Math 174E
Lecture 10

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References

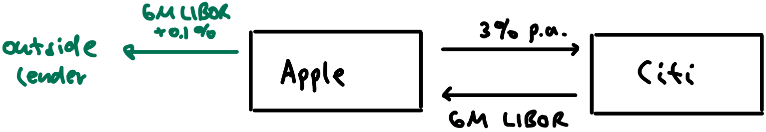
Hull

Chapters 7.1, 7.2, 7.3, 7.4, 7.11, 7.12

Typical Usage of an Interest Rate Swap 1/2

Swap *fixed rate for floating rate* (Apple's perspective) or *floating rate for fixed rate* (Citi's perspective).

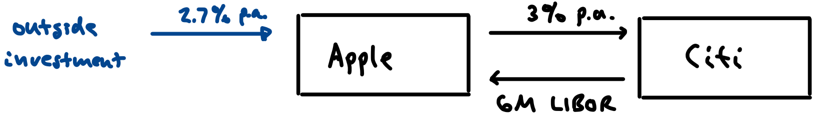
1. Converting a liability:
   - suppose Apple has arranged to **borrow** \$100 million for three years at a floating rate of LIBOR + 10 basis points (= LIBOR + 0.10%) from an **outside lender**
   - Apple can use the *swap contract* with Citigroup to transform a **floating rate loan** into a **fixed rate loan**
   - net interest rate paid by Apple: 3.1% p.a.
   - Apple transforms **borrowings** of \$100 million at a floating rate of LIBOR + 0.10% into borrowings at a fixed rate of 3.1%



net interest paid: 3% + LIBOR + 0.1% − LIBOR = 3.1%

Typical Usage of an Interest Rate Swap 2/2

2. Transform an asset:
   - suppose Apple owns \$100 million in bonds that provide interest at 2.7% p.a. over the next three years
   - Apple can use the *swap contract* with Citigroup to transform an **asset** earning a **fixed rate** of interest into an asset earning a **floating rate** of interest
   - net interest rate earned by Apple: LIBOR−0.3%
   - Apple transforms **asset** earning 2.7% into an asset earning LIBOR−30 basis points



net interest received: 2.7% + LIBOR − 3% = LIBOR − 0.3%

Organization of Trading 1/3

- regulators in the United States require that standard swaps must be traded on electronic platforms and must be cleared through **central counterparties (CCPs)** (see Chapter 1.2 in Hull)
- the swaps are then treated like futures contracts with initial and variation margins being posted by both sides (*collateralization*)
- the rules do not apply if one of the party's main activities are not financial but uses the swap to hedge or mitigate commercial risk (e.g., like in the example above where Apple is a non-financial company)
- in this case Apple could directly enter into the trade with an investment bank (Citigroup) and the trade is cleared bilaterally (and is *uncollateralized*)

Organization of Trading 2/3

- in above illustration Citigroup is likely to act as an **intermediary**
- Citigroup might have *another opposite swap contract* with *another company*, e.g., Intel, where it receives LIBOR and pays 2.97%
- in this case Citigroup (as a **market maker** in interest rate swaps) earns a **spread of 3 basis points** with both swap contracts it transacts (2.97% bid rate and 3% offer/ask rate)
- the spread is to compensate Citigroup for its overheads and for potential losses in the event of a default by a counterparty
- ideally Citigroup would try to enter into offsetting trades with two different counterparties (such as Apple and Intel)
- alternatively, Citigroup manages its risk from the swap contract with Apple by entering into the opposite trade with another investment bank (e.g., Deutsche Bank)

Organization of Trading 3/3

net gain : 0.03 %

bid rate : 2.97 %

ask rate : 3 %

Comparative Advantage Argument

- explanation for the popularity of swaps
- it is argued that some companies have **comparative advantages** when borrowing in fixed rate markets, whereas others have advantages in floating rate markets
- an interest rate swap allows them to change the nature of their liability (fixed to floating or floating to fixed) and to be better off
- the comparative advantage argument for interest rate swaps is controversial (see end of Chapter 7.4 in Hull), but makes more sense for currency swaps

Illustration: Comparative Advantage Argument 1/3

- two companies, AAACorp and BBBCorp, wish to borrow \$10 million for five years
  - AAACorp wants to borrow at a floating rate (linked to 6-month LIBOR)
  - BBBCorp wants to borrow at a fixed rate
- BBBCorp has a worse credit rating than AAACorp

Offered rates:

	fixed	floating
AAACorp	4.0%	6-month LIBOR − 0.1%
BBBCorp	5.2%	6-month LIBOR + 0.6%

Observe:

- BBBCorp pays 1.2% more than AAACorp in fixed-rate markets but only 0.7% more in floating-rate markets
  - BBBCorp has a comparative advantage in floating-rate markets
  - AAACorp has a comparative advantage in fixed-rate markets

Illustration: Comparative Advantage Argument 2/3

Idea:

- AAACorp borrows at fixed rate 4% p.a.
- BBBCorp borrows at floating rate 6-month LIBOR $+ 0.6\%$
- AAACorp and BBBCorp then enter into an interest rate swap to convert the liability
  - from fixed to floating for AAACorp (as desired by AAACorp)
  - from floating to fixed for BBBCorp (as desired by BBBCorp)

Structure of a swap (**without** financial intermediary) which is equally attractive to AAACorp and BBBCorp:

- total gain for both parties in the swap is 1.2%-0.7% $= 0.5\%$
- best scenario for both companies (compared to their offered rates)
  - net gain for AAACorp: $\frac{0.5\%}{2} = 0.25\%$
  - net gain for BBBCorp: $\frac{0.5\%}{2} = 0.25\%$

Illustration: Comparative Advantage Argument 3/3

Structure of the swap (**with** financial intermediary) which is equally attractive to AAACorp and BBBCorp:

- swap brokered by a financial institution which wants to earn a spread of 0.04%
- total gain to all three parties: 0.5% (as before)
- total gain for both companies AAACorp and BBBCorp in the swap is 0.5%-0.04% $=$ 0.46%
- best scenario for both companies (compared to their offered rates)
  - net gain for AAACorp: $\frac{0.46\%}{2} = 0.23\%$
  - net gain for BBBCorp: $\frac{0.46\%}{2} = 0.23\%$

Other Types of Swaps

- **currency swaps:** exchange principal and interest payments in different currencies
- **equity swap:** exchange total return realized on an equity index for fixed or floating rate of interest
- **commodity swap**
- **volatility swap:** exchange realized volatility against preagreed volatility during a certain time period
- **credit default swap (CDS)**
  - allows companies/investors to hedge **credit risk** (credit protection)
  - like an "insurance contract" that pays off if a particular company or country defaults (*reference entity*)
  - buyer of a CDS (*default protection buyer*) pays an insurance premium (*CDS spread*)
  - seller of a CDS (*default protection seller*) pays if reference entity defaults
  - see Hull, Chapter 25 ("Credit Derivatives")
- **swaption:** option on a swap

Chapter 10: Mechanics of Options Markets

Hull

Chapter 10.1–10.9, 10.12

Plain Vanilla Options

calls and puts = **"plain vanilla"** options

More "exotic" options (see Math 179):

- Asian options
- barrier options
- lookback options

- basket options
- spread or exchange options

- forward-start options

Types of Options

Recall Definition 1.8 (Lecture 2)!

- Types of options:
  - a **call** is an option to buy
  - a **put** is an option to sell
  - a **European option** can be exercised only on the expiration date
  - an **American option** can be exercised at any time up to the expiration date (*early exercise*)
- Option positions:
  - **long** position in a **call** option (buyer of option)
  - **long** position in a **put** option (buyer of option)
  - **short** position in a **call** option (seller/writer of option)
  - **short** position in a **put** option (seller/writer of option)
- writer of option receives a **premium** from buyer up front, but has potential liabilities later

Notation

Recall Lecture 2, slide 17:

- K = strike price ($K > 0$)
- T = maturity ($T > 0$)
- $(S_t)_{0 \leq t \leq T}$ = underlying's price process

- $C_t(K, T)$ = price/value of a **European call** option at time $t \in [0, T]$ with strike K and maturity T
- $P_t(K, T)$ = price/value of a **European put** option at time $t \in [0, T]$ with strike K and maturity T

- $C_t^{\mathrm{am}}(K, T)$ = price/value of an **American call** option at time $t \in [0, T]$ with strike K and maturity T
- $P_t^{\mathrm{am}}(K, T)$ = price/value of an **American put** option at time $t \in [0, T]$ with strike K and maturity T

American vs. European

For options written on the same underlying:

Lemma 10.1

We have

$$C_t^{\mathrm{am}}(K, T) \geq C_t(K, T) \qquad \text{and} \qquad P_t^{\mathrm{am}}(K, T) \geq P_t(K, T)$$

for all $t \in [0, T]$.

Payoffs & Net Profits

Recall Lecture 2, slides 18–20:

- Payoff at maturity T:
  - long call: $(S_T - K)^+$
  - long put: $(K - S_T)^+$
  - short call: $-(S_T - K)^+$
  - short put: $-(K - S_T)^+$

- Net profits at maturity T:
  - long call: $(S_T - K)^+ - C_0(K, T)$
  - long put: $(K - S_T)^+ - P_0(K, T)$
  - short call: $C_0(K, T) - (S_T - K)^+$
  - short put: $P_0(K, T) - (K - S_T)^+$

Some Terminology 1/2

- **Intrinsic value** at time $t \in [0, T]$: payoff function at time t
  - call: $(S_t - K)^+$
  - put: $(K - S_t)^+$
- **At-the-money (ATM)**:
  - $S_t = K$
- **In-the-money (ITM)**:
  - call: $S_t > K$
  - put: $S_t < K$
- **Out-of-the money (OTM)**:
  - call: $S_t < K$
  - put: $S_t > K$

Definition 10.2

The **time value** at time $t \in [0, T]$ is the value of an option arising from the time left to maturity. It is defined as

$$\text{time value}_t = \text{option price}_t - \text{intrinsic value}_t.$$

Note: option price$_t$ = time value$_t$ + intrinsic value$_t$

Some Terminology 2/2

Comments:

- note that the intrinsic value is the payoff to the holder of an American option when the option is exercised
- an *in-the-money American option* must be worth at least as much as its intrinsic value because the holder has the right to exercise it immediately
- often it is optimal for the holder of an in-the-money American option to wait rather than exercise immediately
- in this case, the excess of the option's value over its intrinsic value is the option's time value

Underlying Assets & Exchanges 1/2

- Stock Options
  - mostly traded on exchanges (electronically)
    - *Examples* in U.S.: Chicago Board Options Exchange (CBOE), NYSE Euronext, International Securities Exchange, Boston Options Exchange
  - traded on several thousand different stocks
  - contract size 100 shares (convenient because the shares themselves are usually traded in multiples of 100)
- ETP Options
  - CBOE trades many options on ETPs (= exchange-traded products)
  - ETPs are listed on an exchange and traded like a share of a company's stock
  - most common ETPs: ETFs (= exchange-traded funds)
  - ETFs are usually designed to replicate the performance of a particular market, often by tracking an underlying benchmark index (equity index like S&P 500 or bond index)
  - other ETPs are designed to track the performance of commodities or currencies

Underlying Assets & Exchanges 2/2

- Foreign Currency Options
  - most currency options trading is in the OTC market
  - some exchange trading (e.g., NASDAQ OMX in the U.S.)
- Index Options
  - exchange-traded (e.g., CBOE) and OTC
  - most popular exchange-traded contracts in the U.S. are on S&P 500 Index (SPX), S&P 100 Index (OEX), Nasdaq-100 Index (NDX), Dow Jones Industrial Index (DJX)
  - one contract is usually to buy or sell 100 times the index at the specified strike price
  - settlement is always in cash (rather than by delivering the portfolio underlying the index)
- Futures Options
  - exchanges trading futures often trade options on the futures as well
  - the life of a futures option normally ends a short period of time before the expiration of trading in the underlying futures contract
- Options on Cryptocurrencies

Specification of Stock Options

- standard exchange-traded stock option in the U.S. is an American-style option contract to buy/sell 100 shares
- other contract details specified by the exchange:
  - expiration dates
  - strike prices
  - adjustments in case of dividend payments (cash and stock dividends) as well as stock splits (see Math 179)
  - position limits and exercise limits (e.g., options on the largest and most frequently traded stocks have position limits of 250,000 contracts)

Stock Options Trading 1/2

- most derivatives exchanges are fully electronic (with market and limit orders)
- market makers quote bid and ask prices, provide liquidity and make profits from the bid-ask spread
- investors can close out their positions by issuing an offsetting order (similar to futures markets)

Stock Options Trading 2/2

- **commissions** are charged by a broker for placing orders at the exchange
- **margin requirements** when selling/writing an option (but not for buying an option)
  - in particular for writing a *naked option* (= an option that is not combined with an offsetting position in the underlying stock)
  - in contrast, a *covered call option* (writing an option when the shares that might have to be delivered are already owned) does not require margins
- trades are cleared through the exchange's Options Clearing Corporation (OCC) (similar to the clearing house in futures markets)
- exchange-traded options markets are regulated in a number of different ways (by the exchanges and by state and federal authorities)

Chapter 11: Properties of Stock Options

Hull
Chapter 11.1–11.3

Introduction

Standing assumptions (cf. Chapter 5, Lecture 8, slide 3)

1. There are no transaction costs.
2. All trading profits (net of trading losses) are subject to the same tax rate.
3. Borrowing and lending are possible at the (nominal) risk-free interest rate $r > 0$ (p.a. and continuously compounded).
4. Market participants use arbitrage opportunities as they occur (**“no-arbitrage assumption”** or **“absence of arbitrage assumption”**).

Goal: Derive basic properties of stock option prices based on no-arbitrage arguments (i.e., properties of **“arbitrage-free”** stock option prices).

Two Simple Properties

Under the **no-arbitrage assumption** following properties hold true (all options are written on the **same stock**):

Lemma 11.1

1. For all $t \in [0, T]$, if $K_1 \leq K_2$ then
$$C_t(K_1, T) \geq C_t(K_2, T) \quad \text{and} \quad P_t(K_1, T) \leq P_t(K_2, T).$$
2. For all $t \in [0, T]$ the mappings $K \mapsto C_t(K, T)$ and $K \mapsto P_t(K, T)$ are convex. That is, e.g.,
$$C_t\left(\frac{K_1 + K_2}{2}, T\right) \leq \frac{1}{2}C_t(K_1, T) + \frac{1}{2}C_t(K_2, T)$$

Proof: See lecture notes.
1.) "Bull/Bear spread" (see Hull, Chapter 12);
2.) "Butterfly spread" (see Hull, Chapter 12);

Arbitrage-free Bounds: European Call

For arbitrage-free **European call option** prices on a **non-dividend** paying stock:

Lemma 11.2

We have

$$(S_t - Ke^{-r(T-t)})^+ < \boldsymbol{C_t(K, T)} < S_t$$

for all $t \in [0, T)$.

Proof: See lecture notes (for the upper bound, see also Lecture 3, Example 1.19).

Remarks:

- note that $C_t(K, T) > 0$
- note that $(S_t - K)^+ < (S_t - Ke^{-r(T-t)})^+$ and hence $C_t(K, T) > (S_t - K)^+$
    - price of a European call is always greater than its intrinsic value

Arbitrage-free Bounds: European Put

For arbitrage-free **European put option** prices on a **non-dividend** paying stock:

Lemma 11.3

We have
$$(Ke^{-r(T-t)} - S_t)^+ < \boldsymbol{P_t(K, T)} < Ke^{-r(T-t)}$$

for all $t \in [0, T)$.

Proof: See lecture notes for a sketch and Assignment 4 (for the upper bound, see also Assignment 2, Exercise 2.9).

Remarks:

- note that $P_t(K, T) > 0$
- note that $(Ke^{-r(T-t)} - S_t)^+ < (K - S_t)^+$
  - price of a European put can be lower than its intrinsic value

Numerical Example

Example 11.4 (Arbitrage-free price intervals)

- European call option (on a non-dividend paying stock): Current stock price is \$51, strike price is \$50, time to maturity is 6 months, risk-free interest rate is 12% p.a.

$$3.91 < C_0(K, T) < 51$$

- European put option (on a non-dividend paying stock): Current stock price is \$38, strike price is \$40, time to maturity is 3 months, risk-free interest rate is 10% p.a.

$$1.01 < P_0(K, T) < 39.01$$