

- GEMODELTOOLS -

TWO-SECTOR I-HANK MODEL

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Model

We consider a *small open economy* with heterogeneous agents and *sticky wages*.

The *foreign* economy is thus taken as exogenously given.

Households. The home economy has a continuum of infinitely lived households indexed by $i \in [0, 1]$. Households are *ex ante* heterogeneous in terms of which sector they work in, $s_i \in \{TH, NT\}$, where *TH* is the *tradeable* sector (in the home country), and $s_i = NT$ is the *non-tradeable* sector. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, z_{it} , and their (end-of-period) savings, a_{it-1} . The distribution of households over idiosyncratic states is denoted \underline{D}_t before shocks are realized and \underline{D}_t afterwards. Households supply labor, $n_{s_i, it}$, chosen by a union in each sector, and choose consumption, c_{it} , on their own. Households are not allowed to borrow. The return on savings is r_t^a , the sector-specific real wage is $w_{s_i, t}$, and labor income is taxed with the rate $\tau_t \in [0, 1]$.

The household problem in real terms is

$$\begin{aligned} v_t(s_i, z_t, a_{t-1}) &= \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} + \beta_t \mathbb{E}_t [v_{t+1}(s_i, z_{t+1}, a_t)] \\ \text{s.t. } a_{it} + c_{it} &= (1 + r_t^a) a_{it-1} + (1 - \tau_t) w_{s_i, t} n_{s_i, t} z_{it} \\ \log z_{it+1} &= \rho_z \log z_{it} + \psi_{it+1}, \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \mathbb{E}[z_{it}] = 1 \\ a_{it} &\geq 0. \end{aligned} \tag{1}$$

where β_t is the time-varying discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frisch elasticity.

Aggregate quantities are

$$A_t^{hh} = \int a_{it} d\mathbf{D}_t \quad (2)$$

$$C_t^{hh} = \int c_{it} d\mathbf{D}_t \quad (3)$$

$$s \in \{TH, NT\} : S_x^{hh} = \int 1\{s_{it} = s\} d\mathbf{D}_t. \quad (4)$$

An *outer* CES demand system implies that the consumption of tradeable goods, $C_{T,t}$, and non-tradeable goods, $C_{NT,t}$, are given by

$$C_{T,t} = \alpha_T \left(\frac{P_{T,t}}{P_t} \right)^{-\eta_{T,NT}} C_t^{hh} \quad (5)$$

$$C_{NT,t} = (1 - \alpha_T) \left(\frac{P_{NT,t}}{P_t} \right)^{-\eta_{T,NT}} C_t^{hh}, \quad (6)$$

where α_T is the share of tradeable goods and $\eta_{T,NT}$ is the substitution elasticity. The corresponding price index is

$$P_t = \left[\alpha_T P_{T,t}^{1-\eta_{T,NT}} + (1 - \alpha_T) P_{NT,t}^{1-\eta_{T,NT}} \right]^{\frac{1}{1-\eta_{T,NT}}}. \quad (7)$$

An *inner* CES demand system implies that consumption of tradeable goods produced at home, $C_{TH,t}$, and tradeable goods produced in the foreign country, $C_{TF,t}$, are given by

$$C_{TF,t} = \alpha_F \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\eta_{F,H}} C_{T,t} \quad (8)$$

$$C_{TH,t} = (1 - \alpha_F) \left(\frac{P_{TH,t}}{P_{T,t}} \right)^{-\eta_{F,H}} C_{T,t}. \quad (9)$$

where α_F is the share of foreign tradeable goods and $\eta_{F,H}$ is the substitution elasticity. The corresponding price index is

$$P_{T,t} = \left[\alpha_F P_{F,t}^{1-\eta_{F,H}} + (1 - \alpha_F) P_{TH,t}^{1-\eta_{F,H}} \right]^{\frac{1}{1-\eta_{F,H}}} \quad (10)$$

Firms. A representative firm in each sector, $s \in \{TH, NT\}$, hires labor, $N_{s,t}$, to produce goods, with the production function

$$Y_{s,t} = Z_{s,t} N_{s,t}, \quad (11)$$

where Z_t^s is the exogenous technology level. Profits are

$$\Pi_{s,t} = P_{s,t}Y_{s,t} - W_{s,t}N_{s,t}. \quad (12)$$

where $P_{TH,t}$ and P_{NT} are the price levels and $W_{s,t}$ are the nominal wage levels. The first order condition for labor implies that

$$P_{s,t} = W_{s,t}/Z_{s,t}. \quad (13)$$

The real wage is

$$w_{s,t} = \frac{W_{s,t}}{P_t}. \quad (14)$$

Unions. A union in each sector chooses the labor supply of each household and sets wages. Each household is chosen to supply the same amount of labor,

$$n_{s,t} = N_{s,t}^{hh}, \quad s \in \{T, NT\}. \quad (15)$$

Unspecified adjustment costs imply *New Keynesian Wage Philips Curves*,

$$\pi_{s,t}^w = \kappa \frac{\int \left(\varphi n_{s,t}^{-\nu} - \frac{1}{\mu} (1 - \tau_t) w_{s,t} z_{it} c_{it}^{-\sigma} \right) 1\{s_{it} = s\} d\mathbf{D}_t}{\int 1\{s_{it} = s\} d\mathbf{D}_t} + \beta \pi_{s,t+1}^w, \quad (16)$$

where $1 + \pi_{s,t}^w = W_{s,t}/W_{s,t-1}$, κ is the slope parameter and μ is a wage mark-up.

Central bank. The central bank follows a standard Taylor rule

$$1 + i_t = (1 + i_{ss}) \left(\frac{1 + \pi_{t+1}}{1 + \pi_{ss}} \right)^\phi + i_t^{\text{shock}}, \quad (17)$$

where i_t is the nominal return from period t to period $t + 1$, $1 + \pi_{t+1} = P_{t+1}/P_t$, i_t^{shock} is a monetary policy shock and ϕ is the Taylor coefficient on inflation.

The *ex ante* real interest rate from t to $t + 1$ is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}. \quad (18)$$

The *ex post* real interest rate from $t - 1$ to t is

$$1 + r_t^a = \frac{1 + i_{t-1}}{1 + \pi_t}. \quad (19)$$

Government. The government chooses spending, G_t , and the labor income tax rate, τ_t . The budget constraint for the government then is

$$B_t = (1 + r_t^a)B_{t-1} + \frac{P_{NT,t}}{P_t}G_t - \tau_t \sum_{s \in \{TH, NT\}} w_{s,t} N_{s,t}.$$

where government consumption is fully in terms of non-tradeable goods.

The tax rule is

$$\tau_t = \tau_{ss} + \omega \frac{B_{t-1} - B_{ss}}{Y_{TH,ss} + Y_{NT,ss}}. \quad (20)$$

Foreign economy. The nominal exchange in home currency units per foreign currency unit is denoted E_t . The foreign price level in foreign currency is $P_{F,t}^*$. In home currency, the foreign price level is

$$P_{F,t} = P_{F,t}^* E_t. \quad (21)$$

The price of home tradeable goods in foreign currency is

$$P_{TH,t}^* = \frac{P_{TH,t}}{E_t}. \quad (22)$$

The real exchange is

$$Q_t = \frac{P_{F,t}}{P_t} = \frac{E_t P_{F,t}^*}{P_t}. \quad (23)$$

The foreign demand for the home tradeable goods is

$$C_{TH,t}^* = \left(\frac{P_{TH,t}^*}{P_{F,t}^*} \right)^{-\eta^*} M_t^* = \left(\frac{1}{Q_t} \frac{P_{TH,t}}{P_t} \right)^{-\eta^*} M_t^*, \quad (24)$$

where M_t^* is the foreign market size and η^* is the elasticity of foreign demand.

Capital markets are free such that the uncovered interest parity must hold,

$$1 + i_t = \left(1 + i_t^f \right) \frac{E_{t+1}}{E_t}. \quad (25)$$

where i_t^f is the foreign nominal interest rate. In real terms this is

$$1 + r_t = \left(1 + r_t^f \right) \frac{Q_{t+1}}{Q_t}. \quad (26)$$

where $1 + r_t^f = \frac{1 + i_t^f}{1 + \pi_{t+1}^f}$ and $1 + \pi_{t+1}^f = P_{F,t+1}^* / P_{F,t}^*$.

Market clearing. The market for home tradeable goods and the market for non-tradeable goods both clear

$$Y_{T,t} = C_{TH,t} + C_{TH,t}^* \quad (27)$$

$$Y_{NT,t} = C_{NT,t} + G_t. \quad (28)$$

Accounting. We define the following variables,

$$\text{Gross domestic product: } GDP_t = \frac{P_{TH,t}Y_{TH,t} + P_{NT,t}Y_{NT}}{P_t} \quad (29)$$

$$\text{Net exports: } NX_t = GDP_t - C_t^{hh} - \frac{P_{NT,t}}{P_t}G_t \quad (30)$$

$$\text{Net foreign assets: } NFA_t = A_t^{hh} - B_t \quad (31)$$

$$\text{Current account: } CA_t = NX_t + r_t^a NFA_{t-1}. \quad (32)$$

Walras' law then implies

$$NFA_t - NFA_{t-1} = CA_t, \quad (33)$$

as shown by

$$\begin{aligned} \int a_{it} d\mathbf{D}_t &= \int (1 + r_t^a) a_{it-1} + (1 - \tau_t) w_{s_i,t} n_{s_i,t} z_{it} - c_{it} d\mathbf{D}_t \\ A_t^{hh} &= (1 + r_t^a) A_{t-1}^{hh} + (1 - \tau_t) \sum_{s \in \{TH, NT\}} w_{s,t} N_{s,t} - C_t^{hh} \\ &= (1 + r_t^a) A_{t-1}^{hh} + GDP_t - C_t^{hh} - \tau_t \sum_{s \in \{TH, NT\}} w_{s,t} N_{s,t} \\ &= (1 + r_t^a) A_{t-1}^{hh} + GDP_t - C_t^{hh} + \left(B_t - (1 + r_t^a) B_{t-1} + \frac{P_{NT,t}}{P_t} G_t \right) \\ &= (1 + r_t^a) NFA_{t-1} + NX_t + B_t \Leftrightarrow \\ NFA_t - NFA_{t-1} &= r_t^a NFA_{t-1} + NX_t. \end{aligned}$$