$\mathbb{X}\ (X_1, \dots, X_d) \overset{iid}{\sim} N($

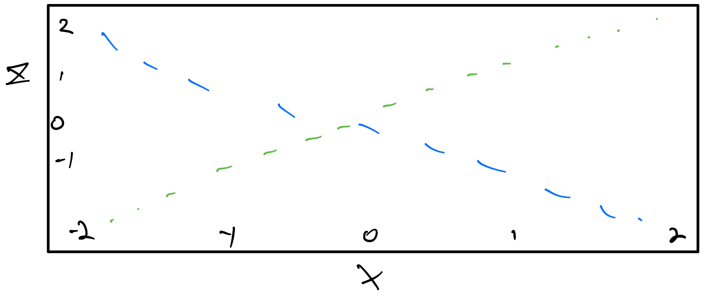
$Y_1 \cdots Y_d$ are jointly normal if
$\mathbb{Y} \doteq (Y_1 \dots Y_d)$ is MVN

Def Explain

$X \sim N(0,1)$ Let $W \overset{d}{\sim}$ Below condition
$\Rightarrow P(W=1) = \frac{1}{2} \qquad P(W=-1) = \frac{1}{2}$
$\Rightarrow W \perp\!\!\!\perp X$
$\Rightarrow Z \doteq WX$

$Z \sim N(0,1)$ (compute mgf of Z using
$E(E(e^{tZ}(W)) = E(e^{tZ})$

$\boxed{\text{is mv Normal?}}$ $(X, Z) \sim N(\mu, \sigma^2)$



Show a vector $a^T \binom{X}{Z}$ is not normally dist'd

Take $a = \binom{1}{1}$ then

$$a^T \binom{X}{Z} = X + Z = X + WX = (1+W)X.$$

How to show $X+Z$ is NOT normal?

$$\boxed{\begin{aligned} &P(X+Z=0) = P(W=-1) = \tfrac{1}{2} \\ &P(X+Z>0) = P(W=1,\ X>0) \\ &\qquad\qquad \leq P(X>0) = \tfrac{1}{2} \\ &\therefore\ X+Z \text{ is not normal RV.} \end{aligned}}$$

Let $\binom{Z}{0}$ be MVN but has no density as Σ^{-1} exist

How to Construct muti Random variable normal.

THM:

Theorem 4.6. *Let $Z_1, Z_2 \overset{iid}{\sim} \mathcal{N}(0,1)$. Then, the bivariate random variable (X, Y) defined by*

$$X \doteq \sigma_X Z_1 + \mu_X, \quad Y \doteq \sigma_Y(\rho Z_1 + \sqrt{1-\rho^2} Z_2) + \mu_Y,$$

follows a $\mathcal{N}_2(\boldsymbol{\mu}, \Sigma)$ distribution, where

$$\boldsymbol{\mu} = (\mu_X, \mu_Y), \quad \Sigma = \begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix}.$$

Theorem 4.7. *Suppose that* (X, Y) *are jointly normal. Then* X *and* Y *are independent if and only if* $Cov(X, Y) = 0$.

Browian Bridge

Pf: if $X \perp\!\!\!\perp Y$ $\quad Cov(X,Y) =$

$$= E(XY) - E(X)E(Y)$$

$$\overset{X \perp\!\!\!\perp Y}{=} E(X)E(Y) - E(X)E(Y)$$

$$= 0$$

Suppose $Cov(X,Y) = 0$

Denote mean vector $(X\ Y)$ by (μ_x, μ_y)

Sigma $\to \Sigma$ by $\begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}$

$Z_1, Z_2 \overset{iid}{\sim} N(0,1)$ define

$\bar{X} = \sigma_x Z_1 + \mu_x$

$\bar{Y} = \sigma_y Z_2 + \mu_y$

THM $\Rightarrow (\bar{X}, \bar{Y}) \overset{d}{=} (X, Y)$

$\bar{X} \perp\!\!\!\perp \bar{Y} \Rightarrow X \perp\!\!\!\perp Y$

Proposition 4.8. *Let $\boldsymbol{X} \sim \mathcal{N}_d(\boldsymbol{\mu}, \Sigma)$, and let $A \in \mathbb{R}^{n\times d}$ be a deterministic matrix. Then, $A\boldsymbol{X} \sim \mathcal{N}_n(A\boldsymbol{\mu}, A\Sigma A^T)$.*

$\Rightarrow$ Linear Transformation of MVN is also MVN.

Definition 4.9. *An $\mathbb{R}$-valued stochastic process $\{W_t\} = \{W_t\}_{t\geq 0}$ is said to be a* ***standard Brownian motion (SBM)*** *or* ***Wiener process*** *if:*

(1) *The increments of $\{W_t\}$ are stationary and independent.*
(2) *For each $t \geq 0$, $W_t \sim \mathcal{N}(0,t)$.*
(3) *$\mathbb{P}(W_t$ is continuous at all $t \geq 0) = 1$.* Continous Sample Path Property

has Continous State space, Continous Everywhere but not differniable

PP State space = discrete

is there a Stochastic Process satisfy all 3 propeties?

There is no process $\{X_t\}$ satisfying all 3 properties.

Ⓐ $\{X_t\}$ iid RVS

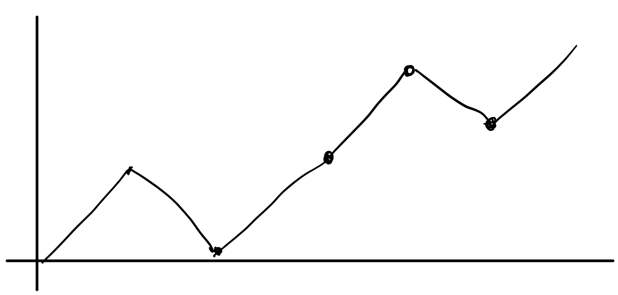
Ⓑ $Var(X_t) > 0$

Ⓒ $P(X_t$ is a Cont @ all $t > 0) = 1$

Simple Random Walk w/ $\{X_i\}$ iid $P(X_i = 1) = P(X_i = -1) = \frac{1}{2}$

Discrete

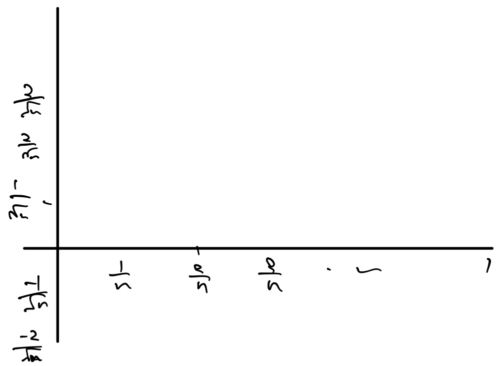
Continous



CLT: $S_n \approx N(0, n)$

$\hookrightarrow$ Var

$\frac{S_n}{\sqrt{n}} \approx N(0,1)$



if n is large

Discrete

$S^n_m = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \xi_i$

Continuous

X_t = Connecting S^n_m

n Jumps in $[0,1]$

Each w/ Size = $\frac{1}{\sqrt{n}}$