- GEMODELTOOLS -

A HANK-SAM MODEL

Jeppe Druedahl

Model

Households. The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Time is discrete and indexed by $t \in \{0,1,\dots\}$. Each period is one month.

The households are *ex ante* heterogeneous in terms of their discount factor β_i. There are three types of households:

1. Hands-too-mouth households with $\beta_i = \beta^{\text{HtM}}$.
2. Buffer-stock households with $\beta_i = \beta^{\text{BS}}$.
3. Permanent income hypothesis households with $\beta_i = \beta^{\text{PIH}}$.

Households are *ex post* heterogeneous in terms of their unemployment status, u_{it}, and lagged end-of-period savings, a_{it-1}. If $u_{it} = 0$ the household is employed. If $u_{it} > 0$ the household is in its u_{it}'th month of unemployment.

Each period the household chooses consumption, c_{it}, and savings, a_{it}. Borrowing is not allowed and the utility function is CRRA.

The recursive household problem is

$$
\begin{aligned}
v_t(\beta_i, u_{it}, a_{it-1}) = &\max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t \left[v_{t+1}\left(\beta_i, u_{it+1}, a_{it}\right)\right] \\
&\text{s.t. } a_{it} + c_{it} = (1+r_t)a_{it-1} + (1-\tau_t)y_t(u_{it}) + \text{div}_t + \text{transfer}_t \\
&a_{it} \geq 0
\end{aligned}
\tag{1}
$$

where r_t is the ex post return from period $t-1$ to t, $y_t(u_{it})$ is labor market income (including unemployment insurance), τ_t is the tax rate on labor market income, div_t is dividends, and transfer_t is a transfer from the government (or a lump-sum tax if negative).

The employment/unemployment transition probabilities are

$$
\begin{aligned}
\Pr[u_{it+1} = 0 \,|\, u_{it} = 0] &= 1 - \delta_t \\
\Pr[u_{it+1} = 1 \,|\, u_{it} = 0] &= \delta_t \\
\Pr[u_{it+1} > 1 \,|\, u_{it} = 0] &= 0 \\
\Pr[u_{it+1} = 0 \,|\, u_{it} > 0] &= \lambda_t^{u,s} s(u_{it-1}) \\
\Pr[u_{it+1} = u_{it} + 1 \,|\, u_{it} > 0] &= 1 - \lambda_t^{u,s} s(u_{it-1}) \\
\Pr[u_{it+1} \notin \{0, u_{it} + 1\} \,|\, u_{it} > 0] &= 0
\end{aligned} \tag{2}
$$

where δ_t is the separation rate, $\lambda_t^{u,s}$ is the job-finding rate per effective searcher, and $s(u_{it-1})$ determines the effectiveness of search conditional on unemployment status.

When employed the households earn a fixed wage w_{ss}. When unemployed they get unemployment insurance. For the first $\overline{u}$ months this is $\overline{\phi}$. Afterwards it is $\underline{\phi}$. The income function thus is

$$
y_{it}(u_{it}) = w_{ss} \cdot \begin{cases} 1 & \text{if } u_{it} = 0 \\ \overline{\phi}\mathrm{UI}_{it} + (1 - \mathrm{UI}_{it})\underline{\phi} & \text{else} \end{cases} \tag{3}
$$

$$
\mathrm{UI}_{it} = \begin{cases} 0 & \text{if } u_{it} = 0 \\ 1 & \text{else if } u_{it} < \overline{u}_t \\ 0 & \text{else if } u_{it} > \overline{u}_t + 1 \\ \overline{u} - (u_{it} - 1) & \text{else} \end{cases}
$$

where $\mathrm{UI}_{it} \in [0, 1]$ is the share of high unemployment insurance in period t.

The aggregate quantities of central interest are

$$
C_t^{hh} = \int c_{it} d\boldsymbol{D}_t \tag{4}
$$

$$
A_t^{hh} = \int a_{it} d\boldsymbol{D}_t \tag{5}
$$

$$
U_t^{hh} = \int 1\{u_{it} > 0\} d\boldsymbol{D}_t \tag{6}
$$

$$
\mathrm{UI}_t^{hh} = \int \mathrm{UI}_{it} d\boldsymbol{D}_t \tag{7}
$$

$$
S_t^{hh} = \int s(u_{it-1}) d\underline{\boldsymbol{D}}_t \tag{8}
$$

Intermediate-good producers. Intermediate-good producers hire labor in a frictional labor market with search and matching frictions. Matches produce a homogeneous good sold in a perfectly competitive market. The Bellman equation for the value of a job is

$$V_t^j = p_t^x Z_t - w_{ss} + \beta^{\text{firm}} \mathbb{E}_t \left[(1 - \delta_{ss}) V_{t+1}^j \right] \tag{9}$$

where p_t^x is the intermediary goods price, Z_t is aggregate TFP, w_{ss} is the wage rate, β^{firm} is the firm discount factor, and δ_{ss} is the exogenous separation rate. The value of a vacancy is

$$V_t^v = -\kappa + \lambda_t^v V_t^j + (1 - \lambda_t^v)(1 - \delta_{ss}) \beta^{\text{firm}} \mathbb{E}_t \left[V_{t+1}^v \right] \tag{10}$$

where κ is flow cost of posting vacancies, and λ_t^v is the job-filling rate. The assumption of free entry implies

$$V_t^v = 0 \tag{11}$$

Whole-sale and final-good producers. Wholesale firms buy intermediate goods and produce differentiated goods that they sell in a market with monopolistic competition. The wholesale firms set their prices subject to a Rotemberg adjustment cost. Final-good firms buy goods from wholesale firms and bundle them in a final good, which is sold in a perfectly competitive market. Together this implies a New Keynesian Phillips Curve,

$$1 - \epsilon + \epsilon p_t^x = \phi \pi_t (1 + \pi_t) - \phi \beta^{\text{firm}} \mathbb{E}_t \left[\pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] \tag{12}$$

where ϵ is the elasticity of substitution between the differentiated goods, ϕ is the Rotemberg adjustment cost, π_t is the inflation rate from period $t-1$ to t, and Y_t is aggregate output given by

$$Y_t = Z_t (1 - u_t) \tag{13}$$

The adjustment costs are assumed to be virtual such that total dividends are

$$\text{div}_t = Z_t (1 - u_t) - w_t (1 - u_t) \tag{14}$$

Labor market dynamics. Labor market tightness is given by

$$\theta_t = \frac{v_t}{S_t} \tag{15}$$

A Cobb-Douglas matching function implies that the job-finding and job-finding rates are

$$\lambda_t^v = A \theta_t^{-\alpha} \tag{16}$$

$$\lambda_t^{u,s} = A \theta_t^{1-\alpha} \tag{17}$$

The law of motion for unemployment is

$$u_t = u_{t-1} + \delta_t(1 - u_{t-1}) - \lambda_t^{u,s} S_t \tag{18}$$

Central bank.

The central bank controls the nominal interest rate from period t to $t+1$, and follows a standard Taylor rule,

$$1 + i_t = (1 + i_{ss}) \left(\frac{1 + \pi_t}{1 + \pi_{ss}} \right)^{\delta_\pi} \tag{19}$$

Government.

The government can finance its expenses with long-term bonds, B_t, with a geometrically declining payment stream of $1, \delta, \delta^2, \ldots$ for $\delta \in [0, 1]$. The bond price is q_t. The expenses on unemployment insurance is

$$\Phi_t = w_{ss} \left(\overline{\phi}_t \mathrm{UI}_t^{hh} + \underline{\phi} \left(u_t - \mathrm{UI}_t^{hh} \right) \right) \tag{20}$$

Total expenses thus are

$$X_t = \Phi_t + G_t + \mathrm{transfer}_t \tag{21}$$

Total taxes are

$$\mathrm{taxes}_t = \tau_t \left(\Phi_t + w_{ss}(1 - u_t) \right) \tag{22}$$

The government budget is

$$q_t B_t = (1 + q_t \delta_q) B_{t-1} + X_t - \mathrm{taxes}_t \tag{23}$$

The government adjust taxes to so that the value of government debt returns to its steady state value,

$$\tilde{\tau}_t = \frac{\left(1 + q_t \delta_q\right) B_{t-1} + X_t - q_{ss} B_{ss}}{\Phi_t + w_{ss}(1 - u_t)} \tag{24}$$

$$\tau_t = \omega \tilde{\tau}_t + (1 - \omega) \tau_{ss} \tag{25}$$

Financial markets. Arbitrage between government bonds and reserves implies that

$$\frac{1 + \delta_q q_{t+1}}{q_t} = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{26}$$

The ex post realized return on savings is

$$1 + r_t = \begin{cases} \frac{(1+\delta_q q_0)B_{-1}}{A_{-1}^{hh}} & \text{if } t = 0 \\ \frac{1+i_{t-1}}{1+\pi_t} & \text{else} \end{cases} \tag{27}$$

Market clearing.

Asset and goods market clearing implies

$$A_t^{hh} = q_t B_t \tag{28}$$

$$Y_t = C_t^{hh} + G_t \tag{29}$$