

# A HANK-SAM MODEL

## Model

**Households.** The model has a continuum of infinitely lived households indexed by  $i \in [0, 1]$ . Time is discrete and indexed by  $t \in \{0, 1, \dots\}$ . Each period is one month.

The households are *ex ante* heterogeneous in terms of their discount factor  $\beta_i$ . There are three types of households:

1. Hands-too-mouth households with  $\beta_i = \beta^{\text{HtM}}$ .
2. Buffer-stock households with  $\beta_i = \beta^{\text{BS}}$ .
3. Permanent income hypothesis households with  $\beta_i = \beta^{\text{PIH}}$ .

Households are *ex post* heterogeneous in terms of their unemployment status,  $u_{it}$ , and lagged end-of-period savings,  $a_{it-1}$ . If  $u_{it} = 0$  the household is employed. If  $u_{it} > 0$  the household is in its  $u_{it}$ 'th month of unemployment.

Each period the household chooses consumption,  $c_{it}$ , and savings,  $a_{it}$ . Borrowing is not allowed and the utility function is CRRA.

The recursive household problem is

$$\begin{aligned} v_t(\beta_i, u_{it}, a_{it-1}) &= \max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t [v_{t+1}(\beta_i, u_{it+1}, a_{it})] \\ \text{s.t. } a_{it} + c_{it} &= (1 + r_t)a_{it-1} + (1 - \tau_t)y_t(u_{it}) + \text{div}_t + \text{transfer}_t \\ a_{it} &\geq 0 \end{aligned} \tag{1}$$

where  $r_t$  is the ex post return from period  $t - 1$  to  $t$ ,  $y_t(u_{it})$  is labor market income (including unemployment insurance),  $\tau_t$  is the tax rate on labor market income,  $\text{div}_t$  is dividends, and  $\text{transfer}_t$  is a transfer from the government (or a lump-sum tax if negative).

The employment/unemployment transition probabilities are

$$\begin{aligned}
\Pr[u_{it+1} = 0 \mid u_{it} = 0] &= 1 - \delta_t \\
\Pr[u_{it+1} = 1 \mid u_{it} = 0] &= \delta_t \\
\Pr[u_{it+1} > 1 \mid u_{it} = 0] &= 0 \\
\Pr[u_{it+1} = 0 \mid u_{it} > 0] &= \lambda_t^{u,s} s(u_{it-1}) \\
\Pr[u_{it+1} = u_{it} + 1 \mid u_{it} > 0] &= 1 - \lambda_t^{u,s} s(u_{it-1}) \\
\Pr[u_{it+1} \notin \{0, u_{it} + 1\} \mid u_{it} > 0] &= 0
\end{aligned} \tag{2}$$

where  $\delta_t$  is the separation rate,  $\lambda_t^{u,s}$  is the job-finding rate per effective searcher, and  $s(u_{it-1})$  determines the effectiveness of search conditional on unemployment status.

When employed the households earns a fixed wage  $w_{ss}$ . When unemployed they get unemployment insurance. For the first  $\bar{u}$  months this is  $\bar{\phi}$ . Afterwards it is  $\underline{\phi}$ . The income function thus is

$$\begin{aligned}
y_{it}(u_{it}) &= w_{ss} \cdot \begin{cases} 1 & \text{if } u_{it} = 0 \\ \bar{\phi}\text{UI}_{it} + (1 - \text{UI}_{it})\underline{\phi} & \text{else} \end{cases} \\
\text{UI}_{it} &= \begin{cases} 0 & \text{if } u_{it} = 0 \\ 1 & \text{else if } u_{it} < \bar{u}_t \\ 0 & \text{else if } u_{it} > \bar{u}_t + 1 \\ \bar{u} - (u_{it} - 1) & \text{else} \end{cases}
\end{aligned} \tag{3}$$

where  $\text{UI}_{it} \in [0, 1]$  is the share of high unemployment insurance in period  $t$ .

The aggregate quantities of central interest are

$$C_t^{hh} = \int c_{it} d\mathbf{D}_t \tag{4}$$

$$A_t^{hh} = \int a_{it} d\mathbf{D}_t \tag{5}$$

$$U_t^{hh} = \int 1\{u_{it} > 0\} d\mathbf{D}_t \tag{6}$$

$$\text{UI}_t^{hh} = \int \text{UI}_{it} d\mathbf{D}_t \tag{7}$$

$$S_t^{hh} = \int s(u_{it-1}) d\mathbf{D}_t \tag{8}$$

**Intermediate-good producers.** Intermediate-good producers hire labor in a frictional labor market with search and matching frictions. Matches produce a homogeneous good sold

in a perfectly competitive market. The Bellman equation for the value of a job is

$$V_t^j = p_t^x Z_t - w_{ss} + \beta^{\text{firm}} \mathbb{E}_t \left[ (1 - \delta_{ss}) V_{t+1}^j \right] \quad (9)$$

where  $p_t^x$  is the intermediary goods price,  $Z_t$  is aggregate TFP,  $w_{ss}$  is the wage rate,  $\beta^{\text{firm}}$  is the firm discount factor, and  $\delta_{ss}$  is the exogenous separation rate. The value of a vacancy is

$$V_t^v = -\kappa + \lambda_t^v V_t^j + (1 - \lambda_t^v)(1 - \delta_{ss}) \beta^{\text{firm}} \mathbb{E}_t [V_{t+1}^v] \quad (10)$$

where  $\kappa$  is flow cost of posting vacancies, and  $\lambda_t^v$  is the job-filling rate. The assumption of free entry implies

$$V_t^v = 0 \quad (11)$$

**Whole-sale and final-good producers.** Wholesale firms buy intermediate goods and produce differentiated goods that they sell in a market with monopolistic competition. The wholesale firms set their prices subject to a Rotemberg adjustment cost. Final-good firms buy goods from wholesale firms and bundle them in a final good, which is sold in a perfectly competitive market. Together this implies a New Keynesian Phillips Curve,

$$1 - \epsilon + \epsilon p_t^x = \phi \pi_t (1 + \pi_t) - \phi \beta^{\text{firm}} \mathbb{E}_t \left[ \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] \quad (12)$$

where  $\epsilon$  is the elasticity of substitution between the differentiated goods,  $\phi$  is the Rotemberg adjustment cost,  $\pi_t$  is the inflation rate from period  $t - 1$  to  $t$ , and  $Y_t$  is aggregate output given by

$$Y_t = Z_t (1 - u_t) \quad (13)$$

The adjustment costs are assumed to be virtual such that total dividends are

$$\text{div}_t = Z_t (1 - u_t) - w_t (1 - u_t) \quad (14)$$

**Labor market dynamics.** Labor market tightness is given by

$$\theta_t = \frac{v_t}{S_t} \quad (15)$$

A Cobb-Douglas matching function implies that the job-finding and job-finding rates are

$$\lambda_t^v = A \theta_t^{-\alpha} \quad (16)$$

$$\lambda_t^{u,s} = A \theta_t^{1-\alpha} \quad (17)$$

The law of motion for unemployment is

$$u_t = u_{t-1} + \delta_t(1 - u_{t-1}) - \lambda_t^{u,s} S_t \quad (18)$$

### Central bank.

The central bank controls the nominal interest rate from period  $t$  to  $t + 1$ , and follows a standard Taylor rule,

$$1 + i_t = (1 + i_{ss}) \left( \frac{1 + \pi_t}{1 + \pi_{ss}} \right)^{\delta_\pi} \quad (19)$$

### Government.

The government can finance its expenses with long-term bonds,  $B_t$ , with a geometrically declining payment stream of  $1, \delta, \delta^2, \dots$  for  $\delta \in [0, 1]$ . The bond price is  $q_t$ . The expenses on unemployment insurance is

$$\Phi_t = w_{ss} \left( \bar{\phi}_t \text{UI}_t^{hh} + \underline{\phi} (u_t - \text{UI}_t^{hh}) \right) \quad (20)$$

Total expenses thus are

$$X_t = \Phi_t + G_t + \text{transfer}_t \quad (21)$$

Total taxes are

$$\text{taxes}_t = \tau_t (\Phi_t + w_{ss}(1 - u_t)) \quad (22)$$

The government budget is

$$q_t B_t = (1 + q_t \delta_q) B_{t-1} + X_t - \text{taxes}_t \quad (23)$$

The government adjust taxes to so that the value of government debt returns to its steady state value,

$$\tilde{\tau}_t = \frac{(1 + q_t \delta_q) B_{t-1} + X_t - q_{ss} B_{ss}}{\Phi_t + w_{ss}(1 - u_t)} \quad (24)$$

$$\tau_t = \omega \tilde{\tau}_t + (1 - \omega) \tau_{ss} \quad (25)$$

**Financial markets.** Arbitrage between government bonds and reserves implies that

$$\frac{1 + \delta_q q_{t+1}}{q_t} = \frac{1 + i_t}{1 + \pi_{t+1}} \quad (26)$$

The ex post realized return on savings is

$$1 + r_t = \begin{cases} \frac{(1+\delta_g q_0)B_{-1}}{A_{-1}^{hh}} & \text{if } t = 0 \\ \frac{1+i_{t-1}}{1+\pi_t} & \text{else} \end{cases} \quad (27)$$

### **Market clearing.**

Asset and goods market clearing implies

$$A_t^{hh} = q_t B_t \quad (28)$$

$$Y_t = C_t^{hh} + G_t \quad (29)$$