Exam II PSTAT 126 5/19/2022

1(a) $SSreg = \hat{\beta}' X' y - n\bar{y}^2 = 226.3 - 12 \times 4^2 = 34.3$

$SS_{Tot} = S_{yy} = y'y - n\bar{y}^2 = 264 - 12 \times 4^2 = 72$

$RSS = y'y - \hat{\beta}' X' y = 264 - 226.3 = 37.7$

ANOVA

source of variation	SS	df	MS
Regression	34.3	3	11.43
Error	37.7	8	4.7125
Total	72	11	

(b) Reject H_0 if $F > F_{3,8,0.05} = 4.07$

$$F = \frac{MS\,reg}{MSE} = \frac{11.43}{4.7125} = 2.42$$

$\Rightarrow$ Do not reject H_0 $\Rightarrow$ The overall regression is not significant

(c) $$R^2 = \frac{SSreg}{S_{yy}} = \frac{34.3}{72} = 0.4764 = 47.64\%$$

$\Rightarrow$ 47.64% of y-variability was explained by the linear relation

(d) $H_0: \beta_1 = 0$ vs $H_1: \beta_1 < 0$

under $H_0: \beta_1 = 0 \Rightarrow t = \frac{\hat{\beta}_1}{\sqrt{\widehat{Var}(\hat{\beta}_1)}} \sim t_{n-p-1}$

$\Rightarrow$ Reject H_0 if $t < -t_{8,0.05} = 1.86$

$$t = \frac{-0.14}{\sqrt{4.7125}\sqrt{0.088}} = -0.217 \not< -1.86$$

$\Rightarrow$ Do not reject H_0

(e) $H_0: \beta_2 = \beta_3 \quad H_1: \beta_2 \neq \beta_3$

Reject H_0 if $|t| > t_{8, 0.025} = 2.306$

$$t = \frac{(\hat{\beta}_2 - \hat{\beta}_3)}{\sqrt{\widehat{var}(\hat{\beta}_2) + \widehat{var}(\hat{\beta}_3) - 2\,\widehat{cov}(\hat{\beta}_2, \hat{\beta}_3)}}$$

$$= \frac{(0.24 - 1.01)}{\sqrt{4.7125}\sqrt{0.049 + 0.055 - 2\times(-0.003)}}$$

$$= 1.069 \not> 2.306 \Rightarrow \text{Do not reject } H_0$$

(f) $\text{rank}(C) = 2 \qquad \text{row 2} + \text{row 3} = \text{row 1}$

$$C\underset{\sim}{\beta} = \underset{\sim}{0} \iff \begin{aligned} 2\beta_1 + \beta_2 &= 0 \quad (1) \\ 4\beta_2 - \beta_3 &= 0 \quad (2) \\ 2\beta_1 - 3\beta_2 + \beta_3 &= 0 \quad (3) \end{aligned} \Rightarrow 2\beta_1 + \beta_2 = 0$$

use (1) & (2) $\Rightarrow \quad \beta_3 = 4\beta_2 \qquad \beta_2 = -2\beta_1$

$$\Rightarrow \underline{\beta_2 = -2\beta_1} \qquad \underline{\beta_3 = 4\beta_2 = -8\beta_1}$$

$$E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

$$= \beta_0 + \beta_1 X_1 - 2\beta_1 X_2 - 8\beta_1 X_3$$

$$= \beta_0 + \beta_1 (X_1 - 2X_2 - 8X_3)$$

(8)

$$X = \begin{bmatrix} 1 & -1-(2)(2)-(8)(3) \\ 1 & -1-(2)(2)-(8)(3) \\ 1 & -1-(2)(2)-(8)(3) \\ 1 & 0-(2)(1)-(8)(2) \\ 1 & 0-(2)(1)-(8)(5) \\ 1 & 1-(2)(-1)-(8)(5) \\ 1 & 1-(2)(-1)-(8)(5) \\ 1 & (-(2)(-1)-(8)(5) \\ 1 & -1-(2)(-1)-(8)(3) \\ 1 & 2-(2)(2)-(8)(5) \\ 1 & 2-(2)(2)-(8)(9) \\ 1 & -3-(2)(0)-(8)(3) \end{bmatrix} = \begin{bmatrix} 1 & -29 \\ 1 & -29 \\ 1 & -29 \\ 1 & -18 \\ 1 & -42 \\ 1 & -37 \\ 1 & -37 \\ 1 & -37 \\ 1 & -23 \\ 1 & -42 \\ 1 & -74 \\ 1 & -27 \end{bmatrix}$$

(b) Reject H_0 if $F > F_{2,8,0.05} = 4.46$

$$F = \frac{(66.79 - 37.7)/2}{4.9125} = 3.086 \not> 4.46$$

$\Rightarrow$ Do not reject H_0

2. $x_2 = \begin{cases} 1 & \text{if B is in effect} \\ 0 & \text{if not} \end{cases}$ $x_3 = \begin{cases} 1 & \text{if C is in effect} \\ 0 & \text{if not} \end{cases}$

$E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

$\Rightarrow$ Program A: $E(Y) = \beta_0 + \beta_1 x_1$

Program B: $E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 = (\beta_0 + \beta_2) + \beta_1 x_1$

Program C: $E(Y) = \beta_0 + \beta_1 x_1 + \beta_3 = (\beta_0 + \beta_3) + \beta_1 x_1$

(a) $H_0: \beta_2 = \beta_3 = 0$

$RSS = RSS(X_1, X_2, X_3) = 1000 \quad df = n - p - 1 = 9 - 3 - 1 = 5$

$RSS(X_2, X_3 | X_1) = RSS(X_1) - RSS(X_1, X_2, X_3)$

$= 3600 - 1000 = 2600$

Reject H_0 if $F > F_{2,5,0.05} = 5.79$

$$F = \frac{RSS(X_2, X_3 | X_1)/2}{RSS/5} = \frac{2600/2}{1000/5} = \frac{1300}{200} = 6.5 > 5.79$$

$\Rightarrow$ Reject H_0 $\Rightarrow$ Not the same for all 3 programs

(b) $H_0: \beta_2 = 0$ vs $H_1: \beta_2 \neq 0$

$RSS(X_3 | X_1) = RSS(X_1) - RSS(X_1, X_3)$

$1400 = 3600 - RSS(X_1, X_3)$

$\Rightarrow RSS(X_1, X_3) = 2200$

$$RSS(X_2|X_1,X_3) = RSS(X_1,X_3) - RSS(X_1,X_2,X_3)$$

$$= 2200 - 1000 = 1200$$

Reject H_0 if $F > F_{1,5,0.05} = 6.61$

$$F = \frac{RSS(X_2|X_1,X_2)}{MSE} = \frac{1200}{200} = 6 \not> 6.61$$

$\Rightarrow$ Do not reject H_0

(c) $H_0: \beta_3 = 0$ vs $H_1: \beta_3 \neq 0$

$$RSS(X_2|X_1) = RSS(X_1) - RSS(X_1,X_2)$$

$$600 = 3600 - RSS(X_1,X_2)$$

$$\Rightarrow RSS(X_1,X_2) = 3000$$

$$\Rightarrow RSS(X_3|X_1,X_2) = RSS(X_1,X_2) - RSS(X_1,X_2,X_3)$$

$$= 3000 - 1000 = 2000$$

Reject H_0 if $F > F_{1,5,0.05} = 6.61$

$$F = \frac{RSS(X_3|X_1,X_2)}{MSE} = \frac{2000}{200} = 10 > 6.61$$

$\Rightarrow$ Reject $H_0 \Rightarrow$ A & C are different.