Math 174E
Lecture 3

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References

Hull

Chapters 1.6, 1.7, 1.8, 1.9, 1.10

Hedging Example 1/4

Hedging using a **forward contract** (recall also Example 1.7):

Example 1.13 (Forward contract)

On May 3, 2016, a U.S. company (based in the U.S.) knows that it will receive £30 million 3 months later, on August 3, 2016, for exporting goods to the U.K.

The company can hedge its foreign exchange risk by selling £30 million in the 3-month forward market at an exchange rate of \$1.4547 per GBP (see the forward quotes in the table in Example 1.6 above).

This has the effect of **locking in** the U.S. dollars to be received for the sterling at $\$1.4547 \cdot 30,000,000 = \$43,641,000$.

Hedging Example 2/4

Remarks:

- Forward contracts are designed to neutralize (financial) risk by **fixing the price** the company (= hedger) will pay or receive for the underlying asset (price fluctuations = **market risk**).
- Note that the company might do better if it chooses not to hedge; but it also might do worse!
- The purpose of hedging is to reduce risk! There is no guarantee that the outcome with hedging will be better than the outcome without hedging.
- The *purpose of hedging* should also not be confused with *speculating*.

Hedging Examples 3/4

Hedging using **stock options**:

Example 1.14 (Put options)

Suppose an **investor owns 1,000 shares** of a U.S. company currently worth \$28 per share.

She is concerned about a possible share price decline in the next 2 months and wants **protection**. The investor can buy (today at time $t = 0$) ten 2-month put option contracts on the stock with a strike price of \$27.50. Each contract is on 100 shares. Suppose the option price is \$1.

Draw the **market value** of her **stock portfolio** ...

(i) ...without buying puts (*without hedging*),

(ii) ...with buying puts (*with hedging*) and net the premium paid

as a function of the underlying stock price S_T in $T = 2$ months.

See Lecture Notes.

Hedging Example 4/4

Remarks:

- In contrast to hedging with forward contracts, option contracts provide **insurance**.
- They offer a way for investors to protect themselves against adverse price movements in the future while still allowing them to benefit from favorable price movements.
- Recall that options involve the payment of an **up-front fee/premium** (unlike forward contracts).

Speculation Example 1/2

Speculating using **stock options**:

Example 1.15 (Call option)

An investor with **$2,000 to invest** considers that a stock is likely to increase in value over the next 2 months.

The stock price is currently $20, and a 2-month call option with a $22.50 strike price is currently selling for $1.

What are **possible investment strategies** for the speculator? Draw the corresponding investor's **profit and loss (P&L)** in $T = 2$ months as a function of the underlying stock price S_T.

See Lecture Notes.

Speculation Example 2/2

Remarks:

- The call option strategy can be much more profitable than directly buying the stock.
- However, options also give rise to a greater potential loss (maximum possible realized loss = amount paid for the options, i.e., the total money invested, **total loss**)!
- Options provide a form of **leverage**:
  - they magnify the financial consequences (good outcomes become very good, while bad outcomes result in the whole initial investment being lost)
  - they offer the ability to acquire a position (= exposure to an asset) for a fraction of the cost of the actual position (e.g., call option much cheaper than the underlying stock; see, e.g., the quotes in Example 1.9)
- Forward/Futures contracts provide *even more* leverage than options (potential losses/gains can be much bigger).

Arbitrage Example 1/5

Recall from Lecture 1, slide 9:

- **Arbitrage:** to take offsetting positions in two or more instruments to lock in a *riskless profit*.

Example 1.16 (see Chapter 1.9 in Hull)

Consider a stock that is traded on both the New York Stock Exchange and the London Stock Exchange. Suppose that the stock price is \$140 in New York and £100 in London at a time when the exchange rate is \$1.4300 per pound.

Is there a **risk-free profit**, i.e., is there an **arbitrage opportunity**?

This is called **price arbitrage**.

Arbitrage Example 2/5

Remarks:

- Transaction costs (fees, commissions, bid-ask spread) would probably eliminate the profit for a small trader.
- However, a large investment bank faces very low transaction costs in both the stock market and the foreign exchange market, and can try to take as much advantage of it as possible.

- Arbitrage opportunities such as the one described above cannot last for long!
- The very existence of "arbitrageurs" means that in practice only very small arbitrage opportunities are observed in the prices that are quoted in most financial markets.

Arbitrage Example 3/5

Another very simple example of an **arbitrage opportunity**:

Example 1.17

Suppose you can borrow money at an interest rate of 5% per year and you are able to invest money for a guaranteed return of 10% per year.

Is there a **risk-free profit**, i.e., is there an **arbitrage opportunity**?

This example is *extremely unrealistic* but it reveals/summarizes the *basic idea* of *all* arbitrage strategies which can arise in practice.

Important Notion of Arbitrage

Definition used in financial mathematics:

Definition 1.18

An **arbitrage opportunity** is a trading strategy with following two properties:

1. The strategy does not require any initial external capital to implement (**no upfront costs**, **no value**).
2. The strategy always leads to a *non-negative profit* which is also greater than 0 with strictly positive probability (**riskless profit**)

Arbitrage opportunities in practice:

- exploit "mispricing/misvaluation": taking advantage of two or more financial securities being **mispriced relative to each other**;
- once detected and exploited arbitrage opportunities disappear ("like a $10 dollar bill found on the sidewalk").

Fundamental Principle in Derivatives Pricing

- **arbitrage-free pricing** or **no-arbitrage pricing** ("relative pricing principle")
- prices of financial derivatives are determined such that they do **not** allow for **arbitrage** opportunities (**"no free lunch"** or **"no free lunch without any risk"**)

Specifically: As we will see in the upcoming chapters...

- ... **forward prices** of forward contracts (futures prices of futures contracts) are determined in an arbitrage-free manner;
- ... **premiums** of stock **options** are computed as arbitrage-free prices.

The "No-arbitrage principle" is like a "physical law" which needs to be obeyed when determining the "fair" value/price of financial derivatives in financial markets.

Arbitrage Example 4/5

A first glimpse on the idea of **arbitrage-free pricing** of a call option:

Example 1.19

Suppose at time 0 the price of a call option $C_0(K, T)$ on a stock equals the current spot price S_0 of the stock, i.e, $C_0(K, T) = S_0$.

Is this an **arbitrage-free price** for the call option? Or does this price lead to an **arbitrage opportunity** (in the sense of Definition 1.18)?

Arbitrage Example 5/5

Example 1.19 (continued)

Arbitrage strategie if $C_0(K,T) = S_0$:

	$t=0$	$t=T$	
		$S_T > K$	$S_T < K$
sell call (short position)	$-C_0(K,T)$	$-(S_T - K)$	0
buy stock (long position)	$+S_0$	$+S_T$	$+S_T$
net value	0	$+K$	$+S_T$

Compare with Definition 1.18: Strategy does not require any upfront capital and *always* leads to the strictly positive payoff (= value) $\min\{S_T, K\} > 0$.

Note: Call option price is *too high* relative to the stock price.

More on "arbitrage-free properties" of stock option prices in **Chapter 11**.

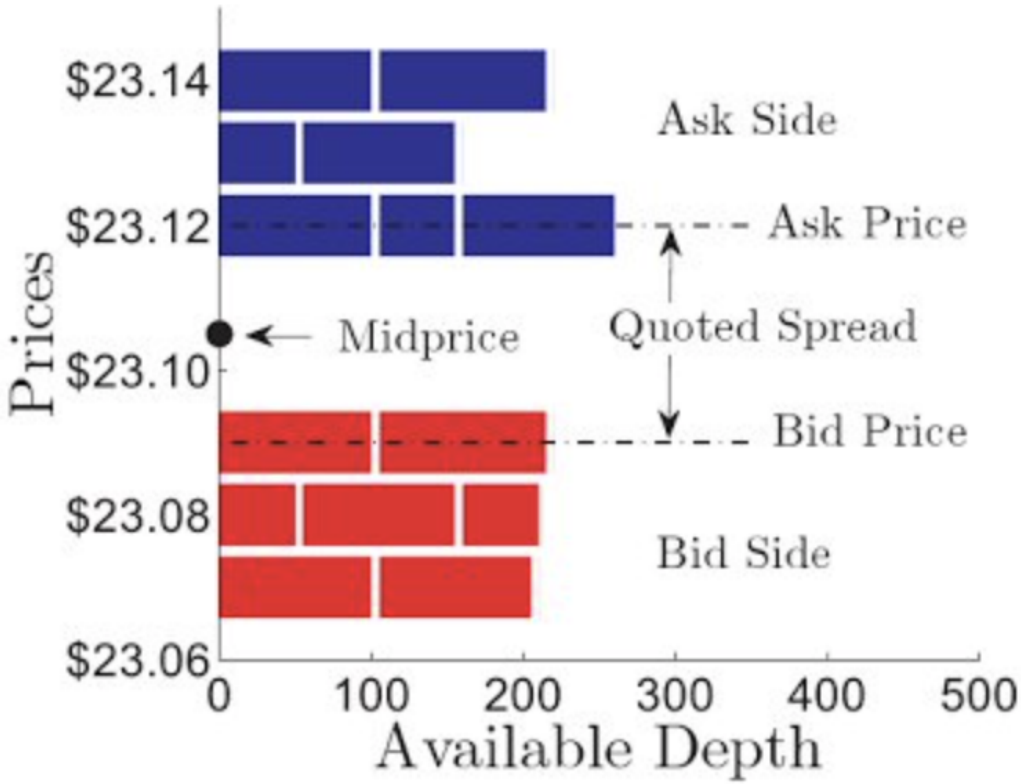
A Short Intro to Limit Order Books 1/2

How does trading in **electronic markets** actually work? **Price discovery** and **liquidity provision** on electronic exchanges:

- two types of orders: **limit order** and **market order**
  - market order = buy/sell a certain quantity *immediately* at the *best available* price (aggressive order)
  - limit order = buy/sell a certain quantity at a *given* price (passive order)
- a **limit order book** (LOB) collects all outstanding limit orders (price quotes plus available volume)
- market orders are executed immediately at the best available prices in the LOB
  - market buy order at the **best offer/ask price**
  - market sell order at the **best bid price**
- **bid-ask spread**: best ask $>$ best bid
- limit orders are not guaranteed to be executed immediately (this depends on their limit price) and they may have to wait in the LOB until they are matched or cancelled

A Short Intro to Limit Order Books 2/2

An illustration (snapshot) of a **limit order book**:



Source of figure: Cartea, A., Sebastian, J. and Penalva, J. (2015) *Algorithmic and High-Frequency Trading*. Cambridge University Press.

Film on the Financial Crisis of 2007–2008

Check out the wikipedia articles:

- `https://en.wikipedia.org/wiki/Financial_crisis_of_2007-2008`
- `https://en.wikipedia.org/wiki/The_Big_Short_(film)`

See also Hull, Chapter 8, on the "Credit Crisis of 2007".