- GEMODELTOOLS -

HANC WITH GOVERNMENT

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Model

We consider a *endowment* economy with heterogeneous agents.

Time is discrete and indexed by t. There is a continuum of households indexed i.

Households. Households are *ex ante* homogeneous, but *ex post* with respect to their productivity, z_{it}, and assets, a_{it-1}. Each period household get stochastic endowment z_{it} of consumption good, and choose consumption c_{it} subject to a no-borrowing constraint, $a_{it} \geq 0$. The households pay proportional taxes τ_t and can trade in government bonds at price p_t^B. Households have *perfect foresight* wrt. to the aggregate variables, $\{p_t^B, \tau_t\}_{t=0}^{\infty}$, and solve the problem

$$\begin{aligned} v_t(z_{it}, a_{it-1}) &= \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v_{it+1}(z_{it+1}, a_{it}) \right] \qquad (1) \\ \text{s.t. } p_t^B a_{it} + c_{it} &= a_{it-1} + (1-\tau_t) z_{it} \geq 0 \\ \log z_{it+1} &= \rho_z \log z_{it} + \psi_{it+1} \ \ , \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_{it}] = 1, \end{aligned}$$

where implicitly $v_t(z_{it}, a_{it-1}) = v(z_{it}, a_{it-1}, \{p_s^B, \tau_s\}_{s=t}^{\infty})$.

We denote optimal policy functions by $a_t^*(\beta_i, z_{it}, a_{it-1})$, and $c_t^*(\beta_i, z_{it}, a_{it-1})$. The distribution of households *before* the realization of idiosyncratic shocks, i.e. over z_{it-1} and a_{it-1}, is denoted $\underline{\boldsymbol{D}}_t$. The distribution of households *after* the realization idiosyncratic shocks, i.e. over z_{it} and a_{it-1}, is denoted $\boldsymbol{D}_t$.

Central aggregate variables are

$$
\begin{aligned}
A_t^{hh} &= \int a_t^*(\beta_i, z_{it}, a_{it-1}) d\boldsymbol{D}_t \qquad (2)\\
&= \boldsymbol{a}_t^{*\prime} \boldsymbol{D}_t \\
C_t^{hh} &= \int c_t^*(\beta_i, z_{it}, a_{it-1}) d\boldsymbol{D}_t \qquad (3)\\
&= \boldsymbol{c}_t^{*\prime} \boldsymbol{D}_t,
\end{aligned}
$$

To solve the model, we define the beginning-of-period value function as

$$
\underline{v}_t \left(z_{it-1}, a_{it-1}\right) = \mathbb{E}_t \left[v_t(z_{it}, a_{it-1})\right], \qquad (4)
$$

The *envelope condition* implies

$$
\underline{v}_{a,t}(z_{it-1}, a_{it-1}) = c_{it}^{-\sigma}, \qquad (5)
$$

The *first condition* for consumption implies

$$
c_{it}^{-\sigma} = \beta \frac{\underline{v}_{a,t+1}(z_{it}, a_{it})}{p_t^B}, \qquad (6)
$$

Government. The government chooses government spending, collect taxes, τ_t, proportional to endowments, and issues bonds, which pays 1 consumption good in the next period. The government budget constraint is

$$
p_t^B B_t = B_{t-1} + G_t - \int \tau_t z_{it} d\boldsymbol{D}_t \qquad (7)
$$

Taxes are set to ensure convergence to steady state debt, as

$$
\tau_t = \tau_{ss} + \eta_t + \varphi \left(B_{t-1} - B_{ss}\right) \qquad (8)
$$

where η_t is a tax-shifter.

Market clearing. Market clearing requires

$$
\text{Bonds: } B_t = A_t^{hh} \qquad (9)
$$

$$
\text{Goods: } C_t^{hh} + G_t = \int z_{it} d\boldsymbol{D}_t = 1 \qquad (10)
$$

Social welfare. Utilitarian social welfare can be calculated as the average expected discounted utility,

$$\mathrm{SWF}_t = \sum_{t=0}^{\infty} \beta^t \int u\left(c_t^*\left(\beta_i, z_{it}, a_{it-1}\right)\right) d\boldsymbol{D}_t = U_t^{hh}. \tag{11}$$

where $\underline{\boldsymbol{D}}_0$ (and therefore B_{-1}) is given.

In steady state this simplifies to

$$\mathrm{SWF}_{ss} = \frac{1}{1-\beta} U_{ss}^{hh}. \tag{12}$$

Equation system

1. Shocks: $\boldsymbol{Z} = \{\boldsymbol{G}\}$
2. Unknowns: $\boldsymbol{U} = \{\boldsymbol{p}^B\}$
3. Targets: $\{B_t - A_t^{hh}\}$ (bond market clearing)
4. Aggregate variables: $\boldsymbol{X} = \{\boldsymbol{G}, \boldsymbol{p}^B, \boldsymbol{\tau}, \boldsymbol{A}^{hh}, \boldsymbol{C}^{hh}, \boldsymbol{U}^{hh}, \boldsymbol{B}\}$
5. Household inputs: $\boldsymbol{X}_t^{hh} = \{\boldsymbol{p}^B, \boldsymbol{\tau}\}$
6. Household outputs: $\boldsymbol{Y}_t^{hh} = \{\boldsymbol{A}^{hh}, \boldsymbol{C}^{hh}, \boldsymbol{U}^{hh}\}$

This implies the equation system

$$\begin{gathered}\boldsymbol{H}(\boldsymbol{p}^B, \boldsymbol{G}) = \boldsymbol{0} \Leftrightarrow \\ \left[\begin{array}{c} B_t - A_t^{hh} \end{array}\right] = \left[\begin{array}{c} 0 \end{array}\right], \ \forall t \in \{0, 1, \ldots, T-1\},\end{gathered} \tag{13}$$

where we have

$$\left[\begin{array}{c} \tau_t \\ B_t \end{array}\right] = \left[\begin{array}{c} \tau_{ss} + \phi \frac{B_{t-1} - B_{ss}}{Y_{ss}} \\ \frac{B_{t-1} + G_t - \tau_t}{p_t^B} \end{array}\right] \text{ (forwards).}$$