# Foundations of Agents: Practical Assignment 1

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September 26, 2018

### 1 Tower of Hanoi problem

In a Tower of Hanoi problem the agent needs to move disks of different sizes from one pin to another pin. Furthermore, the disks need to be in the order that a smaller disk needs to be on top of a bigger one. Only one disk can be moved at a time and only the topmost disk can be moved. In our problem we have 3 pins and two disks. The starting position is pin 1 and the smaller disk is on the bigger disk. The goal is to move the disks to pin 3.

#### 1.1 Description of States

Notation:

- a = small disk,

• 2 = pin2,

• b = big disk,

• 3 = pin3,

• 1 = pin1,

 $\bullet$  ab = a is on b

We have 12 different possible states:

State		Pin	
$s_0$	ab1	2	3
$s_1$	1	ab2	3
$s_2$	1	2	ab3
$s_3$	ba1	2	3
$s_4$	1	ba2	3
$s_5$	1	2	ba3
$s_6$	b1	a2	3
$s_7$	a1	b2	3
$s_8$	b1	2	a3
$s_9$	a1	2	b3
$s_{10}$	1	a2	b3
$s_{11}$	1	b2	a3

### 1.2 Description of Actions

We have 6 different actions that the agent can take.

Action	effect
$\overline{a_1}$	move a to pin1
$a_2$	move a to pin2
$a_3$	move a to pin3
$b_1$	move b to pin1
$b_2$	move b to pin2
$b_3$	move b to pin3

### 2 Optimal policy

The optimal policy describes for every state the best action the agent should take.

• 
$$\pi(s_0) = a_2$$

• 
$$\pi(s_1) = a_1$$

• 
$$\pi(s_2) = a_2$$

• 
$$\pi(s_3) = b_3$$

• 
$$\pi(s_4) = b_3$$

• 
$$\pi(s_5) = b_1$$
 according to PI and  $b_2$  according to VI

• 
$$\pi(s_6) = b_3$$

• 
$$\pi(s_7) = b_3$$

• 
$$\pi(s_8) = a_2$$

• 
$$\pi(s_9) = a_3$$

• 
$$\pi(s_{10}) = a_3$$

• 
$$\pi(s_{11}) = a_1$$

The difference in  $\pi(s_5)$  arises because of different utilities of the states. Using action  $b_1$  would transition into state  $s_8$  and using action  $b_2$  would transition into state  $s_{11}$ . Below we can see that policy iteration gives the state  $s_8$  a higher utility than  $s_{11}$ , vice versa for value iteration.

## 3 Utility

The utility of each state given the optimal policy. The utility is calculated the following formula:

$$u(s,\pi) = r(s,\pi(s)) + \gamma \cdot \sum_{s' \in S} t(s,\pi(s),s') \cdot u(s',\pi)$$
 (1)

Using the values of the policy-iteration:

• 
$$u(s_0) = 73.59$$

• 
$$u(s_1) = 62.27$$

• 
$$u(s_2) = 0.00$$

• 
$$u(s_3) = 86.00$$

• 
$$u(s_4) = 86.63$$

• 
$$u(s_5) = 53.27$$

• 
$$u(s_6) = 85.91$$

• 
$$u(s_7) = 85.81$$

• 
$$u(s_8) = 74.31$$

• 
$$u(s_9) = 98.79$$

• 
$$u(s_{10}) = 98.79$$

• 
$$u(s_{11}) = 74.11$$

Using value-iteration the following utility-values arise:

• 
$$u(s_0) = 75.31$$

• 
$$u(s_1) = 75.39$$

• 
$$u(s_2) = 0.00$$

• 
$$u(s_3) = 86.75$$

• 
$$u(s_4) = 86.75$$

• 
$$u(s_5) = 66.77$$

• 
$$u(s_6) = 85.93$$

• 
$$u(s_7) = 85.93$$

• 
$$u(s_8) = 74.48$$

• 
$$u(s_9) = 98.79$$

• 
$$u(s_{10}) = 98.79$$

• 
$$u(s_{11}) = 75.39$$

### 4 Conclusion

The policies for value-iteration and policy-iteration differed in one state  $(s_5)$ . But apart from that they were similar. Interestingly, the policy-iteration was significantly faster than value-iteration: value-iteration needed about 0.016s to converge whereas policy-iteration only needed 0.0036s to converge. Furthermore, policy-iteration delivers a more accurate result.

#### 5 Note

The Hanoi.py file requires Python 3.6.