

Homework 1

PSTAT 120C

Summer 2022 Session B

Reading

The purpose of this portion of the assignment is to guide your reading and help you generate concise reading notes that list the key concepts – generally, terminology, definitions, and theorems. For the submission, treat each bullet point as an exercise and submit your ‘answers’ as you would a problem set.

- Define **deterministic** and **probabilistic** mathematical models. Give an example of each.
- Write the general equation for a **simple linear regression** model.
- Describe, in your own words, the overall concept of the **method of least squares**.
- State the **least-squares estimators** for the simple linear regression model.
- State the means and variances of the least-squares estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ in simple linear regression.
- State a pair of null and alternative hypotheses for making inferences about **single regression parameters** and **linear functions of the parameters**.

Practice

The purpose of this portion of the assignment is to help you practice applying concepts in the reading, and in some cases, establish results that will be used later on. Remember that you will be graded on problem attempts, not solutions; do your best and ask questions if you get stuck.

1. Auditors are often required to compare the audited (or current) value of an inventory item with the book (or listed) value. If a company is keeping its inventory and books up to date, there should be a strong linear relationship between the audited and book values. A company sampled ten inventory items and obtained the audited and book values given in the accompanying table.

Item	Audit Value (y_i)	Book Value (x_i)
1	9	10
2	14	12
3	7	9
4	29	27
5	45	47
6	109	112
7	40	36
8	238	241
9	60	59
10	170	167

- (a) Fit the model $Y = \beta_0 + \beta_1 x + \epsilon$ to these data, using least squares.
 - (b) Plot the 10 data points and graph the line representing the model.
 - (c) Calculate SSE and S^2 .
 - (d) Do the data present sufficient evidence to indicate that the slope β_1 differs from zero? Conduct a hypothesis test at the 5% significance level.
 - (e) What is your model's estimate for the expected change in audited value per one-unit change in book value?
 - (f) What does your model predict the audited value to be for an item with a book value of \$100?
2. Let β_0 and β_1 be the least-squares estimates for the intercept and slope in a simple linear regression model. Show that the least-squares equation $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ will always go through the point (\bar{x}, \bar{y}) .
 3. Suppose that the model $y = \beta_0 + \beta_1 x + \epsilon$ is fit to the n data points $(y_1, x_1), \dots, (y_n, x_n)$. At what value of x will the length of the prediction interval for y be minimized?