Lecture 5 : 08/10/22

Example 3.1 Short hedge

Today : May 15 (time t_1)

Oil producer : will sell 1 million barrels of crude oil on August 15 (time t_2)

at the then prevailing market price S_{t_2} (spot price at t_2)

Today's spot price : $S_{t_1} = \$50$ per barrel

Today's August futures price : $F_{t_1} = \$49$ per barrel (one contract = 1,000 barrel)

Short hedge :

At time t_1 : take short position in 1,000 futures contracts

At time t_2 : close out futures position at F_{t_2}

F_{t_2} = August futures price at t_2

S_{t_2} = spot price at time t_2

$\Big\}\ F_{t_2} = S_{t_2}$

Effect of short hedge: lock in a price of $F_{t_1} = \$49$ per barrel

Two possible scenarios at t_2 : (prices in USD per barrel)

	$F_{t_2} = S_{t_2} = 45$	$F_{t_2} = S_{t_2} = 50$
sell price	45	50
P&L on future	$\underbrace{49}_{=F_{t_1}} - \underbrace{45}_{=F_{t_2}} = 4$	$\underbrace{49}_{=F_{t_1}} - \underbrace{50}_{=F_{t_2}} = -1$
net sell price	$45 + \underbrace{4}_{\text{futures P\&L}} = 49 = F_{t_1}$	$50 + \underbrace{(-1)}_{\text{futures P\&L}} = 49 = F_{t_1}$

Example 3.2: Long hedge

Today: January 15 (time t_1)

Copper fabricator: will buy 100,000 pounds of copper on May 15 (time t_2)

at the then prevailing market price S_{t_2} (spot price at t_2)

Today's spot price: $S_{t_1} = \$3.40$ per pound

Today's May futures price: $F_{t_1} = \$3.20$ per barrel (one contract = 25,000 pounds)

Long hedge:

At time t_1: take a long position in 4 futures contracts

At time t_2: close out futures position at F_{t_2}

F_{t_2} = May futures price at t_2

S_{t_2} = spot price at t_2

$\Big\} \; F_{t_2} = S_{t_2}$

that's a simplifying assumption here !!!

Effect of long hedge: lock in a price close to $F_{t_1} = 3.20$ per pound

Two possible scenarios at t_2: (prices in USD per pound)

	$F_{t_2} = S_{t_2} = 3.25$	$F_{t_2} = S_{t_2} = 3.05$
purchase price	3.25	3.05
P & L on future	$\underbrace{3.25}_{=F_{t_2}} - \underbrace{3.20}_{=F_{t_1}} = 0.05$	$\underbrace{3.05}_{=F_{t_2}} - \underbrace{3.20}_{=F_{t_1}} = -0.15$
net purchase price	$3.25 - \underbrace{0.05}_{\text{future's P\&L}} = 3.20 = F_{t_1}$	$3.05 - \underbrace{(-0.15)}_{\text{future's P\&L}} = 3.2 = F_{t_1}$

In any case: Copper fabricant pays $\$320{,}000$. (locked in price $F_{t_1} = 3.2$!!)

Illustration:

buy / sell asset

buy asset at S_{t_2}

sell asset at S_{t_2}

long T-futures contracts w/ F_{t_1}

short T-futures contracts w/ F_{t_1}

P&L from closing out: $F_{t_2} - F_{t_1}$

P&L from closing out: $F_{t_1} - F_{t_2}$

actual price paid: $S_{t_2} - (F_{t_2} - F_{t_1}) = F_{t_1} + \underbrace{(S_{t_2} - F_{t_2})}_{= b_{t_2}}$

actual price received: $S_{t_2} + (F_{t_1} - F_{t_2}) = F_{t_1} + \underbrace{(S_{t_2} - F_{t_2})}_{= b_{t_2}}$

If t_2 is close to T: $F_{t_2} \approx S_{t_2} \iff b_{t_2} \approx 0$

If t_2 is far from T: $b_{t_2} \neq 0$ ($\rightsquigarrow$ basis risk!)

Example 3.5:

March 1: U.S. company expects to sell 50 million Japanese yen end of July

Short hedge: short position in 4 September yen futures contracts (each 12.5 million)

t_1 = March 1

F_{t_1} = 1.0800 cents per yen (September yen futures price)

t_2 = end of July (September yen futures contract closed out)

F_{t_2} = 1.0250 cents per yen

S_{t_2} = 1.0200 cents per yen

P&L on future at t_2: $F_{t_1} - F_{t_2} = 0.0550$ cents per yen

basis at t_2: $b_{t_2} = S_{t_2} - F_{t_2} = -0.0050$

Hedger's effective price received for selling her yen:

$$F_{t_1} + b_{t_2} = 10800 + (-0.0050) = 1.0750 \quad \text{(cents per yen)}$$

(alternatively: $S_{t_2} + \underbrace{(F_{t_1} - F_{t_2})}_{\text{gain of short position in futures contracts}} = 1.0200 + 0.0550 = 1.0750$)

Total amount received by company for selling 50 million yen:

$$50{,}000{,}000 \cdot 0.010750 = \$\,537{,}500$$

Example 3.6:

June 8: company expects to buy 20,000 barrels of crude oil in October or November

Long hedge: Long position in 20 December crude oil futures contracts (each 1,000 barrels)

t_1 = June 8

F_{t_1} = 48 per barrel (December crude oil futures price)

t_2 = November 10 (December crude oil futures contract closed out)

F_{t_2} = 49.10 per barrel

S_{t_2} = 50 per barrel

P&L on future at t_2: $F_{t_2} - F_{t_1} = 1.10$ per barrel

basis at t_2: $b_{t_2} = S_{t_2} - F_{t_2} = 0.9$ per barrel

Hedger's effective price paid for buying her crude oil:

$$F_{t_1} + b_{t_2} = 48 + 0.9 = 48.90 \text{ per barrel}$$

(alternatively: $S_{t_2} - \underbrace{(F_{t_2} - F_{t_1})}_{\text{gain of long position in futures contracts}} = 50 - 1.10 = 48.90$)

Total price paid for 20,000 barrels:

$$20{,}000 \cdot 48.90 = 978{,}000$$