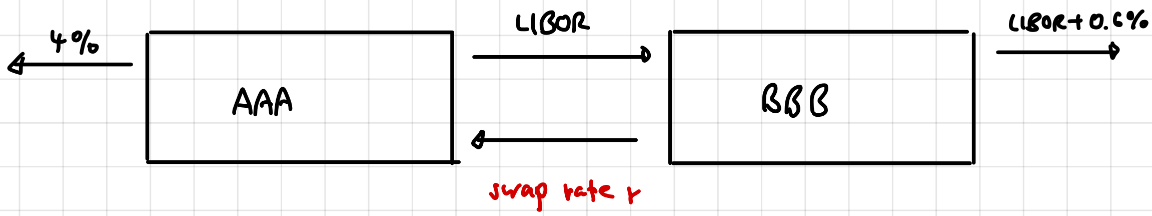
Lecture 10: 08/22/22

Illustration: Comparative advantage argument



Swap rate r equally attractive to both:

AAA's net interest : $4\% + LIBOR - r \leq LIBOR - 0.1\% \iff r \geq 4.1\%$

BBB's net interest : $LIBOR + 0.6\% + r - LIBOR \leq 5.2\% \iff r \leq 4.6\%$

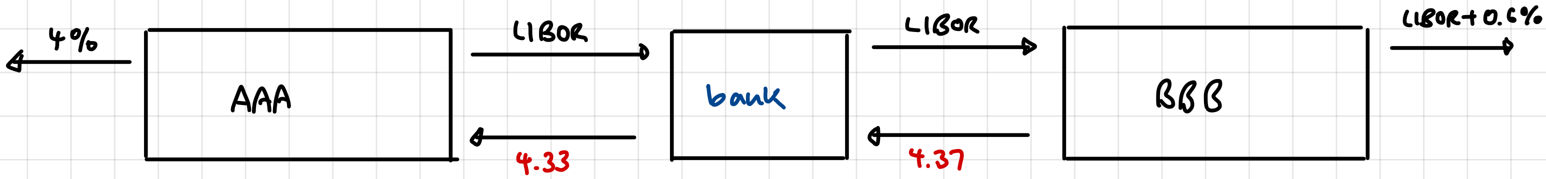
Choose midpoint: $\frac{4.1 + 4.6}{2} = 4.35 = r$

Hence: AAA's net interest : $LIBOR - 0.35\%$ (0.25 % less than LIBOR - 0.1 %)

BBB's net interest : 4.95% (0.25 % less than 5.2 %)

Total gain: $1.2\% - 0.7\% = 0.5\%$

With financial intermediary:



Spread earned: 0.04

Hence: AAA's net interest: LIBOR − ~~0.35~~ 0.33 % (~~0.25~~ 0.23 % less than LIBOR−0.1%)

BBB's net interest: ~~4.95~~ 4.97 % (~~0.25~~ 0.23 % less than 5.2%)

Total gain: 1.2 % − 0.7 % − 0.04 % = 0.46 %

Proof of Lemma 11.1:

1.) Let $t=0$ (w.l.o.g.) and $0 < K_1 \leq K_2$

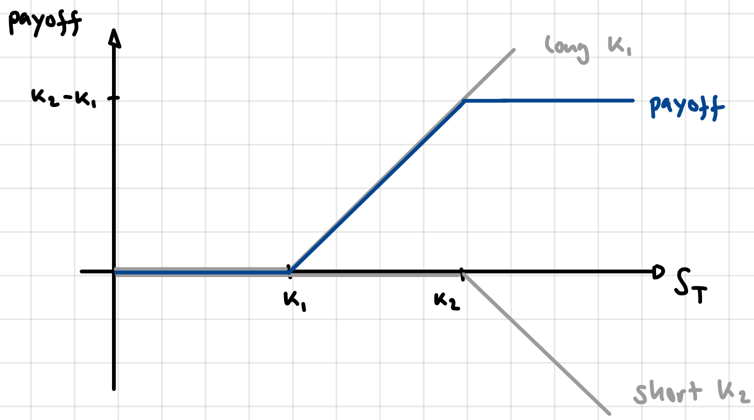
Show: $C_0(K_1,T) \geq C_0(K_2,T) \iff C_0(K_1,T) - C_0(K_2,T) \geq 0$

Idea: $C_0(K_1,T) - C_0(K_2,T)$ is the value at time 0 of following strategy:

$$\left.\begin{array}{l} \text{(i) long position in call w/ strike } K_1 \\ \text{(ii) short position in call w/ strike } K_2 \end{array}\right\} \text{bull spread}$$

Payoff at maturity T:

$$(S_T-K_1)^+ - (S_T-K_2)^+ = \left\{\begin{array}{ll} 0 & , S_T \leq K_1 \\ S_T-K_1 & , K_1 \leq S_T \leq K_2 \\ K_2-K_1 & , S_T \geq K_2 \end{array}\right\} \geq 0$$



$\Rightarrow$ value of this position at time 0 must be non-negative, otherwise there is an arbitrage opportunity (recall Def. 1.18 from Lecture 3)

$\Rightarrow$ $C_0(K_1,T) - C_0(K_2,T) \geq 0$

Similar argument for showing: $P_0(K_1,T) \leq P_0(K_2,T)$

2.) let $t=0$ (w.l.o.g.) and $K_1 \leq K_2$ (w.l.o.g.):

Show: $$C_0\left(\frac{K_1+K_2}{2}, T\right) \leq \frac{1}{2} C_0(K_1,T) + \frac{1}{2} C_0(K_2,T)$$

$$\Leftrightarrow \quad C_0(K_1,T) + C_0(K_2,T) - 2\,C_0\left(\frac{K_1+K_2}{2}, T\right) \geq 0$$

Idea: $C_0(K_1,T) + C_0(K_2,T) - 2\,C_0\left(\frac{K_1+K_2}{2}, T\right)$ is the value at time 0 of following strategy:

(i) long position in one call w/ strike K_1

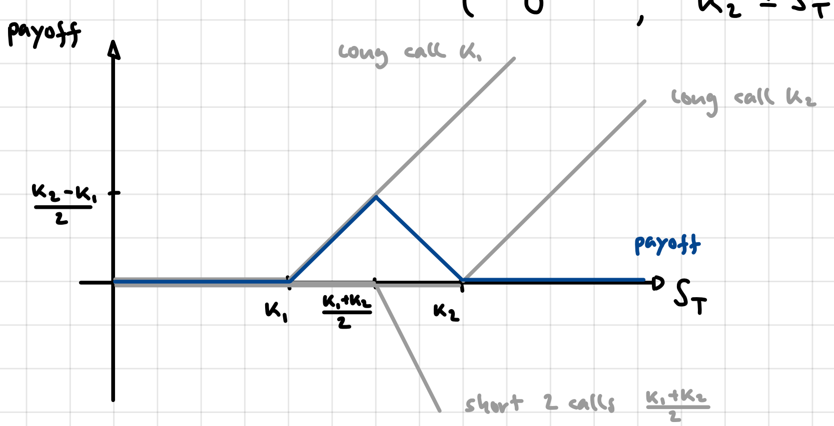
(ii) long position in one call w/ strike K_2

(iii) short position in two calls w/ strike $\frac{K_1+K_2}{2}$

butterfly spread

Payoff at maturity T:

$$(S_T-K_1)^+ + (S_T-K_2)^+ - 2\left(S_T - \frac{K_1+K_2}{2}\right)^+ = \left\{\begin{array}{ll} 0 & , S_T \leq K_1 \\ S_T - K_1 & , K_1 \leq S_T \leq \frac{K_1+K_2}{2} \\ K_2 - S_T & , \frac{K_1+K_2}{2} \leq S_T \leq K_2 \\ 0 & , K_2 \leq S_T \end{array}\right\} \geq 0$$



$\Rightarrow$ value of this position at time 0 must be non-negative, otherwise there is an arbitrage opportunity (recall Def. 1.18 from Lecture 3)

$$\Rightarrow \quad C_0(K_1,T) + C_0(K_2,T) - 2\,C_0\left(\frac{K_1+K_2}{2}, T\right) \geq 0$$

Similar argument for puts