ND-MIT QCS Lecture Series

Software and Algorithmic Approaches to Quantum Error Mitigation

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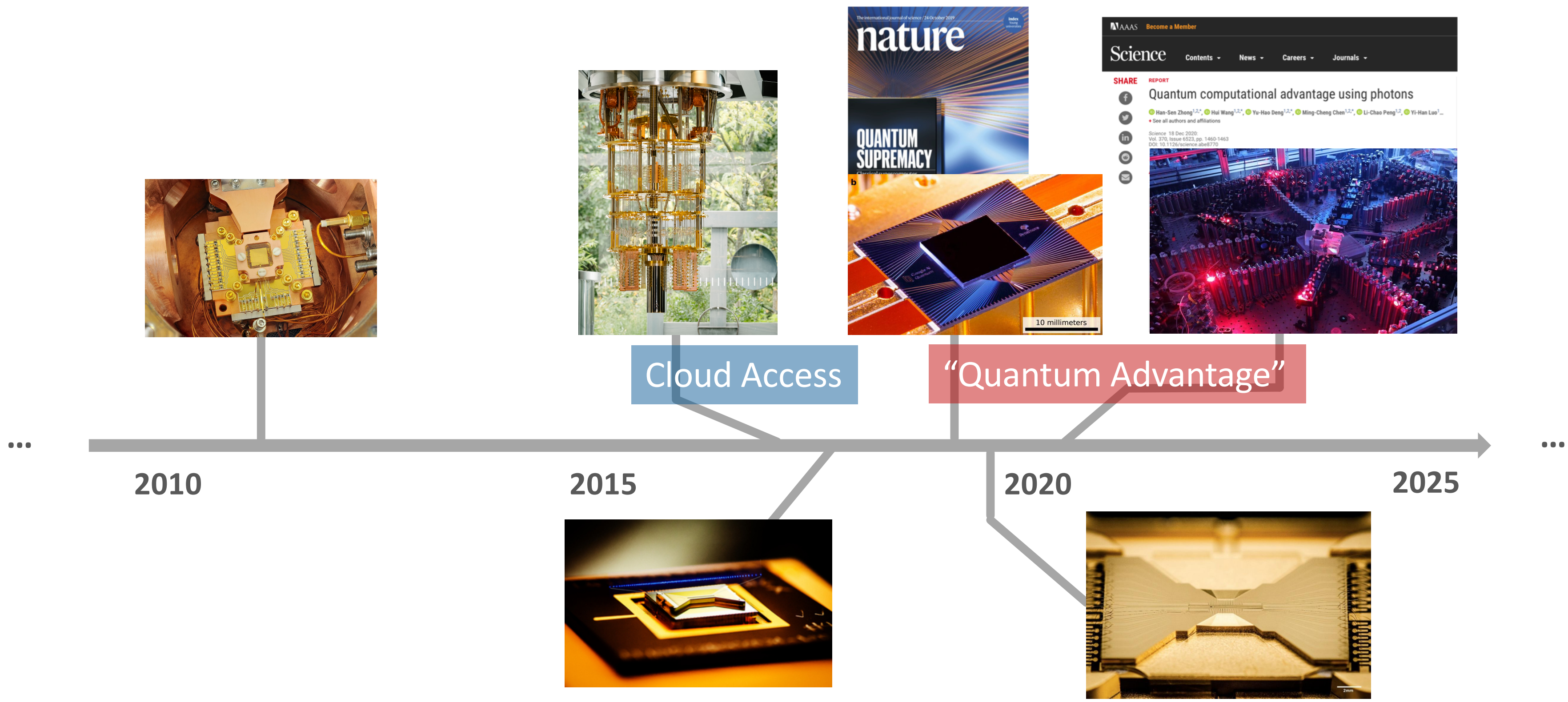
Yale

OUTLINE

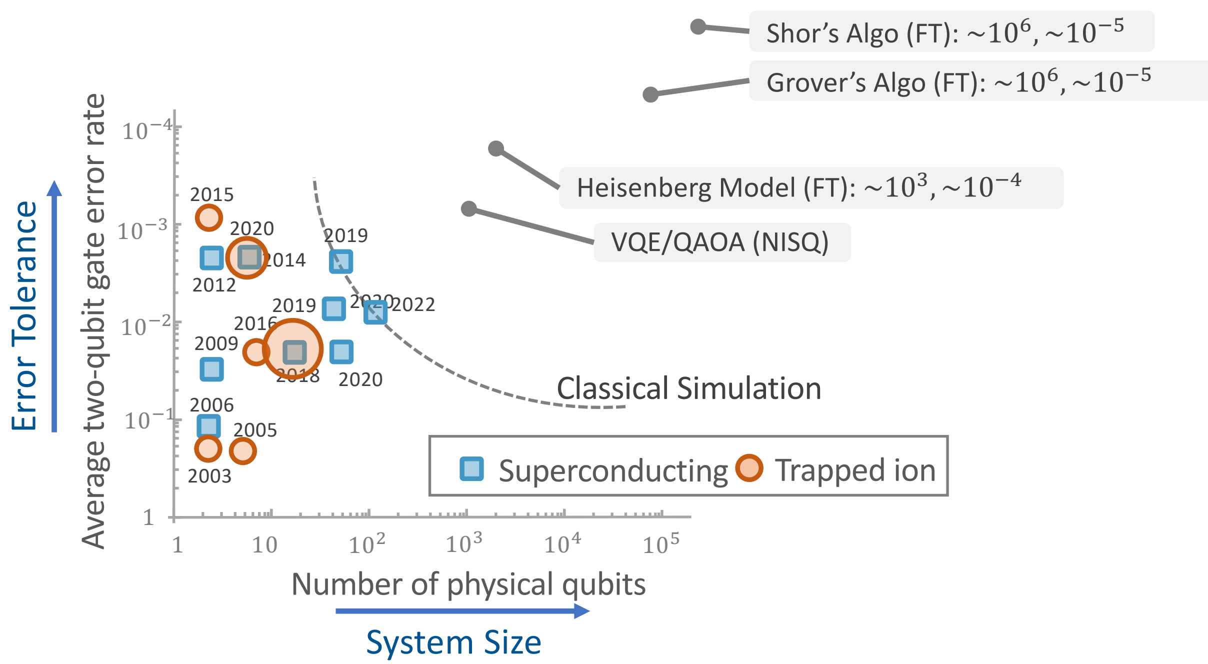
1. The State of QC and Noisy Quantum Systems
2. Software and Algorithmic Approaches to Noise Mitigation
3. Discussion

Quantum Computers are Getting Accessible and Powerful



Photos from: NIST, IBM Q Graham Carlow, Google AI Quantum, Kai Hudek IonQ, E. Edwards JQI, Hansen Zhong USTC

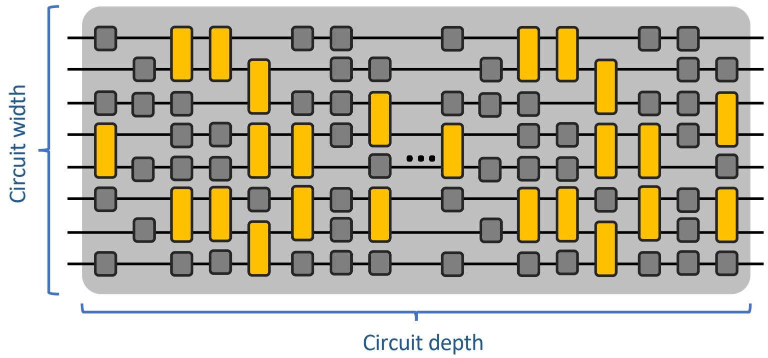
Experimental Progress – where we are today



*Size of data point indicates connectivity; larger means denser connectivity.

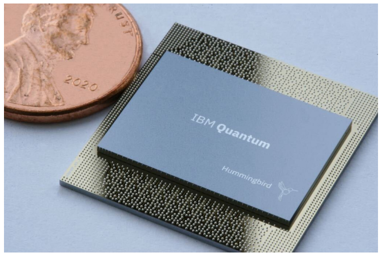
Data: Ding & Chong

Technological Challenge: System Size



Large Quantum Circuits (thousands of qubits/gates)

Deploy



Small Systems (10-100 qubits)

Hard to execute large quantum programs on systems with small number of qubits.

Photo from: medium.com by IBM Qiskit

Technological Challenge: System Fidelity

Quantum gate error and qubit decoherence severely limit program success rate.

Technology	Gate Error Rate	Qubit Lifetime/Operation Time
Superconducting Circuits	$\sim 10^{-3} - 10^{-4}$	$\sim 10^{4} - 10^{5}$
Ion, Atom, Molecule	$\sim 10^{-3} - 10^{-5}$	$\sim 10^{3} - 10^{4}$
Electron Spin	$\sim 10^{-3}$	$\sim 10^{5} - 10^{8}$
Nuclear Spin	$\sim 10^{-3}$	$\sim 10^{5} - 10^{6}$

Common sources of error:

Qubit idle error, Coherent gate error, Incoherent gate error, Measurement/readout error

Data: Ding & Chong (2020)

Key Ideas in Noise Mitigation

A Quantum Engineer's Survival Kit

(Software side: compile/run time)

- Cancel Error Coherently
- Avoid Noisy Operations
- Synthesize Robust Operations
- Engineering Errors
- Classical Post-Processing

(Hardware side: fabrication time)

- Discover stable qubit materials
- Improve material synthesis and fabrication
- Optimize Device Parameters

Key Ideas in Noise Mitigation

A Quantum Engineer's Survival Kit

(Software side: compile/run time)

Focus of this talk

- Cancel Error Coherently
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(Hardware side: fabrication time)

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Key Ideas in Noise Mitigation

A Quantum Engineer's Survival Kit

(Software side: compile/run time)

Cancel Error Coherently	Avoid Noisy Operations	Synthesize Robust Operations	Engineering Errors	Classical Post-Processing

- Dynamical Decoupling [Viola et. al. Arxiv: 9803057]
- Composite Pulses [Brown et. al. Arxiv: 0407022]

State of a Qubit on Bloch Sphere

Quantum state of a qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$

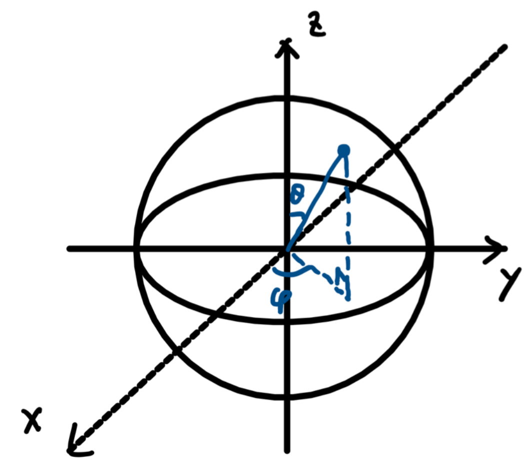
$= \cos(\theta/2)\,|0\rangle + \sin(\theta/2)\,e^{i\varphi}|1\rangle$ where $0 \le \theta \le \pi, 0 \le \varphi < 2\pi$

Unitary transformation of a qubit: $|\psi\rangle \rightarrow U|\psi\rangle$

Rotation about $\hat{n}$-axis for θ: $R_{\hat{n}}(\theta) = \exp\left(-\frac{i\theta}{2}\sigma_{\hat{n}}\right)$, $\sigma_{\hat{n}} = r_x\sigma_x + r_y\sigma_y + r_z\sigma_z$

Single-qubit unitary can be decomposed into rotations:

$$U = e^{i\alpha}R_x(\theta_2)R_z(\theta_1)R_x(\theta_0)$$



In practice, rotations can have systematic errors

Ideal: $R_{\hat{n}}(\theta)$
Actual: $R_{\hat{n}}(\theta + \epsilon)$ for some unknown ϵ.
can depend on θ: ϵ_θ

Technique 1: Composite Pulses

(Cancel Errors Coherently)

Motivation: Unitary/coherent errors are reversible.

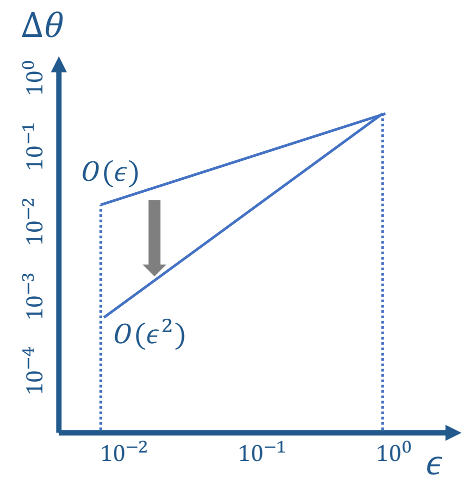
Example: Target rotation $R_x(\theta)$

Actual rotation $R_x' = R_x(\theta + \Delta\theta) = R_x(\theta)R_x(\theta\epsilon)$,

$\Delta\theta = \theta\epsilon$ for some unknown ϵ

$$\boxed{R_x'(2\pi) = R_x(2\pi + 2\pi\epsilon) = R_x(2\pi\epsilon)}$$

Can we use this to implement $R_x(-\theta\epsilon)$?



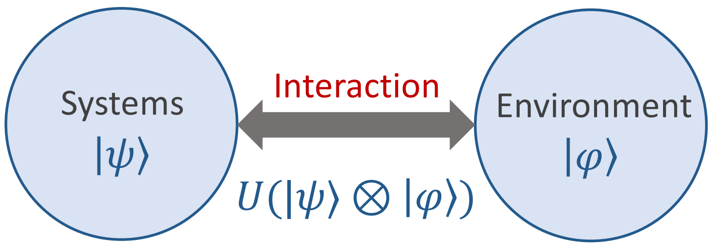
Trotter-Suzuki formula:

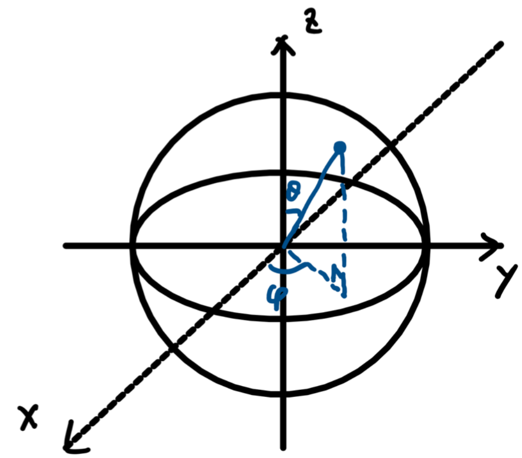
$$\exp\left(-\frac{i\theta\epsilon}{2}\sigma_{\hat{x}}\right) = \exp(-i(H_1 + H_2 + H_3)\epsilon)$$
$$= \underbrace{\exp(-iH_1\epsilon)\,\exp(-iH_2\epsilon)\,\exp(-iH_3\epsilon)}_{\text{Three composite rotations}} + O(\epsilon^2)$$

$$H_1 = H_3 = 2\pi(\sigma_x \cos\phi + \sigma_y \sin\phi)$$
$$H_2 = 4\pi(\sigma_x \cos\phi - \sigma_y \sin\phi)$$
$$\phi = \cos^{-1}(\theta/8\pi)$$

References:
Composite Pulses [Brown et. al. Arxiv: 0407022]

Decoherence of a Qubit





System not perfectly isolated from the environment.

Ideal: $|\psi\rangle \otimes |\varphi\rangle$
Actual: $U(|\psi\rangle \otimes |\varphi\rangle)$ for some unknown U.

Hard to undo U: We cannot control environment $|\varphi\rangle$ like Nature does.

System $|\psi\rangle$ decoheres quickly.

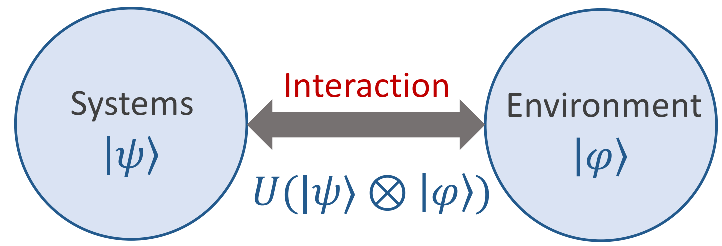
$$|\psi\rangle\langle\psi| \to Tr_{env}(U(|\psi\rangle \otimes |\varphi\rangle)) = \rho$$

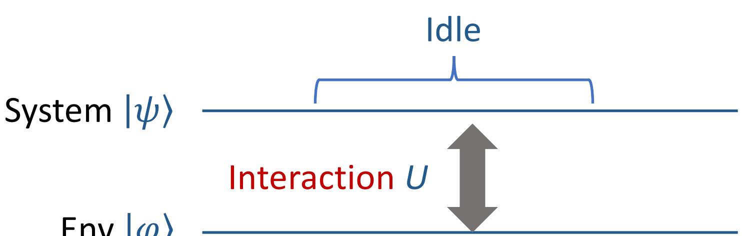
T1 relaxation: loss of energy
T2 relaxation: loss of quantum phase

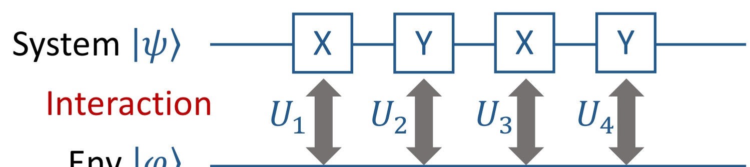
Technique 2: Dynamical Decoupling

(Cancel Errors Coherently)

Motivation: Reverse the coupling between systems and environment.







Here the gate sequence: $YXYX = I$ acts trivially on $|\psi\rangle$.
Also the coupling with environment U_1, U_2, U_3, U_4 cancels out.

Mitigate decoherence using coherent error cancelation!

References:
Dynamical Decoupling [Viola et. al. Arxiv: 9803057]
Adaptive DD [Das et. al. Arxiv: 2109.05309]

Key Ideas in Noise Mitigation

A Quantum Engineer's Survival Kit

(Software side: compile/run time)

- Cancel Error Coherently
- Avoid Noisy Operations
- Synthesize Robust Operations
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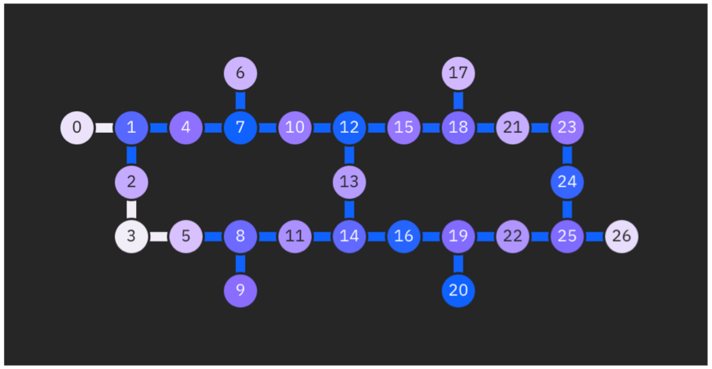
System-Level Performance

Qubit Connectivity and Frequencies:



IBM Quantum Systems (ibmq_toronto)

Qubits have limited lifetime, fidelity, and connectivity.

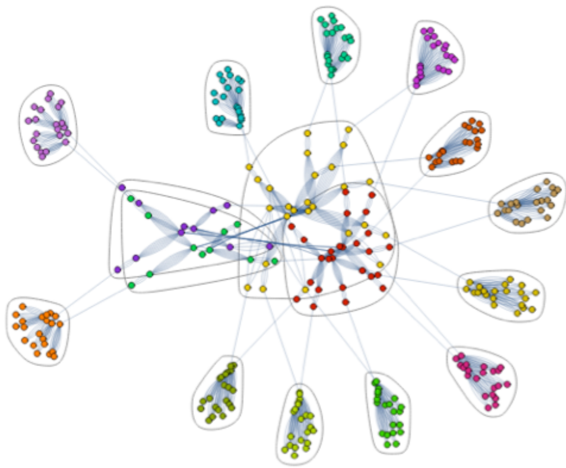
Ideal: All qubits are identical and any pair of qubits can interact.
Actual: Each qubit is different and only some pair of qubits can interact.

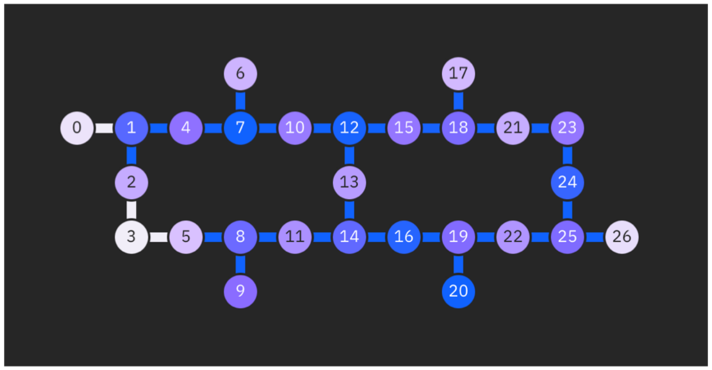
Data: IBM Q (2022)

Technique 3: Noise-aware qubit mapping

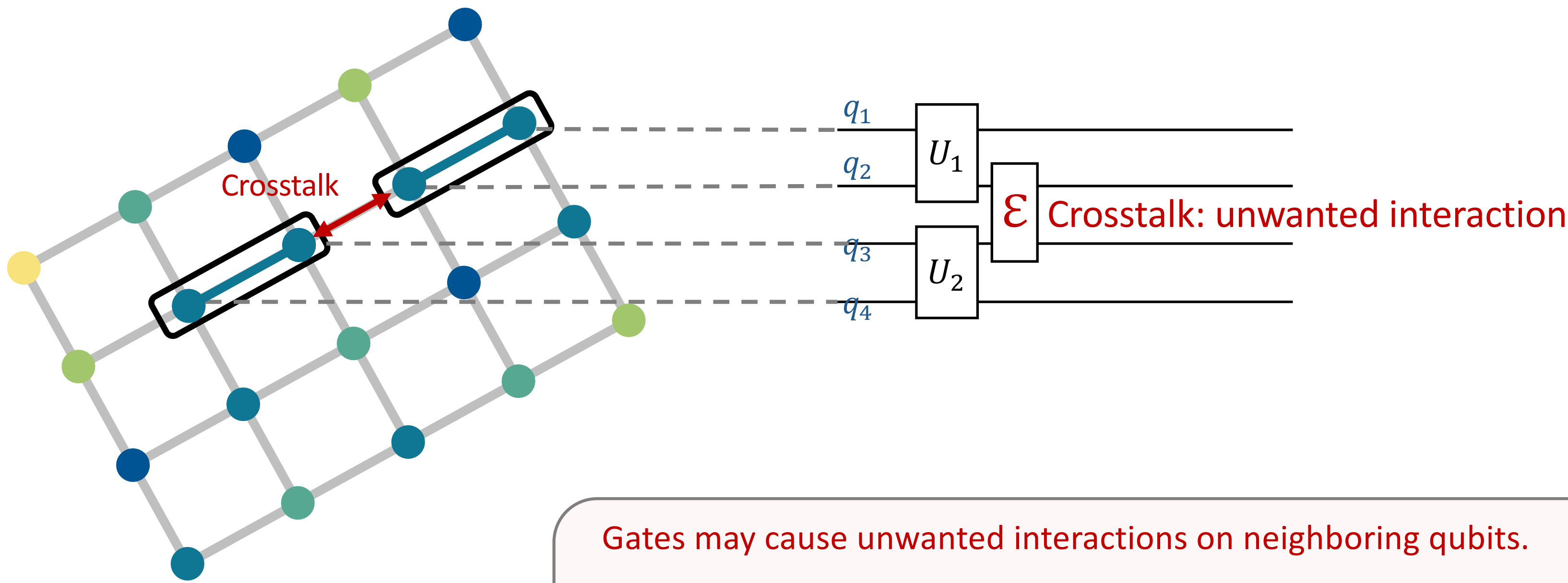
(Avoid Noisy Operations)

Motivation: Some qubits are noisier than others.





Individually Addressed Gates

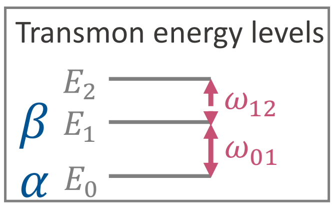


Gates may cause unwanted interactions on neighboring qubits.

Ideal: Each gate is perfectly isolated.
Actual: Run U_1 and U_2 in parallel increases gate error rates!

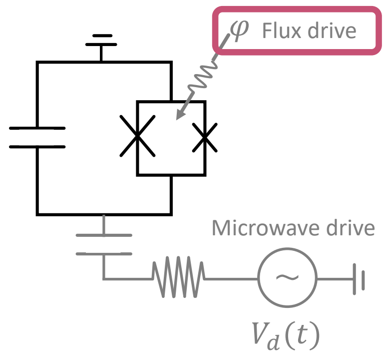
Tunable Superconducting Qubit 101

Qubit State



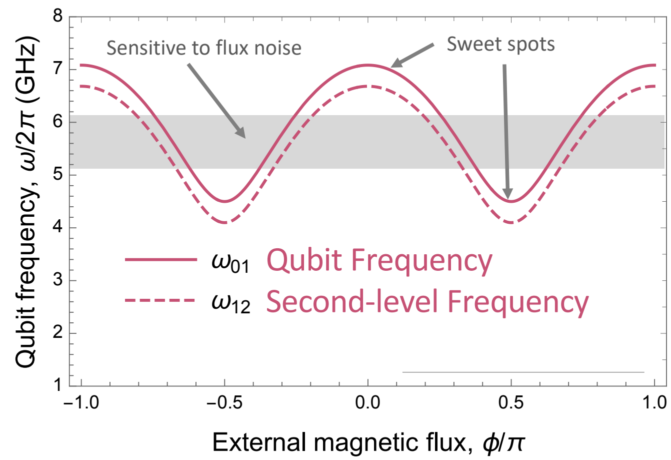
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Qubit Implementation



Frequency-tunable
Transmon Qubit

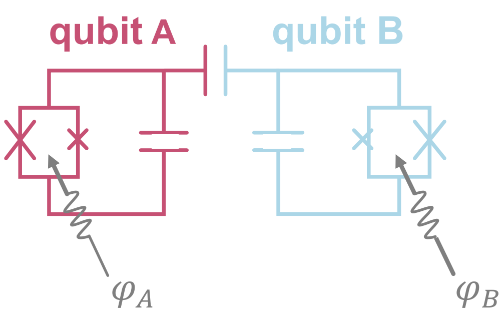
Frequency Tuning



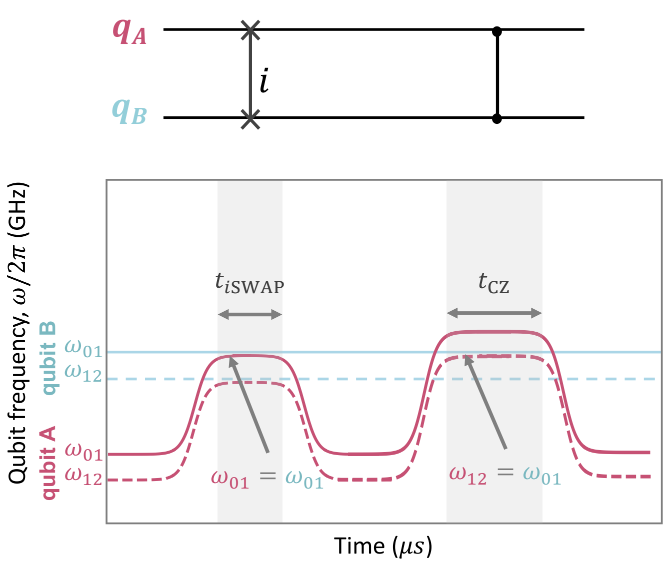
Software control parameters: $\alpha, \beta, (\omega_{01}, \omega_{12})$

Two-Qubit Interactions via Resonance

Interaction through resonance.

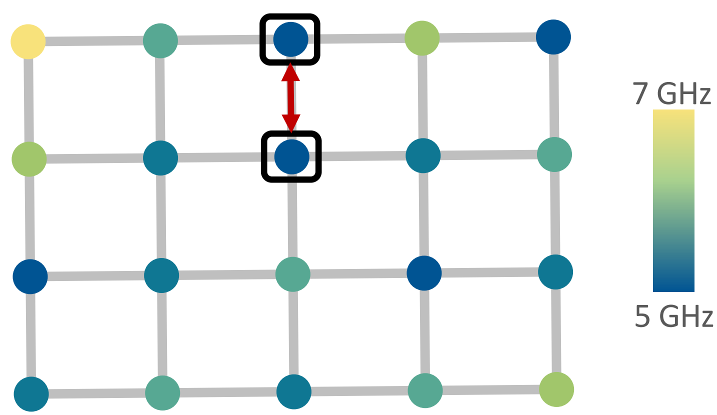


Two capacitively coupled qubits

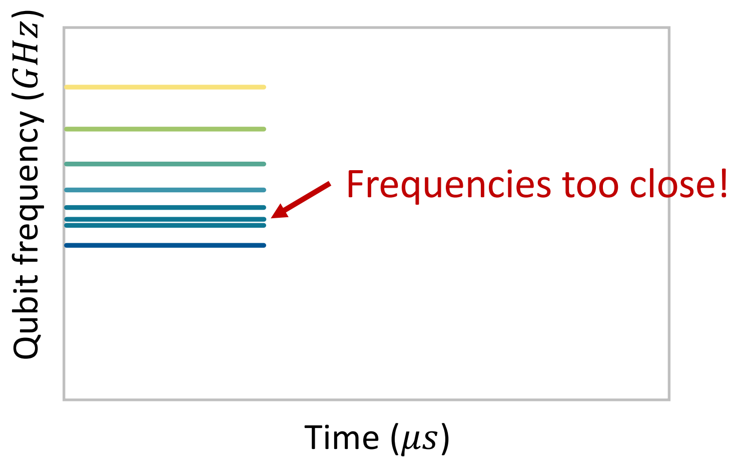


Source of Error – Frequency Crowding

20-Qubit Quantum Processor:



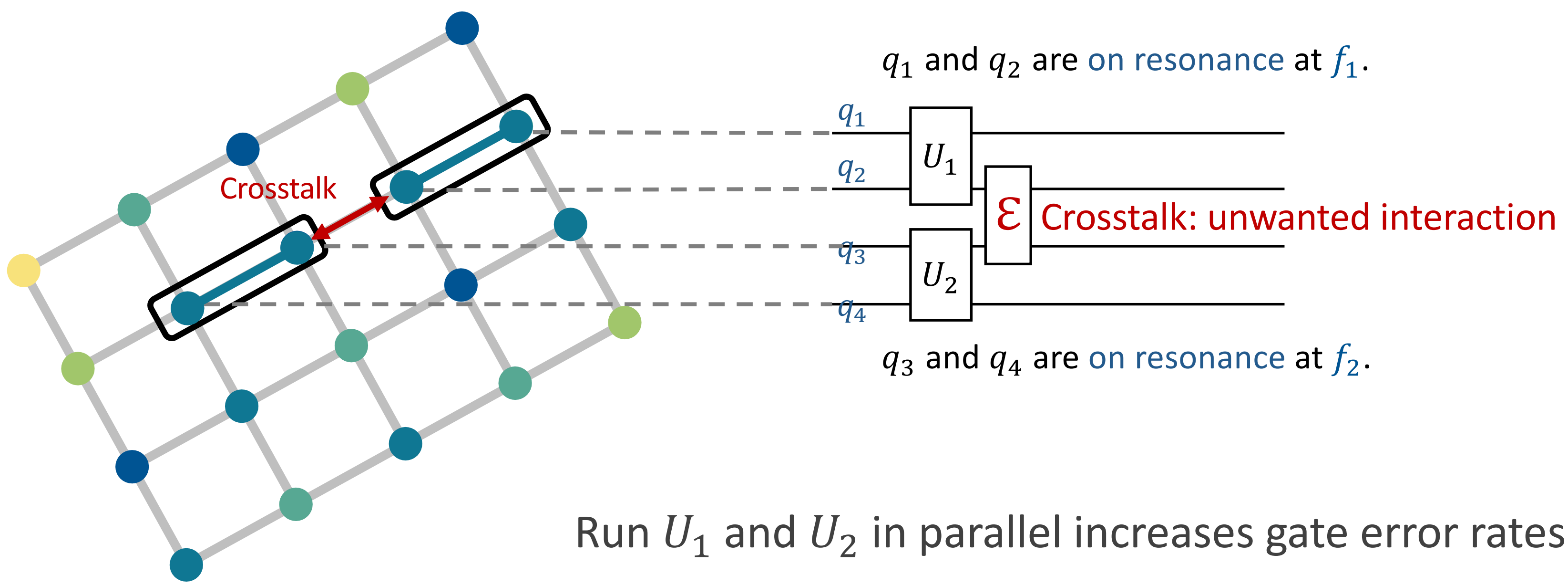
Initial Frequency of Each Qubit



Accidental qubit interactions due to frequency crowding.

Source of Error – Parallel Quantum Gates

q_1 and q_2 are on resonance at f_1.



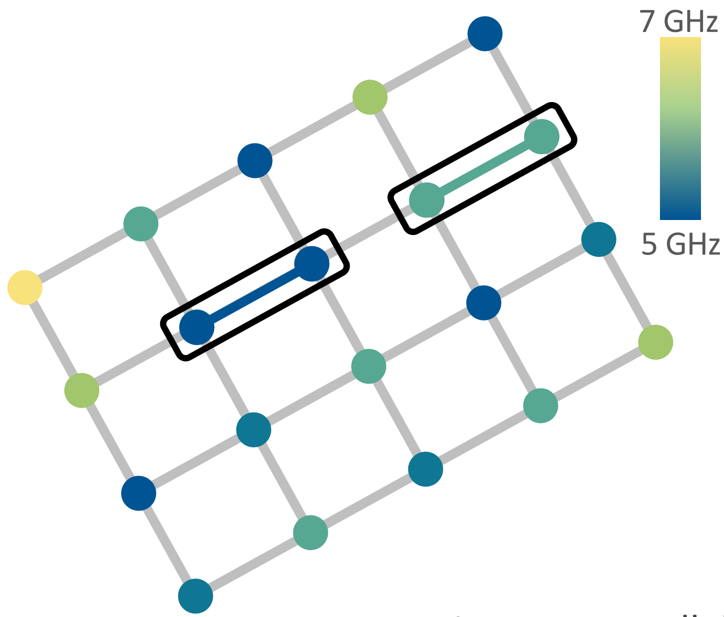
q_3 and q_4 are on resonance at f_2.

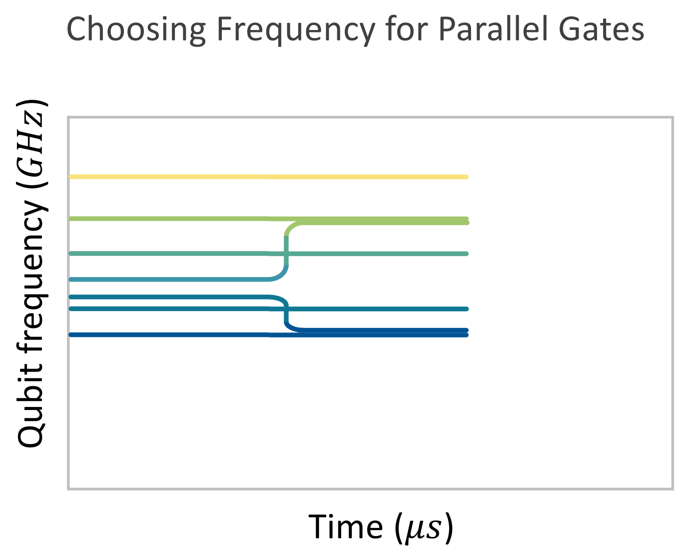
Run U_1 and U_2 in parallel increases gate error rates!

Technique 4: Software-tuned gates

(Avoid Noisy Operations)

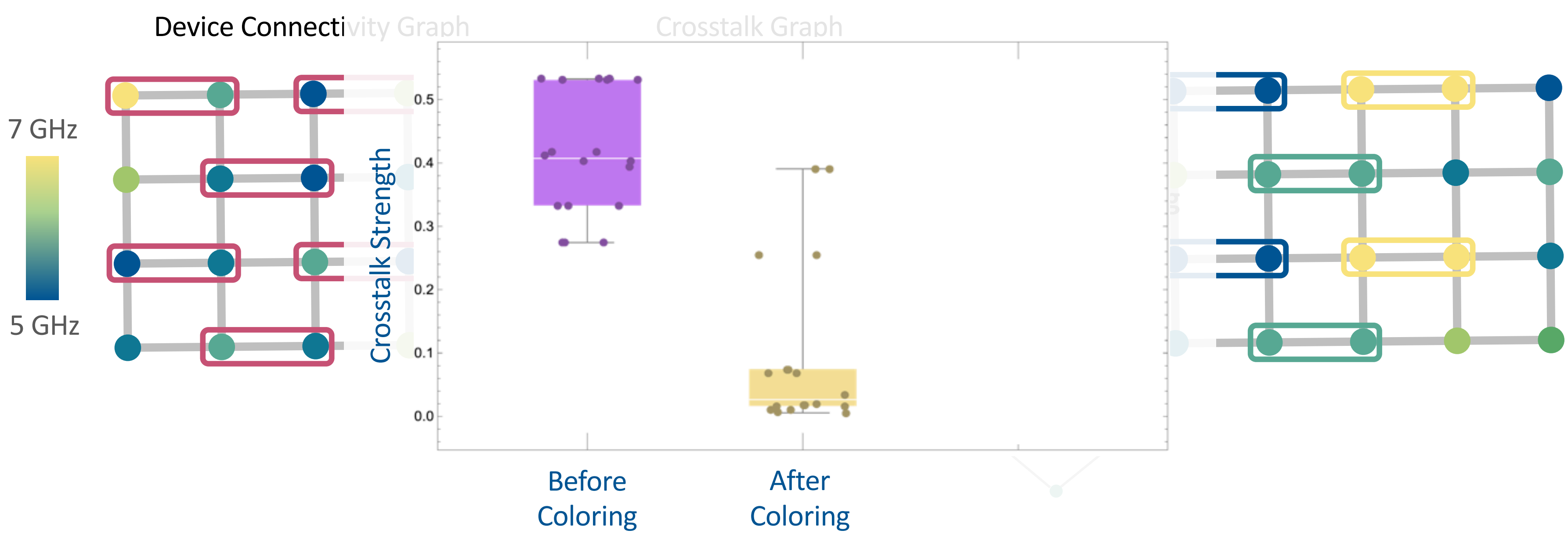
Motivation: Crosstalk can be avoided by changing frequencies of qubits.



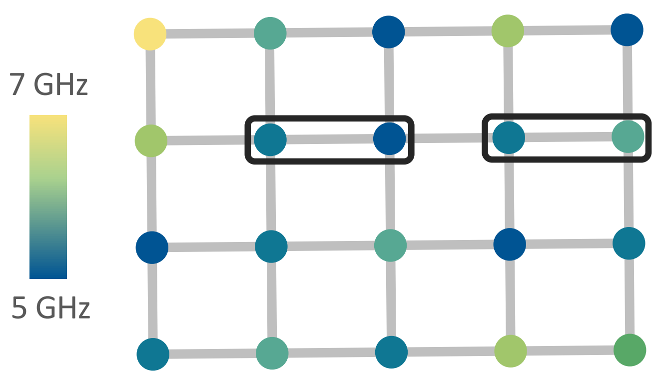


Improve parallel gates by allowing software to control qubit frequencies.

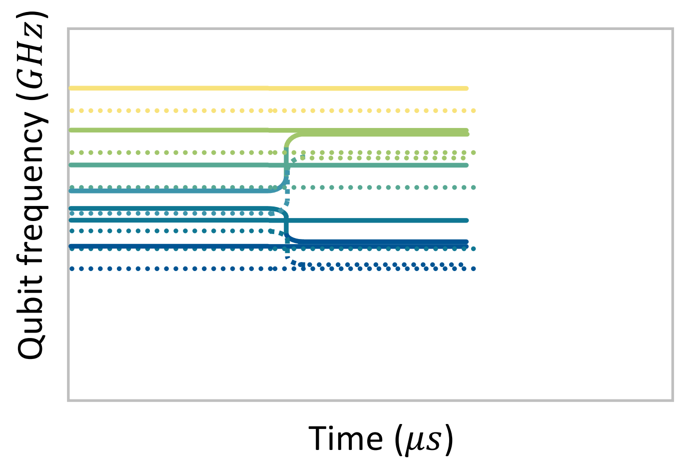
Avoiding Crosstalk by Graph Coloring



Pushing Further to Avoid Leakage

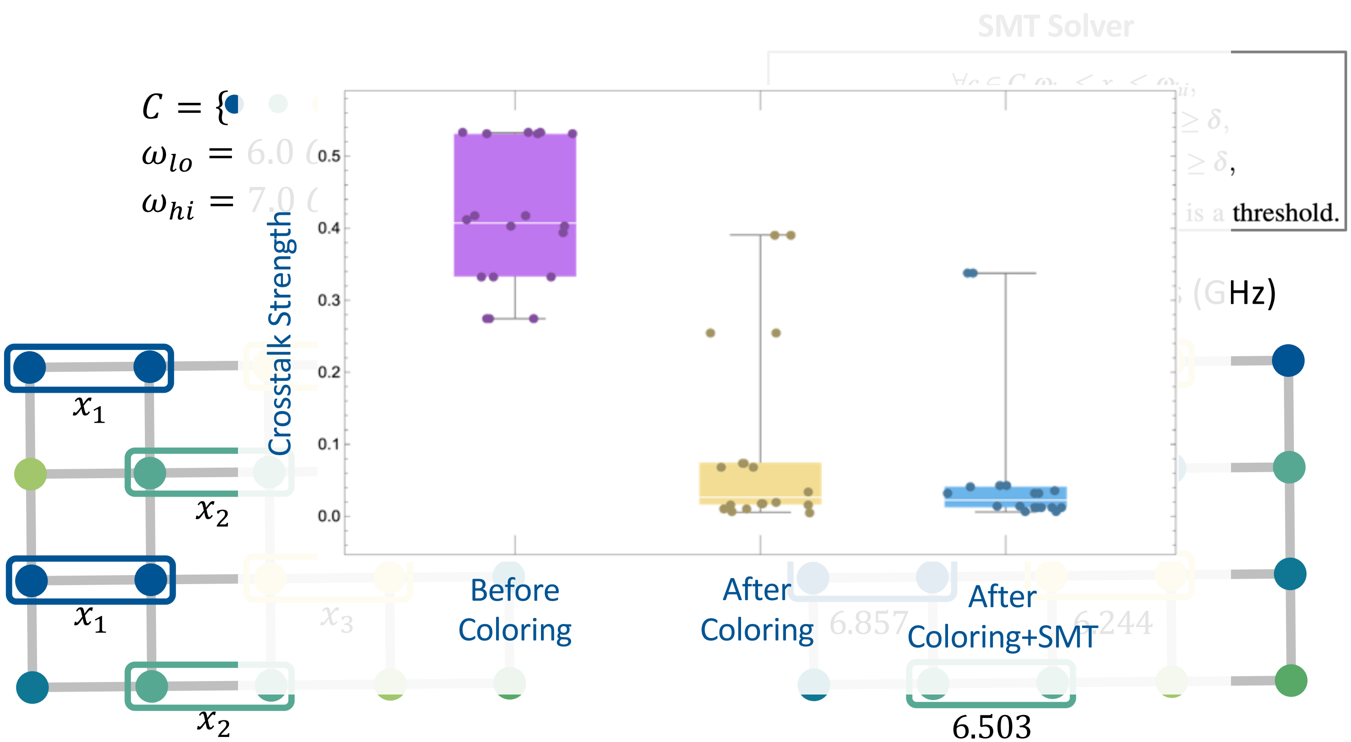


Avoiding resonance with second-level frequencies.

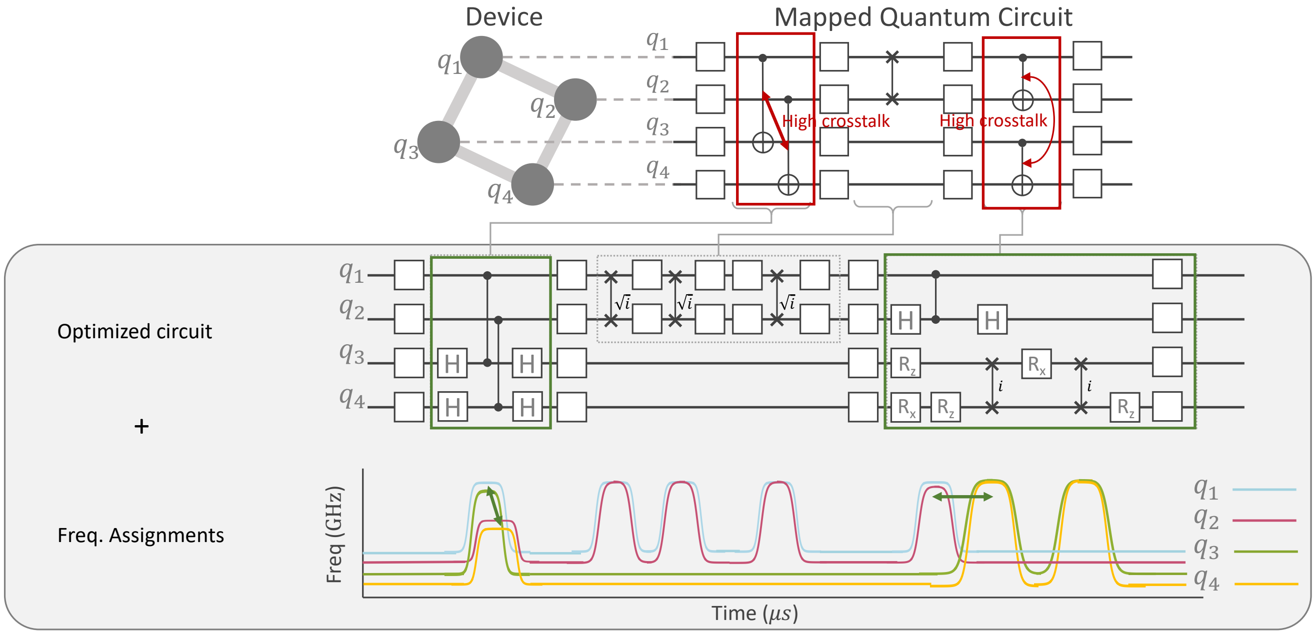


Leakage error happens when second-level frequencies are on resonance.

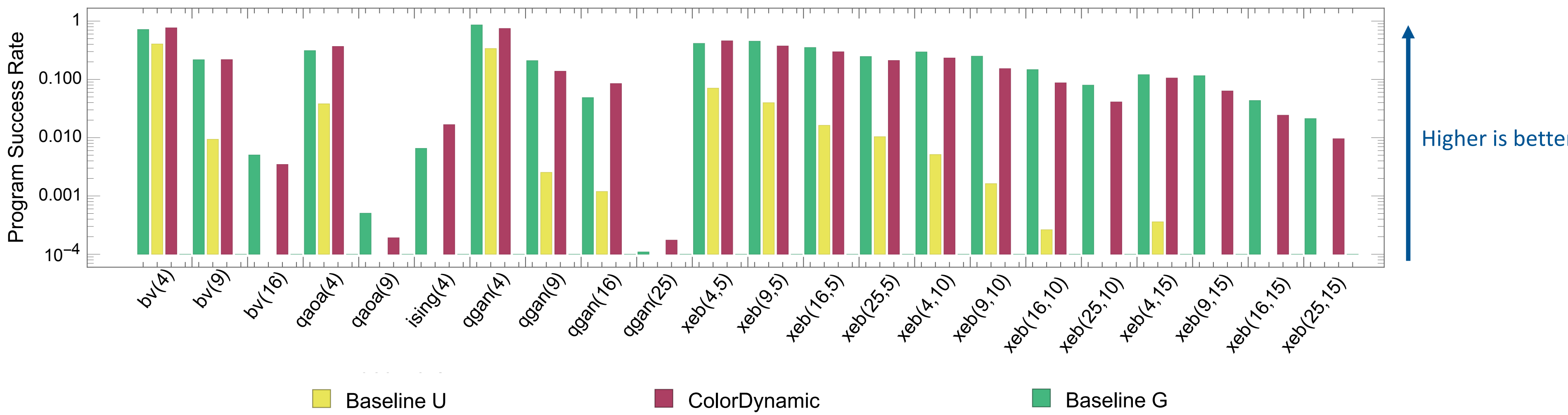
Constraint Satisfaction Problem



Circuit-Frequency Co-Optimization



Results: Program Success Rate



Architecture:	Fixed Qubit Fixed Coupler	→ 13X	Tunable Qubit Fixed Coupler	≈	Tunable Qubit Tunable Coupler
	IBM Q		OUR WORK		GOOGLE SYCAMORE
Crosstalk Mitigation:	Scheduling		Tuning qubit frequencies		Tuning qubit+coupler frequencies

Key Ideas in Noise Mitigation

A Quantum Engineer's Survival Kit

(Software side: compile/run time)

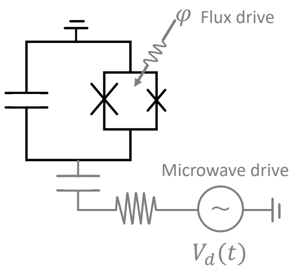
Cancel Error Coherently | Avoid Noisy Operations | **Synthesize Robust Operations** | Engineering Errors | Classical Post-Processing

- Quantum optimal control [Khaneja et. al. J. Magn. Reson. 172, 296]
- Noise-aware variational ansatz [Wang et. al. Arxiv: 2107.10845, Ravi et. al. Arxiv: 2112.05821]

Technique 5: Quantum optimal control

(Synthesize Robust Operations)

Motivation: Finding for the best pulse operation is an optimization problem.



Frequency-tunable Transmon Qubit

External control parameters for the quantum system: $\boldsymbol{\theta}(t)$

Initial quantum state $\xrightarrow{\text{Schrödinger Equation}}$ Final quantum state

$|\psi_0\rangle$ $\qquad$ $|\psi(t)\rangle$

$$ih\frac{d}{dt}|\psi(t)\rangle = H(\boldsymbol{\theta}(t), t)|\psi(t)\rangle$$

Hamiltonian $H(\boldsymbol{\theta}(t), t)$ describes the quantum systems.

QOC: Optimization problem with objective function $f(\theta(t))$ and constraints.

References:
QOC [Khaneja et. al. J. Magn. Reson. 172, 296]

Technique 5: Quantum optimal control

(Synthesize Robust Operations)

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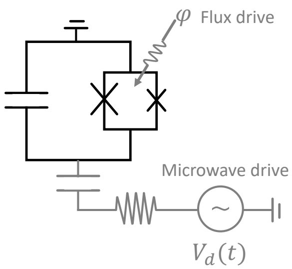


Frequency-tunable Transmon Qubit

Minimize objective function $f\big(\boldsymbol{\theta}(t)\big) = 1 - |\langle\psi_{ideal}|\psi(T)\rangle|^2$ with some constraints.

1. Discretize $t_k = t_{k-1} + \Delta t$, for $k = 1, \ldots, N$

   At k^{th} step, use constant control: $\boldsymbol{\theta}(t_k)$, state: $|\psi(t_k)\rangle$

2. Compute derivative of the objective function $\nabla f(\boldsymbol{\theta})$

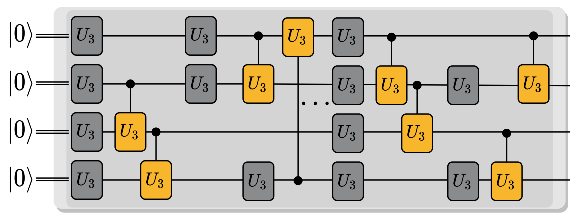
3. Gradient descent search

References:
QOC [Khaneja et. al. J. Magn. Reson. 172, 296]

Technique 6: Variational Ansatz

(Synthesize Robust Operations)

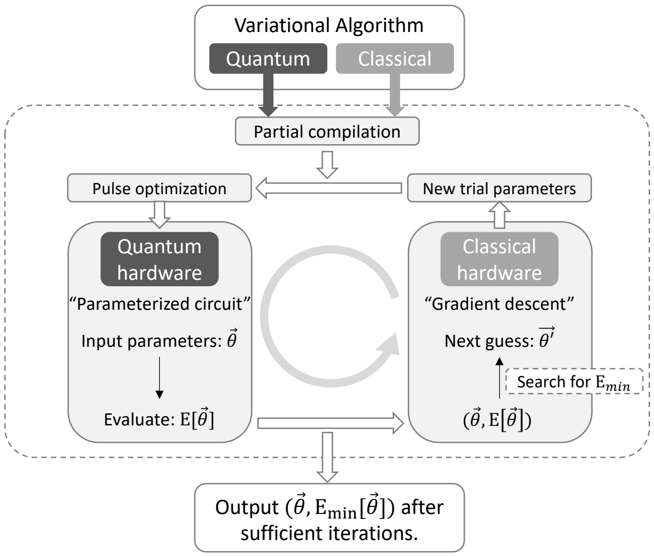
Motivation: Learning a robust quantum circuit with trial-and-error.



Parametrized quantum circuit

Prepares $|\psi(\boldsymbol{\theta})\rangle$

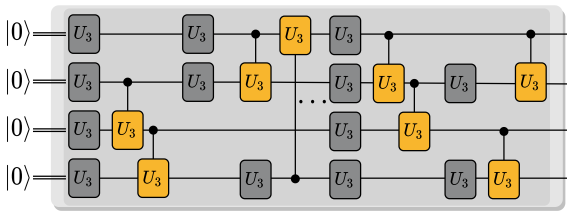
Circuit/Unitary $U(\boldsymbol{\theta})$, parametrized by $\boldsymbol{\theta}$

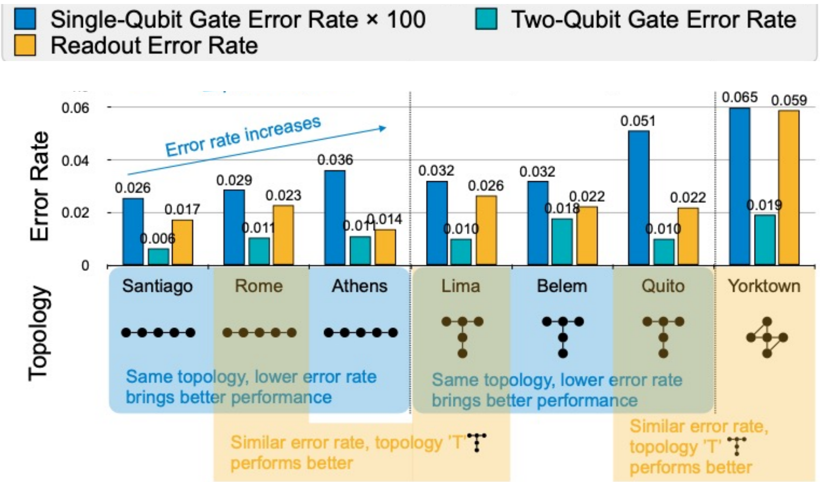


Variational approach: $\langle\psi|H|\psi\rangle \geq E_{min}$ for any trial ansatz $|\psi\rangle$.

References:
VQE [Peruzzo et. al. Arxiv: 1304.3061]
Partial compilation [Gokhale et. al. Arxiv: 1909.07522]
QuantumNAS [Wang et. al. Arxiv: 2107.10845]

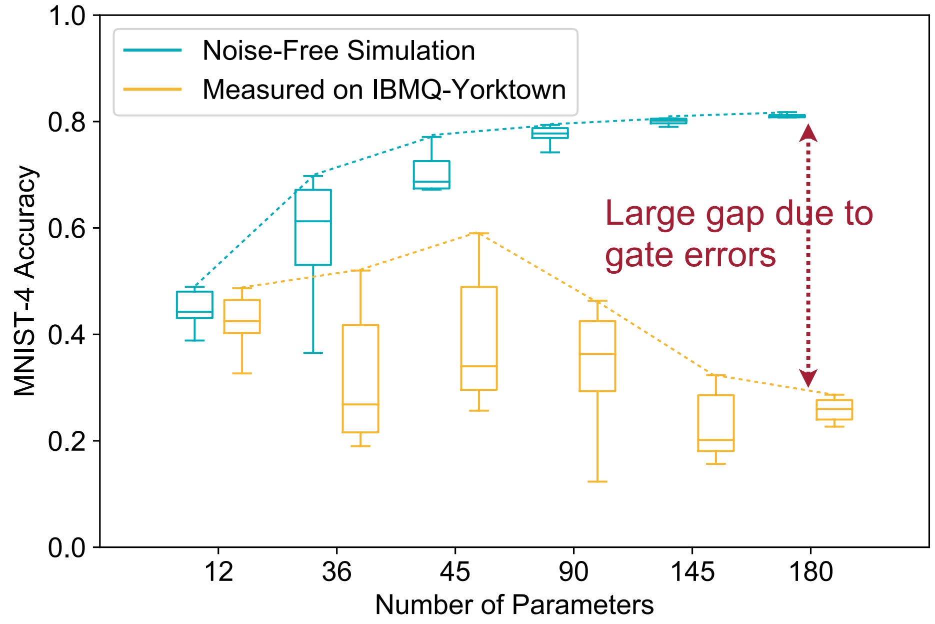
Parametrized Quantum Circuits on Noisy Hardware





Ansatz that works the best in simulation may not work in practice.

Variational ansatz in the presence of noise

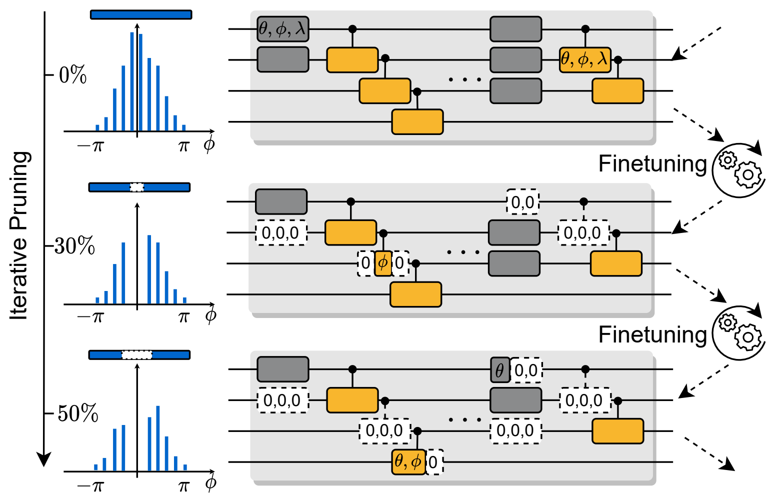


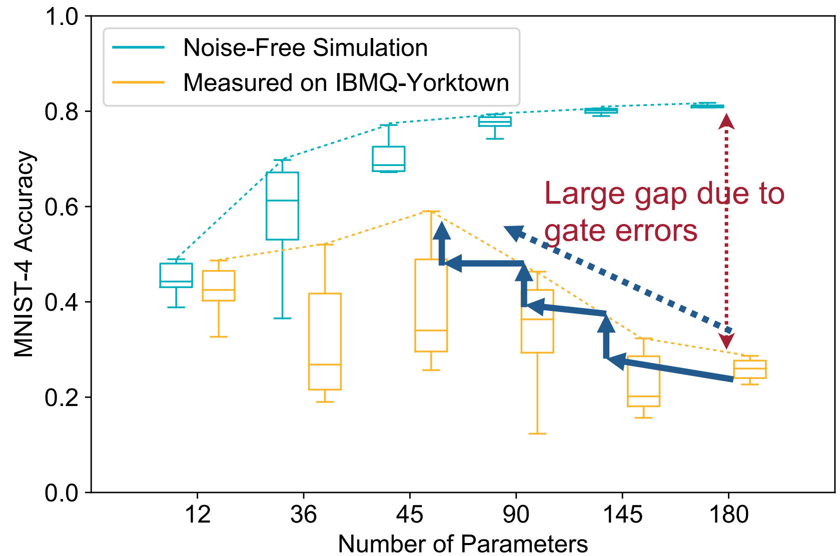
Choosing the Right Ansatz Model

How to get to peak performance?

Iterative Gate Pruning

(Strategically compress the circuit)





Searching And Training for A Best-Performing PQC

Idea: Rank the performance of ansatz circuits:

A naïve approach:

```
For devices:
   For search episodes:
      For circuit training iterations:
         update_parameters();
      If good_circuit: break;
```

Too expensive!

An Efficient Ansatz Search Approach

Idea: Rank the performance of ansatz circuits:

A naïve approach:

```
For devices:
   For search episodes:
      For circuit training iterations:
         update_parameters();
      If good_circuit: break;
```

Too expensive!

A SuperCircuit Approach:

Construct a SuperCircuit to estimate ranking of its SubCircuits:

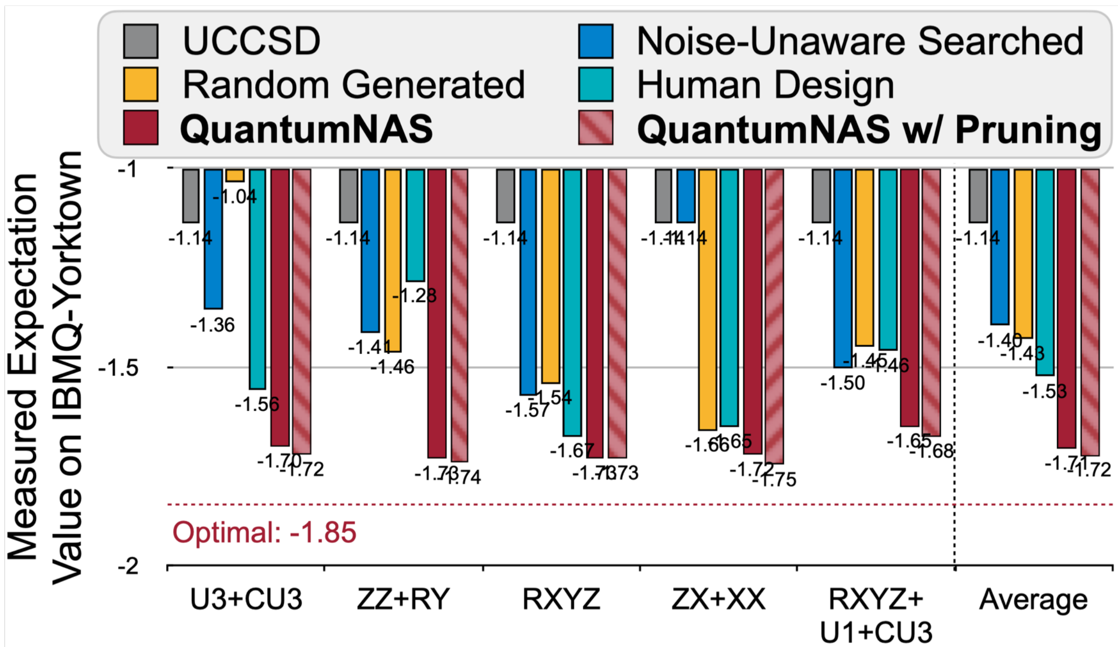
```
For SuperCircuit training iterations:
   update_parameters();
```

decouple search from training

```
For q_devices:
   For search episodes:
      sample from SuperCircuit;
      If good_circuit: break;
```

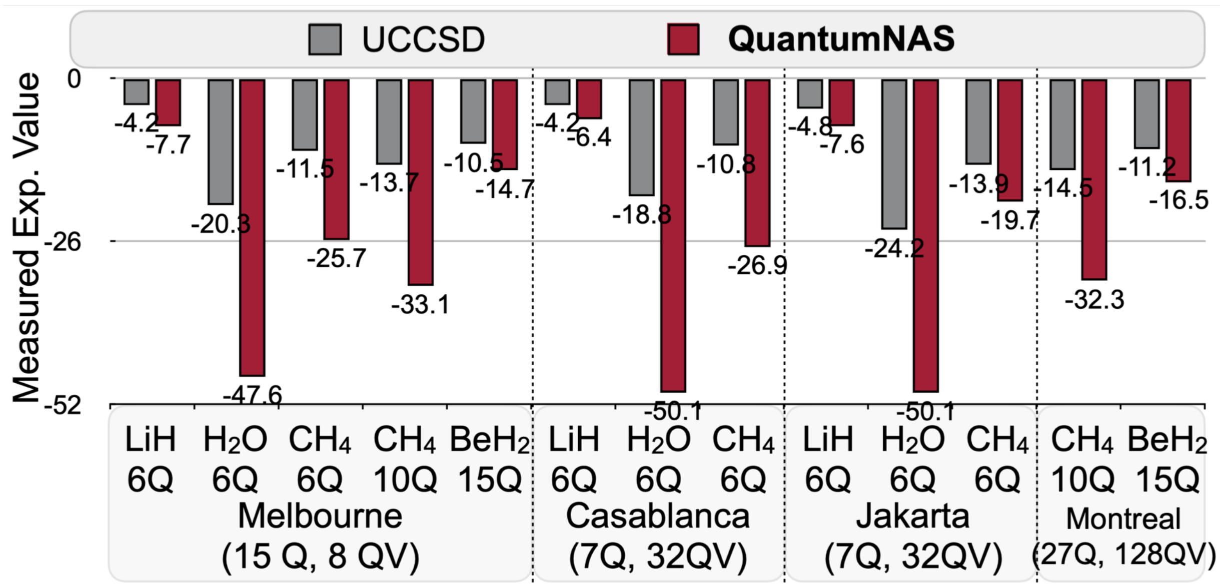
VQE Results

- H2 in different design spaces on IBMQ-Yorktown



VQE Results

- VQE on different devices



Key Ideas in Noise Mitigation

A Quantum Engineer's Survival Kit

(Software side: compile/run time)

Cancel Error Coherently	Avoid Noisy Operations	Synthesize Robust Operations	**Engineering Errors**	Classical Post-Processing

- Bias-noise qubits and bias-preserving gates [Puri et. al. Arxiv: 1905.00450]
- Randomized compiling [Wallman et. al. Arxiv: 1512.01098]

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- Zero-Noise Extrapolation [Temme et. al. Arxiv: 1612.02058]
- Probabilistic Error Cancellation [Endo et. al. Arxiv: 1712.09271]

Summary - Quantum Engineer's Survival Kit

Gate-Level Error Cancellation:
- Dynamical Decoupling [Viola et. al. Arxiv: 9803057]
- Composite Pulses [Brown et. al. Arxiv: 0407022]

Circuit-Level Optimization:
- Qubit Mapping and Gate Scheduling [Murali et. al. Arxiv: 1901.11054, Wu et. al. Arxiv: 2010.15876]

Pulse-Level Shaping:
- Quantum Optimal Control [Werschnik et. al. Arxiv: 0707.1883]
- Quantum Signal Processing [Low et. al. Arxiv: 1606.02685]

Error Engineering:
- Bias-Preserving Gates [Puri et. al. Arxiv: 1905.00450]
- Randomized Compiling [Wallman et. al. Arxiv: 1512.01098]

Learning-Based:
- Variational Ansatz [Peruzzo et. al. Arxiv: 1304.3061, Gokhale et. al. Arxiv: 1909.07522]
- Parametrized Quantum Circuits [Ravi et. al. Arxiv: 2112.05821, Wang et. al. Arxiv: 2107.10845, 2110.11331]

Data-Processing:
- Zero-Noise Extrapolation [Temme et. al. Arxiv: 1612.02058]
- Probabilistic Error Cancellation [Endo et. al. Arxiv: 1712.09271]

Thank You!