

- Define the ANOVA procedure in your own words. How does the model detect a difference in means by comparing variances?

1. List control variables and observable variable.
2. Find variance of observable variables.
3. Compare SSE of the Between groups

- Write equations for the following in a general one-way ANOVA
 - The total sum of squares:
 - The sum of squares for treatment:
 - The sum of squares for error:

$$\begin{aligned} \text{Total SS} &: \sum_{i=1}^k \sum_{j=1}^{h_i} (Y_{ij} - \bar{Y})^2 \\ SST &: \sum_{i=1}^k n_i (\bar{Y}_i - \bar{Y})^2 \\ SSE &: \sum_{i=1}^k \sum_{j=1}^{h_i} (Y_{ij} - \bar{Y}_i)^2 \end{aligned}$$

- What are the null and alternative hypotheses for a one-way ANOVA?

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 = \dots = \mu_k \\ H_A &: \mu_i \neq 0, \quad i \in [1, k] \end{aligned}$$

- What assumptions should be met when we conduct an ANOVA F-test?

The Assumption is Var is same of 2 groups

- Write the general statistical model for a one-way ANOVA.

$$\begin{aligned} Y_{ij} &= \mu_i + \epsilon_{ij} \quad \text{where } \epsilon_{ij} \sim N(0, \sigma^2) \\ \mu_i &= \mu_{GM} + \gamma_i \quad \text{where } \gamma_1 + \gamma_2 + \dots + \gamma_k = 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} Y_{ij} &= \mu_i + \epsilon_{ij} \\ \mu_i &= \mu_{GM} + \gamma_i \end{aligned}} \right\} Y_{ij} = \mu_{GM} + \gamma_i + \epsilon_{ij}$$

- Write the general statistical model for a two-way ANOVA.

$$Y_{ij} = \mu_{GM} + \gamma_i + \beta_j + \epsilon_{ij}$$

1. The Florida Game and Fish Commission desires to compare the amounts of residue from three chemicals found in the brain tissue of brown pelicans. Independent random samples of ten pelicans each yielded the accompanying results (measurements in parts per million). Is there evidence of sufficient differences among the mean residue amounts at the 5% level of significance?

	Chemical		
Statistic	DDE	DDD	DDT
Mean	.032	.022	.041
Standard deviation	.014	.008	.017

$$MST = \frac{SST}{k-1} = \frac{0.00199}{2} = 0.000945$$

$$MSE = \frac{SSE}{n-k} = \frac{0.00494}{27} = 0.000182$$

$$F = \frac{MST}{MSE} = 5.192$$

$$F_{\alpha=0.05, \nu_1=2, \nu_2=27} = 3.35 \Rightarrow F > F_{\alpha} \quad \text{Reject } H_0$$

There is sufficient amount
evidence of difference among
the mean residue amounts

2. It has been hypothesized that treatments (after casting) of a plastic used in optic lenses will improve wear. Four different treatments are to be tested. To determine whether any differences in mean wear exist among treatments, 28 casting from a single formulation of the plastic were made and 7 castings were randomly assigned to each of the treatments. Wear was determined by measuring the increase in "haze" after 200 cycles of abrasion (better wear being indicated by smaller increases). The data collected are reported in the accompanying table.

Treatment			
A	B	C	D
9.16	11.95	11.47	11.35
13.29	15.15	9.54	8.73
12.07	14.75	11.26	10.00
11.97	14.79	13.66	9.75
13.31	15.48	11.18	11.71
12.32	13.47	15.03	12.45
11.78	13.06	14.86	12.38

- (a) Is there evidence of a difference in mean wear among the four treatments? Use $\alpha = 0.05$.
 (b) Estimate the mean difference in haze increase between treatments B and C using a 99% confidence interval.
 (c) Find a 90% confidence interval for the mean wear for lenses receiving treatment A.

a) $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
 $H_a: \mu_i \neq \mu_j \quad j, i \in [1, 4]$

From R, $F = 4.85 \quad \Pr(>|t|) = 0.0089$

Since P-value smaller than 0.05 \Rightarrow Reject H_0

b) $\bar{y}_2 = 14.09 \quad \bar{y}_3 = 12.429 \quad S = 2.5118$

99% CI given by

$$\begin{aligned}
 & (\bar{y}_2 - \bar{y}_3) \pm t_{0.005, 24} \sqrt{S^2 \left(\frac{1}{7} + \frac{1}{7} \right)} \\
 & = 1.66429 \pm 2.369 \\
 & = (-0.705, 4.034)
 \end{aligned}$$

c) $\bar{y}_1 = 11.9857 \quad 90\% \text{ CI given by } \bar{y}_1 \pm t_{0.05, 24} \sqrt{\frac{S}{n_1}}$
 $= 11.9857 \pm 1.02492$
 $= (10.9608, 13.0106)$

3. Fill in the blanks in the following two-way ANOVA table, using the information provided:

Source	SS	df	MS	F
Block		7		14.5
Treatment	5797.5		1932.5	
Interaction	11363.1			
Error	14841.6		154.6	
Total		127		

	SS	df	MS	F
Block	15691.9	7	2241.7	14.5
Treatment	5797.5	3	1932.5	12.5
Interaction	11363.1	21	541.1	3.5
Error	14841.6	96	154.6	
Total	47694.1	127		