RV	PMF/PDF	$\mathbb{E}[X]$	$\mathrm{Var}(X)$	MGF
$\mathrm{Bin}(n,p)$	$\binom{n}{k} p^k(1-p)^{n-k}$	np	$np(1-p)$	$[(1-p)+pe^t]^n$
$\mathrm{Poiss}(\lambda)$	$e^{-\lambda}\frac{\lambda^k}{k!}$	λ	λ	$\exp(\lambda(e^t-1))$
$\mathrm{Geom}(p)$	$(1-p)^{k-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}, \quad t<-\ln(1-p)$
$\mathrm{Exp}(\lambda)$	$\begin{cases}\lambda e^{-\lambda t} & t\geq 0\\ 0 & t<0\end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}, \quad t<\lambda$
$\mathrm{Gamma}(n,\lambda)$	$\frac{\beta^n}{(n-1)!}x^{n-1}e^{-\beta x}, \quad x\geq 0$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^n, \quad t<\lambda$

COUNTING PROCESS ➠ $\{N_t\}_{t\geq 0}$ ➠ $N_t\in\mathbb{N}\cup\{0\}$ ➠ $0\leq s\leq t\Rightarrow N_s\leq N_t$

MEMORYLESS ➠ $\mathbb{P}(X>s+t\mid X>s)=\mathbb{P}(X>t), \quad s,t\geq 0$
$\mathrm{Geom}(p)$, $\mathrm{Exp}(\lambda)$ are memoryless.

MINIMUM EXP ➠ $X_i\sim\mathrm{Exp}\left(\lambda_i\right)_n$ ➠ $\lambda_i>0$ ➠ $M=\min\{X_1,X_2,\ldots,X_n\}$ ➠ $N\doteq\sum_{i=1}^n i1_{\{M=X_i\}}$

$$\boxed{\mathbb{P}(M>x,N=i)=\frac{\lambda_i}{\lambda}e^{-\lambda x}} \quad \lambda\doteq\sum_{i=1}^n\lambda_i \text{ ➠ Corollary ➠ } \boxed{\mathbb{P}(N=i)=\frac{\lambda_i}{\lambda}}$$

$X_i\overset{\perp\!\!\!\perp}{\sim}\mathrm{Exp}(\lambda)$ ➠ $S\doteq\sum_{i=1}^n X_i$ ➠ $S\sim\mathrm{Gamma}(n,\lambda)$

POISSON PROCESS ➠ $\mathbb{P}\left(N_t=k\right)=e^{-\lambda t}\frac{(\lambda t)^k}{k!}$

$\{N_t\}_{t\geq 0}\sim PP(\lambda)$ ➠ shifted ➠ $\boxed{N_t^s\doteq N_{t+s}-N_s}$ $\quad N_t^s\doteq\#$ of events that occur in the t time-units after s. $N_t\perp\!\!\!\perp N_t^s$

STATIONARY ➠ $X_{t_2}-X_{t_1}\overset{d}{=}X_{s_2}-X_{s_1}$ INDEPENDENT ➠ $X_{t_1}\perp\!\!\!\perp X_{t_4}-X_{t_3}$ $\quad \{N_t\}_{t\geq 0}\sim PP(\lambda)\iff$ stationary and indep.

$$\boxed{\mathbb{P}\left(N_{t_1}=k_1,N_{t_2}=k_2,\ldots,N_{t_n}=k_n\right)=e^{-\lambda t_n}\prod_{i=1}^n\frac{\left(\lambda\left(t_i-t_{i-1}\right)\right)^{k_i-k_{i-1}}}{\left(k_i-k_{i-1}\right)!}}$$

SUPERPOSITION $\left\{N_t^1\right\}_{t\geq 0}\sim PP\left(\lambda_1\right),\left\{N_t^2\right\}_{t\geq 0}\sim PP\left(\lambda_2\right),\ldots,\left\{N_t^n\right\}_{t\geq 0}\sim PP\left(\lambda_n\right)$ independent PP
➠ $N_t\doteq\sum_{i=1}^n N_t^i$ ➠ $N_t\sim PP(\lambda)$ ➠ $\lambda\doteq\sum_{i=1}^n\lambda_i$

SPLITTING $\{N_t\}\sim PP(\lambda)$ ➠ r events $\perp\!\!\!\perp$ s.t. $\sum_{i=1}^r p_i=1$.
$N_t\doteq\sum_{i=1}^r N_t^i$ ➠ $\left\{N_t^i\right\}\sim PP\left(\lambda p_i\right)$, and the r processes $\left\{N_t^1\right\},\ldots,\left\{N_t^r\right\}$ are independent.

NON HOMOGENOUS Let $\lambda:\mathbb{R}_+\to\mathbb{R}_+$ ➠ $\Lambda(t)\doteq\int_0^t\lambda(s)ds<\infty$ ➠ Poisson process with rate function $\lambda(\cdot)$
(1) $\{N_t\}$ has independent increments, (2) $N_t\sim\mathrm{Poisson}(\Lambda(t))$
➠ $\{N_t\}\sim NPP(\lambda(\cdot))$ ➠
(1) $\mathbb{P}\left(N_0=0\right)=1$, (2) $N_{s+t}-N_s\sim\mathrm{Poisson}(\Lambda(s+t)-\Lambda(s))$

COMPOUND ➠ $\{N_t\}\sim PP(\lambda)$ ➠ $\{X_n\}$ *iid* RV that are $\perp\!\!\!\perp$ $\{N_t\}$ ➠ $Z_t=\sum_{n=1}^{N_t}X_n$ ➠ $\mathbb{E}\left(X_n\right)=\mu$, and $\mathbb{E}\left(X_n^2\right)=\theta^2$

$$\mathbb{E}\left(Z_t\right)=\lambda\mu t, \quad \mathrm{Var}\left(Z_t\right)=\lambda\theta^2 t \quad t\geq 0 \quad \text{Wald's Identity}$$

SPATIAL Let $\{N_A\}_{A\subseteq\mathbb{R}^d}$ be a spatial Poisson process with parameter $\lambda>0$. Then for each $A\subseteq\mathbb{R}^d$, and each $B\subseteq A$,

$$\mathbb{P}\left(N_B=k\mid N_A=n\right)=\binom{n}{k}p^k(1-p)^{n-k} \quad p=\frac{|B|}{|A|}$$

CTMC MARKOV PROPERTY ➠ $(X_t)_{t\geq 0}$ ➠ w/ discrete state space S ➠ is CTMC if

$$P\left(X_{t+s}=j \mid X_s=i, X_u=x_u, 0\leq u<s\right)=P\left(X_{t+s}=j \mid X_s=i\right) \quad \forall\, s,t,\geq 0, i,j,x_u\in S,\, 0\leq u<s$$

TIME-HOMOGENEOUS ➠ $P\left(X_{t+s}=j \mid X_s=i\right)=P\left(X_t=j \mid X_0=i\right),$ for $s\geq 0$

C-K EQUATIONS ➠ CTMC $(X_t)_{t\geq 0}$ with transition function $\boldsymbol{P}(t)$,

$$\boldsymbol{P}(s+t)=\boldsymbol{P}(s)\boldsymbol{P}(t),$$

for $s,t\geq 0$. That is,

$$P_{ij}(s+t)=[P(s)P(t)]_{ij}=\sum_k P_{ik}(s)P_{kj}(t), \text{ for states } i,j, \text{ and } s,t\geq 0.$$

PP ARE CTMCS $\{N_t\}\sim PP(\lambda)$ ➠ is a CMTC on $S=\{0,1,2,\ldots\}$ with transition function $P_{x,y}(t)=e^{-\lambda t}\frac{(\lambda t)^{y-x}}{(y-x)!}$

HOLDING TIMES The length of time that a CTMC started in i stays in i before transitioning to a new state ➠ $T_i\sim \operatorname{Exp}(q_i)$

$$\text{HOLDING TIME } q_i=\sum_k q_{ik} \quad ✦ \quad \text{TRANSITION } p_{ij}=\frac{q_{ij}}{\sum_k q_{ik}}=\frac{q_{ij}}{q_i} \quad ✦ \quad \text{GENERATOR } q_{ij}=q_i p_{ij}$$

$\mathbb{P}\left(X_{\text{holding time}}= \text{ new } S \mid X_0=\text{old } S\right)=\frac{Q_{\text{old,new}}}{\left|Q_{\text{old,old}}\right|}$

If $\{X_t\}$ is a stochastic process that has the stationary increments property and the independent increments property, then it is time homogeneous and has the Markov property.
If $\{X_t\}$ is a stochastic process that has the independent increments property, then it also has the Markov property.
If $\{X_t\}$ has the Markov property, then it does not necessarily have the independent increments property.
If $\{X_t\}$ has the Markov property and the stationary increments property, then it is not necessarily time-homogeneous.
If $\{X_t\}$ has the Markov property and is time-homogeneous, it does not necessarily have the stationary increments property.

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law of total expectation $E(Y)=\sum_{i=1}^k E\left(Y \mid A_i\right) P\left(A_i\right)$ JOINT $E(Y)=\sum_x E(Y \mid X=x)P(X=x)$
SPECIAL CASES $P(B)=\sum_{i=1}^{\infty} P\left(B \mid A_i\right) P\left(A_i\right)$ $E[Y]=\sum_{i=1}^{\infty} E\left[Y \mid X=x_i\right] P\left(X=x_i\right)$

CONDITIONAL EXPECTATION ➠ LINEARITY $\mathbb{E}[a\cdot Y+b\cdot Z \mid X=x]=a\cdot\mathbb{E}[Y \mid X=x]+b\cdot E[Z \mid X=x]$
INDEPENDENT $\mathbb{E}[Y \mid X=x]=\mathbb{E}[Y]$ ✦ TAKING OUT $\mathbb{E}[Y \mid X=x]=\mathbb{E}[g(X) \mid X=x]=\mathbb{E}[g(x) \mid X=x]=g(x)$

RV ➠ LINEARITY $\mathbb{E}[a\cdot Y+b\cdot Z \mid X]=a\cdot\mathbb{E}[Y \mid X]+b\cdot\mathbb{E}[Z \mid X]$
INDEPENDENCE $\mathbb{E}[Y \mid X]=\mathbb{E}[Y]$ ✦ TAKING OUT $\mathbb{E}[g(X)Y \mid X]=g(X)\mathbb{E}[Y \mid X]$

CONDITIONAL VARIANCE ➠ $\operatorname{Var}(Y \mid X)=\mathbb{E}\left[Y^2 \mid X\right]-(\mathbb{E}[Y \mid X])^2$
TOTAL VARIANCE ➠ $\operatorname{Var}(Y)=\mathbb{E}[\operatorname{Var}(Y \mid X)]+\operatorname{Var}(\mathbb{E}[Y \mid X])$

TOTAL EXPECTATION ➠ $\mathbb{E}[\mathbb{E}[X \mid Y]]=\mathbb{E}[X]$ $\mathbb{E}[\underbrace{\mathbb{E}[X \mid Y]}_{=h(Y)}]=\begin{cases}\sum_y \mathbb{E}[X \mid Y=y]\cdot\mathbb{P}[Y=y] & Y \text{ DISCRETE}\\ \int_{-\infty}^{\infty}\mathbb{E}[X \mid Y=y]\cdot f_Y(y)dy & Y \text{ CNTS}\end{cases}$

BAYES ➠ $P(B \mid A)=\dfrac{P(A \mid B)P(B)}{P(A \mid B)P(B)+P\left(A \mid B^c\right)P\left(B^c\right)}$ $\left[\dfrac{P\left(A \mid B_i\right)P\left(B_i\right)}{\sum_{k=1}^n P\left(A \mid B_k\right)P\left(B_k\right)}\right]$