

$$dX_t = 3X_t^{2/3} dW_t + 3X_t^{1/3} dt$$

Idea: choose f so that

$$f'(X_t) 3X_t^{2/3} dW_t = dW_t.$$

This f is $f(X_t) = X_t^{1/3}$,

$$\text{then } f'(X_t) = \frac{1}{3} X_t^{-2/3}, \quad f''(X_t) = -\frac{2}{9} X_t^{-5/3}$$

Then: apply Itô's formula to this f :

$$d(X_t^{1/3}) = df(X_t)$$

$$= f'(X_t) 3X_t^{2/3} dW_t + f''(X_t) 3X_t^{1/3} dt$$

$$\begin{aligned}
& + \frac{1}{2} f''(X_t) (3X_t^{2/3})^2 dt \\
& = \left(\frac{1}{3} X_t^{-2/3} \right) (3X_t^{2/3}) dW_t + \left(\frac{1}{3} X_t^{-2/3} \right) (3X_t^{1/3}) dt \\
& \quad + \left(\frac{1}{2} \right) \left(-\frac{2}{9} X_t^{-5/3} \right) (9X_t^{4/3}) dt
\end{aligned}$$

$$= dW_t + X_t^{-1/3} dt + X_t^{-1/3} dt$$

$$= dW_t.$$

We've shown $dX_t^{1/3} = dW_t$, so

$$(*) \quad X_t^{1/3} = W_t$$

This suggests that $X_t = W_t^3$.

Now: to verify this, apply Itô's formula
to $g(W_t) = W_t^3$.