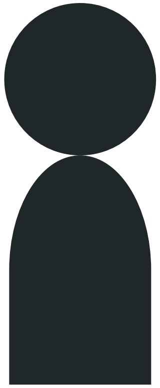
Classical Verification of Quantum Depth

Nai-Hui Chia (Rice University)

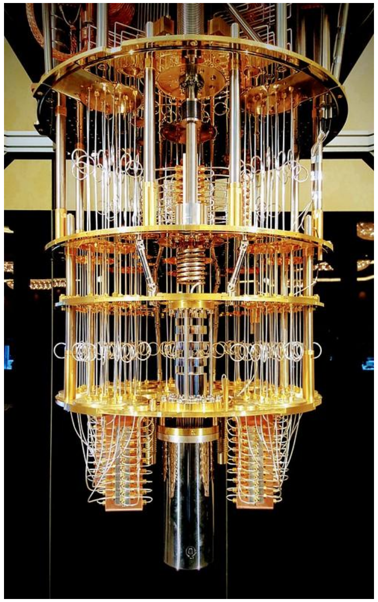
Based on joint work with Shih-Hang Hung

Certifying Quantum Resources

Can classical clients verify that a remote server has claimed quantum resources?



Classical client



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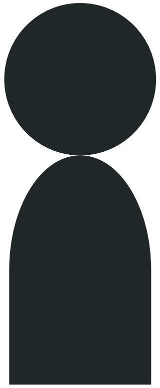

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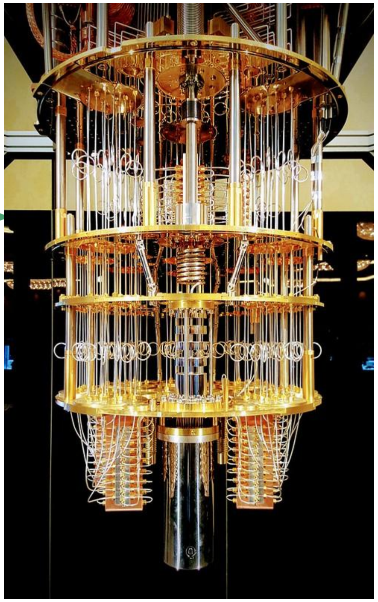

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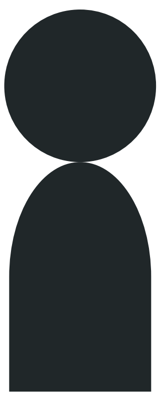

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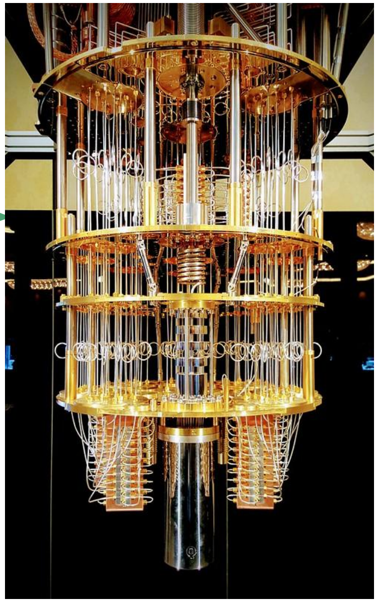

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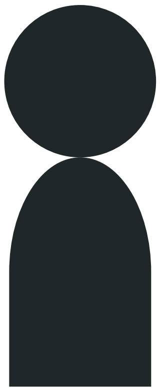

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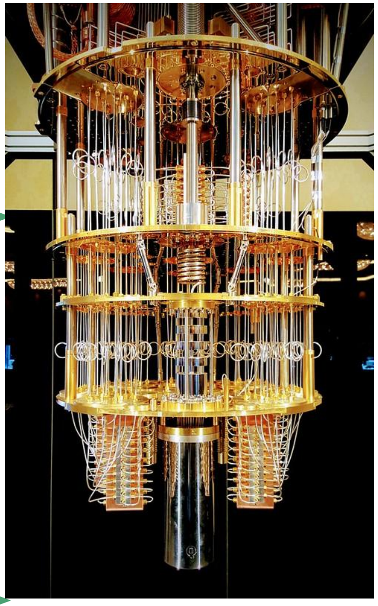

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Actually, I only have 50 qubits and 10 depths. But you never know -- I can use some supercomputer to complete you task!

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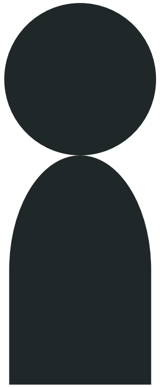

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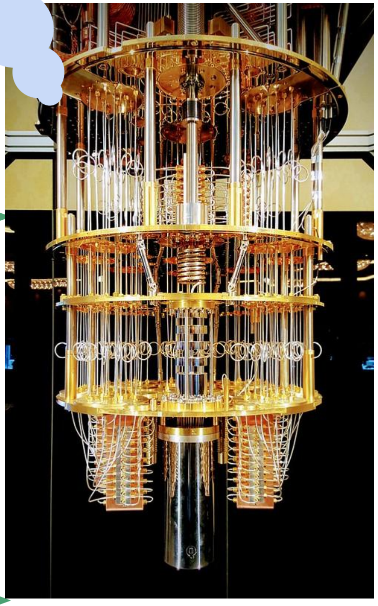

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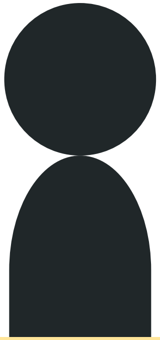

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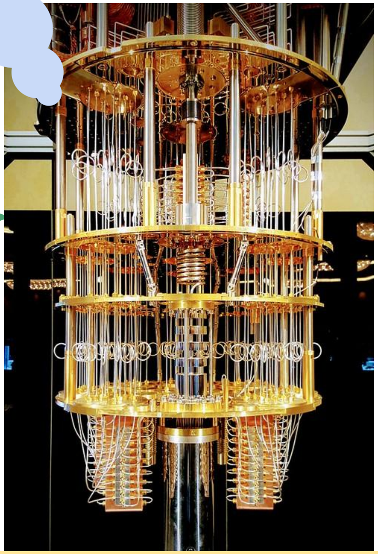

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We are especially interested in protocols for certifying **quantum depth**

Why do we care about quantum depth?

Near-term QC:

- > 100 qubits
- Noisy gates
  (not fault-tolerant)
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- Many qubits
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- Can do all poly-time (quantum) algorithms

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Goal: A method to check quantum depth of a remote server

Classical Verification of Quantum Depth (CVQD)

A protocol which lets a classical verifier check the quantum depth of a remote server

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Three requirements for CVQD

1. **Classical** verifier and **classical** protocol
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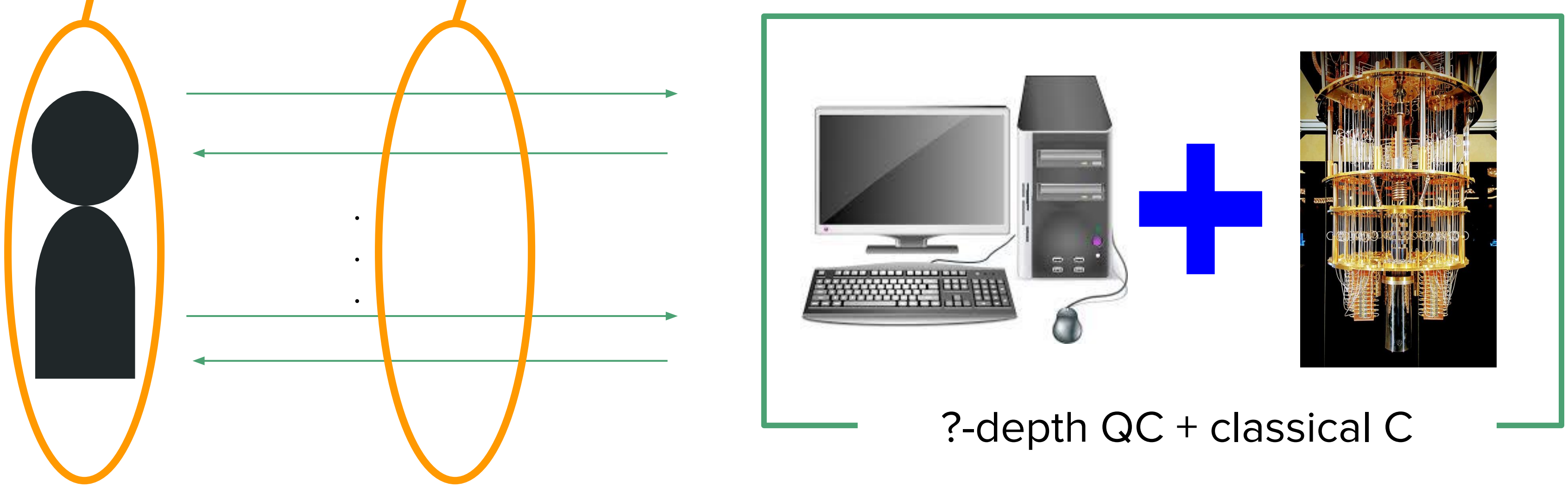
3.



Classical Verification of Quantum Depth (CVQD)

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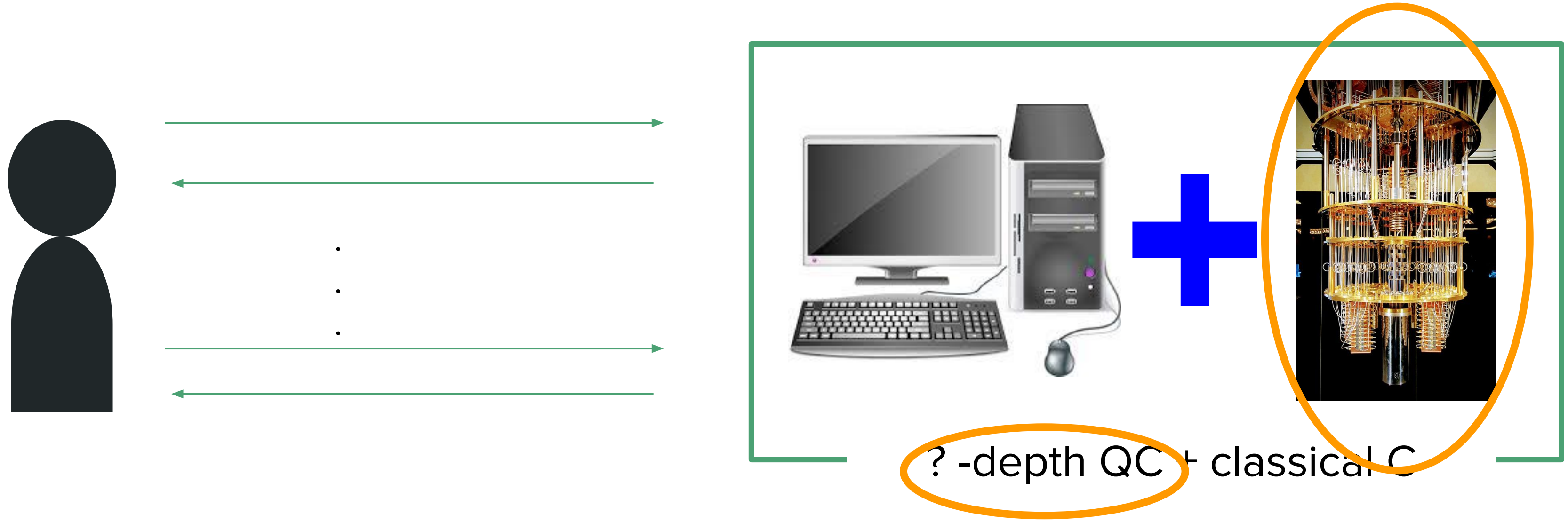

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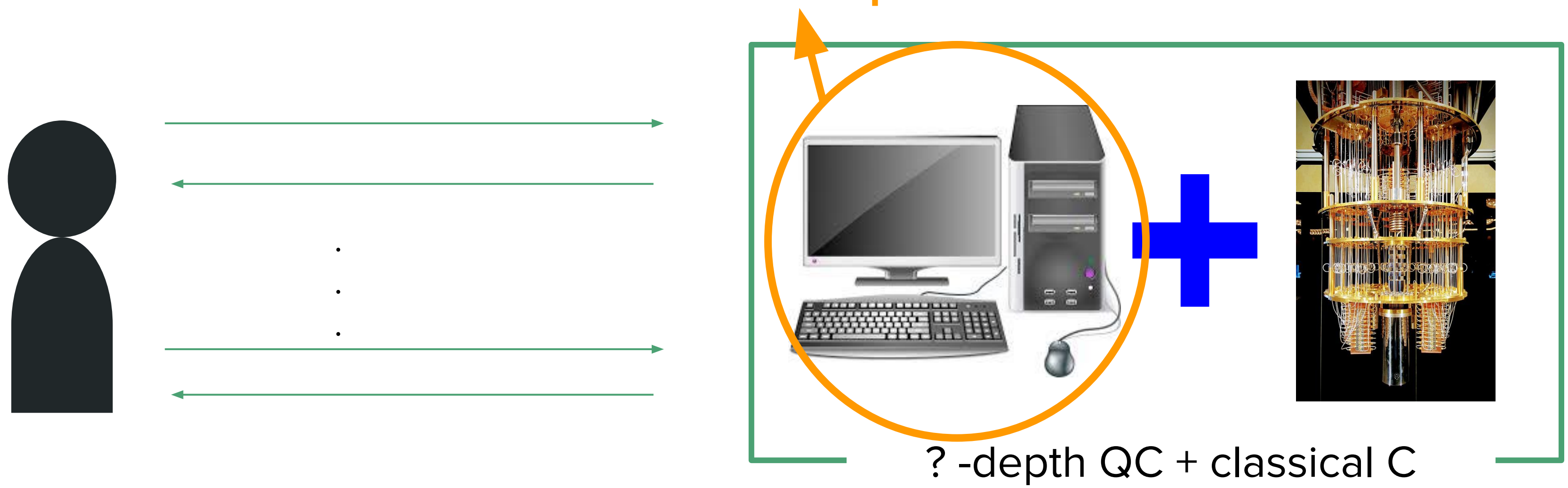
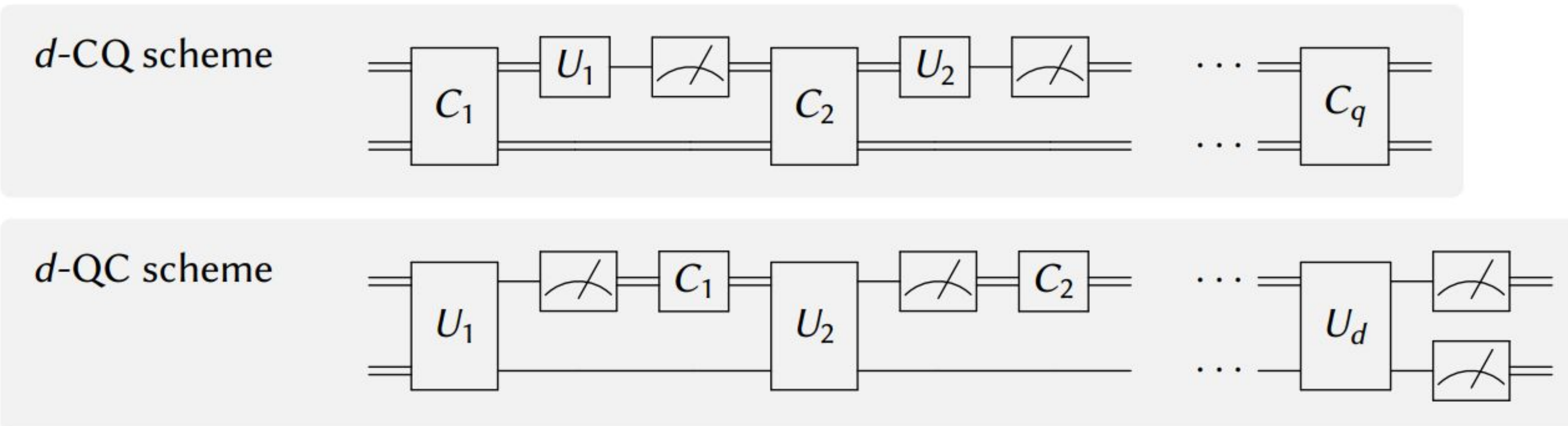


Two Models of Hybrid Quantum-Classical Comp.



- *d*-CQ scheme: classical computer can access a *d*-depth quantum circuit
- *d*-QC scheme: *d*-depth quantum circuits can access classical computer after each layer of gates.

First Attempt for CVQD protocols

First Attempt for CVQD Protocols

Theorem [**C**CL20, CH22, HG22]: ∃ a problem (called d-SSP) such that

- **[(d+3)**-depth QC + Classical C] solves the problem
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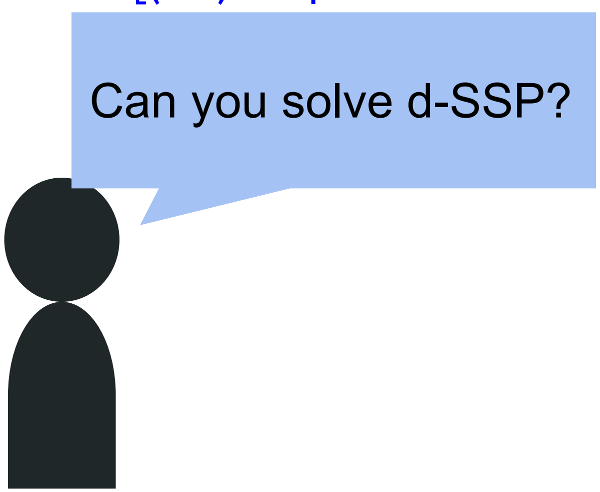

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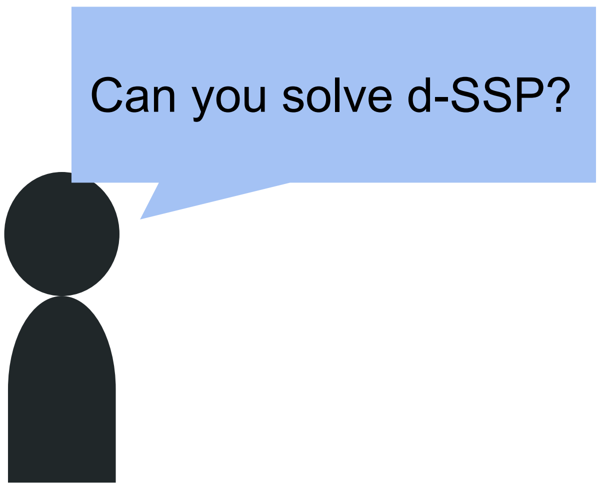

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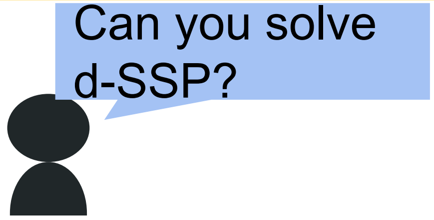

Verifier

Server

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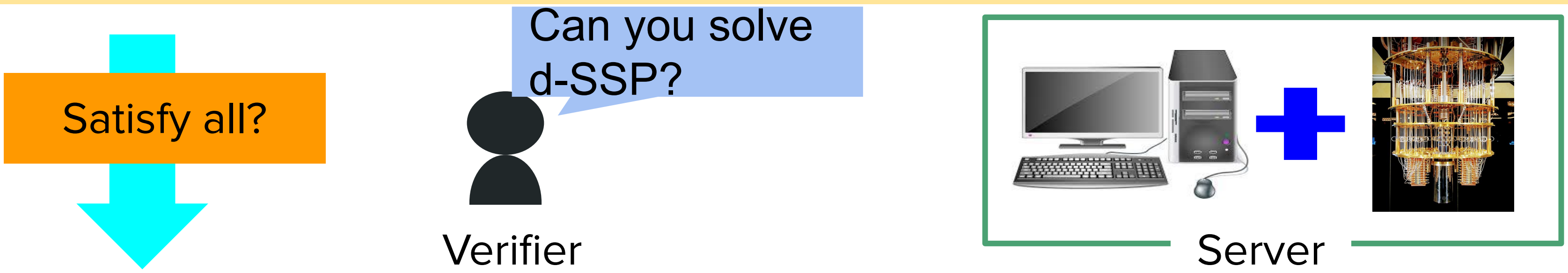

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Satisfy all?

Separate d+3 quantum depth from d.

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Server without sufficient quantum depth cannot convince the verifier even using classical computer

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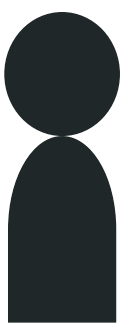

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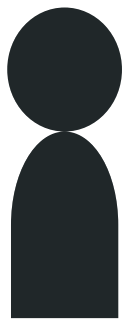

Algorithm

? -depth QC + classical C

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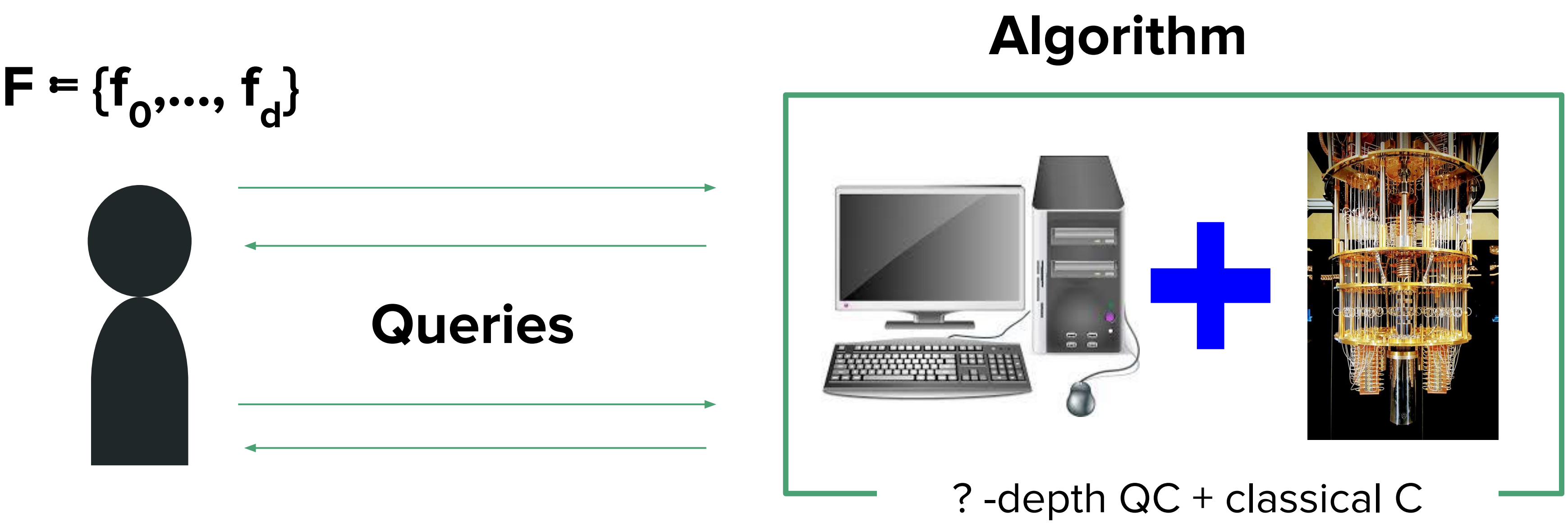

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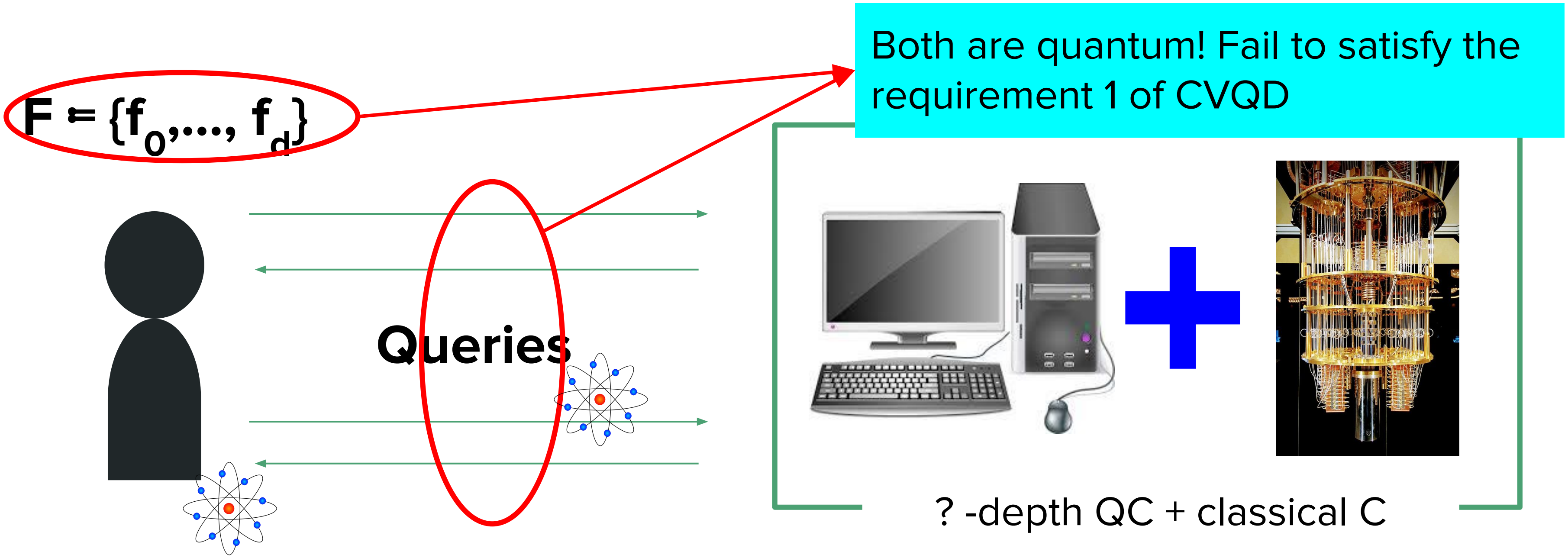

Our CVQD Protocols

Two CVQD protocols:

Theorem 1 [Chia-Hung 22]: ∃ two CVQD protocols where

- **Protocol A** distinguishes **d-depth quantum circuit** from **d+c*** for any d

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Protocol A

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Remarks of Protocol A

- The separation **(d v.s. d+c*) is not optimal**
- Honest server **requires (>c*)-depth** to implement the protocol
- Other improvements that we will discuss later

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Protocol B

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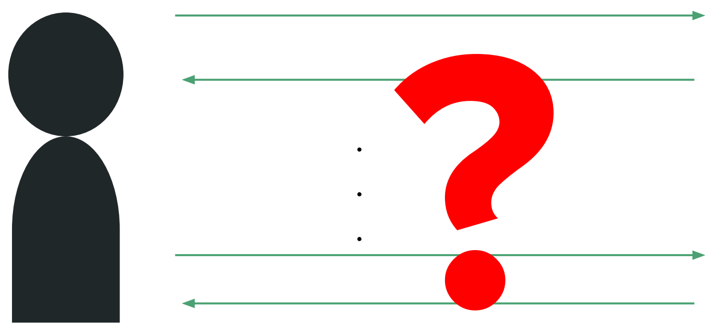
Remarks of Protocol B

Better separation (d versus d+3), but protocol B **requires another dishonest quantum prover** to help.

What is our protocol?



? -depth QC + classical C

Protocol A

Idea:

Proof of Quantumness Protocol + “Pointer chasing”

Proof of Quantumness (PoQ)

BCMVV’s proof of quantumness protocol (PoQ)

- Goal: Let a classical client to distinguish the following two cases
  a. A remote server has specific quantum power
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[Brakerski, Christiano, Mahadev, Vazirani, Vidick ’18]

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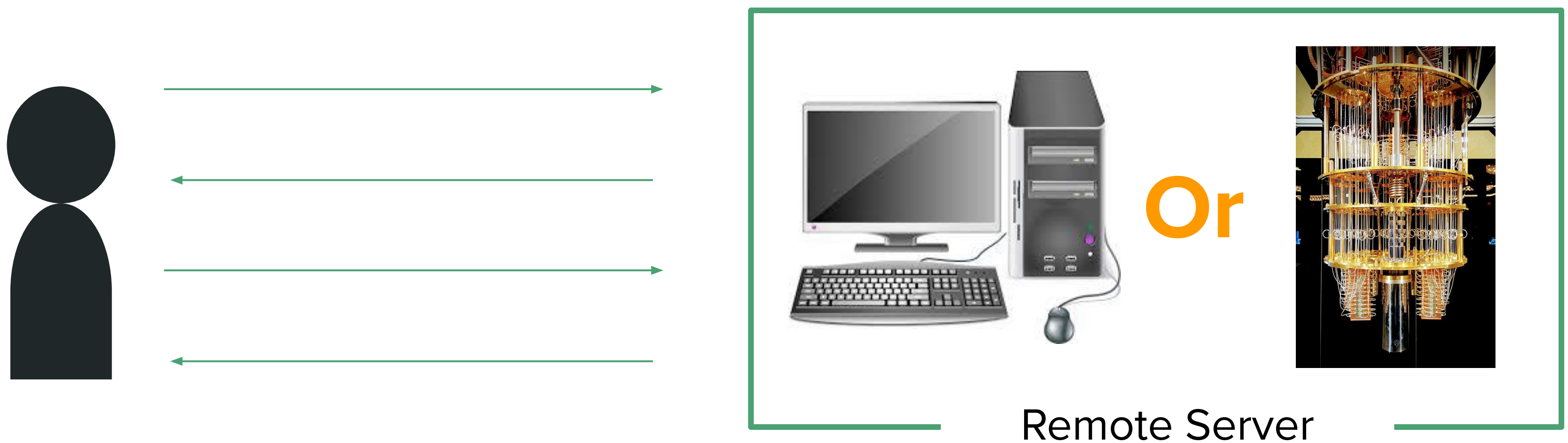

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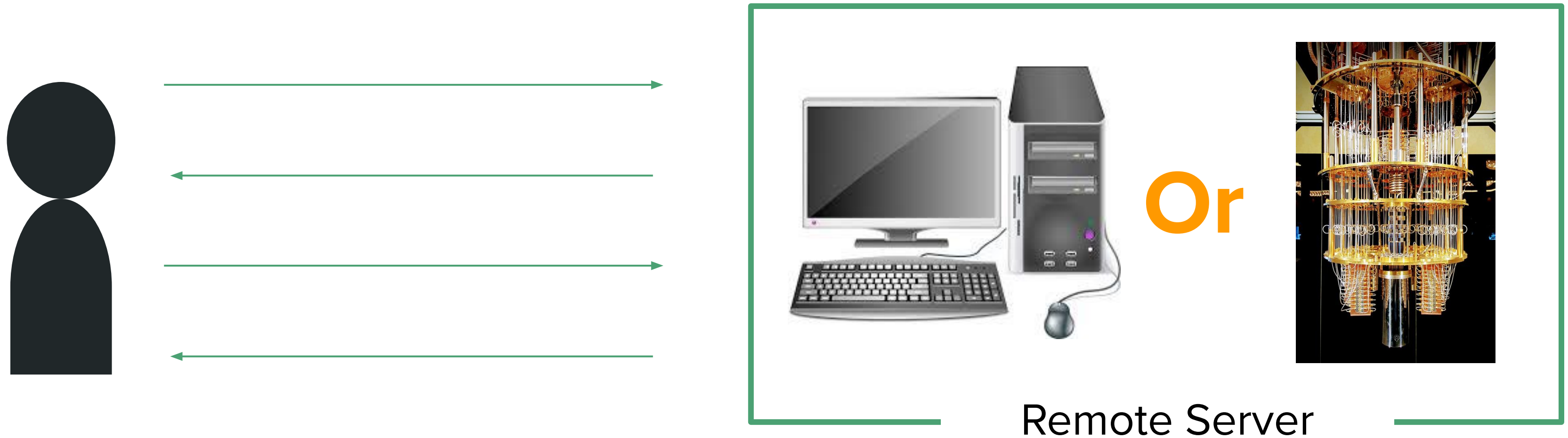

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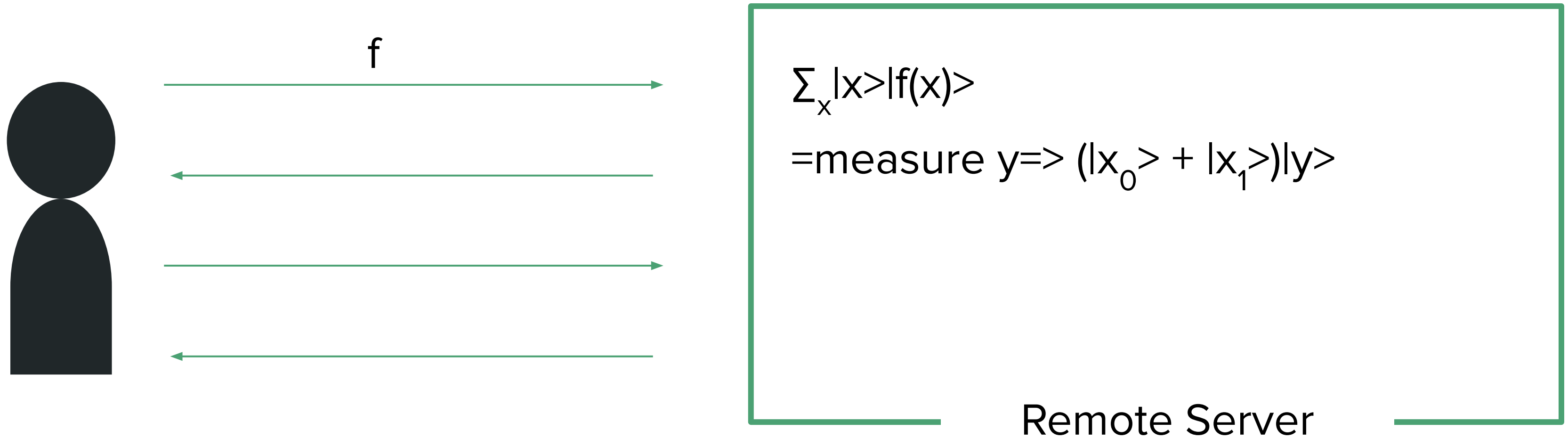

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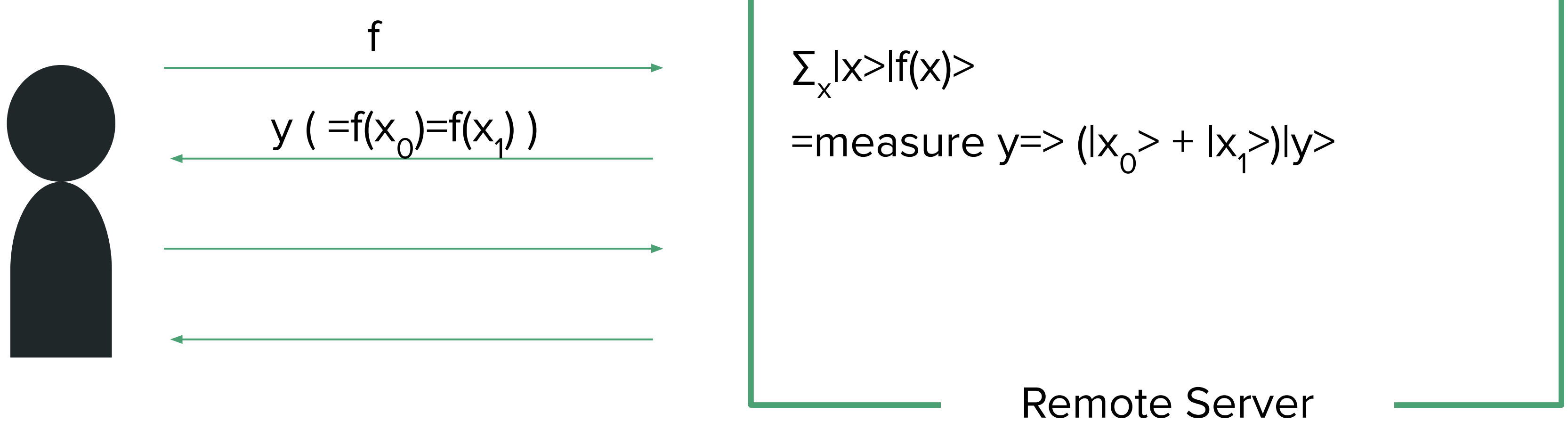

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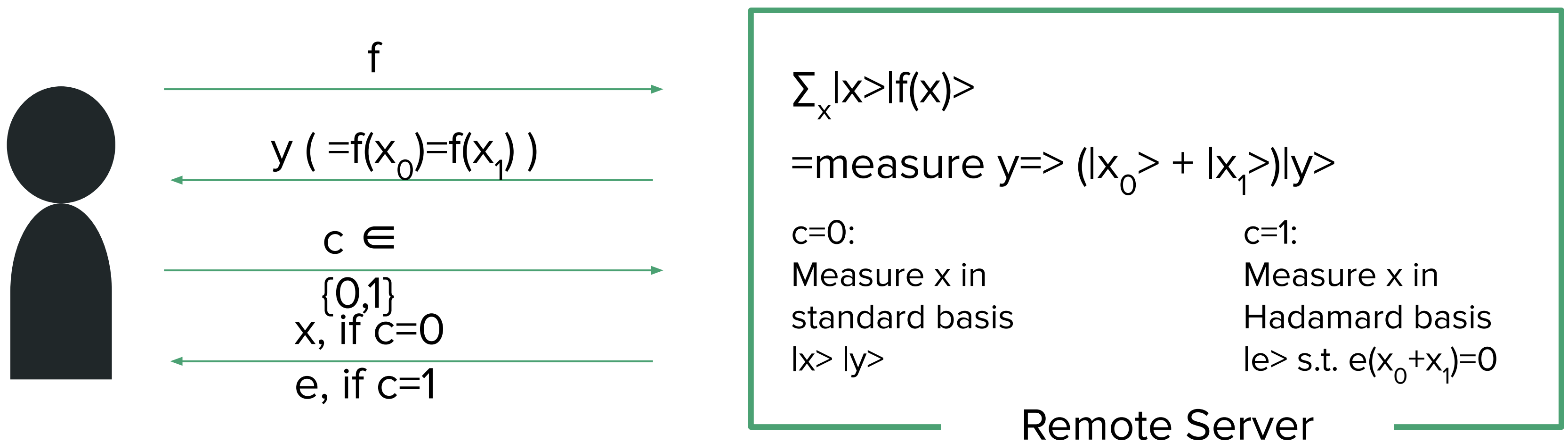

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- A classical server who can win the game can break AHB by rewinding!
- [Hirahara-Le Gall '21] and [Liu-Gheorghiu '21] showed that f can be evaluated in depth-O(1) (i.e., c*) ⇒ separates depth-O(1) from depth-0 (i.e., classical) device.

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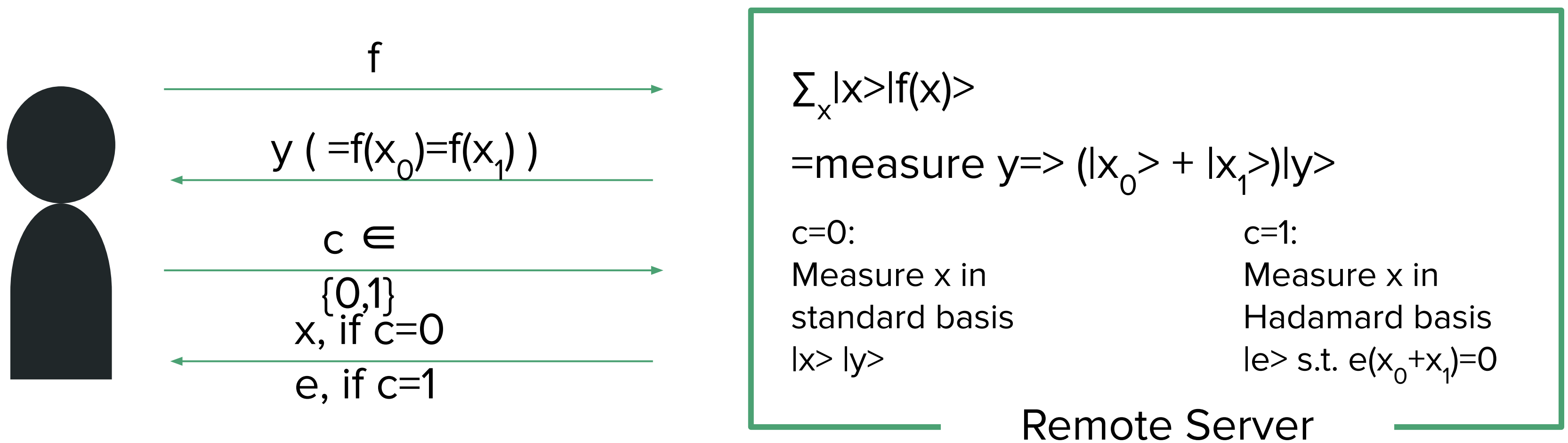

[Brakerski, Christiano, Mahadev, Vazirani, Vidick '18]

PoQ + “Pointer Chasing” = CVQD!

- Goal: Distinguish a depth-$(d + c^*)$ from a depth-d QC scheme

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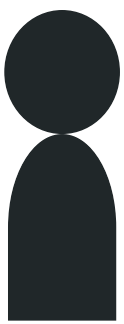

$\Sigma_x|x\rangle|f_1(x)\rangle, \Sigma_x|x\rangle|f_2(x)\rangle, \ldots, \Sigma_x|x\rangle|f_d(x)\rangle$

$\Rightarrow (|x_0^1\rangle + |x_1^1\rangle)|y_1\rangle, \ldots, (|x_0^d\rangle + |x_1^d\rangle)|y_d\rangle$

Remote Server

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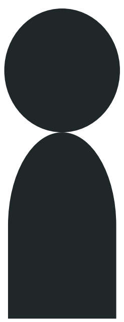

$\Sigma_x|x\rangle|f_1(x)\rangle$, $\Sigma_x|x\rangle|f_2(x)\rangle$,..., $\Sigma_x|x\rangle|f_d(x)\rangle$

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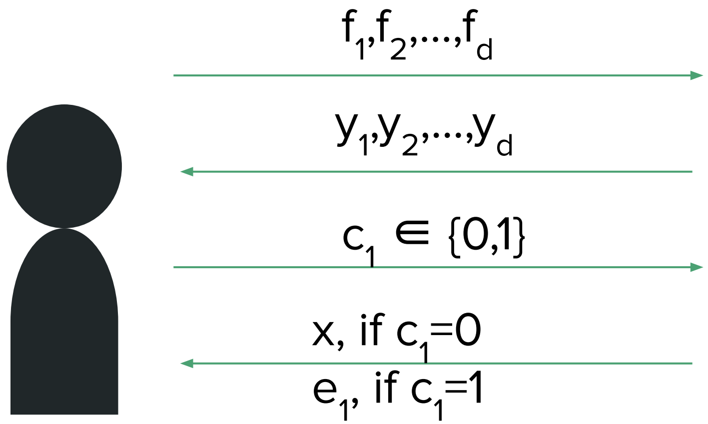

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$c_1=0$:
Measure x in standard basis
$|x\rangle\ |y_1\rangle$

$c_1=1$:
Measure x in Hadamard basis
$|e_1\rangle$ s.t. $e(x_0+x_1)=0$

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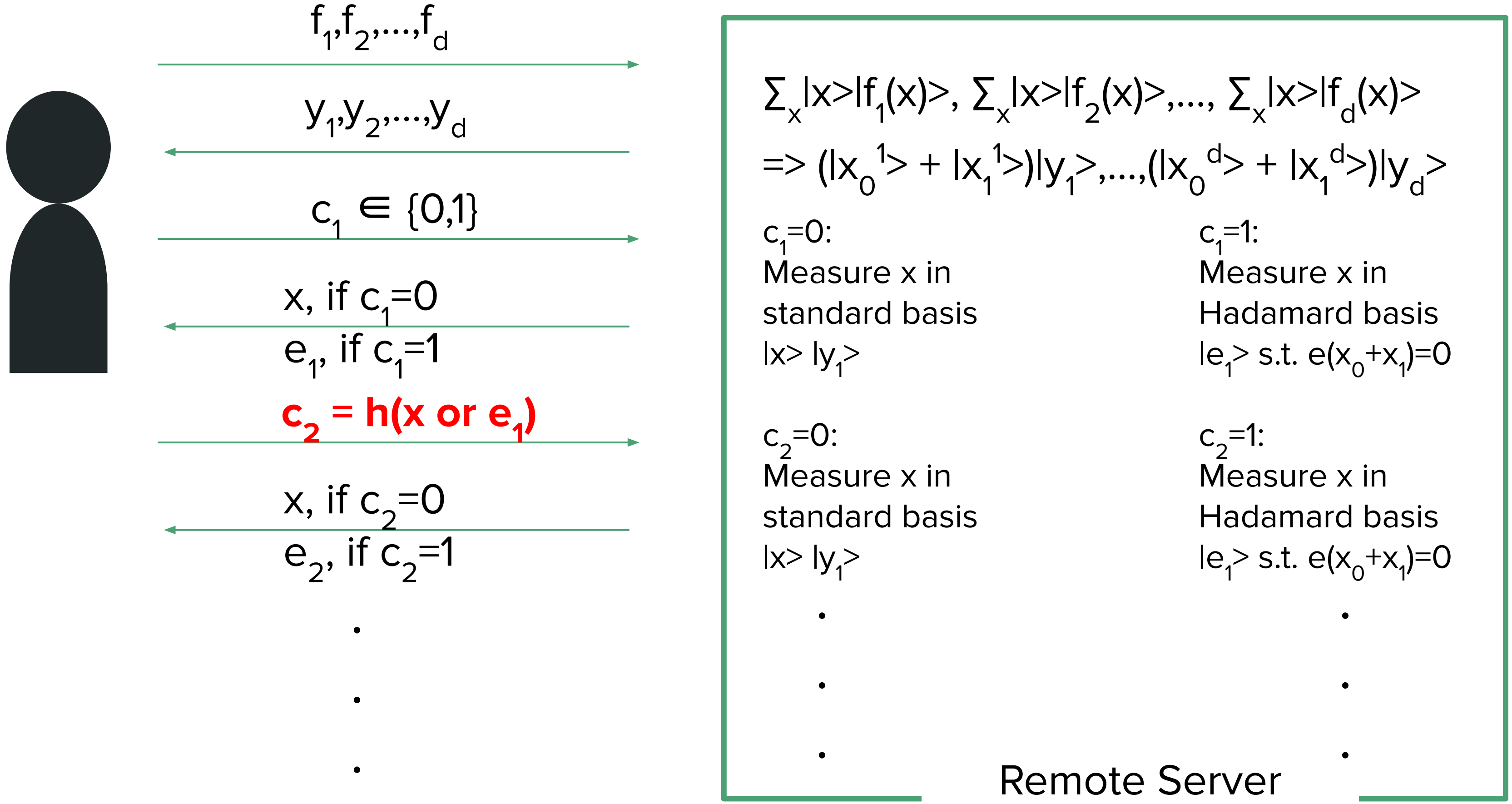

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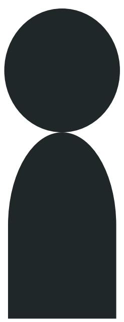

$f_1, f_2, \ldots, f_d$ →

← $y_1, y_2, \ldots, y_d$

$c_1 \in \{0,1\}$ →

← x, if $c_1=0$ / e_1, if $c_1=1$

$\mathbf{c_2 = h(x \text{ or } e_1)}$ →

← x, if $c_2=0$ / e_2, if $c_2=1$

.

.

.

c_i is determined by the answer of the (i-1)-th round

Remote Server

$\Sigma_x |x\rangle|f_1(x)\rangle, \Sigma_x |x\rangle|f_2(x)\rangle, \ldots, \Sigma_x |x\rangle|f_d(x)\rangle$

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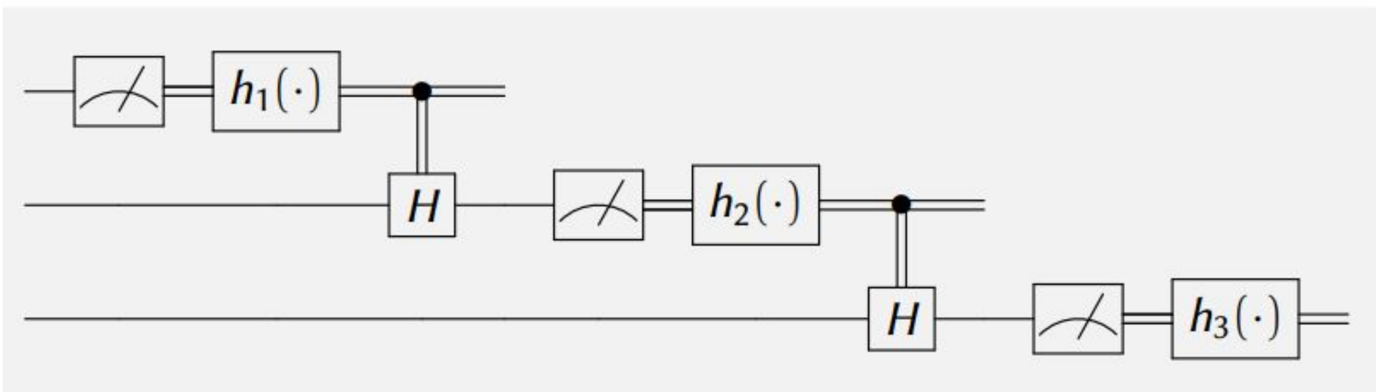
Measure x in standard basis	Measure x in Hadamard basis
$|x\rangle\ |y_1\rangle$	$|e_1\rangle$ s.t. $e(x_0+x_1)=0$
.	.
.	.
.	.

Completeness and Soundness

Completeness: ∃(d+c*)-QC that can win the game!

- Need c* depth to evaluate $\sum_x |x\rangle|f(x)\rangle$
- (i-1)-th answer determines i-th challenge
  - i-th qubit keeps coherence until all previous i-1 rounds complete



Soundness: ∀d'-QC with d'<d can't win the game breaks AHB

- Pigeon holes:
  - **1** quantum depth can only be used in **1** round
  - **d'** depth vs **d** rounds => ∃ rounds that are executed classically =>one can use these rounds to break AHB

First CVQD Protocol

Theorem 1 [Chia-Hung 22]: ∃ CVQD protocols which distinguishes **d-depth quantum circuit** from **d+c*** for any d

Protocol A

1. **Classical** verifier and **classical** protocol
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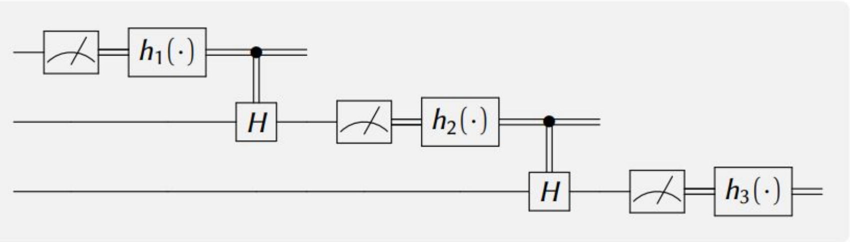

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- The separation **(d v.s. d+c*) is not optimal**
- Honest server **requires >c*-depth** to implement the protocol
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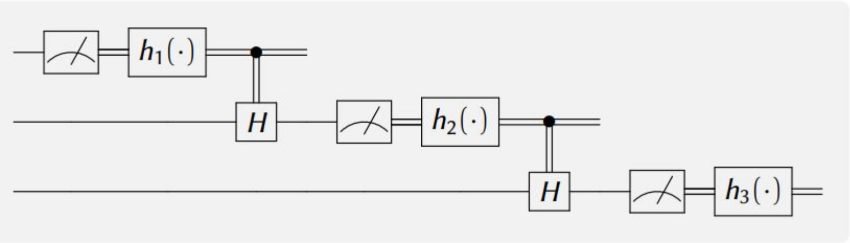

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Question: Can we solve these issues?

Protocol B

Disadvantages of Protocol A

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Protocol B

Protocol B

- The separation **(d v.s. ~~d+c*~~) is not optimal** **d+3**
- ~~Honest server **requires >c*-depth** to implement the protocol~~
- ~~Honest server needs to be able to implement **d-QC scheme**~~ **d-depth circuit with classical post processing**

Protocol B

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d+3

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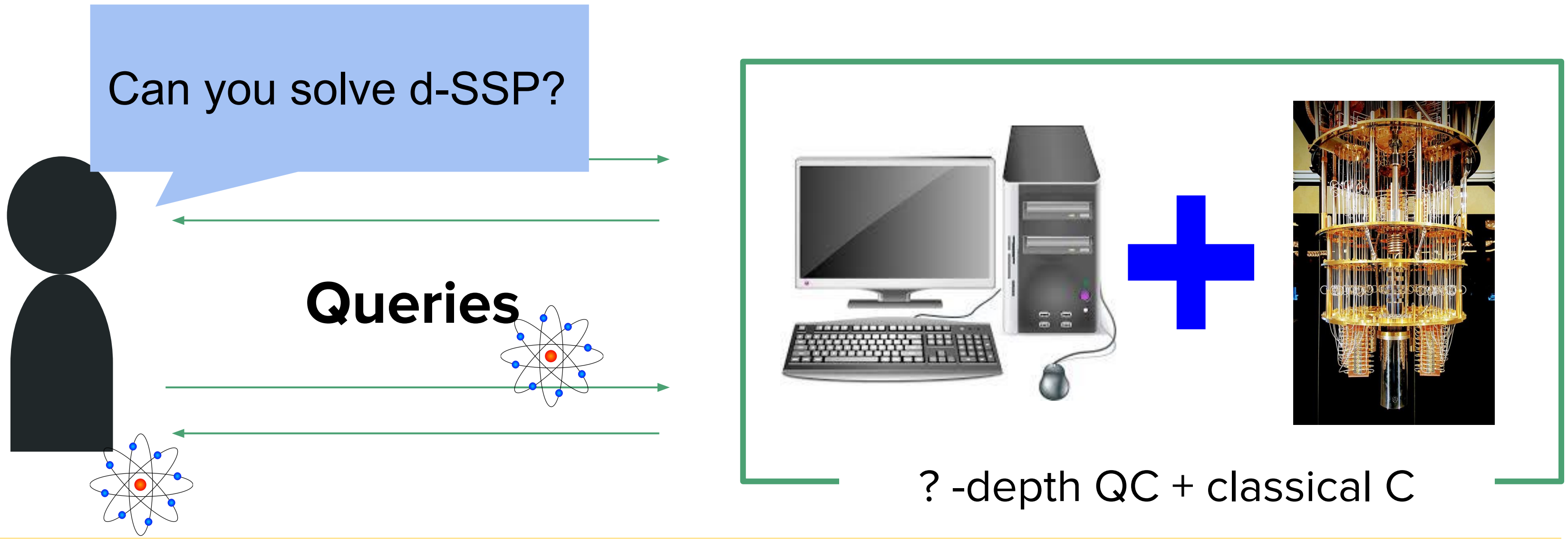
Idea:

Revisit the first failed attempt (the protocol from d-SSP problem) and dequantize the protocol via **non-local game**

Need to “Dequantize” the Protocol



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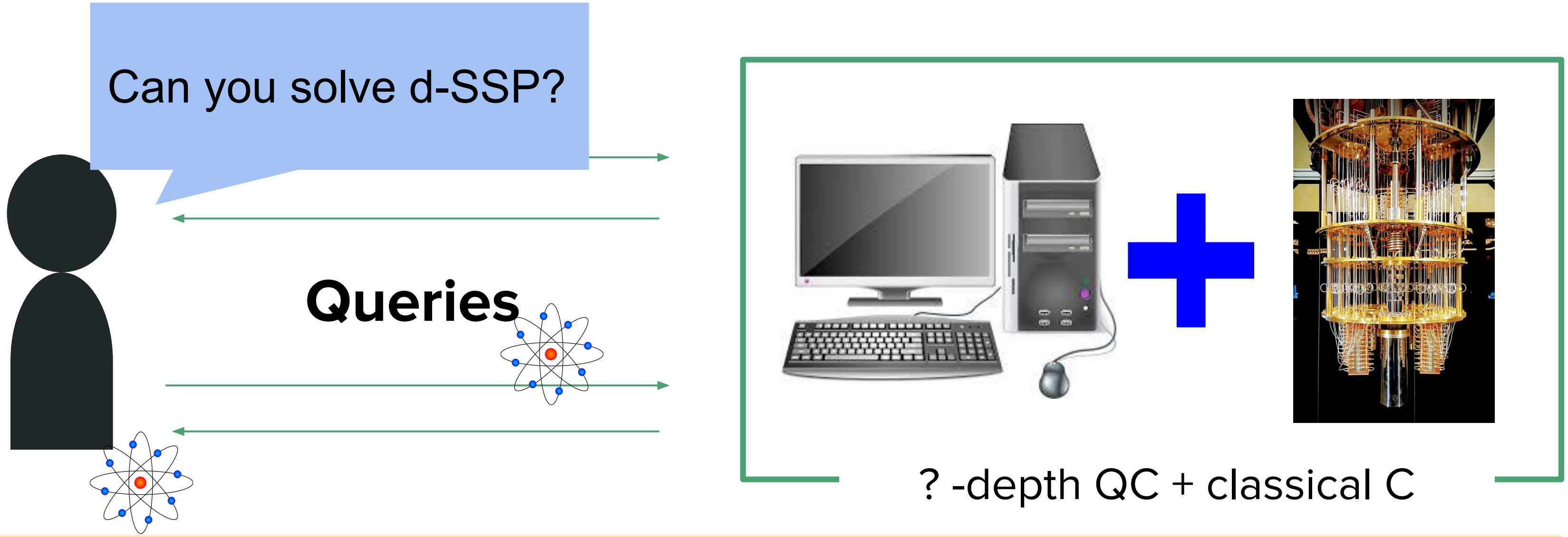

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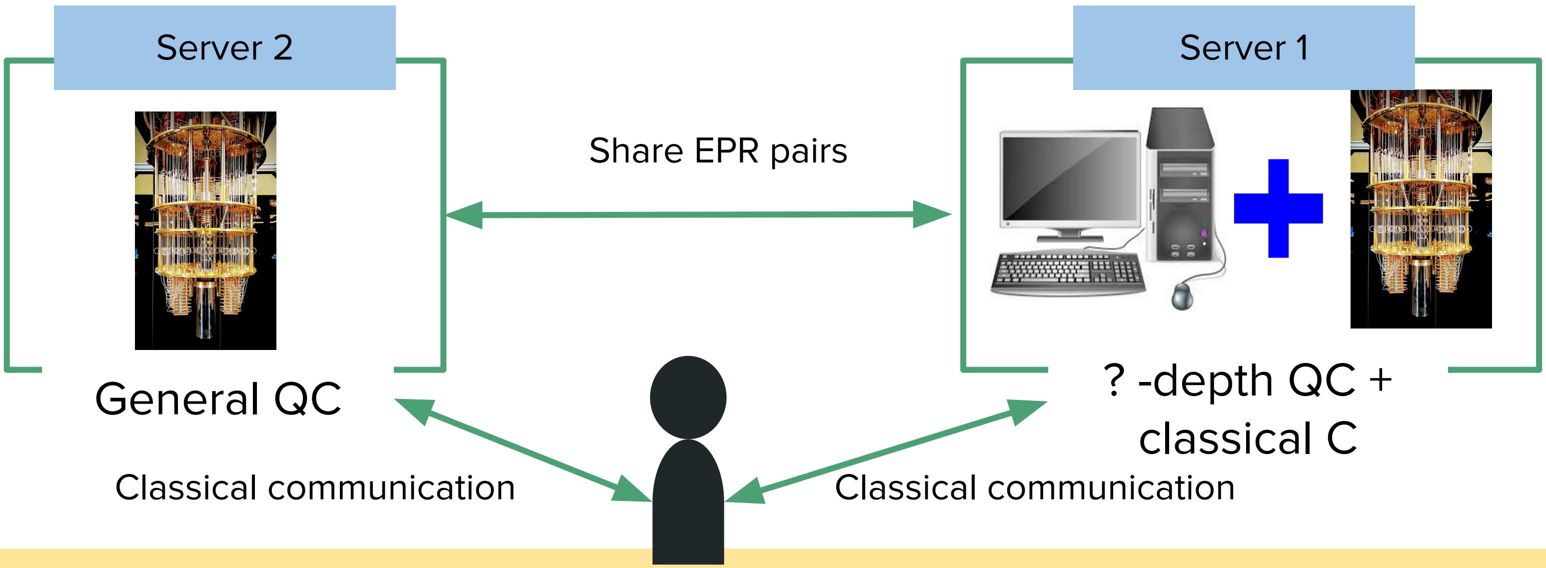

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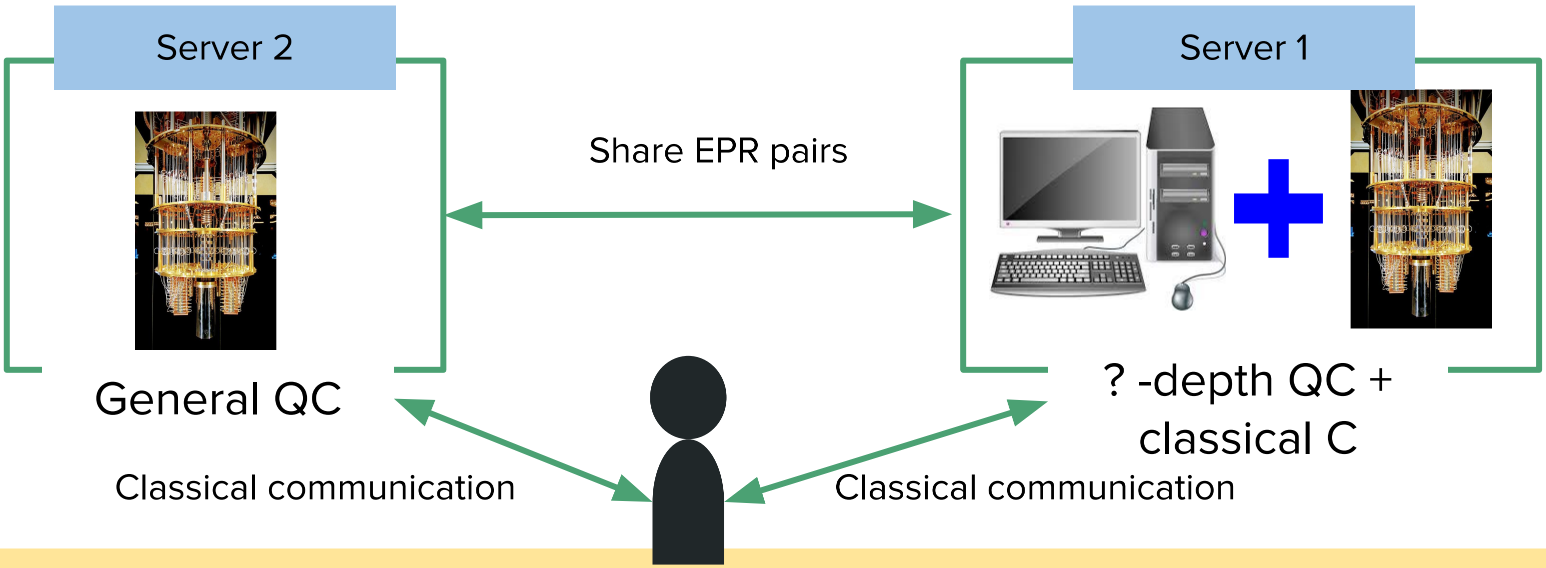

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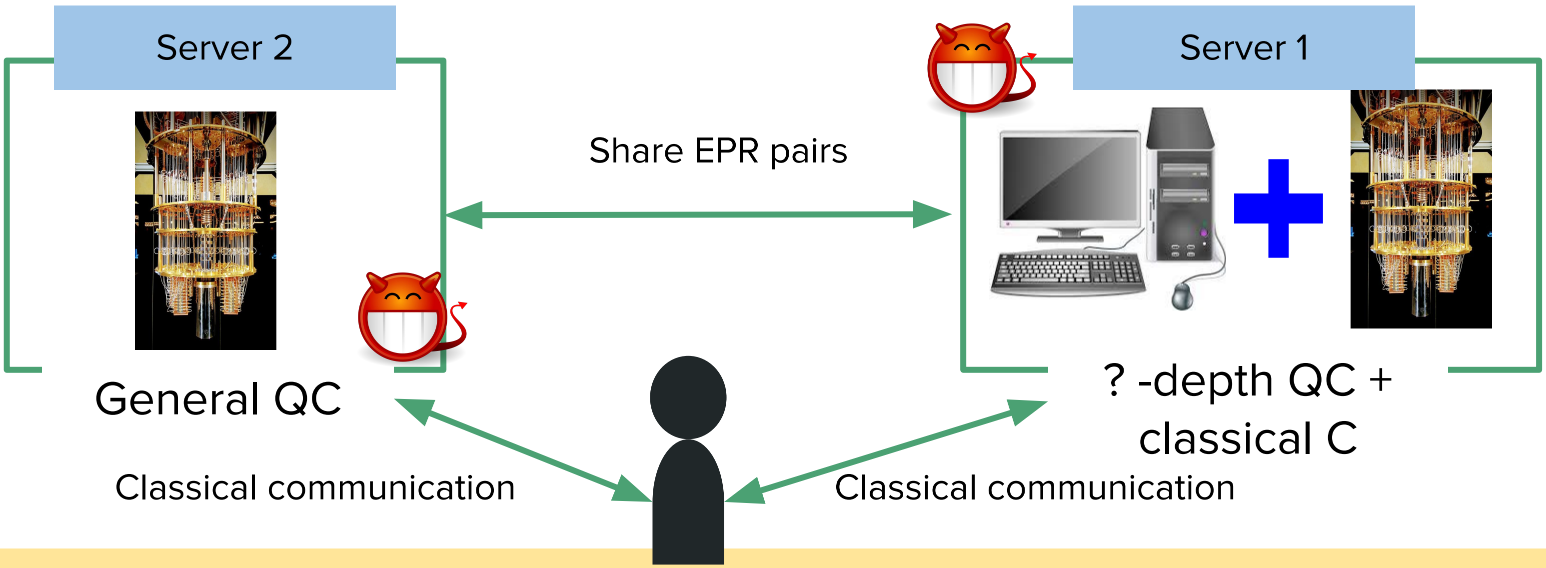

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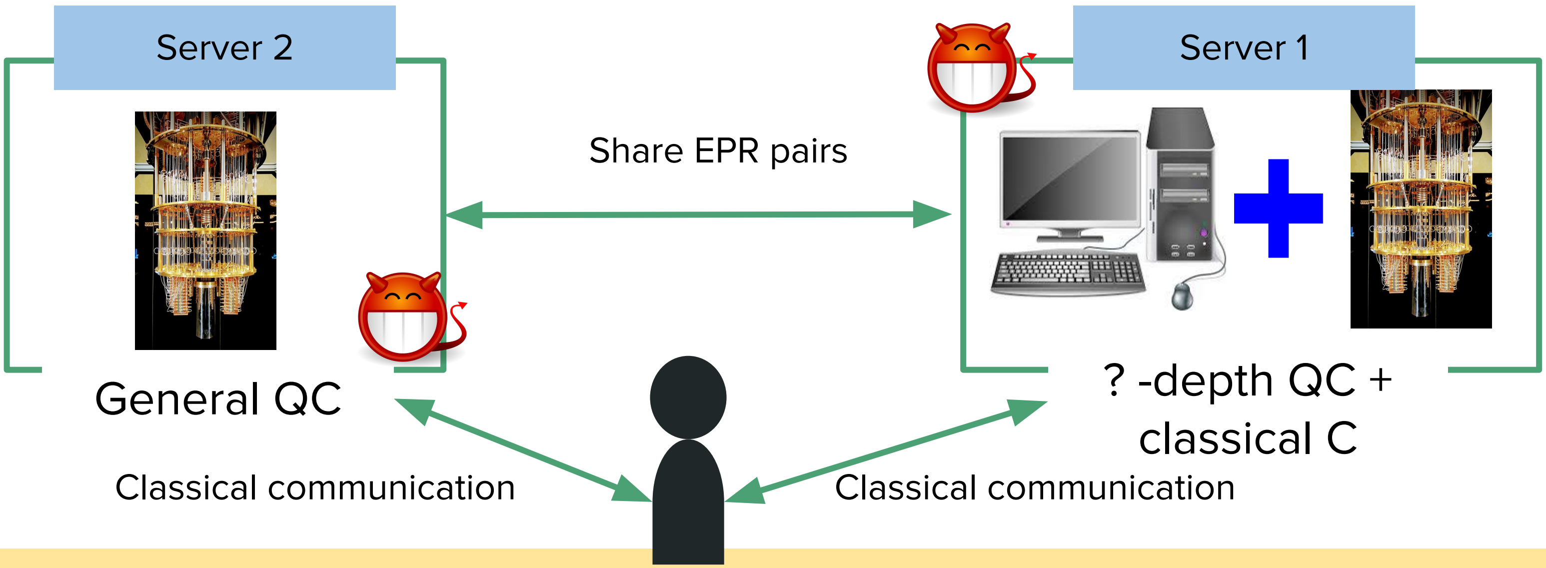

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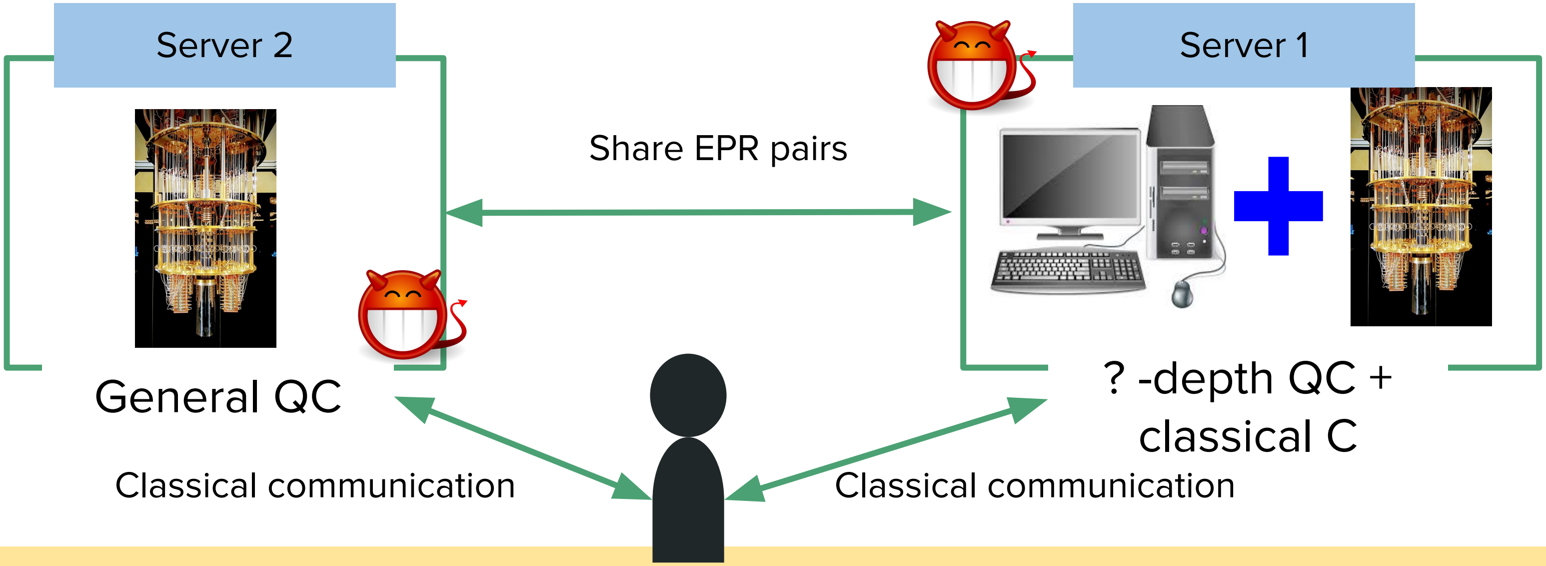

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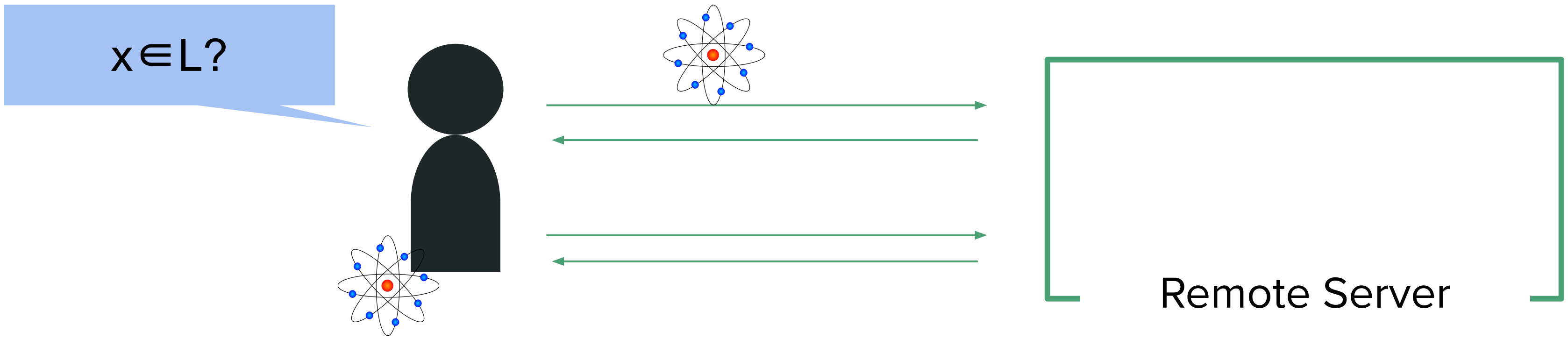
Sketch of the Proof

- The Broadbent protocol for **BQP** [Broadbent '18]
  - Three types: Comp, X-, and Z-test **(indistinguishable from server's view)**
  - Tests for detecting bit-flip (X-test) and phase-flip (Z-test).



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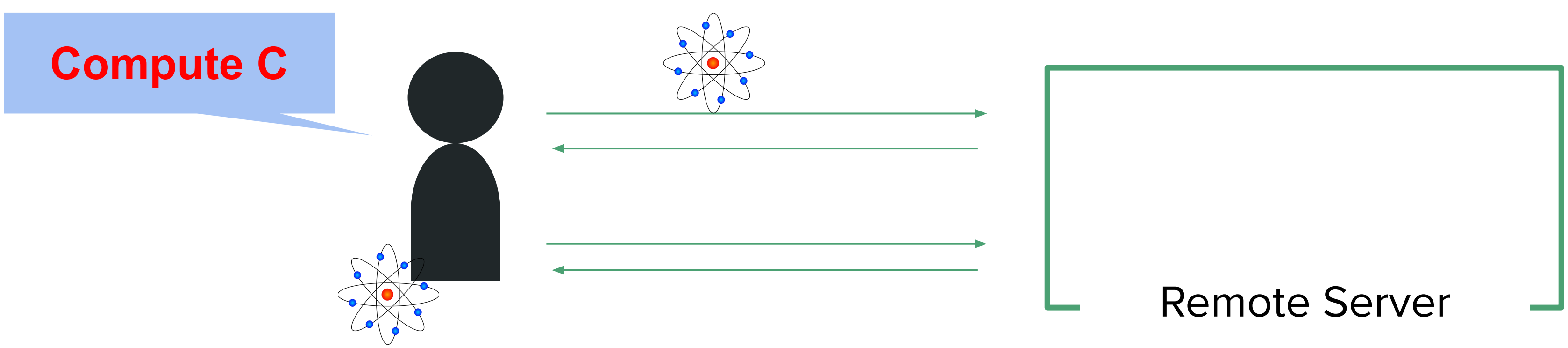

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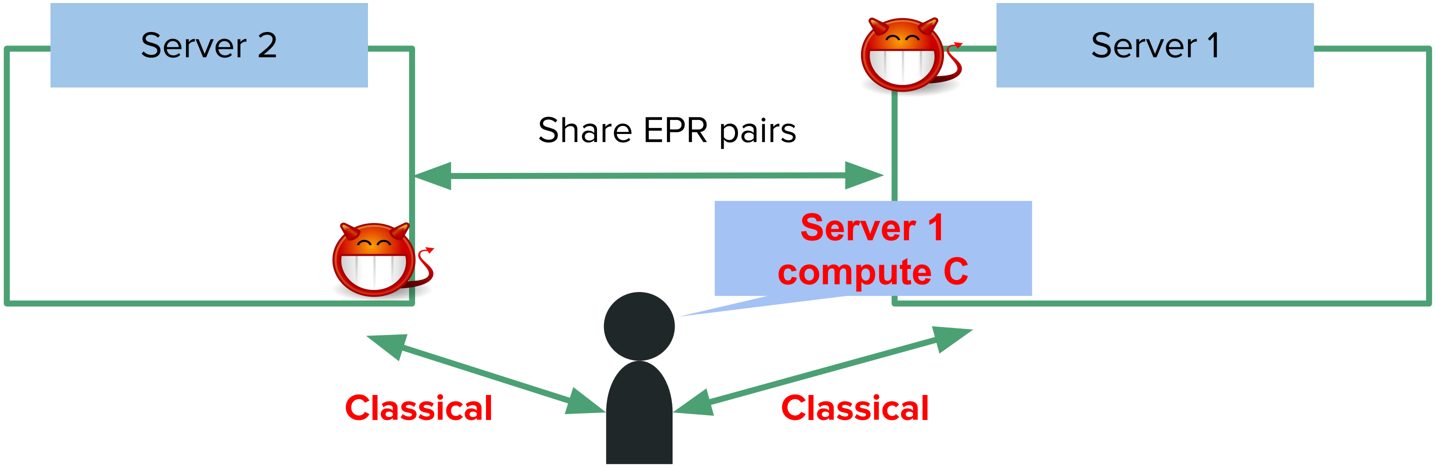
- Theorem [Chia-Hung '22]
  - For efficient circuit C, if server wins w.h.p., it implements a channel C' ≍ C.
  - Use X-test and Z-test to check if the entire output state has error.
- Apply the verifier-on-a-leash protocol [Coladangelo, Grilo, Jeery, Vidick '19]:
  - ∃ two-player test which certifies Clifford measurements: if the player succeeds w.h.p., then the measurements is correct.



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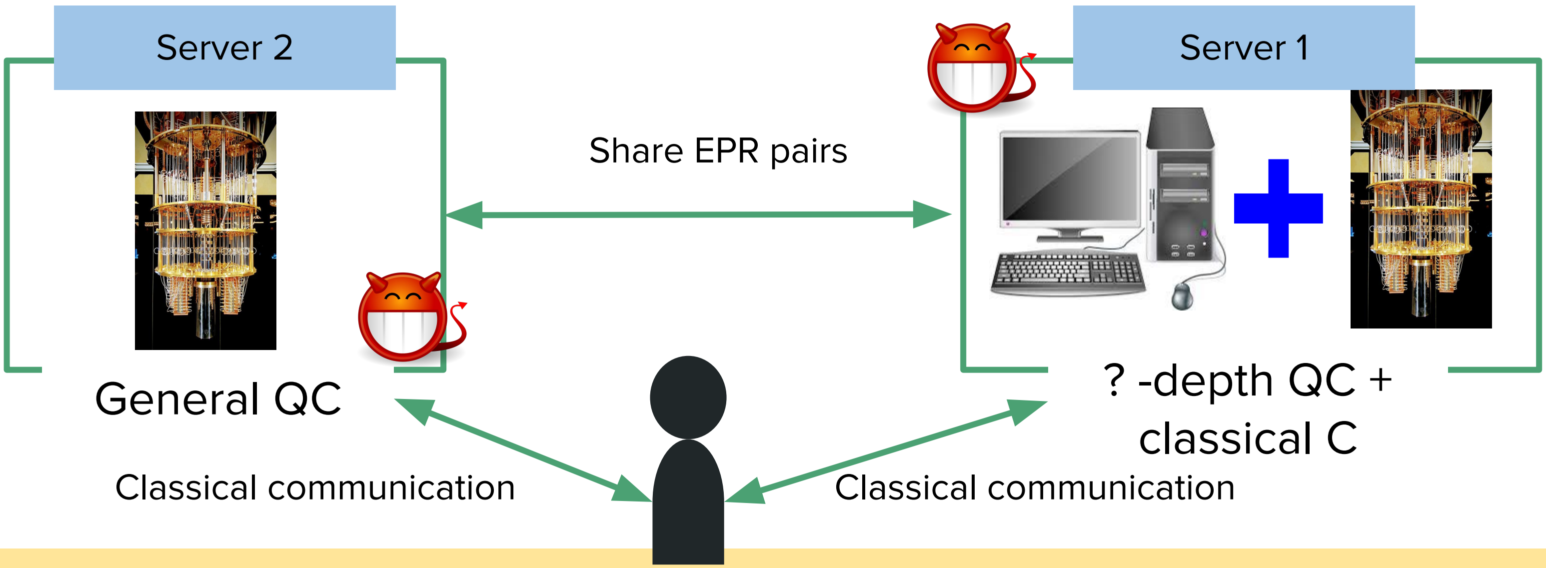

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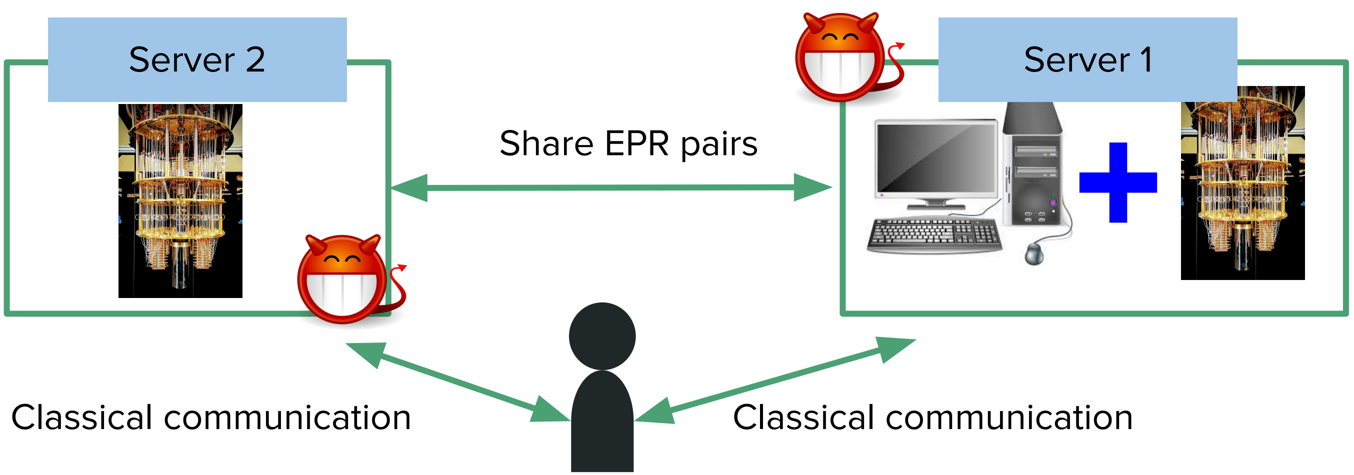
Recap: Classical Verification of Quantum Depth

Theorem 2 [Chia-Hung 21]: There exists a two-prover classical protocol that satisfies the following:

1. The classical verifier accepts if the server has ≥ **d+3** quantum depth
2. The classical verifier rejects if the server has ≤ d quantum depth

- Unconditionally secure (inefficient) / Nearly optimal separation



Two-Prover CVQD

Theorem 1 [Chia-Hung 22]: ∃ two CVQD protocols where

- **Protocol A** distinguishes **d-depth quantum circuit** from **d+c*** for any d
- **Protocol B, a two-prover protocol,** distinguishes **d-depth quantum circuit** from **d+3** for any d

Protocol B

1. **Classical** verifier and **classical** protocol
2. Recognize server's quantum depth
    a. Server's quantum depth > **d+3** => Verifier accepts
    b. Server's quantum depth ≤ **d** => Verifier rejects
3. The server 1 only needs **d-depth circuit + classical post processing**

Unconditionally secure (inefficient) / Nearly optimal separation

Disadvantage of Protocol B
Protocol B **requires another dishonest quantum prover** to help...

Open Questions

Open question 1: Classical verification of quantum **resources**, e.g., quantum memory, quantum volume, etc.

Open question 2: Improved CVQD protocol that can be implemented **on near-term devices**

Open question 3: Separations between hybrid quantum-classical computation (d-CQ and d-QC schemes) **in the plain model**

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Thank you!