Lecture 8: 08/16/22

Proof of Lemma 5.2:

Claim: $F_0(T) = S_0 e^{rT}$ today's arbitrage-free forward price

Case 1: Suppose $F_0(T) > S_0 e^{rT}$

Arbitrage: short position in forward

long position in asset + borrow S_0 @ r

Strategy	today $(t=0)$	maturity (T)
short position in forward @ $F_0(T)$	0	$F_0(T) - S_T$
borrow money (short position)	$-S_0$	$-S_0 e^{rT}$
buy one unit of asset (long position)	$+S_0$	$+S_T$
net value	0	$F_0(T) - S_0 e^{rT} > 0$

$\Rightarrow$ arbitrage strategy

Case 2 : Suppose $F_0(T) < S_0 e^{rT}$

Arbitrage : long position in forward and

short position in asset + investing S_0 @ r

strategy	today ($t=0$)	maturity ($t=T$)
long position in forward @ $F_0(T)$	0	$-(F_0(T) - S_T)$
invest money (long position)	$+S_0$	$+S_0 e^{rT}$
short sell one unit of asset (short position)	$-S_0$	$-S_T$
net value	0	$S_0 e^{rT} - F_0(T) > 0$

$\Rightarrow$ arbitrage opportunity