MIDTERM EXAM

Math 174E – Summer Session C 2022
Moritz Voss

Instructions:

- Please submit your exam together with this cover sheet and signature below via **Gradescope** and match your pages with the problems. Alternatively, you can also copy the honor statement below on a separate sheet of paper, sign and date, and upload it on Gradescope together with your exam.
- Submission deadline: **Saturday, August 20, 2022, at 8 a.m. (PT).**
- This is an open notes and open book exam and you are welcome to refer to our lecture slides, lecture notes, discussion notes and assignment sheets.
- This is **not a collaborative exam**. You are expected to take this exam in isolation with no communication with other people. **Communicating with others about the exam problems is considered cheating**.
- You must show all of your work and justify your answers to receive any credit. You may justify your answers by either writing brief sentences explaining your reasoning or annotating your math work with brief explanations.
- You must write legibly and clearly mark the answers you want graded. If the work is illegible or it is unclear which answers you intend to be graded, credit may not be given. I strongly suggest circling, boxing, or writing a sentence indicating your answer.
- Use a calculator to compute numerical values as final answers (if applicable). Round your numerical values to 4 decimal places.
- Academic dishonesty: Any student caught cheating will get 0 points for the midterm. According to University rules, cheating must be reported to the College of Physical Sciences. Further action may be taken.

Please sign the honor statement:

> *I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.*

Signature & Date

Good Luck!

1. (10 points) Today's price of the Apple Inc. (AAPL) stock is $S_0 = \$175$.

   An investor instructs a broker to sell 100 European call options (= 1 call option contract) as well as 100 European put options (= 1 put option contract) written on Apple stock with a strike price of $K = \$175$ and maturity T on September 16, 2022 (that is, in exactly four weeks).

   The European call options are currently trading for \$5.00 per option and the European put options are currently trading for \$4.50 per option.

   (a) What is the payoff function of the investor's position at maturity T as a function of the then prevailing Apple stock price S_T? Sketch the graph of the function and add suitable annotations.

   (b) What is the profit and loss of the investor's position at maturity T as a function of the then prevailing Apple stock price S_T? Sketch the graph of the function and add suitable annotations.

   (c) What is the maximum possible profit and what is the maximum possible loss for the investor at maturity T?

   (d) For which stock prices S_T does the investor make a strictly positive profit?

   (e) What is the investor speculating on with her option strategy?

2. (10 points) Suppose today a corporate treasurer from a U.S. company is saying the following to her investment banker: *"I will have 1 million euros to buy in 6 months. If the exchange rate is more than 1.05 USD per one EUR, I want you to sell me euros for 1.05 USD per one EUR. If it is less than 0.95 USD per one euro, I will accept to pay you 0.95 USD per one EUR. If the exchange rate is between 0.95 and 1.05, I will buy the euros for the exchange rate".*

   Explain how options written on the EUR/USD exchange rate can be used to satisfy the treasurer.

3. (10 points) Today American Airlines (AAL) wishes to hedge its exposure to changes in the jet fuel price by using heating oil futures. It expects to purchase 800,000 gallons of jet fuel in 3 months (on November 15, 2022) in the spot market and decides to use December 2022 heating oil futures for hedging. Historical market price data shows that (over a period of 3 months) changes in the jet fuel spot price have a 0.8 correlation with heating oil futures price changes, and the changes in the heating oil futures price have a standard deviation that is 15% lower than the standard deviation of spot price changes in jet fuel.

   The contract size of heating oil futures is 42,000 gallons, and December 2022 heating oil futures currently trade at \$3.48 per gallon.

   (a) If December 2022 heating oil futures are used to hedge AAL's exposure to changes in jet fuel price, what is the minimum variance hedge ratio?

   (b) What position should AAL take in the heating oil futures? What is the optimal number of futures contracts?

   Suppose that on November 15, 2022, the jet fuel spot price is \$3.50 per gallon, and AAL closes out its position in the futures contracts at the then prevailing December 2022 heating oil futures price of \$4.00 per gallon.

   (c) What would be AAL's total P&L from its trade in the futures contracts on November 15, 2022, in this scenario?

(d) How much money would AAL effectively pay (including its profit or loss from the hedge) to purchase the 800,000 gallons of jet fuel on November 15, 2022, in the spot market in this scenario? Is AAL better off with the hedge?

4. (10 points) The following table gives current prices of U.S. Treasury bonds:

Bond principal ($)	Time to maturity (months)	Annual coupon ($)	Bond price ($)
100	6	0.0	98.49
100	12	0.0	96.87
100	18	3.0	99.64
100	24	3.0	99.38

Half the stated coupon is assumed to be paid every six months.

(a) Calculate the implied zero spot rates (p.a. and continuously compounded) for maturities of 6 months, 12 months, 18 months, and 24 months.

(b) Suppose you can borrow and lend $100 at the zero spot rates computed in (a). Today, someone is offering you to invest money in USD at 4% (p.a. and continuously compounded) for a 6 month period starting in 18 months. Is there an arbitrage opportunity? If yes, explain carefully how the arbitrage strategy looks like and what the arbitrage gain in 24 months would be.

5. (10 points) Consider a forward contract on a non-dividend-paying stock with maturity 12 months. Assume the current stock price is $50 per share and the 1-year risk-free interest rate is 5% per annum (continuously compounded).

(a) Compute today's arbitrage-free forward price of the stock per share.

(b) Show that if the forward price is $55 per share then there exists an arbitrage opportunity. What is the arbitrage gain at maturity?

(c) Show that if the forward price is $45 per share then there exists an arbitrage opportunity. What is the arbitrage gain at maturity?