Exam Solution

Course: AE4870B Re-Entry Systems
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1 Question 1: True-False

a The total energy of a vehicle returning from space is equally divided over potential and kinetic energy. **FALSE.**

b Hazard detection is also possible without knowing the DEM of the surface. **TRUE. See 9-8-2 of the reader.**

c For landing on Mars, main parachutes trailing at least five meters behind the descent module are not exposed to its wake. **FALSE?**

d For entry from Low Earth Orbit, a re-entry vehicle is not exposed to rarefied gas flow. **FALSE.**

e The gravity turn cannot be used for a precision landing. **TRUE.**

f An inherently safe landing region does not contain any slopes. **FALSE.**

g In the altitude-velocity plane, the heat-flux constraint moves downwards with increasing flux value. **TRUE?**

h An Inertial Measurement Unit operates well during the so-called "black-out" phase. **TRUE.**

i For Lunar entry, on average thicker heat shields than for Mars are required. **FALSE.**

j Active sensors for hazard detection are superior to passive sensors in every aspect. **FALSE.**

2 Question 2

2.1 2a

To facilitate a precision landing in an otherwise safe environment the addition of multiple advanced systems to the standard GNC system is necessary. Describe (in detail) at least three systems that are either completely new or need con-siderable changes with respect to the standard GNC system.

The three systems are Terrain Absolute Navigation (TAN), Terrain Relative Navigation (TRN) and Hazard Detection and Avoidance (HDA).

A vehicle equipped with **Terrain Absolute Navigation** has a pre-determined map of the surface of the body on board. For example, if this body is the moon, the vehicle has a map of the lunar surface. TAN then uses sensors that can detect features on the surface, and then compare these with the on-board map to determine the location. Sensors may be optical, but radar or LIDAR sensors can also be used. The inputs for TAN are therefore the data from the surface sensors and the data from the on-board map as inputs, and can produce altitude, position, relative speed, and heading as outputs.

Alternatively, if no detailed map of the surface is available, **Terrain Relative Navigation** can be used. This method uses similar sensors to look at the surface below, and take a image. However, it then compares this image to the previously taken image. By using algorithms that can detect features of the landscape, it can determine speeds and headings relative to the surface. This can also be seen in the figure.

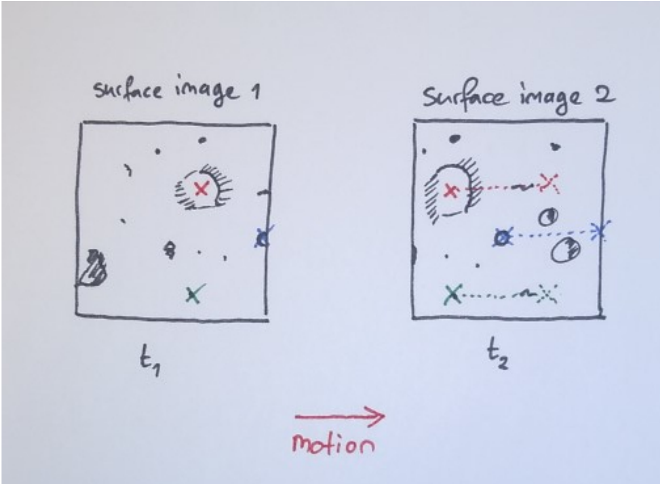

Figure 1: The principle behind TRN

Lastly, **Hazard Detection and Avoidance** is a more advanced system to ensure a safe landing and provide the lander with a higher level of autonomy. This system includes sensory equipment that can accurately provide a 3D map of the prospected landing area. Usually LIDAR is used, but radio doppler sensors work also. Using computer algorithms, potential hazards can be identified, and poor landing zones can be rejected in favour of better suited ones. The outputs of the system can therefore be, besides velocity and relative position, coordinates for better landing locations.

2.2 2b

Draw the standard GNC flowchart and add the systems described in (a) to the chart. Clearly indicate which blocks are from the standard system and which are only needed for a precision landing.
See lecture notes section 9-9.

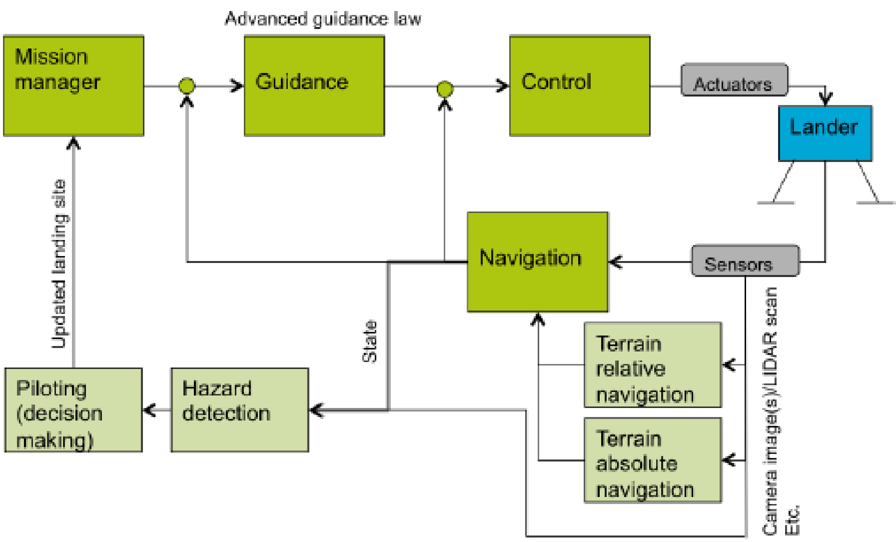

Figure 2: A GNC system with TAN, TRN, and HDA

2.3 2c

The topic of E-guidance is no longer part of the study material for this course, and is therefore omitted from this guide.

3 Question 3

3.1 3a

Given radiation equilibrium for a laminar boundary layer, derive an expression for the wall temperature. Give explicit formulations for the fluxes involved.
In the case of equilibrium, the radiative heat flux q_{rad} and the conductive heat flux q_c are in balance. So let's first find expressions for these quantities, and then solve for the wall temperature.
First, the radiative heat flux is defined using:

$$q_{rad} = \epsilon\sigma T_{w,eq}^4 \tag{1}$$

Where ϵ is the emissivity and σ the Boltzmann constant.

Next, the conductive heat flux can be found using:

$$q_s = \alpha(T_{w,ad} - T_w) \tag{2}$$

This is where we make our first assumption: We assume that the adiabatic wall temperature is far larger than T_w, and furthermore that it can be approximated by:

$$T_{w,ad} \approx \frac{V_\infty}{2C_{p_\infty}} \tag{3}$$

If we combine this with equation 2, we now get:

$$q_c = \alpha\frac{V_\infty}{2c_{p_\infty}} \tag{4}$$

Now, we further expand α:

$$\alpha = St \cdot c_p \cdot \rho \cdot V \tag{5}$$

So we obtain:

$$q_c = Stc_p\rho V\frac{V_\infty}{2c_{p_\infty}} \tag{6}$$

At this point, we can recognise that the question is about an equilibrium situation and we can equate the two fluxes:

$$\frac{1}{2}St\rho_\infty V_\infty^3 = \epsilon\sigma T_{w,eq}^4 \tag{7}$$

And solve for $T_{w,eq}$:

$$T_{w,eq} = \sqrt[4]{\frac{St\rho_\infty V_\infty^3}{2\epsilon\sigma}} \tag{8}$$

3.2 3b

Starting with the hydrostatic equation, derive the exponential atmosphere model for the density. State your assumptions.
Start with the hydrostatic equation:

$$dp = -\rho g dh \tag{9}$$

Assume the atmosphere is made of a perfect gas so we can use the perfect gas law:

$$p = \rho RT \tag{10}$$

Now divide the left side of the hydrostatic equation by the left side of the perfect gas law. Do the same for the right sides, so you get:

$$\frac{dp}{p} = -\frac{g}{RT}dh \tag{11}$$

Now assume two things:

- Constant T in the atmosphere.
- Ignore variation in the gravitational acceleration: $g = g_0$

This enables us to formulate the previous equation as:

$$\frac{dp}{p} = \frac{d\rho}{\rho} = -\frac{g_0}{RT}dh \tag{12}$$

This we can integrate to find an expression for the density as function of altitude:

$$\begin{aligned} \int \rho_0 \rho \frac{d\rho*}{\rho*} &= -\frac{g_0}{RT} \int h_0 h dh* \\ ln\left(\frac{\rho}{\rho_0}\right) &= -\frac{g_0}{RT}(h - h_0) \quad with \quad h_0 = 0 \\ ln\left(\frac{\rho}{\rho_0}\right) &= -\frac{g_0}{RT} h \\ \frac{\rho}{\rho_0} &= e^{-\frac{g_0}{RT}h} = e^{-\beta h} \end{aligned} \tag{13}$$

Where parameter β is defined as:

$$\beta = \frac{g_0}{RT} \tag{14}$$

Solving for ρ gives us the answer to the question:

$$\rho = \rho_0 e^{-\frac{g_0}{RT}h} = \rho_0 e^{-\beta h} \tag{15}$$

3.3 3c

Provide a clear sketch of a step response, and indicate in this figure the delay time, td, rise time, tr, peak time, tp, maximum percentage overshoot, Mp, and settling time, ts

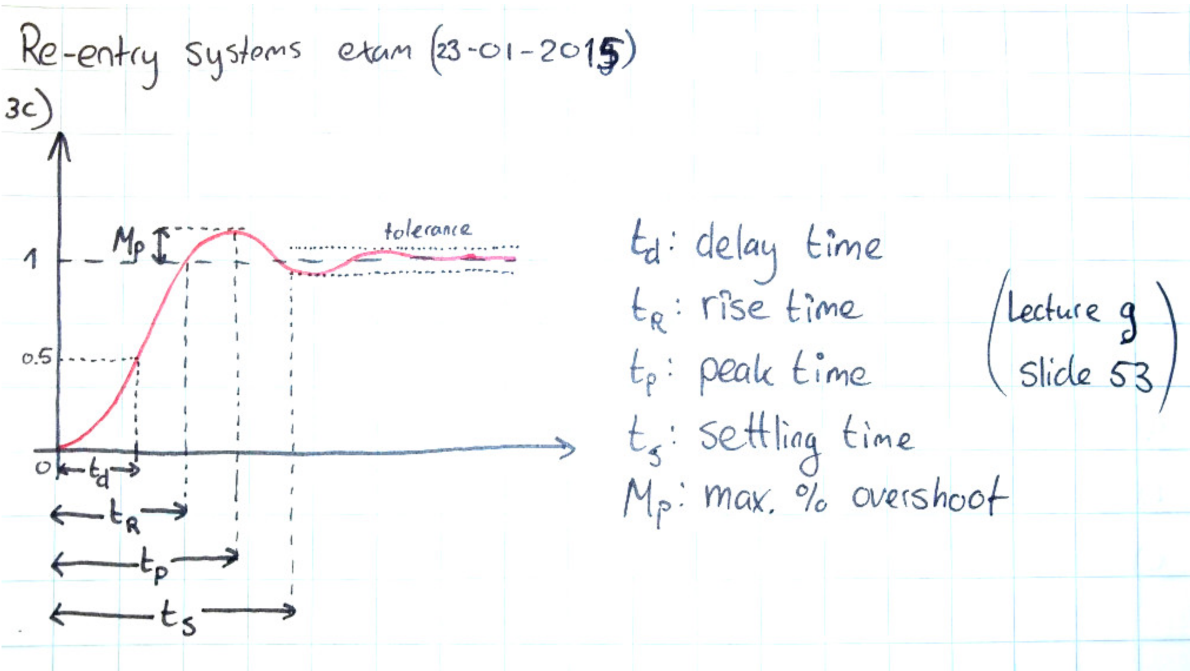

Figure 3: A beautiful Renaissance painting with the features of a step function

3.4 3d

Provide a clear drawing of a parachute/payload system and indicate the following elements: cover, deployment bag, pilot chute, bridle, canopy, main parachute, suspension lines, riser, payload.

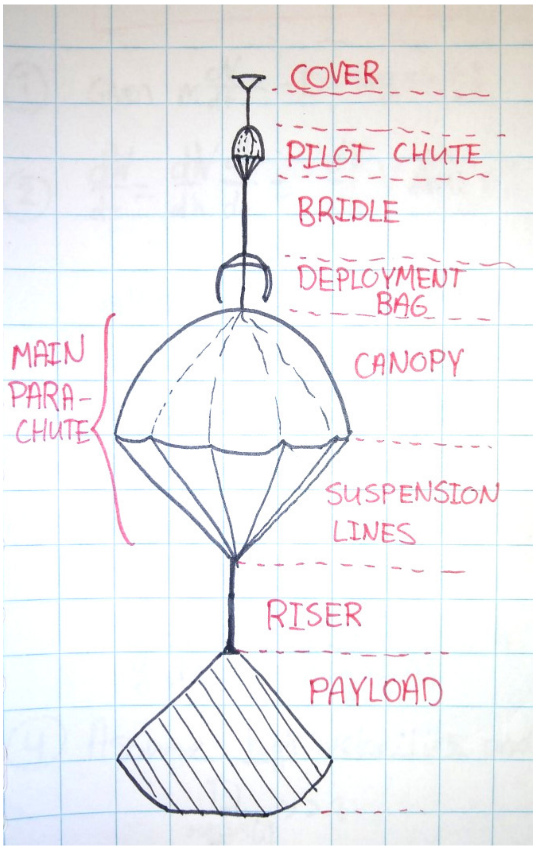

Figure 4: The anatomy of a parachute

4 Question 4

4.1 4a

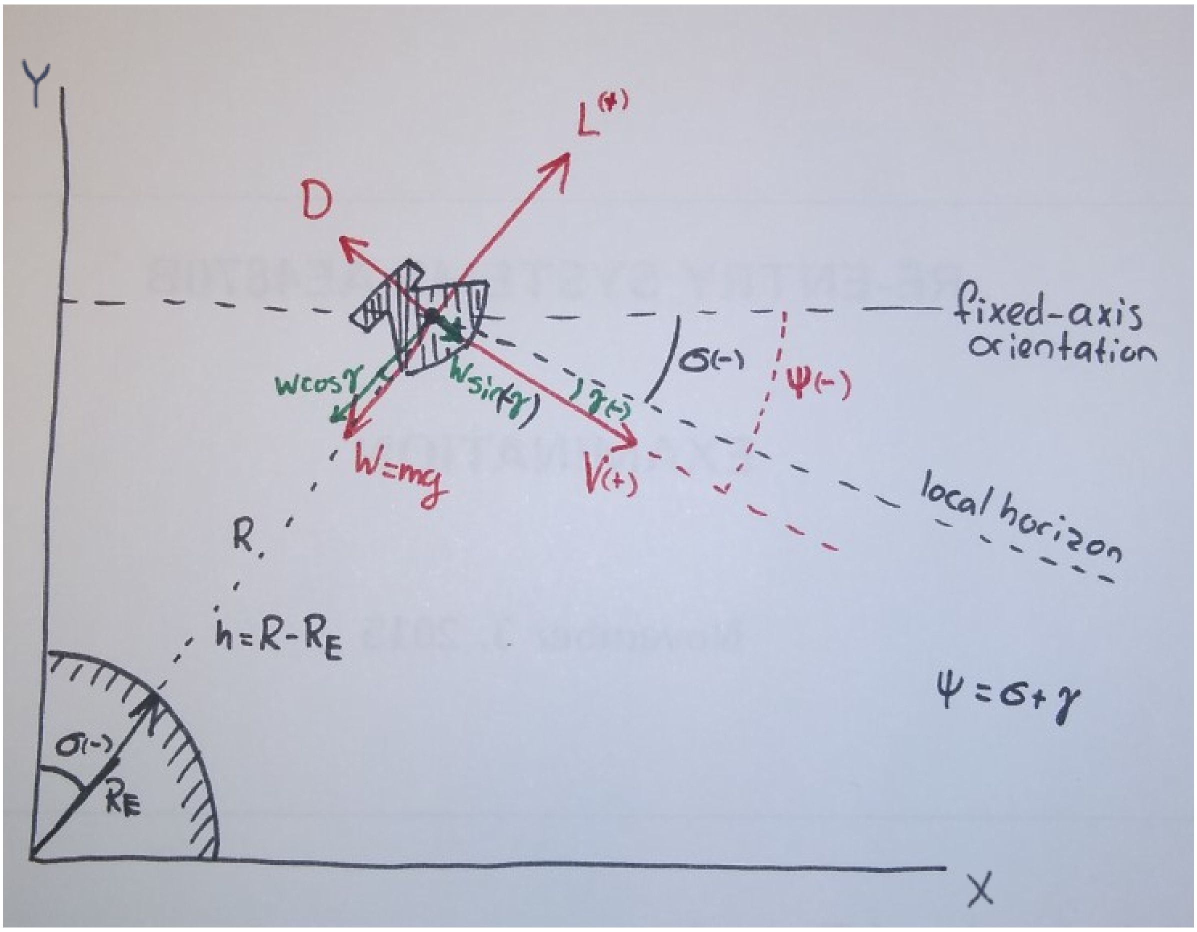

Figure 5: Framework from which to derive equations of motion

4.2 4b

We start with linear momentum parallel to velocity V. We can then find using Newton's second law that:

$$m\frac{dV}{dt} = -D - mgsin(\gamma) \tag{16}$$

This is the first of the equations of motion that we need to find. The second one we can find by looking at how the length of R changes:

$$\frac{dh}{dt} = Vsin(\gamma) \tag{17}$$

Finally, we look at the normal acceleration, from which we obtain that:

$$mV\frac{d\psi}{dt} = L - mgcos(\gamma) \tag{18}$$

We would like to have things in terms of γ rather than ψ, so first we need to transform this. First, we can see from the figure that there is a direct relation between these two angles.

$$\psi = \sigma + \gamma \tag{19}$$

and therefore:

$$\frac{d\psi}{dt} = \frac{d\sigma}{dt} + \frac{d\gamma}{dt} \tag{20}$$

Great, but now we also need an expression for $\frac{d\sigma}{dt}$. Due to the fact that position vector R is describing an instantaneous circular motion, we can relate the tangential velocity of this motion with the projection of V onto the local horizon, as they are equal:

$$R\frac{d\sigma}{dt} = -Vcos(\gamma) \tag{21}$$

If we combine this with equation 20, we find:

$$\frac{d\psi}{dt} = -\frac{V}{R}cos(\gamma) + \frac{d\gamma}{dt} \tag{22}$$

This we can now substitute into 18:

$$\begin{aligned}
mV\left(-\frac{V}{R}cos(\gamma) + \frac{d\gamma}{dt}\right) &= L - mgcos(\gamma) \\
mV\frac{d\gamma}{dt} &= L - mgcos(\gamma) + m\frac{V^2}{R}cos(\gamma) \\
mV\frac{d\gamma}{dt} &= L - mgcos(\gamma) \cdot \left(1 - \frac{V^2}{gR}\right) \\
mV\frac{d\gamma}{dt} &= L - mgcos(\gamma) \cdot \left(1 - \frac{V^2}{V_c^2}\right)
\end{aligned} \tag{23}$$

where $V_c = \sqrt{gR}$. This is our third equation of motion.

4.3 4c

Our starting point is the given expression for V/V_E:

$$\frac{V}{V_E} = exp\left(\frac{g\rho}{2K\beta sin(\gamma_E)}\right) \tag{24}$$

and the first equation of motion:

$$m\frac{dV}{dt} = -D - mgsin(\gamma) \tag{25}$$

First, we define the deceleration $\bar{a}$ as:

$$\bar{a} = -\frac{dV}{dt} \tag{26}$$

Furthermore, we assume that since re-entry motions generally occur at high velocities, the drag forces far exceed the weight components of the vehicle (i.e. $D \gg W$). This allows us to ignore the weight-related term of equation 25. Now we do some recreational algebra to find an expression for the deceleration.

$$\begin{aligned}
m\bar{a} = m - \frac{dV}{dt} &= +D - mgsin(\gamma) = D \\
\bar{a} = \frac{D}{m} &= \frac{C_D\rho SV^2}{2m}\frac{g}{g} = \frac{\rho V^2 g}{2K}
\end{aligned} \tag{27}$$

Where we introduced the weight-referenced ballistic coefficient, defined as:

$$K = \frac{mg}{C_D S} \tag{28}$$

The relation we have found so far has a term for density, so it is helpful to unpack this variable to see how it influences the deceleration. But we can solve equation 24 for ρ and put that in 27.

$$\begin{aligned}
\frac{V}{V_E} &= exp\left(\frac{g\rho}{2K\beta sin(\gamma_E)}\right) \\
ln\left(\frac{V}{V_E}\right) &= \frac{g\rho}{2K\beta sin(\gamma_E)} \\
\rho &= \frac{2K\beta sin(\gamma_E) ln(\frac{V}{V_E})}{g}
\end{aligned} \tag{29}$$

We now substitute this into 27:

$$\bar{a} = \beta sin(\gamma_E)V^2 ln\left(\frac{V}{V_E}\right) \tag{30}$$

We now apply a trick by multiplying and dividing this by a factor V_E^2. We do this because it will be easier to differentiate if we use V/V_E as differentiation variable.

$$\bar{a} = \beta sin(\gamma_E)V_E^2\left(\frac{V}{V_E}\right)^2 ln\left(\frac{V}{V_E}\right) \tag{31}$$

Ultimately, we want to know when the deceleration is maximal. The approach is to differentiate equation 31 and figure out when this is zero. As mentioned, we differentiate with respect to V/V_E:

$$\begin{aligned}\frac{d\bar{a}}{d(V/V_E)} &= 2\beta sin(\gamma_E)V_E^2\frac{V}{V_E}ln\left(\frac{V}{V_E}\right) + \beta sin(\gamma_E)V_E^2\left(\frac{V}{V_E}\right)^2\left(\frac{1}{V/V_E}\right) \\ &= \beta sin(\gamma_E)V_E^2\left[2 + ln\left(\frac{V}{V_E}\right)\right]\frac{V}{V_E} = 0\end{aligned} \tag{32}$$

This is only equal to zero if:

$$\begin{aligned}2ln\left(\frac{V'}{V_E}\right) &= -1 \\ \frac{V'}{V_E} &= e^{-1/2} = \frac{1}{\sqrt{e}}\end{aligned} \tag{33}$$

This means that the maximum deceleration occurs when the velocity of the vehicle is about 61% of the entry velocity. We can substitute this result into 31 to find the corresponding deceleration.

$$\bar{a}_{max} = \beta sin(\gamma_E)V_E^2 \cdot (e^{-1/2})^2 \cdot ln(e^{-1/2}) = -\frac{\beta sin(\gamma_E)}{2e}V_E^2 \tag{34}$$

4.4 4d

Just a matter of plugging the right values into equation 34. Use $\beta = 1/H_s$ to find β.

$$(\bar{a}_{max})_♀ = -\frac{15900^{-1} \cdot sin(-32.4°)}{2e} \cdot 11670^2 = 844.2\, m/s^2 \tag{35}$$

Similarly, we can find the other decelerations:

$$\begin{aligned}(\bar{a}_{max})_\oplus &= 1903.9\, m/s^2 \\ (\bar{a}_{max})_♂ &= 1525.3\, m/s^2\end{aligned} \tag{36}$$

4.5 4e

For this question, we use equation 29 and fill in the velocity ratio for maximal deceleration. This density ρ' is the density of the air when maximum deceleration occurs. Then we can extract the corresponding altitude h' from this density. So we start with:

$$\rho' = \frac{2K\beta sin(\gamma_E)ln(\frac{V}{V_E})_{\bar{a}_{max}}}{g} = \frac{2K\beta sin(\gamma_E)ln(\frac{1}{\sqrt{e}})}{g} = -\frac{K\beta sin(\gamma_E)}{g} \tag{37}$$

Since we also assume that density varies exponentially with altitude, we have:

$$\begin{aligned}\rho' = -\frac{K\beta sin(\gamma_E)}{g} &= \rho_0 e^{-\beta h'} \\ -\beta h' &= ln\left(-\frac{K\beta sin(\gamma_E)}{g\rho_0}\right) \\ h' &= \frac{1}{\beta}ln\left(-\frac{g\rho_0}{K\beta sin(\gamma_E)}\right)\end{aligned} \tag{38}$$

4.6 4f

This result from the previous question can be used to find the three altitudes we need. However, the weight-referenced ballistic coefficient K for the lander on each planet is still unknown, so we have to find those first:

$$K_♀ = \frac{mg}{c_D S} = \frac{8.87 \cdot 315}{1.1 \cdot \pi/4 \cdot 1.4228^2} = 1598\left[\frac{kg}{m \cdot s^2}\right] \tag{39}$$

And for the other planets:

$$\begin{aligned}K_\oplus &= 1767\left[\frac{kg}{m \cdot s^2}\right] \\ K_♂ &= 671.3\left[\frac{kg}{m \cdot s^2}\right]\end{aligned} \tag{40}$$

Now for the corresponding altitudes:

$$
\begin{aligned}
h'_{♀} = 15900 ln \left(- \frac{8.87 \cdot 65}{15900^{-1} 1598 sin(-32.4°)} \right) &= 147.5\, km \\
h'_{\oplus} &= 31.68\, km \\
h'_{♂} &= 3.75\, km
\end{aligned}
\tag{41}
$$

4.7 4g

To answer this question, we want an equation that relates parachute diameter (since a circular planform is used) to a landing velocity V_f. We can find this relation by first looking at the forces acting on the vehicle and parachute.

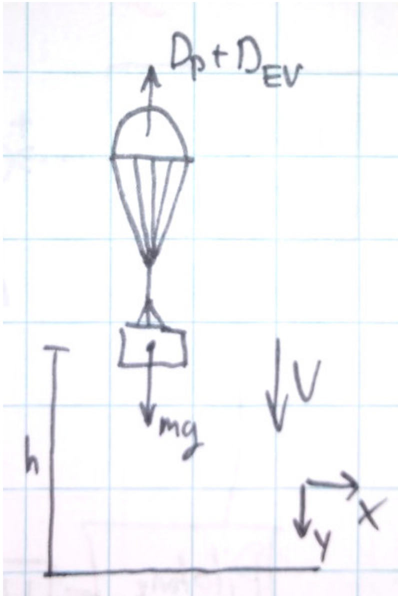

Figure 6: Simple FBD of the vehicle and parachute.

Since we can neglect the drag of the vehicle, $D_{EV} = 0$ and all the drag is generated by the parachute, i.e. $D = D_p$. Applying Newton's second law to the diagram in the positive Y-direction, we get:

$$
m\frac{dV}{dt} = -D_p + mg = -\frac{1}{2} C_{D_p} \rho S_p V^2 + mg \tag{42}
$$

When the parachute is near the ground, we should be in a situation where the vehicle is no longer decelerating but drifting down at constant speed. So we assume that we have $\frac{dV}{dt} = 0$, and then solve the previous equation for S_p:

$$
\begin{aligned}
\frac{1}{2} C_{D_p} \rho S_p V_f^2 &= mg \\
S_p &= \frac{2mg_0}{C_{D_p} \rho V_f^2}
\end{aligned}
\tag{43}
$$

Note that we have changed V to the final velocity V_f, as the speed no longer changes. For a circular parachute planform, can find the parachute diameter:

$$
d_p = \sqrt{\frac{8}{\pi} \frac{mg_0}{C_{D_p} \rho V_f^2}} \tag{44}
$$

And so on Earth you would need:

$$
(d_p)_{\oplus} = \sqrt{\frac{8}{\pi} \frac{315 \cdot 9.81}{0.512 \cdot 1.225 \cdot 10^2}} = 11.2\, m \tag{45}
$$

4.8 4h

Using the exact same relation as in the previous question:

$$
\begin{aligned}
(d_p)_{♀} &= 1.46\, m \\
(d_p)_{♂} &= 59.0\, m
\end{aligned}
\tag{46}
$$

4.9 4i

In terms of deceleration, the Earth seems the most demanding case, owing to the high value of $\bar{a}_{max}$ compared to Mars and Venus. This means for an Earth re-entry, the structure will have to be the strongest, and this may increase the structural mass of the re-entry vehicle. It also has other consequences if there are people on board, who will have to survive these decelerations, and so if the peak deceleration is too high, a redesign may have to be in order. Venus is the most favourable planet in terms of maximum deceleration (so if astronauts ever want to travel to its surface, at least it won't be the re-entry deceleration peak that kills them).

Owing to its dense atmosphere, a Venus lander also requires a vastly smaller parachute size, which can reduce vehicle mass because the parachute will be lighter, and potentially less complex. In comparison, the same lander on Mars would require about forty times the parachute diameter compared to the one needed for a Venus landing. For Earth, this factor is about ten times, and so it fits somewhere in between.