, \quad \mathbf{w} \in \{0,1\}^M.
\end{aligned}$$

2) Optimality Cuts：

$$\bar{t}(\mathbf{w}) + (u^k)^\top A\mathbf{x} \le b^\top u^k, \text{ for } k \in \hat{K}.$$

$$\Rightarrow P_k \left(\bar{t}(\mathbf{w}) + (u^k)^\top A\mathbf{x} + \sum_{l=-\underline{m}}^{\bar{l}^K} 2^l s_{kl}^K - b^\top u^k \right)^2,$$

$$\text{where } \bar{l}^K = \left\lceil \log_2 \left(b^\top u^k - \min_{\mathbf{w},\mathbf{x}} \left(\bar{t}(\mathbf{w}) + (u^k)^\top A\mathbf{x} \right) \right) \right\rceil$$

3) Feasibility Cuts：

$$(r^j)^\top A\mathbf{x} \le b^\top r^j, \quad \text{for } j \in \hat{J}.$$

$$\Rightarrow P_j \left((r^j)^\top A\mathbf{x} + \sum_{l=0}^{\bar{l}^J} 2^l s_{kl}^J - b^\top r^j \right)^2,$$

$$\text{where } \bar{l}^J = \left\lceil \log_2 \left(b^\top r^j - \min_{\mathbf{x}} \left((r^j)^\top A\mathbf{x} \right) \right) \right\rceil$$

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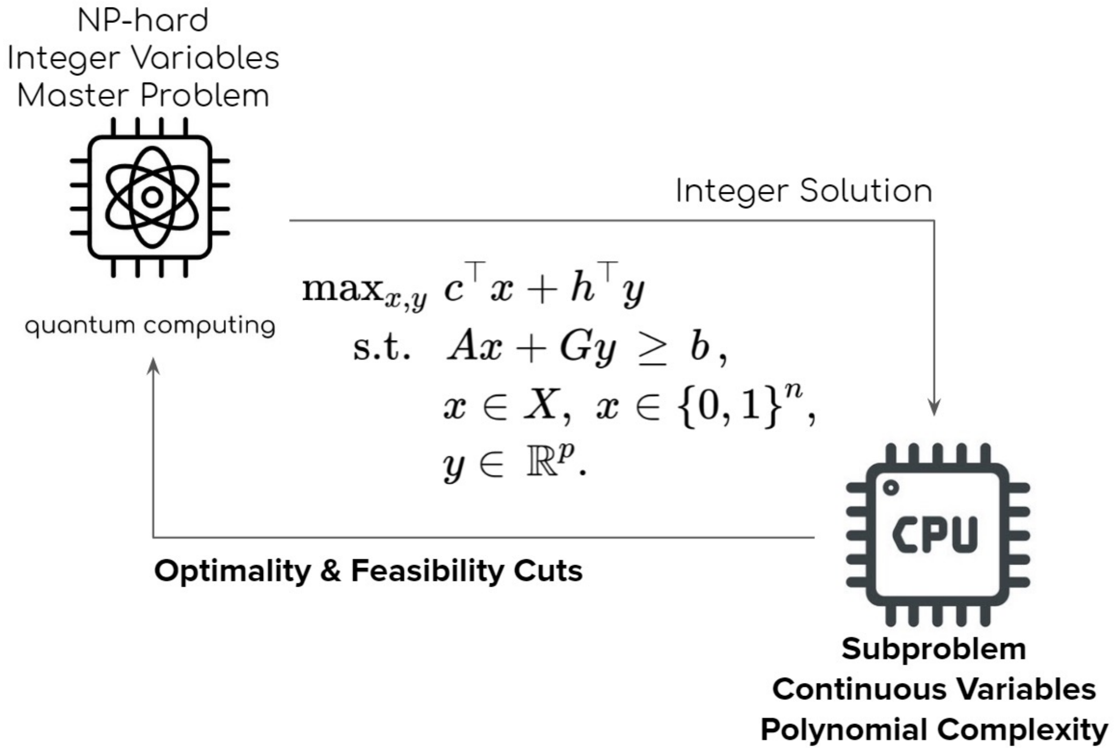
Hybrid Quantum Benders' Decomposition for Mixed-integer Linear Programming



$$\begin{aligned} \max_{x,y}\ & c^\top x + h^\top y \\ \text{s.t.}\ & Ax + Gy \geq b, \\ & x \in X,\ x \in \{0,1\}^n, \\ & y \in \mathbb{R}^p. \end{aligned}$$

$$\mathbf{x}' = \{\mathbf{w}, \mathbf{x}, \mathbf{s}\}$$

$$f(\mathbf{x}') = \mathbf{x}'^\top \mathbf{Q}_{\text{QUBO}} \mathbf{x}'$$

$$\begin{aligned} \mathbf{Q}_{\text{QUBO}} = {} & \mathbf{x}^\top \text{diag}(c)\mathbf{x} \\ & + \sum_{i=-\underline{m}}^{\bar{m}_+} w_{i+\underline{m}} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\bar{m}_-} w_{j+(1+\underline{m}+\bar{m}_+)} 2^j w_{j+(1+\underline{m}+\bar{m}_+)} \\ & + \sum_{k\in K} P_k \left(\bar{t}(\mathbf{w}) + (u^k)^\top A\mathbf{x} + \sum_{l=-\underline{m}}^{\bar{l}^K} 2^l s_{kl}^K - b^\top u^k \right)^2 \\ & + \sum_{j\in J} P_j \left((r^j)^\top A\mathbf{x} + \sum_{l=0}^{\bar{l}^J} 2^l s_{kl}^J - b^\top r^j \right)^2 \end{aligned}$$

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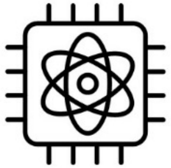
Hybrid Quantum Benders' Decomposition Algorithm

$\mathbf{X}$

NP-hard
Integer Variables
Master Problem



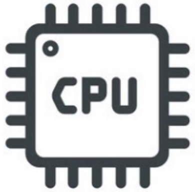
quantum computing

$$\max_{\mathbf{x},\, t} \quad \mathbf{c}^\top \mathbf{x} + t$$
$$\text{s.t.} \quad (\mathbf{b} - \mathbf{A}\mathbf{x})^\top u^k \geq t \quad \text{for } k \in K$$
$$(\mathbf{b} - \mathbf{A}\mathbf{x})^\top r^j \geq 0 \quad \text{for } j \in J$$
$$t \in \mathbb{R},\ \mathbf{x} \in \mathbf{X},\ \mathbf{x} \in \{0,1\}^n$$

$\underline{t} = \bar{t}$?

Integer Solution

$$\max_{x,y}\ c^\top x + h^\top y$$
$$\text{s.t.} \quad Ax + Gy \geq b,$$
$$x \in X,\ x \in \{0,1\}^n,$$
$$y \in \mathbb{R}^p.$$



Subproblem
Continuous Variables
Polynomial Complexity

Optimality & Feasibility Cuts

$$\min_{\mathbf{u}} \quad (\mathbf{b} - \mathbf{A}\mathbf{x})^\top \mathbf{u}$$
$$\text{s.t.} \quad \mathbf{G}^\top \mathbf{u} \geq \mathbf{h}$$
$$\mathbf{u} \in \mathbb{R}^m_+$$

$\underline{t} = (\mathbf{b} - \mathbf{A}\mathbf{x})^\top \mathbf{u}$

u^k or r^k

Algorithm 1 Hybrid Quantum-Classical Benders' Decomposition Algorithm

---

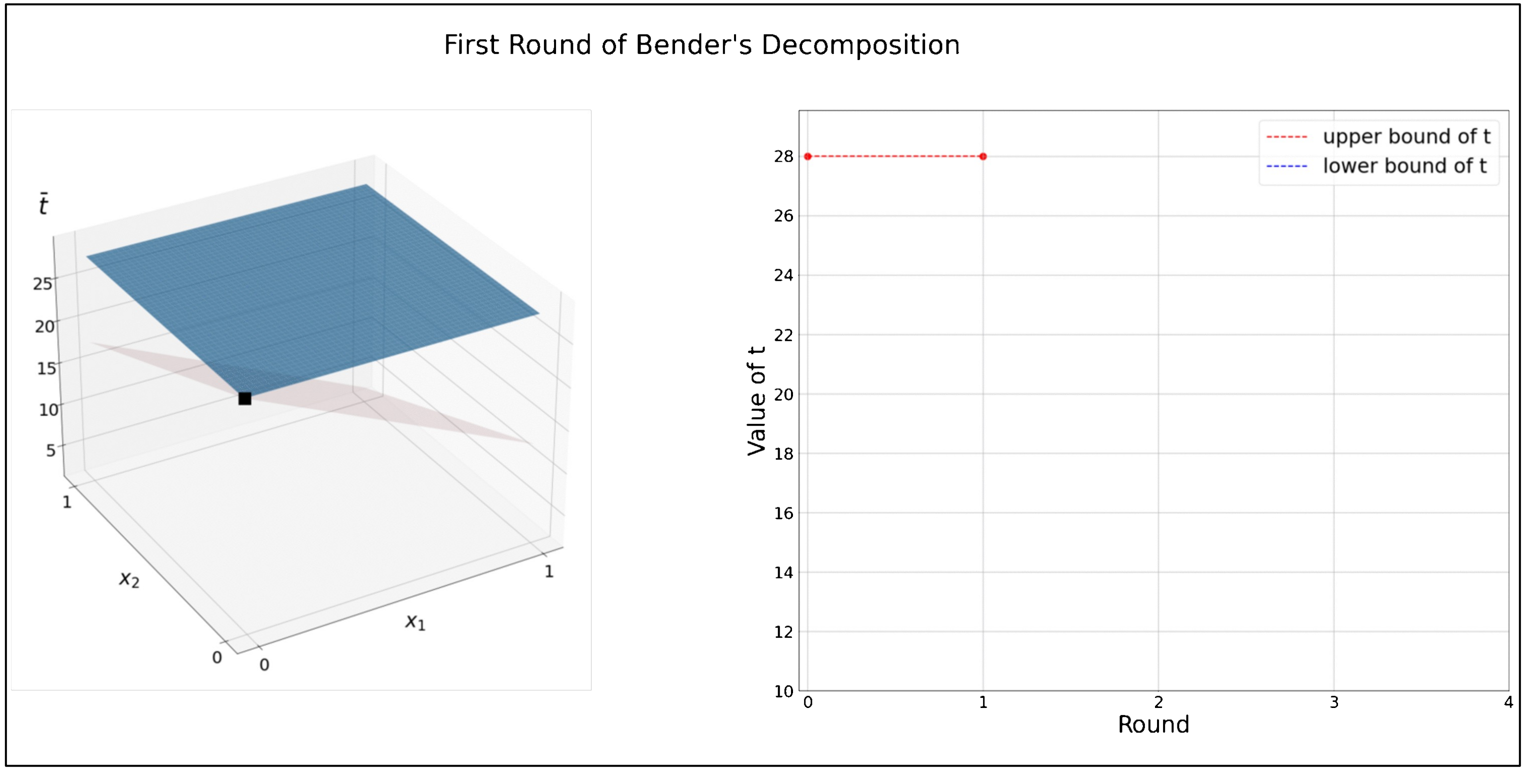
Require: Initial sets $\hat{K}$ of extreme points and $\hat{J}$ of extreme rays of Q
- $\bar{t} \leftarrow +\infty$
- $\underline{t} \leftarrow -\infty$
- **while** $|\ \bar{t} - \underline{t}\ | \geq \epsilon$ **do**
  - $\mathbf{P} \leftarrow$ Appropriate penalties numbers or arrays
  - $\mathbf{Q} \leftarrow$ Reformulate both objective and constraints in (2) and construct the QUBO formulation by using corresponding rules
  - $\mathbf{x}' \leftarrow$ Solve problem (6) by quantum computer.
  - $\bar{t} \leftarrow$ Extract $\mathbf{w}$ and replace the $\bar{t}$ with $\bar{t}\,(\mathbf{w})$ (8)
  - $z_{LP}(\mathbf{x}) \leftarrow$ Solve the problem (3)
  - $\underline{t} \leftarrow z_{LP}(\mathbf{x})$
  - **if** $z_{LP}(\mathbf{x}) = -\infty$ **then**
    - An extreme ray j of Q has been found.
    - $\hat{J} = \hat{J} \cup \{j\}$
  - **else if** $z_{LP}(\mathbf{x}) < \bar{t}$ **and** $\bar{t} \neq +\infty$ **then**
    - An extreme point k of Q has been found.
    - $\hat{K} = \hat{K} \cup \{k\}$
  - **end if**
- **end while**
- **return** $\bar{t}$, $\mathbf{x}$

Hybrid Quantum Benders' Decomposition Algorithm

$$A=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\-1&-1\\-1&0\\-1&0\\0&-1\\0&-1\end{bmatrix},\ G=\begin{bmatrix}1&0&1&0\\1&0&0&1\\0&1&1&0\\0&1&0&1\\0&0&0&0\\1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\ b=\begin{bmatrix}1\\1\\1\\1\\-1\\0\\0\\0\\0\end{bmatrix},$$

$$h^{\mathsf{T}}=[8\quad 9\quad 5\quad 6],\qquad c^{\mathsf{T}}=[-15\quad -10].$$

D-Wave hybrid solver: use classical computation to assist quantum annealing.



Zhongqi Zhao, Lei Fan, and Zhu Han, ``Hybrid Quantum Benders' Decomposition For Mixed-integer Linear Programming," IEEE Wireless Communications and Networking Conference (WCNC), Austin, TX, April 2022.

Lei Fan and Zhu Han, ``Hybrid Quantum-Classical Computing for Future Network Optimization," IEEE Network, special issue on Quantum Communications and Networking.

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- Motivation and Quantum Computing Basics
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Federated Learning in Mobile Edge Computing

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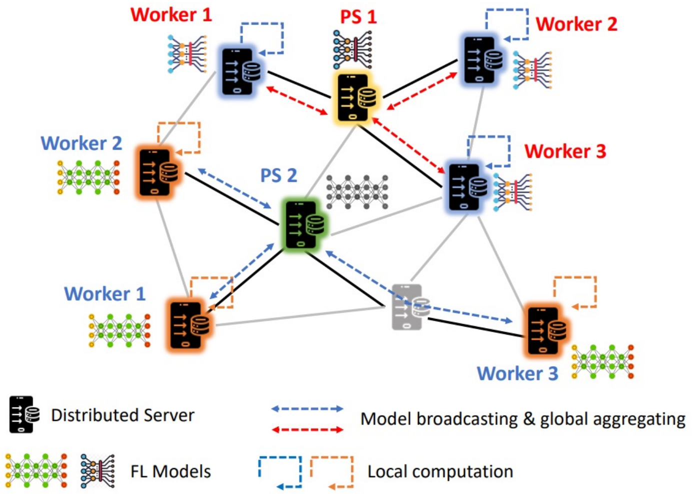
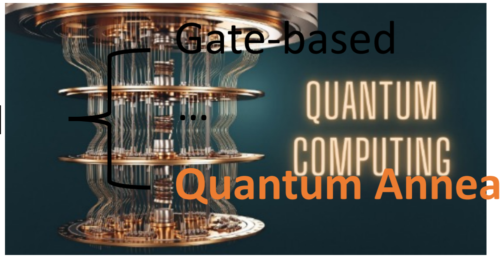
- Federated Edge Learning (FEL)
  - multiple FL models trained **simultaneously** (**resource competion, performance degradation**), scheduling problem is **NP-hard**
- Quantum Computing (QC)
  - has the **parallel computing** capability
  - a powerful tool for **optimization**
- Existing Optimization in Quantum Computing
  - result in **longer computational times** with no guarantee of feasibility[1]
  - focus on **theoretical analysis**, **mathematical optimization**[2]



Computation Model
Gate-based
...
Quantum Annealing (QA)



[1] A. Ajagekar, et al., "Quantum computing based hybrid solution strategies for large-scale discrete-continuous optimization problems", Computers & Chemical Engineering, 2020.
[2] Z, Zhao, et al., "Hybrid quantum Benders' decomposition for mixed-integer linear programming", IEEE WCNC, 2022.

Problem Formulation

➢ **Participant selection and learning scheduling of multi-model FEL**

$$\min_{x,y,\rho} \sum_{j=1}^{W} (C_j^{trans} + C_j^{local} + C_j^{global} + C_j^{rent}) \quad (5)$$

Storage, CPU constraints

$$\text{s.t.} \quad x_{i,j}\mu_j\kappa_j \le c_i, \quad x_{i,j}\chi_j \le f_i, \qquad \forall i,j, \quad (6)$$

$$y_{i,j}\mu_j \le c_i, \quad y_{i,j}\chi_j \le f_i, \qquad \forall i,j, \quad (7)$$

One PS, κ_j FL workers

$$\sum_{i=1}^{N} x_{i,j} = 1, \quad \sum_{i=1}^{N} y_{i,j} = \kappa_j, \qquad \forall j, \quad (8)$$

One edge server can at most work as one of three roles

$$\sum_{j=1}^{W} (x_{i,j} + y_{i,j}) \le 1, \qquad \forall i, \quad (9)$$

$$i \in (1, \ldots, N), j \in (1, \ldots, W), \quad (10)$$

$$x_{i,j}, y_{i,j} \in \{0,1\}, \varrho_j \in [0.01, 0.99]. \quad (11)$$

Participant Selection and learning schedule decision

❑ **Transmission Cost**

Communication cost based on shortest path

$$C_j^{trans} = 2 \cdot \vartheta_j \sum_{k=1}^{N} \sum_{i=1}^{N} x_{k,j} \cdot y_{i,j} \cdot \boxed{\rho_j(v_i, v_k)}.$$

❑ **Local Update Cost**

CPU cycles to process the sample data

$$C_j^{local} = \vartheta_j \cdot \varphi_j \cdot \sum_{i=1}^{N} y_{i,j} \cdot \frac{\boxed{\psi(D_{j,i})}}{f_i}.$$

❑ **Global Aggregation Cost**

$$C_j^{global} = \vartheta_j \cdot \sum_{i=1}^{N} x_{i,j} \cdot \frac{\psi(\mu_j)}{f_i}.$$

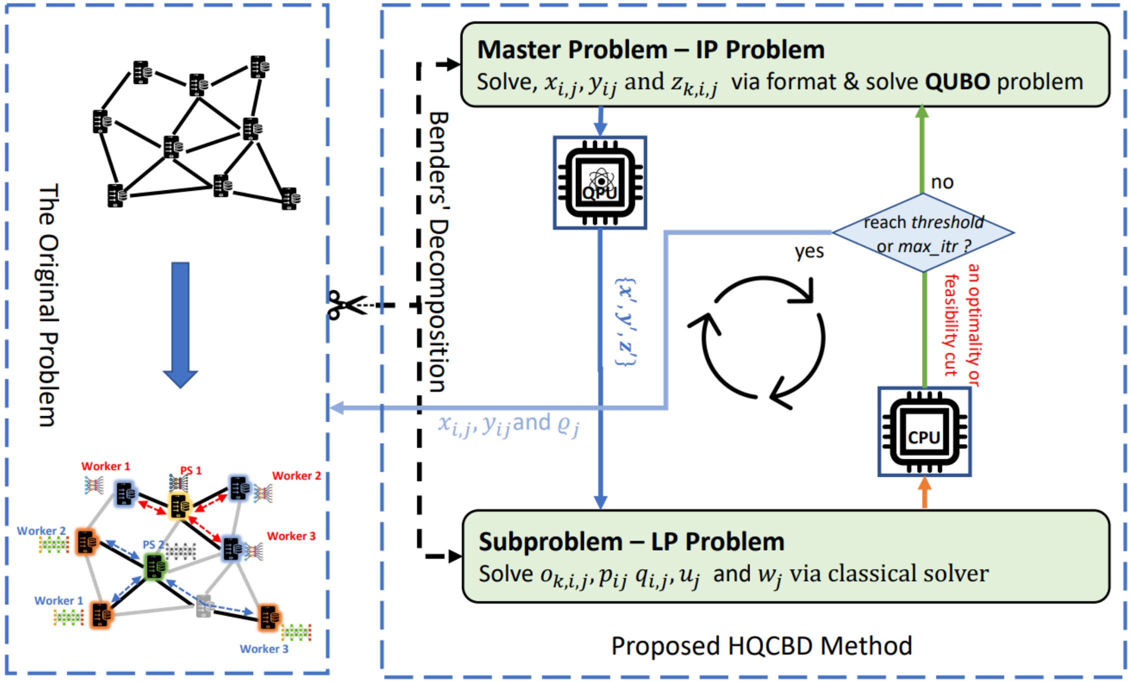
❑ **Participant Cost**

The unit price for a CPU unit

$$C_j^{rent} = \sum_{i=1}^{N} (x_{i,j} + y_{i,j}) \cdot \boxed{p_j} \cdot f_i.$$

➢ Challenges

1. *How to convert our original MINLP problem into an ILP problem that can be recognized by the quantum computer?*
2. *How to further convert the reformulated ILP problem into a QUBO model as the input to the quantum annealer computer?*



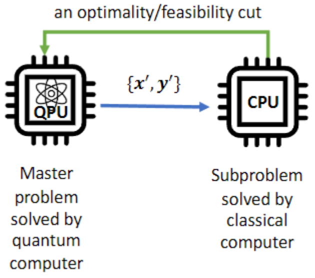
HQCBD Algorithm

Algorithm 1 Hybrid Quantum-Classical Benders' Decomposition (HQCBD) Method

Input: Distributed network with N servers V, W FL models M, Coefficient of the objective function and constraints in master problem and subproblem

Output: PS selection x', worker selection y', and local convergence rate ϱ'

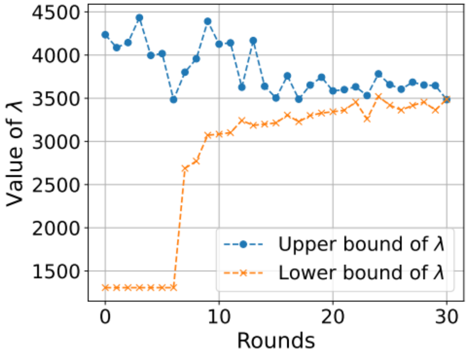
1: Initialize upper/lower bound of λ, $\overline{\lambda} = +\infty$, $\underline{\lambda} = -\infty$
2: Initialize threshold $\epsilon = 0.001$, $max\_itr = 100$, $itr = 1$
3: **while** $|\overline{\lambda} - \underline{\lambda}| > \epsilon$ and $itr < max\_itr$ **do**
4: $\quad$ **P** $\leftarrow$ Appropriate penalty numbers or arrays
5: $\quad$ **Q** $\leftarrow$ Reformulate both objective and constraints in (33) and construct QUBO formulation as (47)
6: $\quad$ x', y', z' $\leftarrow$ Solve problem (47) by quantum computer
7: $\quad$ $\underline{\lambda} \leftarrow$ Extract **w** and replace $\underline{\lambda}$ with $\hat{\lambda}(w)$ as (46)
8: $\quad$ $SUP(x, y, z) \leftarrow$ Solve problem (30) with fixed x', y', z'
9: $\quad$ Extract ϱ' from $SUP(x, y, z)$
10: $\quad$ $\overline{\lambda} \leftarrow SUP(x, y, z)$
11: $\quad$ Add a new benders' cut to the master problem as (44)
12: $\quad$ $itr+ = 1$
13: **end while**
14: **return** x', y', ϱ'



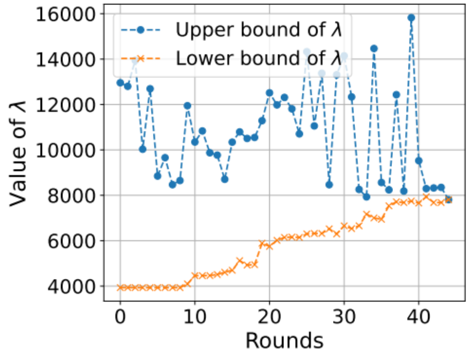
➢ Workflow

1. *Initialize parameters (Lines 1 and 2)*
2. *Generate appropriate penalty numbers or arrays (Line 4)*
3. *Solve the master problem with a quantum annealer (Lines 5-7)*
4. *Solve the subproblem with a classical CPU computer (Lines 8-10)*
5. *Add the Benders' cut to the master problem and continue the next iteration (Lines 11 and 12)*

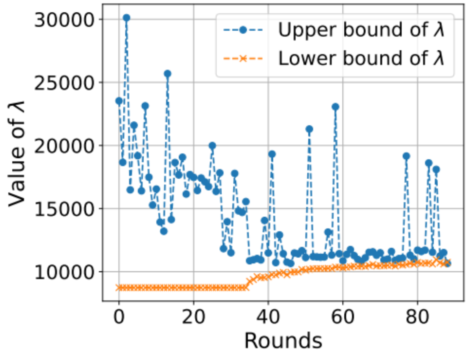
Experiments



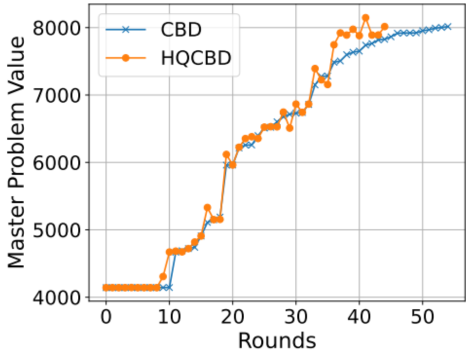
(a) Case 1



(b) Case 2



(c) Case 3



(d) Master problem value

The upper bound and lower bound finally converge.

- Baselines
  - **Classical Benders' Decomposition (CBD):** solve the master problem and subproblem by using classical solver in CPU computer.

TABLE I: Iteration of CBD and HQCBD over three different cases. Here, the set up column shows {# of servers, # of models, # of workers per model} used in each case.

Case #	Set up	# of Variables	Itr. of CBD	Itr. of HQCBD
1	$\{7, 1, 3\}$	63	32	31
2	$\{7, 2, 2\}$	126	55	45
3	$\{9, 2, 3\}$	198	91	89

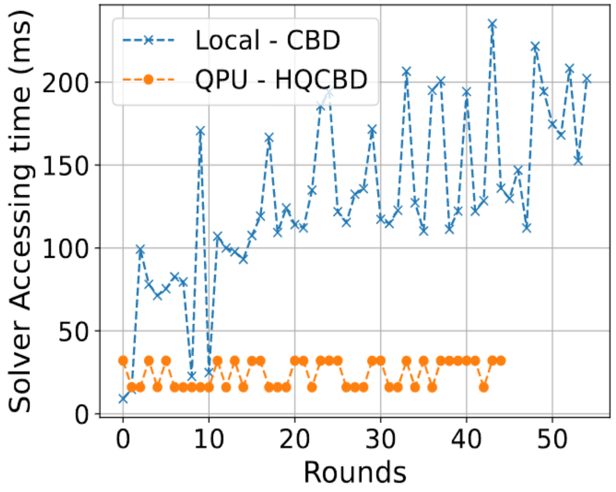
HQCBD takes few iterations to converge.

Experiments

TABLE II: Solver accessing time (ms) of CBD and HQCBD.

Case #	CBD		HQCBD	
	Max./Min.	Mean/Std.	Max/Min.	Mean/Std.
1	190.47/6.71	117.14/50.12	32.10/15.93	31.49/2.79
2	235.29/9.11	129.56/50.04	32.11/15.92	24.10/7.98
3	395.48/14.45	120.25/63.19	32.11/16.00	25.53/7.85

*the **maximal**, **average** accessing time and the **standard deviation** value of CBD is significantly **higher** than HQCBD.*



Solver accessing time (SAT): the real accessing time of QPU solver and local solver without considering other overheads.

CBD in each round/iteration varies significantly, HQCBD in each round keeps stable and is even smaller than that of CBD

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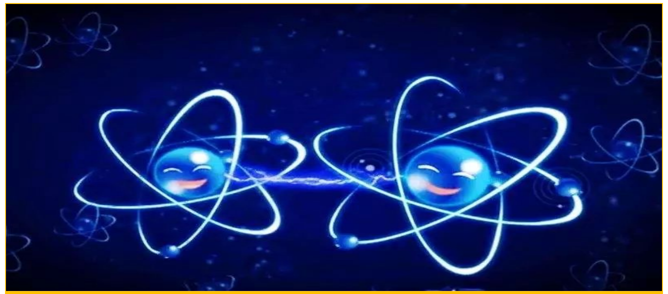
Quantum Machine learning

Machine Learning

Classical learning technicues facing complex tasks



Quantum Computing

Quadradic speed-up

QML

- Quantum Machine Learning
- Machine learning and *fast*
- Not only fast

Feature Selection

Need for feature selection

With feature selection, we can optimize our model in some way

- **Huge data to train**
- **Relevant feature**
- **Irrelevant feature**
- **Redundant feature**

- **Prevent overfitting**
- **Improve accuracy**
- **Reduce training time**

set of features
↓
select best features
↓
learning algorithm
↓
evaluate

BILP Model

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Objective function: maximize relevant feature while minimizing redundant feature

"influence"

$$\sum_i I(f_i, y)x_i$$

α : bias

$$\min_x \left\{ \frac{1-\alpha}{k(k-1)/2} \sum_i \sum_{j<i} I(f_i, f_j)x_i x_j - \frac{\alpha}{k} \sum_i I(f_i, y)x_i \right\} \quad \forall i, j \in S$$

k : number of feature

x : decision variable

$$\sum_i \sum_{j<i} I(f_i, f_j)x_i x_j$$

"independence"

Many Constraints

$$\sum x_i = K_j \quad \forall i \in S_j \quad j \in 1, \ldots, z$$

Basic	1	duration
	2	protocol_type
	3	service
	4	Flag
	5	src_bytes
	6	dst_bytes
	7	Land
	8	wrong_fragment
	9	urgent
Content	10	Hot
	11	num_failed_logins
	12	logged_in
	13	num_compromised
	14	root_shell
	15	su_attempted
	16	num_root
	17	num_file_creations
	18	num_shells
	19	num_access_files
	20	num_outbound_cmds
	21	is_host_login
Content	22	is_guest_login
	23	count
	24	srv_count
	25	serror_rate
	26	srv_serror_rate
	27	rerror_rate
	28	srv_rerror_rate
	29	same_srv_rate
	30	diff_srv_rate
	31	srv_diff_host_rate
Traffic	32	dst_host_count
	33	dst_host_srv_count
	34	dst_host_same_srv_rate
	35	dst_host_diff_srv_rate
	36	dst_host_same_src_port_rate
	37	dst_host_srv_diff_host_rate
	38	dst_host_serror_rate
	39	dst_host_srv_serror_rate
	40	dst_host_rerror_rate
	41	dst_host_srv_rerror_rate
	42	Class

Set K is decided by a wrapper method

Constraints

$$\sum_{i \in \mathbf{C}_g} x_i \leq 1, \quad \forall\, \mathbf{C}_g \in \mathbf{C};$$

Some are conflict

Feature in C_g may have the same information, and are selected at most once

Some rely on others

$$x_i - x_j \leq 0 \quad \forall (i,j) \in \mathbb{E}$$

$$\sum x_i = T \quad \forall i \in \mathbb{D}$$

Some are essential and have priority

Algorithm & Experiments

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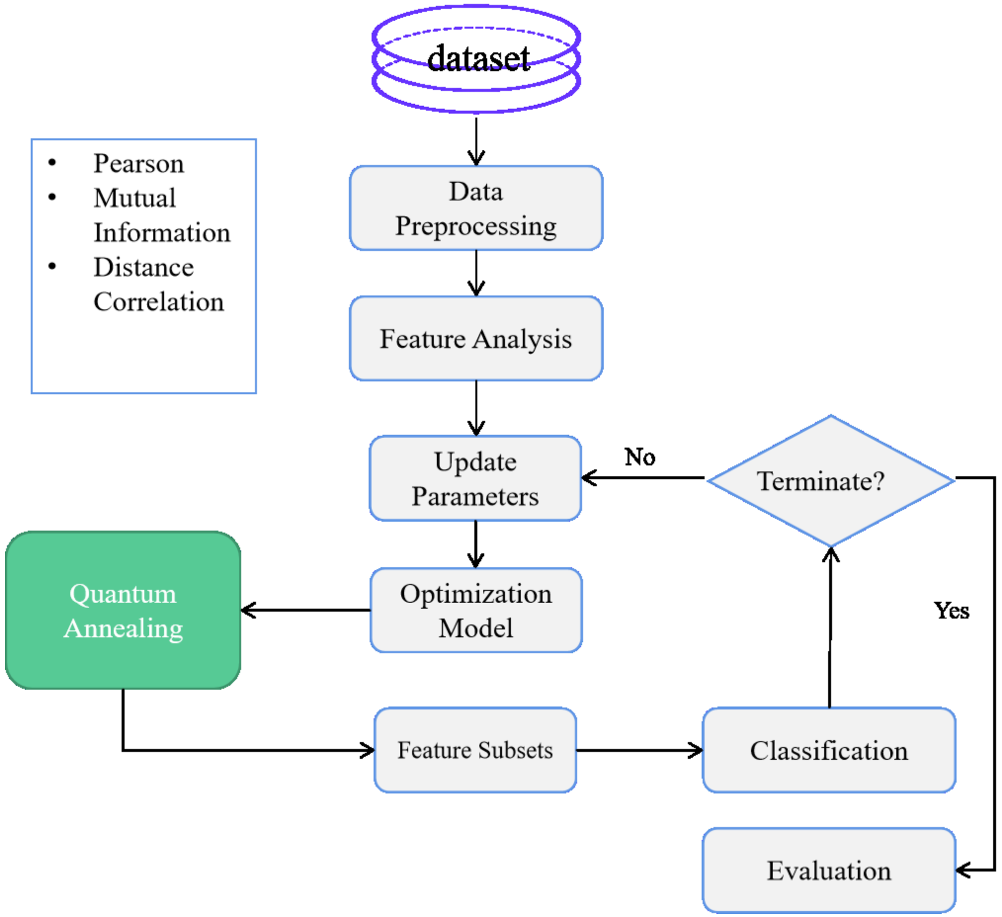

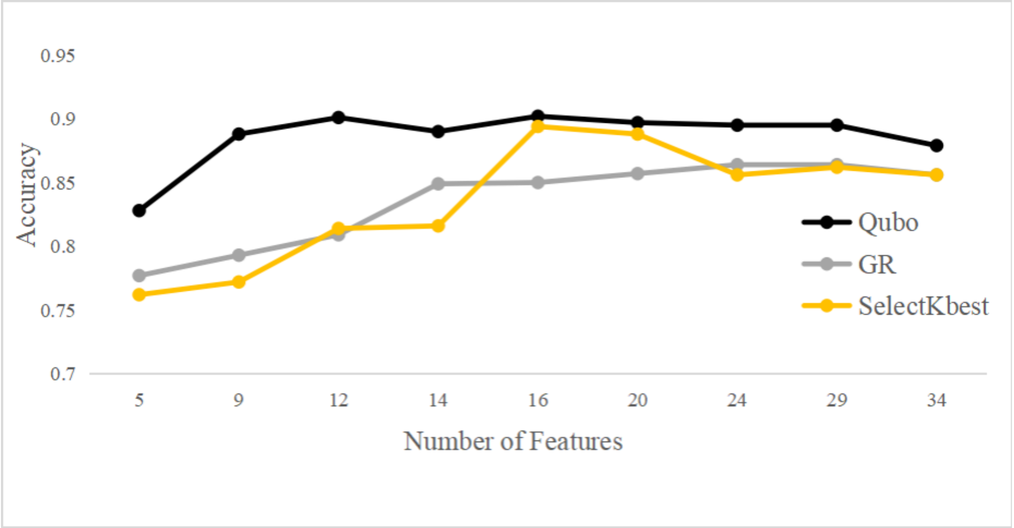

Accuracy of feature selection methods, including Qubo, GR[1]

- QUBO get best performance when k = 12
- QUBO's performance is better than other feature selection methods

Mingze Li, Hongliang Zhang, Lei Fan, and Zhu Han, ``A Quantum Feature Selection Method for Network Intrusion Detection," IEEE 19th International Conference on Mobile Ad Hoc and Smart Systems (MASS), 2022.

Outline

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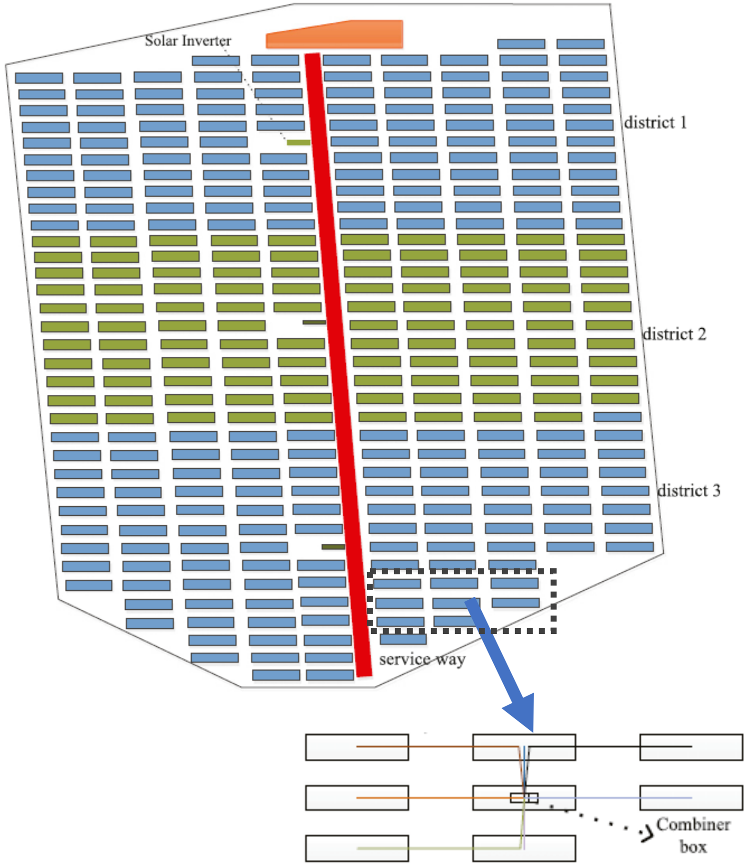
- Motivation and Quantum Computing Basics
- Quantum Annealing
  - Adiabatic Quantum Computing
  - Quadratic Unconstrained Binary Optimization (QUBO)
- Hybrid Quantum Benders' Decomposition
  - Benders Decomposition Basics
  - Quantum and Class Computing Bender Decomposition
- Applications
  - Federated Learning in Mobile Edge Computing
  - Feature Selection for Machine Learning
  - Cable-Routing Problem in Solar Power Plants
- Conclusions

Cable-Routing Problem (CRP) in Solar Power Plants

Quantum Computing for Cable-Routing Problem in Solar Power Plants

Exploring the edge of quantum annealing in the networking problem

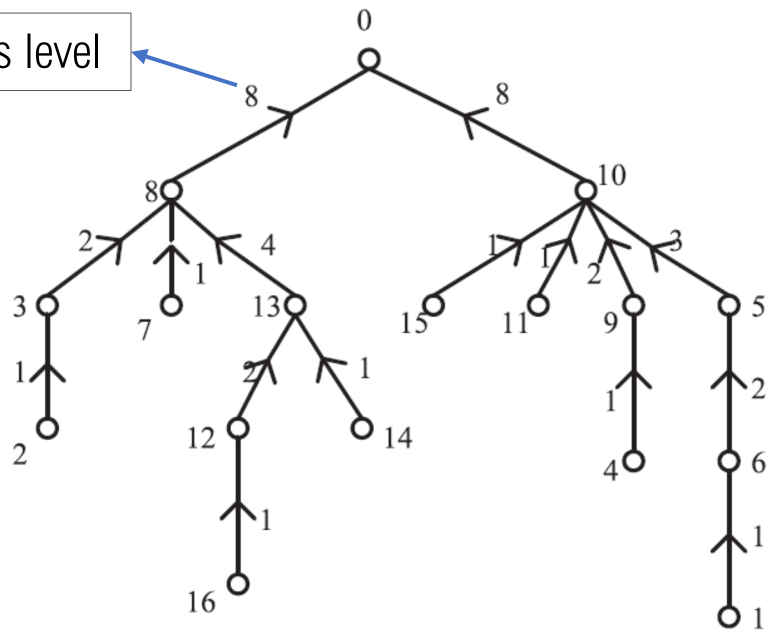
Solar arrays



Fast arrangement with minimum building cost is one objective of the solar power plant design.

Quantum Computing for Cable-Routing Problem in Solar Power Plants

Bus level



the corresponding subtree

Pure integer Programming problem

$$\min_{\mathbf{x}} \quad \sum_{e \in E} c_e \sum_{d=1}^{Q} x_{e,d} + p \sum_{e \in E'} \sum_{d=1}^{Q} (d-1) c_e x_{e,d},$$

s.t.

$$\sum_{e \in \delta^+(i)} \sum_{d=1}^{Q} x_{e,d} = 1, \quad \forall i \in V',$$ Electricity unity leaving

Electricity I/O unity difference

$$\sum_{e \in \delta^+(i)} \sum_{d=1}^{Q} d x_{e,d} - \sum_{e' \in \delta^-(i)} \sum_{d=1}^{Q} d x_{e',d} = 1, \ \forall i \in V',$$

$$m_t^l \leq \sum_{e \in E \setminus E'} x_{e,t} \leq m_t^u, \quad \forall t \in T,$$ Bus level requirements

$$x_{e,t} = 0, \quad \forall e \in E \setminus E', \ \forall t \notin T,$$ Special requirements

$$x_{e,t} \in \{0,1\}, \quad \forall e \in E, \ t \in \{1, 2, \ldots, Q\}.$$

Experiments

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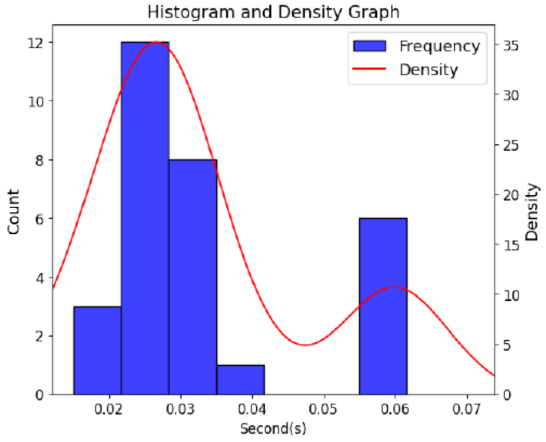
Histogram and Density Graph of Gurobi & D-Wave's Quantum computer in 2 Cases with 30 Attempts

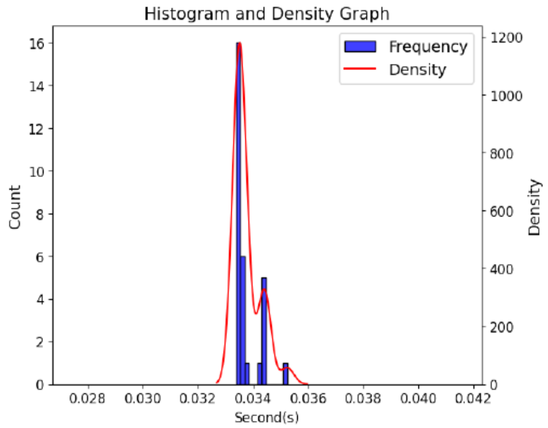
TABLE II: Standard Deviation Comparison

Detail / Model	Standard Deviation *Unit:* 10^{-3}			
	1st Try	***2nd Try***	***3rd Try***	***Average***
W3H3 Gurobi	19.7234	19.3967	19.589	19.5697
W3H3 Quantum	0.4234	0.566	0.2919	0.4271
W3H4 Gurobi	14.0653	14.3928	14.5188	14.3256
W3H4 Quantum*	0.3945	0.5358	0.6339	0.5214

* It excludes 3 failure attempt samples out of 30 samples.



(a) Gurobi 3×3



(b) D-Wave 3×3

The quantum computing approach **outperforms** the classical computing approach in terms of *solution quality*, *average running time*, and *robustness*.

Conclusion

Quantum Computing

Universal Quantum

Analog Quantum

Quantum Annealing

QUBO model

+ Benders' decomposition

VNFs scheduling problem → A heuristic algorithm → The optimal solution

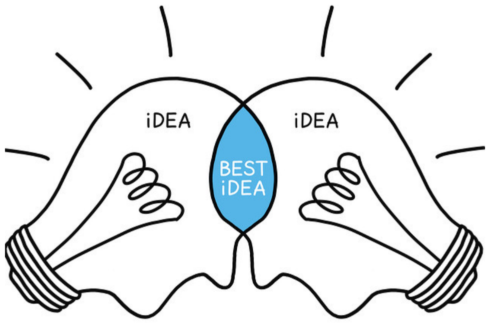
Wireless Networks

Feature selection → A wrapper algorithm → The best combination of features

Machine Learning

MILP problem → A classical-quantum hybrid algorithm → The optimal solution

Smart Grid Networks



Collaboration

More Applications …

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Videos, slides and codes can be found

http://wireless.egr.uh.edu/

http://www2.egr.uh.edu/~zhan2

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