Math 174E
Lecture 2

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References

Hull

Chapters 1.3, 1.4, 1.5

Some More General Terminology 1/3

Definition 1.3

An **ask price** (also called offer price) is a market price at which a *seller* is willing to sell one unit of an asset. The *lowest* ask price is the **best ask price**.

A **bid price** is a market price at which a *buyer* is willing to buy one unit of an asset. The *highest* bid price is the **best bid price**.

The difference between the best ask price and the best bid price is called the **bid-ask spread**:

$$\text{bid-ask spread} = \text{best ask price} - \text{best bid price} > 0$$

Note:

- for *immediate* purchase, buyer has to pay the ask price
- for *immediate* sale, seller receives the bid price

Some More General Terminology 2/3

Crucial for analyzing and valuing financial derivatives are their **payoff functions** at maturity:

Definition 1.4

The **payoff function** of a *position in financial securities* (assets, derivatives) describes the **market value** of the position in terms of the market value of the underlyings (mark-to-market). The value can be positive or negative.

We always formalize positions in financial securities as

- **long positions**
  - e.g. buying and holding an asset, long position in a forward contract, long position in a call option, ...
- **short positions**
  - e.g. short selling assets (discussed later!), borrowing cash, short position in a forward contract, selling/writing a call option,...

Some More General Terminology 3/3

Simple illustrative example:

Example 1.5

An investor holds (= owns) 10 shares of Apple Inc. stock. We say that the investor has a **long position** in Apple stocks. The current share price is \$160. The current market value of the investor's long position is therefore

$$(+10) \cdot \$160 = +\$1,600.$$

This is the **payoff** the investor would receive if she were to sell all her shares in the market.

More generally, denote by S_T the stock price at time T. Then, the mark-to-market value of the investor's long position is given by the **payoff function**

$$(+10) \cdot S_T.$$

Forward Contract on a Currency 1/2

Forward contracts on foreign exchange are very popular (but also commodities, interest rates etc.)

Example 1.6 (Spot and Forward quotes)

Spot and forward quotes for the GBP/USD exchange rate made by a large investment bank on May 3, 2016 (GBP = British pound; USD = U.S. dollar; quote is number of USD per GBP)

	Bid	Ask/Offer
Spot	\$1.4542	\$1.4546
1-month forward	\$1.4544	\$1.4548
3-month forward	\$1.4547	\$1.4551
6-month forward	\$1.4556	\$1.4561

Source of table: Hull, Chapter 1.3, Table 1.1, page 6.

exchange rate quote = price of one unit of the foreign currency (**base currency**) quoted in U.S. dollars (**quote currency**)

Forward Contract on a Currency 2/2

Forward contracts can be used to **hedge foreign currency risk**.

Example 1.7

Suppose that, on May 3, 2016, the treasurer of a U.S. corporation knows that the corporation will pay £1 million in 6 months (i.e., on November 3, 2016).

In order to hedge against exchange rate moves the treasurer can agree to buy £1 million 6 months forward at an exchange rate of \$1.4561 per British pound from the bank (see table in Example 1.6).

The corporation has a long position in the forward contract and the bank has a short position in the forward contract.

What is the **payoff function** of the forward contract in 6 months for both parties?

See Exercise 1.2 and Lecture Notes.

Payoff Functions from Forward Contracts

Recall notation:

- $(S_t)_{0 \le t \le T}$ = underlying asset's spot price process
- S_t = spot price of the asset at time $t \in [0, T]$
- F = forward price at time 0 with maturity T (alternatively, more precisely $F_0(T)$)

Payoff at maturity T of a forward contract on *one unit* of the underling asset (= **value** of the forward contract at maturity, **mark-to-market**):

- **long position** in forward contract: $S_T - F$
- **short position** in forward contract: $-(S_T - F) = F - S_T$

Note: Instead of exchanging the underlying asset at time T at the forward price F (physical delivery), long and short position can also *equivalently* just exchange the payoff (cash settlement).

Example 2: Futures Contracts

- very similar to forward contracts: *agreement between two parties to buy or sell an asset for a certain price* (**futures price**) *at a certain time in the future*
- unlike forward contracts, futures contracts are **traded on an exchange** and positions can be **closed out** prior to maturity
- exchange specifies standardized features of the contract
- there are payments **during the lifetime of the futures contract** (margin requirements, **daily settlement** procedures)
- the **exchange clearing house** matches buyers and sellers and clears the trade between the two parties
- traded on a wide range of assets: commodities, stock indices, Treasury bonds, currencies, cryptocurrencies, ...
- the mechanism of futures markets will be discussed in **Chapter 2**

Example 3: Options

Definition 1.8

A **call option** gives the *holder/buyer of the option* the **right to buy** a certain asset by a certain date for a certain price from the *seller/writer of the option*.

A **put option** gives the *holder/buyer of the option* the **right to sell** a certain asset by a certain date for a certain price to the *seller/writer of the option*.

The price in the contract is called the **exercise price** or **strike price** K; the date in the contract is called the **expiration date** or **maturity** T.

American options can be exercised at any time up to the expiration date. **European options** can be exercised only on the expiration date itself.

Most important example: **Stock options**

Options vs. Forwards/Futures

- a futures/forward contract gives the holder the **obligation** to *buy* or *sell* the underlying asset at a certain price
- an option gives the holder the **right** (but not the obligation) to *buy* or *sell* the underlying asset at a certain price

- as a consequence the *buyer* of an option has to pay a **premium** (**price of the option**) to the seller of the option
- in contrast, recall that it does not cost anything to enter into a forward/futures contract

Trading Stock Options 1/2

- options are traded on exchanges and over-the-counter
- largest exchange in the world for trading stock options (equity options) is the Chicago Board Options Exchange (CBOE; www.cboe.com)
- in the exchange-traded equity option market, one **option contract** is usually an agreement to buy or sell 100 shares of the underlying stock (1 contract = 100 options)

- European options are generally easier to analyse than American options
- most of the options that are traded on exchanges are American

Here, in this introductory Chapter 1, we focus for simplicity only on European options.

Trading Stock Options 2/2

Four types of participants in options markets:

1. buyers of calls;
2. sellers of calls;
3. buyers of puts;
4. sellers of puts.

Some terminology (compare with slide 4 above):

- buyer/holder of the option: **long position**
- seller/writer of the option: **short position**
- selling an option = **writing an option**

Notice:

- long position in call/put has the right to buy/sell (right to exercise)
- short position in call/put must sell/buy if the long position decides to exercise her right

Purpose of Trading Options

Long position in a call:

- *hedging:* the holder of the option has the guarantee that she pays at most K for the underlying asset (protection against increase in price)
- *speculation:* the holder of the option speculates that the price will go up above K

Long position in a put:

- *hedging:* the holder of the option has the guarantee that she can sell the asset for at least K (protection against decrease in price)
- *speculation:* the holder of the option speculates that the price will go down below K

And what about the **short positions** in a call/put?

Options on a Stock 1/2

Example 1.9 (Call option quotes)

Prices of call options on Alphabet Inc. (Google), May 3, 2016; stock price: bid \$695.86, ask \$696.25. (Source: CBOE)

Strike Price	June 2016		September 2016		December 2016	
(\$)	Bid	Ask	Bid	Ask	Bid	Ask
660	43.40	45.10	60.80	62.70	72.70	76.70
680	29.90	30.60	47.70	50.70	60.90	64.70
700	18.30	18.90	37.00	39.20	49.70	52.50
720	9.90	10.50	27.50	29.50	40.10	42.80
740	4.70	5.20	19.80	21.60	31.40	34.40

Source of table: Hull, Chapter 1.5, page 9.

(actual expiration day is the third Friday of the expiration month)

Options on a Stock 2/2

Example 1.10 (Put option quotes)

Prices of put options on Alphabet Inc. (Google), May 3, 2016; stock price: bid \$695.86, ask \$696.25. (Source: CBOE)

Strike Price	June 2016		September 2016		December 2016	
(\$)	Bid	Ask	Bid	Ask	Bid	Ask
660	7.50	8.20	24.20	26.20	35.60	38.10
680	13.30	14.00	31.90	33.80	43.40	46.00
700	21.70	23.00	40.80	42.70	52.40	55.20
720	33.10	34.80	51.10	53.20	62.60	65.20
740	47.70	49.60	63.10	65.20	74.10	76.70

Source of table: Hull, Chapter 1.5, page 9.

(actual expiration day is the third Friday of the expiration month)

Some Notation

- K = strike price ($K > 0$)
- T = maturity ($T > 0$)
- $(S_t)_{0 \leq t \leq T}$ = underlying's price process

- $C_0(K, T)$ = price/value (**premium**) of a European call option at time $t = 0$ with strike price K and maturity T
- $P_0(K, T)$ = price/value (**premium**) of a European put option at time $t = 0$ with strike price K and maturity T

Payoffs of Option Positions 1/2

Payoff (= **option position's market value**) at maturity T:

- long position in a call

$$(S_T - K)^+ = \max\{S_T - K, 0\} = \begin{cases} S_T - K & S_T > K \\ 0 & S_T \leq K \end{cases}$$

- long position in a put

$$(K - S_T)^+ = \max\{K - S_T, 0\} = \begin{cases} K - S_T & S_T < K \\ 0 & S_T \geq K \end{cases}$$

Payoffs of Option Positions 2/2

Payoff (= **option position's market value**) at maturity T:

- short position in a call

$$-(S_T - K)^+ = -\max\{S_T - K, 0\} = \begin{cases} K - S_T & S_T > K \\ 0 & S_T \leq K \end{cases}$$

- short position in a put

$$-(K - S_T)^+ = -\max\{K - S_T, 0\} = \begin{cases} S_T - K & S_T < K \\ 0 & S_T \geq K \end{cases}$$

Net Profit of Option Positions

Definition 1.11

The **net profit** (net profit and loss, net P&L) of a *position in financial securities* (assets, derivatives) is the difference between the payoff and the set-up cost.

Net profit at maturity T:

- long position in a call: $(S_T - K)^+ - C_0(K, T)$
- long position in a put: $(K - S_T)^+ - P_0(K, T)$
- short position in a call:
  $-(S_T - K)^+ - (-C_0(K, T)) = C_0(K, T) - (S_T - K)^+$
- short position in a put:
  $-(K - S_T)^+ - (-P_0(K, T)) = P_0(K, T) - (K - S_T)^+$

Break-even point at maturity T:

- call: $S_T = K + C_0(K, T)$
- put: $S_T = K - P_0(K, T)$

Example: Stock Option

Example 1.12

Draw the **payoff function** and the **net profit** at maturity T as a function of the underlying final stock price S_T for following two trades (assume the options to be European): On May 3, 2016, ...

(a) ...a trader buys one December call option contract (= 100 options) on Google with a strike price of \$700.

(b) ...a trader sells one September put option contract (= 100 options) on Google with a strike of \$660.

Use the values from Example 1.9 and 1.10.

See Lecture Notes.