#2

a. type I error : $\{$ reject H_0 when H_0 is true $\}$

$$\Rightarrow \{ Y \le 12 \mid p = 0.8 \}$$

which mean researchers are right, but the conclusion is a drug dosage level won't induce sleep for 80% of people suffering from insomnia.

b.
$$\alpha = P\{ Y \le 12 \mid p = 0.8 \}$$
$$= 1 - P\{ Y > 12 \mid p = 0.8)$$
$$= 1 - \sum_{Y=0}^{20} \binom{20}{Y} (0.8)^Y (0.2)^{20-Y}$$
$$= 1 - \Big[\binom{20}{13}(0.8)^{13}(0.2)^7 + \binom{20}{14}(0.8)^{14}(0.2)^6 + \binom{20}{15}(0.8)^{15}(0.2)^5 +$$
$$\binom{20}{16}(0.8)^{16}(0.2)^4 + \binom{20}{17}(0.8)^{17}(0.2)^3 + \binom{20}{18}(0.8)^{18}(0.2)^2 +$$
$$\binom{20}{19}(0.8)^{19}(0.2)^1 + \binom{20}{20}(0.8)^{20}(0.2)^0 \Big]$$
$$\approx 0.0321$$

c. type II error : $\{$ Do not reject H_0 when H_0 is not true $\}$

$$\Rightarrow \{ Y > 12 \mid H_1 \text{ is true} \}$$

Reasearchers are not true, but the drug dosage level that she claims will induce sleep for 80% of people suffering from insomnia.

d. $\beta = P[\, Y > 12 \mid p = 0.6 \,]$

$$= \sum_{y=13}^{20} \binom{20}{y} (0.6)^{y} (0.4)^{20-y}$$

$$= \left[\binom{20}{13} (0.6)^{13} (0.4)^{7} + \cdots \binom{20}{20} (0.6)^{20} (0.4)^{0} \right]$$

$$\approx 0.4159$$

e: $\beta = P[\, Y > 12 \mid p = 0.4 \,]$

$$= \sum_{y=13}^{20} \binom{20}{y} (0.4)^{y} (0.6)^{20-y}$$

$$= \left[\binom{20}{13} (0.4)^{13} (0.6)^{7} + \cdots \binom{20}{20} (0.4)^{20} (0.6)^{20} \right]$$

$$\approx 0.021$$

$|Y-18| \leq 3$

#6

a. $\alpha = P[\text{ reject } H_0 \mid H_0 \text{ is true}]$

$= P[\,|Y-18| \geq 4 \mid p = 0.5]$

$= 1 - P[\,|Y-18| < 4 \mid p = 0.5]$

$= 1 - [-3 \leq Y-18 \leq 3 \mid p = 0.5]$

$= 1 - [15 \leq Y \leq 21 \mid p = 0.5]$

$$= 1 - \sum_{Y=15}^{21} \binom{36}{Y} (0.5)^{Y} (0.5)^{36-Y}$$

$$= 1 - \left[\binom{36}{15} (0.5)^{15} (0.5)^{21} + \cdots + \binom{36}{21} (0.5)^{21} (0.5)^{15} \right]$$

$\approx 1 - 0.757$

≈ 0.243

b. $\beta = P[\text{ Accep } H_0 \mid H_1 \text{ is true}]$

$= P[\,|Y-18| < 4 \mid p = 0.7]$

$= P[-4 < Y-18 < 4 \mid p = 0.7]$

$= P[14 < Y < 22 \mid p = 0.7]$

$$= \sum_{Y=15}^{21} \binom{36}{Y} (0.7)^{Y} (0.3)^{36-Y}$$

≈ 0.0916

$$Z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

#18

$$\begin{cases} H_0 : \mu = 13.2 \\ H_1 : \mu < 13.2 \end{cases}$$

$\mu = 13.2$ $\qquad \sigma = 2.5$

$$Z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{12.2 - 13.2}{2.5/\sqrt{40}} \approx -2.5298$$

$\Rightarrow -Z_\alpha = -Z_{0.01} \approx -2.32$

$\Rightarrow \quad Z > -Z_\alpha$

$\Rightarrow \quad Z < -Z_\alpha$

$-2.5298 < -2.32$ $\qquad$ Yes ✓

Since Z is smaller than $-Z_\alpha$, so we reject the Null hypothesis. Then there is sufficient evidence to indicate that this company be accused of playing substandard wage.

#42

n? $\qquad \alpha = 0.01 \qquad \beta = 0.05 \qquad \mu_\alpha = 5.5$

$$n = \frac{(Z_\alpha + Z_\beta)^2 \sigma^2}{(\mu_\alpha - \mu_0)^2}$$

$$n \approx \frac{(Z_{0.01} + Z_{0.05})^2 \cdot (3.1)^2}{(5.5 - 5)^2}$$

$$\approx \frac{(2.33 + 1.64)^2 \cdot (3.1)^2}{0.25}$$

$$\approx 605.84$$

#50

$H_0: \mu \geq 60$ $H_1: \mu < 60$

$\bar{Y} = 58$ $S = 11$ $\alpha = 0.1$

$$Z = \frac{\bar{Y} - \mu_0}{S/\sqrt{n}} = \frac{58-60}{11/\sqrt{120}} \approx -1.992$$

$$P = P(Z < -1.992)$$
$$= 0.0233$$

Since $P = 0.0233$ which is less than $\alpha = 0.1$. then we reject the Null hypothesis. So there is sufficient evdience exists to support the fight is unprofitable.

#51

$\bar{Y}_1 = 74$ $\bar{Y}_2 = 71$

$s_1 = 9$ $s_2 = 10$

$n_1 = 50$ $n_2 = 50$

a. $\begin{cases} H_0: & \mu_1 - \mu_2 = 0 \\ H_1: & \mu_1 - \mu_2 \neq 0 \end{cases}$

$$Z_0 = \frac{\bar{Y}_1 - \bar{Y}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{74 - 71}{\sqrt{\frac{81}{50} + \frac{100}{50}}}$$

$$\approx 1.5767$$

$$P = 2 \cdot P(Z > Z_0)$$
$$= 2\,P(Z > 1.5767)$$
$$= 2[1 - P(Z \leq 1.5767)]$$
$$= 2[1 - 0.9429]$$
$$= 2 \cdot 0.0571$$
$$\approx 0.1141$$

b. Since $P = 0.1141 > 0.05$, we fail to reject the Null hypothesis. So we have sufficient evidence to support there is no difference between the two population means.

#72

a.

$$\begin{cases} H_0: & \mu_1 - \mu_2 = 0 \\ H_1: & \mu_1 - \mu_2 \neq 0 \end{cases}$$

$$T = \frac{\bar{Y}_1 - \bar{Y}_2 - D_0}{S_p \cdot \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{19 \cdot 22^2 + 19 \cdot 20^2}{20+20-2}}$$

$$\approx 21.02$$

$$T = \frac{78-67}{21.02\sqrt{\frac{1}{20}+\frac{1}{20}}} \approx 1.65$$

$$V = 20+20-2 = 38$$

$$|t| > t_{\alpha/2}$$

$$= t_{0.05/2}$$

$$> 2.024 \Rightarrow RR \text{ is } |t| > 2.024$$

Since $T = 1.65$ which is smaller than 2.024, so we fail to reject the Null hypothesis. and there is no sufficient evidence to support that there is a difference in the average amount spent per trip on weekend and weekdays.

b.

$$t_{0.2} = 1.304 < T = 1.65 < t_{0.1} = 1.686$$

$$P = 2P(T \geq t)$$

$$= 2P(T \geq 1.65)$$

$$= 2 \cdot 0.053$$

$$= 0.106$$

#79

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} \qquad v = n-1 \text{ df.}$$

a. $H_0: \sigma^2 = 0.01$ $\quad n = 8$ $\quad \mu = 3.1$

$H_1: \sigma^2 > 0.01$ $\quad v = 7$ $\quad S = 0.018$

RR: $\chi^2 > \chi^2_\alpha$ $\quad \alpha = 0.05$

$\chi^2 > 14.067$

$$\chi^2 = \frac{7 \cdot 0.018}{0.01} = 12.6$$

Since the test value is not in the RR. so we cannot reject the null hypothesis, which requires the sample is drawn from a normol distribution.

b. The p is between (0.05, 0.1)

#94

μ ✓ $\quad \sigma^2$ ✗

$$\begin{cases} H_0: & \sigma^2 = \sigma_0^2 \\ H_a: & \sigma^2 = \sigma_1^2 \end{cases}$$

$$f(y|\sigma^2) = \frac{1}{\sigma\cdot\sqrt{2\pi}} \cdot e^{\left[-\frac{(y-\mu)^2}{2\sigma^2}\right]}$$

$$\hookrightarrow L(\sigma^2) = \prod_{i=1}^{n} f(y_i|\sigma^2) = \left(\frac{1}{\sigma\cdot\sqrt{2\pi}}\right)^n e^{\left[-\sum_{i=1}^{n}\frac{(y_i-\mu)^2}{2\sigma^2}\right]}$$

$$\frac{L(\sigma_0^2)}{L(\sigma_1^2)} < k$$

$$\Rightarrow \frac{\left(\frac{1}{\sigma_0\sqrt{2\pi}}\right)^n e^{\left[-\sum_{i=1}^{n}\frac{(y_i-\mu)^2}{2\sigma_0^2}\right]}}{\left(\frac{1}{\sigma_1\sqrt{2\pi}}\right)^n e^{\left[-\sum_{i=1}^{n}\frac{(y_i-\mu)^2}{2\sigma_1^2}\right]}} < k$$

$$\Rightarrow n\left[\ln\left(\frac{\sigma_1}{\sigma_0}\right)\right] - \frac{1}{2}\left(\frac{\sigma_1^2-\sigma_0^2}{\sigma_0^2\sigma_1^2}\right)\sum_{i=1}^{n}(y_i-\mu)^2 < \ln(k)$$

$$\Rightarrow \sum_{i=1}^{n}(y_i-\mu)^2 > k'$$

$$\Rightarrow k' = \left[\ln k - n\cdot\ln\left(\frac{\sigma_1}{\sigma_0}\right)\right]\frac{2\sigma_0^2\sigma_1^2}{\sigma_1^2-\sigma_0^2}$$

$$P\left(\sum_{i=1}^{n}(y_i-\mu)^2 > k' \,\middle|\, H_0\right) = \alpha$$

$$\Rightarrow P\left(\frac{1}{\sigma_0^2}\sum_{i=1}^{n}(y_i-\mu)^2 > \frac{k'}{\sigma_0^2} \,\middle|\, H_0\right) = \alpha$$

$$\Rightarrow P\left(\chi_n^2 > \frac{k'}{\sigma_0^2}\right) = \alpha$$

$$\Rightarrow P\left(\chi_n^2 \leq \frac{k'}{\sigma_0^2}\right) = 1-\alpha$$

$$\Rightarrow \frac{k'}{\sigma_0^2} = \chi^2_{1-\alpha, n}$$

$$\Rightarrow k' = \chi^2_{1-\alpha, n} \cdot \sigma_0^2$$

$$P\left(\frac{1}{\sigma_0^2} \sum_{i=1}^{n} (y_i - \mu)^2 > \frac{\sigma_0^2 \cdot \chi^2_{1-\alpha, n-1}}{\sigma_0^2} \right) = \alpha$$

$$\Rightarrow P\left(\sum_{i=1}^{n} (y_i - \mu)^2 > \sigma_0^2 \cdot \chi^2_{1-\alpha, n-1} \right) = \alpha$$

Since $RR = \left\{ \sum_{i=1}^{n} (y_i - \mu)^2 > \sigma_0^2 \cdot \chi^2_{1-\alpha, n-1} \right\}$ is independent of σ_1^2, so the test uniformly most powerful for $H_a: \sigma^2 > \sigma_0^2$.

#99

a. $\begin{cases} H_0: & \lambda = \lambda_0 \\ H_a: & \lambda = \lambda_a \end{cases} \quad (\lambda_a > \lambda_0)$

$$P(Y|\lambda) = \frac{e^{-\lambda}\lambda^{Y}}{Y!}$$

$$\Rightarrow L(\lambda) = \frac{e^{-n\lambda}\cdot\lambda^{\sum_{i=1}^{n} Y_i}}{\prod_{i=1}^{n}(Y_i!)}$$

$$\frac{L(\lambda_a)}{L(\lambda_0)} > k$$

$$\Rightarrow \frac{\dfrac{e^{-n\lambda_a}\cdot\lambda_a^{\sum_{i=1}^{n} Y_i}}{\prod_{i=1}^{n}(Y_i!)}}{\dfrac{e^{-n\lambda_0}\cdot\lambda_0^{\sum_{i=1}^{n} Y_i}}{\prod_{i=1}^{n}(Y_i!)}} > k$$

$$\Rightarrow \cancel{e^{-n(\lambda_a-\lambda_0)}}\cdot\left(\frac{\lambda_a}{\lambda_0}\right)^{\sum_{i=1}^{n} Y_i} > k\cdot e^{n(\lambda_a-\lambda_0)}$$

$$\overset{\log}{\Rightarrow} \sum_{i=1}^{n} Y_i\cdot\log\left(\frac{\lambda_a}{\lambda_0}\right) > \log\left(k\cdot e^{n(\lambda_a-\lambda_0)}\right)$$

$$\Rightarrow \sum_{i=1}^{n} Y_i > \underbrace{\frac{\log(k\cdot e^{n(\lambda_a-\lambda_0)})}{\log\left(\frac{\lambda_a}{\lambda_0}\right)}}_{\text{constant}}$$

b. $$\alpha = P\left\{\sum_{i=1}^{n} Y_i > \frac{\log(k\cdot e^{n(\lambda_a-\lambda_0)})}{\log\left(\frac{\lambda_a}{\lambda_0}\right)} \,\middle|\, H_0\right\}$$

$$= 1 - P\left\{\sum_{i=1}^{n} Y_i \le C \,\middle|\, H_0\right\}$$

$$= 1 - \sum_{n=0}^{C}\frac{e^{-n\lambda}\cdot(n\lambda)^{x}}{x!}$$

So by this info we can get the costant $\frac{\log(k\cdot e^{n(\lambda_a-\lambda_0)})}{\log\left(\frac{\lambda_a}{\lambda_0}\right)}$.

c. Yes, it is. They both have the same hypothesis.

d.

$$\frac{L(\lambda_a)}{L(\lambda_0)} > k$$

$$\Rightarrow \frac{\dfrac{e^{-n\lambda_a} \cdot \lambda_a^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} (y_i!)}}{\dfrac{e^{-n\lambda_0} \cdot \lambda_0^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} (y_i!)}} > k$$

$$\lambda_a < \lambda_0$$

$$\lambda_a - \lambda_0 < 0$$

$$\Rightarrow e^{-n(\lambda_a - \lambda_a)} \cdot \left(\frac{\lambda_a}{\lambda_0}\right)^{\sum_{i=1}^{n} y_i} < k$$

$$\Rightarrow \left(\frac{\lambda_a}{\lambda_0}\right)^{\sum_{i=1}^{n} y_i} < k e^{n(\lambda_a - \lambda_0)}$$

$$\overset{\log}{\Rightarrow} \sum_{i=1}^{n} y_i \cdot \log\left(\frac{\lambda_a}{\lambda_0}\right) < \log\left(k \cdot e^{n(\lambda_a - \lambda_0)}\right)$$

$$\Rightarrow \sum_{i=1}^{n} y_i < \underbrace{\frac{\log\left(k \cdot e^{n(\lambda_a - \lambda_0)}\right)}{\log\left(\frac{\lambda_a}{\lambda_0}\right)}}_{\text{constant}}$$

So the RR is $\left\{ \sum_{i=1}^{n} y_i < \dfrac{\log\left(k \cdot e^{n(\lambda_a - \lambda_0)}\right)}{\log\left(\frac{\lambda_a}{\lambda_0}\right)} \,\middle|\, H_0 \right\}$.