EXAM II

PSTAT 126 HSU 5/19/2022 5:00 - 6:15 p.m.

Fully explain your answer. Answers with no explaination will not receive credit.

Please submit your answers online by 6:30 p.m. California time. Late answers will not be accepted.

(70%) 1. The model $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + e$ ($\underset{\sim}{Y} = X\underset{\sim}{\beta} + \underset{\sim}{e}$) is postulated in an experiment. After the data is collected the following partial calculations are made:

$$X = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & -1 & 2 & 3 \\ 1 & -1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 5 \\ 1 & 1 & -1 & 5 \\ 1 & 1 & -1 & 5 \\ 1 & 1 & -1 & 5 \\ 1 & -1 & -1 & 3 \\ 1 & 2 & 2 & 5 \\ 1 & 2 & 2 & 9 \\ 1 & -3 & 0 & 3 \end{bmatrix} \qquad \underset{\sim}{y} = \begin{bmatrix} 0 \\ 6 \\ 3 \\ 2 \\ 5 \\ 1 \\ 7 \\ 4 \\ 3 \\ 5 \\ 9 \\ 3 \end{bmatrix} \qquad \underset{\sim}{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

$$X'X = \begin{bmatrix} 12 & 0 & 8 & 51 \\ 0 & 24 & 0 & 22 \\ 8 & 0 & 26 & 35 \\ 51 & 22 & 35 & 255 \end{bmatrix} \qquad (X'X)^{-1} = \begin{bmatrix} 1.091 & 0.214 & -0.021 & -0.234 \\ 0.214 & 0.088 & 0.002 & -0.051 \\ -0.021 & 0.002 & 0.049 & -0.003 \\ -0.234 & -0.051 & -0.003 & 0.055 \end{bmatrix}$$

$$\underset{\sim}{y}'\underset{\sim}{y} = 264, \qquad \bar{y} = 4, \qquad X'\underset{\sim}{y} = \begin{bmatrix} 48 \\ 19 \\ 38 \\ 240 \end{bmatrix} \qquad \underset{\sim}{\hat{\beta}} = \begin{bmatrix} -0.47 \\ -0.14 \\ 0.24 \\ 1.01 \end{bmatrix}$$

(10%) (a) Complete the ANOVA table

SOURCE	SS	DF	MS
REGRESSION			
ERROR			4.7125
TOTAL			

(10%) (b) Is the overall regression significant? That is, test $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus H_1 : not so, with $\alpha = 0.05$.

(10%) (c) Compute R^2 and interpret your result.

(10%) (d) Test the hypothesis $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 < 0$, with $\alpha = 0.05$.

(10%) (e) Test the hypothesis $H_0 : \beta_2 = \beta_3$ versus $H_1 : \beta_2 \neq \beta_3$ with $\alpha = 0.05$.

(5%) (f) Consider the hypothesis $H_0 : C\underset{\sim}{\beta} = \underset{\sim}{0}$, where

$$C = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 2 & -3 & 1 \end{bmatrix}.$$

Express the reduced model implied by H_0 in terms of X_1, X_2 and X_3.

(5%) (g) Write the design matrix X for the reduced model in (f).

(10%) (h) Suppose that the Residual Sum of Squares, RSS = 66.79 for the reduced model in (f). Test the hypothesis $H_0 : C\underset{\sim}{\beta} = \underset{\sim}{0}$, with $\alpha = 0.05$.

(30%) 2. A company is studying three different safety programs in an attempt to reduce the number of work-hours lost due to accidents. Each program is to be tried at three of the company's nine factories, and the plan is to monitor the lost work-hours, y, for a 1 year period beginning 6 months after the new safety program is instituted. The proposed model is

$$E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

where y = total work-hours lost due to accidents for a 1 year period beginning 6 months after the plan is instituted. x_1 = total work-hours lost due to accidents during the year before the plan was instituted. $x_2 = 1$, if program B is in effect; and 0, otherwise. $x_3 = 1$, if program C is in effect; and 0, otherwise. After the programs have been in effect for 18 months, the model is fit to the data. The following summary statistics are reported:
$RSS(X_1) = 3600$, $RSS(X_2|X_1) = 600$, $RSS(X_3|X_1) = 1400$,
$RSS = RSS(X_1, X_2, X_3) = 1000$, and $SYY = \sum_{i=1}^{9}(y_i - \bar{y})^2 = 4600$.

(10%) (a) Test whether the mean work-hours lost differ for the three programs. Use $\alpha = 0.05$.

(10%) (b) Test whether the mean work-hours lost differ for programs A and B. Use $\alpha = 0.05$.

(10%) (c) Test whether the mean work-hours lost differ for programs A and C. Use $\alpha = 0.05$.