Math 174E
Lecture 5

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References

Hull
Chapters 3.3

Hedging Risks

Risk factors with hedging with futures:

1.) The hedger may not know the exact delivery date and the exact quantity of the asset to be bought or sold.

2.) The hedge may require the futures contract to be closed out before its delivery month.

3.) The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract.
$\rightsquigarrow$ **cross hedging**

These factors give rise to what is called **basis risk**.

Basis and Basis Risk

Definition 3.3

The **basis** in a hedging situation is defined as follows:

$$\begin{aligned}\text{Basis} = &\text{ Spot price of asset to be hedged} \\ &- \text{ Futures price of contract used}^{*}.\end{aligned}$$

* Asset underlying the futures contract is not necessarily the asset to be hedged.

Definition 3.4

The risk to a hedger arising from uncertainty about the **basis** *when the hedge is closed out* is called **basis risk**.

Notation

- t_1 : time when the hedge is put in place
- t_2 : time when hedge is closed out $(t_1 < t_2)$

For $i = 1, 2$:

- S_{t_i} = spot price at time t_i of asset to be hedged
- F_{t_i} = futures price at time t_i of contract used
- $b_{t_i} = S_{t_i} - F_{t_i}$ = basis at time t_i

Important: Basis $b_{t_2} \neq 0$ at time t_2 unless . . .

- . . . the asset to be hedged and the asset underlying the futures contract are the same and . . .
- . . . t_2 is very close to the expiration day of the futures contract.

Formalism 1/2

Short hedge:

- hedger is *selling* an asset at time t_2 at spot price S_{t_2}
- hedger takes a short futures position at time t_1 with futures price F_{t_1}
- hedger closes out futures position at time t_2 with futures price F_{t_2}
- hedger's *effective price received* from selling the asset (with hedging) is

$$S_{t_2} + \underbrace{(F_{t_1} - F_{t_2})}_{\substack{\text{profit on short} \\ \text{futures position}}} = F_{t_1} + (S_{t_2} - F_{t_2}) = F_{t_1} + b_{t_2}$$

Formalism 2/2

Long hedge:

- hedger is *buying* an asset at time t_2 at spot price S_{t_2}
- hedger takes a long futures position at time t_1 with futures price F_{t_1}
- hedger closes out futures position at time t_2 with futures price F_{t_2}
- hedger's *effective price paid* for buying the asset (with hedging) is

$$S_{t_2} - \underbrace{(F_{t_2} - F_{t_1})}_{\substack{\text{profit on long} \\ \text{futures position}}} = F_{t_1} + (S_{t_2} - F_{t_2}) = F_{t_1} + b_{t_2}$$

Comments

Price at time t_2 which is "locked in" at time t_1 with hedging is

$$F_{t_1} + b_{t_2}$$

Notice:

- at time t_1 the value F_{t_1} is known but the basis b_{t_2} is **unknown**
- uncertainty of b_{t_2} = **basis risk**
- perfect hedge: $b_{t_2} = 0$
  (perfect in the sense that the hedger knows at time t_1 that the locked in price for selling/buying is exactly equal to the futures price F_{t_1})

Basis Risk Examples 1/2

Example 3.5 (Short hedge)

It is March 1. A U.S. company expects to receive 50 million Japanese yen at the end of July. Yen futures contracts are traded on an exchange with delivery months March, June, September, and December. One contract is for delivery of 12.5 million yen.

The company decides to short four September yen futures contracts on March 1. When the yen are received at the end of July, the company closes out its position.

See Notes Lecture 5 for a numerical discussion.

Basis Risk Examples 2/2

Example 3.6 (Long hedge)

It is June 8 and a company knows that it will need to purchase 20,000 barrels of crude oil at some time in October or November. Oil futures contracts are traded on an exchange for delivery every month and the contract size is 1,000 barrels.

The company decides to use the December contract for hedging and takes a long position in 20 December futures contracts. On November 10 the company is ready to purchase the crude oil and closes out its futures contracts on that date.

See Notes Lecture 5 for a numerical discussion.