Exam: Couples Time Allocation

Household Behavior over the Life Cycle

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Contents

1 Household Trade-Offs

First and foremost, there is an intra-household trade-off between the sexes in allocating time for home production. The trade-off arises due to the presence of differential productivity in home production. If the female, for instance, has a comparative advantage in home production over the male (i.e. $\alpha > 0.5$), the couple will allocate more of the female's time to home production and more of the male's time to labor work. This sex-based division of labor is captured by the female's absolute advantage in home production, AAH $\equiv \alpha/(1-\alpha)$. This intra-household division of labor is a central element of this model and will be discussed extensively throughout the text.
Note, however, that gains from specialization depend on time allocation being interchangeable between females and males. The parameter σ governs the elasticity of substitution between the inputs $h_{j,t}$ for $j \in \{f, m\}$. A lower value of σ implies that the sexes' home production hours are less interchangeable, i.e., the market and home produced goods are complements rather than substitutes.

Secondly, the couple faces a trade-off between work and leisure. The couple can both choose to offer hours in the labor market ($l_{j,t}$) and in home production ($h_{j,t}$) versus alternatively choosing leisure. Choosing to work more, whether in the labor market ($l_{j,t}$) or home production ($h_{j,t}$), contributes to higher consumption and human capital accumulation but at the expense of leisure time, since there is a disutility of work, denoted by the parameter ν.

Thirdly, there is a trade-off between working in home production and the labor market. Time spent on labor market work provides earned income that can be spent on consumption goods C_t, while time spent on home production contributes to home produced goods, H_t. The relative importance of these two sectors is captured through the parameter ω in the composite good Q_t.

2 Explaining how the Model is Solved

The approach to numerically solve the model is that I first discretize the continuous state variables ($K_{f,t}$,$K_{m,t}$) to form a state space: $\Pi = \{K_{i_k}\}_{i_k=1}^{n_k} \times \{K_{i_k}\}_{i_k=1}^{n_k}$. This is done on a denser grid for lower values, reflecting the greater curvature of the value function at lower values. Next, I solve by backward recursion on the grid since it is a finite horizon problem. However, in order to implement backward recursion, a terminal condition is needed. As there is no bequest motive, consuming everything in the final period is optimal, so I set the value function at $T+1$ to 0

$$V_T\left(K_{f,T}, K_{m,T}\right) = \max_{l_{f,T}, h_{f,T}, l_{m,T}, h_{m,T}} U\left(T_{f,T}, T_{m,T}, Q_T\right) + \beta \overbrace{V_{T+1}\left(K_{f,T+1}, K_{m,T+1}\right)}^{=0}.$$

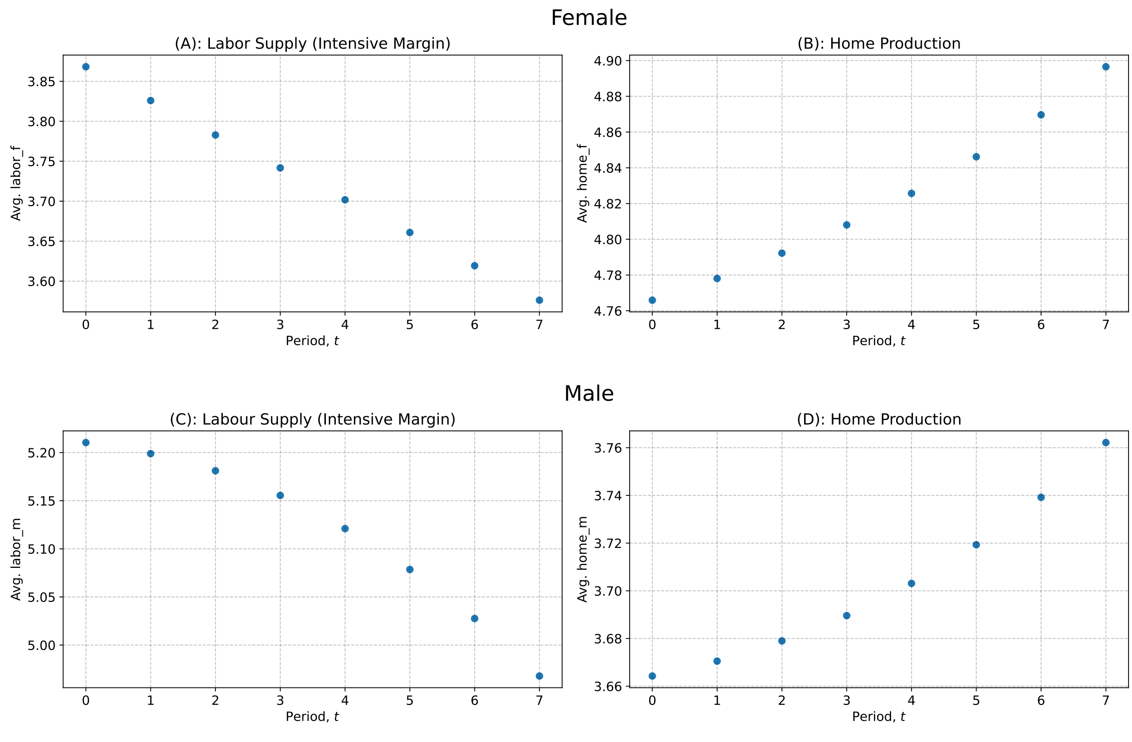
However, the solution may fall off the grid since I have discretized the state space. Hence I use linear interpolation to interpolate between grid points.

I use a simple version of backward recursion: Value Function Iteration (VFI). Specifically for $t \leq T$, I employ a quasi-newton optimization algorithm L-BFGS to do a local search on the bounded interval $[0; 24]$ reflecting 24 hours to maximize the value function.[1] The maximum attainable value function is revealed as the choices of labor hours and home production hours supplied by each sex that maximizes the right-hand side of the Bellman equation provided in the assignment text.

3 Life Cycle Patterns of the Choice Variables

After solving the model, I simulate forwards the average life cycle patterns of the choice variables. In Row 1 of Figure 1, I have plotted the average choice variables $l_{f,t}$ and $h_{f,t}$ for the female. The corresponding variables for the male are plotted in Row 2. Panel A

Figure 1: Average Simulated Age-Profiles of the Choice-Variables



Note: The model was simulated with 10,000 individuals.

reveals a declining trend in the hours the female allocates to the labor market over time. In contrast, Panel B shows an increase in the hours dedicated to home production across her life cycle. When examining the male labor supply in Panel C, the level is significantly higher. However, the male contributes far fewer hours to home production. The observed life cycle

[1]In practice, I minimize the negated value function.

patterns arise from a Beckerian-type home production. I have initialized the parameter α to be 0.7, which encourages a division of labor that yields gains from specialization. Specifically, this parameter setting fosters specialization, with the female primarily engaging in home production due to her relative advantage and the male in the labor market, where his accumulated human capital yields higher earned income. This aligns with the first trade-off mentioned in Question 1.

4 Correlation between Relative Wages and Home Production Hours

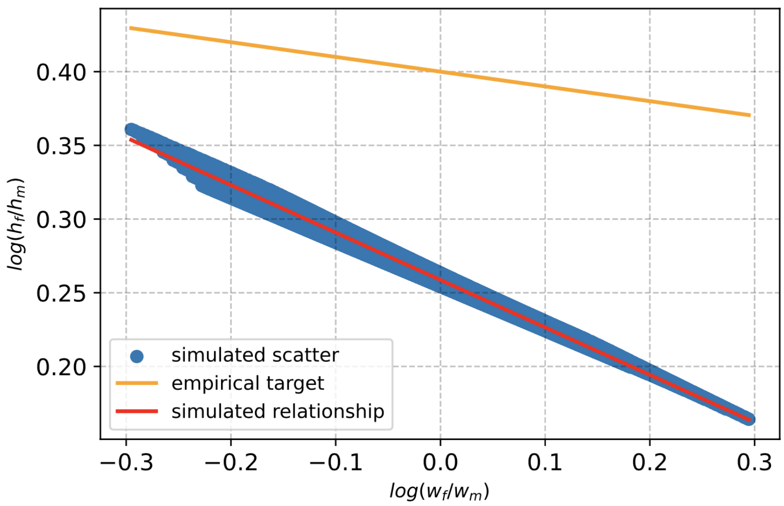
I run a linear regression of log relative home production work time on log relative wages using the specification provided in the assignment text to obtain

$$\log\left(h_{i,f,t}/h_{i,m,t}\right) = 0.259 + (-0.322)\log\left(w_{i,f,t}/w_{i,m,t}\right) + \hat{\varepsilon}_{i,t}. \tag{4.1}$$

The coefficient β_1 in this regression equation shows how the ratio of hours women and men spend on home production responds to the ratio of their wage rates. The estimated coefficient is $\hat{\beta}_1 \approx -0.322$. Ceteris paribus, an increase in a female's relative wage compared to that of a male will lead to a decrease in the relative hours that the female spends on home production compared to the male. Compared to Table 2 in Siminski and Yetsenga, 2022, in wich they find a $\hat{\beta}_1 = -0.1$, the sign in my model is the same, but the (absolute) responsiveness is larger than in their estimated relationship. In Question 9, I fit the estimates to match the data moments from Siminski and Yetsenga, 2022.

Furthermore, In Figure 2, I plot relative domestic work (home production) against the relative wage including the regression line. The figure shows that a female does

Figure 2: Relative Domestic Work Time by Relative Wage



significantly more housework than a male, regardless of the relative wages. The finding is in line with Siminski and Yetsenga, 2022. Even when women have significantly higher wages, they still do more housework than their partners. This is consistent with Becker's gains of specialization in the labor division of the household. The specialization in the labor division should be viewed in the light of the aforementioned parameter setting of $\alpha = 0.7$, which implies that the female has an absolute advantage in home production.

A weak link (i.e. β_1 close to zero) would suggest that changes in relative wages have little or no effect on the relative time a female and male spend on home production. Thus, relative wages cannot predict the intra-household time allocation if β_1 is close to zero.

5 Extending the Model with a Stochastic Child

I extend the model to include a stochastic child. Since a stochastic child alters the expectations formed by the agents, it is necessary to introduce a new state variable, $n_t \in \{0, 1\}$. Below, I provide the updated recursive formulation of the model

$$V_t\left(n_t, K_{f,t}, K_{m,t}\right) = \max_{l_{f,t}, h_{f,t}, l_{m,t}, h_{m,t}} U\left(T_{f,t}, T_{m,t}, Q_t\right) + \beta \mathbb{E}_t\left[V_{t+1}\left(n_{t+1}, K_{f,t+1}, K_{m,t+1}\right)\right] \tag{5.1}$$

s.t.

$$p_{t+1}\left(n_t\right) = \begin{cases} 0.1 & \text{if } n_t = 0 \\ 0.0 & \text{else} \end{cases}, \tag{5.2}$$

$$n_{t+1} = \begin{cases} n_t + 1 & \text{with probability } p_{t+1}(n_t) \\ n_t & \text{with probability } 1 - p_{t+1}(n_t) \end{cases}, \tag{5.3}$$

$$C_t = w_{f,t} l_{f,t} + w_{m,t} l_{m,t} + X_t, \tag{5.4}$$

$$H_t = \left(\alpha h_{f,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) h_{m,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{5.5}$$

$$Q_t = C_t^{\omega(n_t)} H_t^{1-\omega(n_t)}, \tag{5.6}$$

$$\omega(n_t) = \omega + \omega_n n_t, \tag{5.7}$$

$$\log w_{j,t} = \gamma_{j,0} + \gamma_{j,1} K_{j,t}, j \in \{f, m\}, \tag{5.8}$$

$$K_{j,t+1} = (1-\delta) K_{j,t} + l_{j,t}/24, j \in \{f, m\}, \tag{5.9}$$

$$T_{j,t} = l_{j,t} + h_{j,t}, j \in \{f, m\}, \tag{5.10}$$

$$l_{j,t}, h_{j,t} \geq 0, j \in \{f, m\}, \tag{5.11}$$

$$T_{j,t} \leq 24, j \in \{f, m\}. \tag{5.12}$$

Hence, the new equations are the probability of a child arrival in Eq. (5.2), the transition rule for a child in Eq. (5.3), and the new specification of the relative weight in Eq. (5.7). Moreover, the composite good in Eq. (5.6) is modified to take into account the specification of $\omega(n_t)$. Note that the utility is *not* affected by the extension. Hence I do not restate it. Finally, the terminal condition is updated as such

$$V_T\left(n_T, K_{f,T}, K_{m,T}\right) = \max_{l_{f,T}, h_{f,T}, l_{m,T}, h_{m,T}} U\left(T_{f,T}, T_{m,T}, Q_T\right). \tag{5.13}$$

6 Interpretation of a Negative ω_n

Eq. (5.7) in the extended model determines the weight on market purchased and home produced consumption goods. Here, I set the parameter $\omega_n = -0.2$ resulting in a decrease in the relative weight on market purchased goods in favor of home produced goods if the couple has a child.

One interpretation could be that it is as a way to capture the importance of *child care* which requires parental time as input similar to that in Blundell et al., 2018. Consequently, couples with a child must produce an additional amount of home production as opposed to couples without a child. In conclusion, a negative ω_n reflects that raising a child requires time.

7 Simulated Life Cycle Behavior in the Extended Model

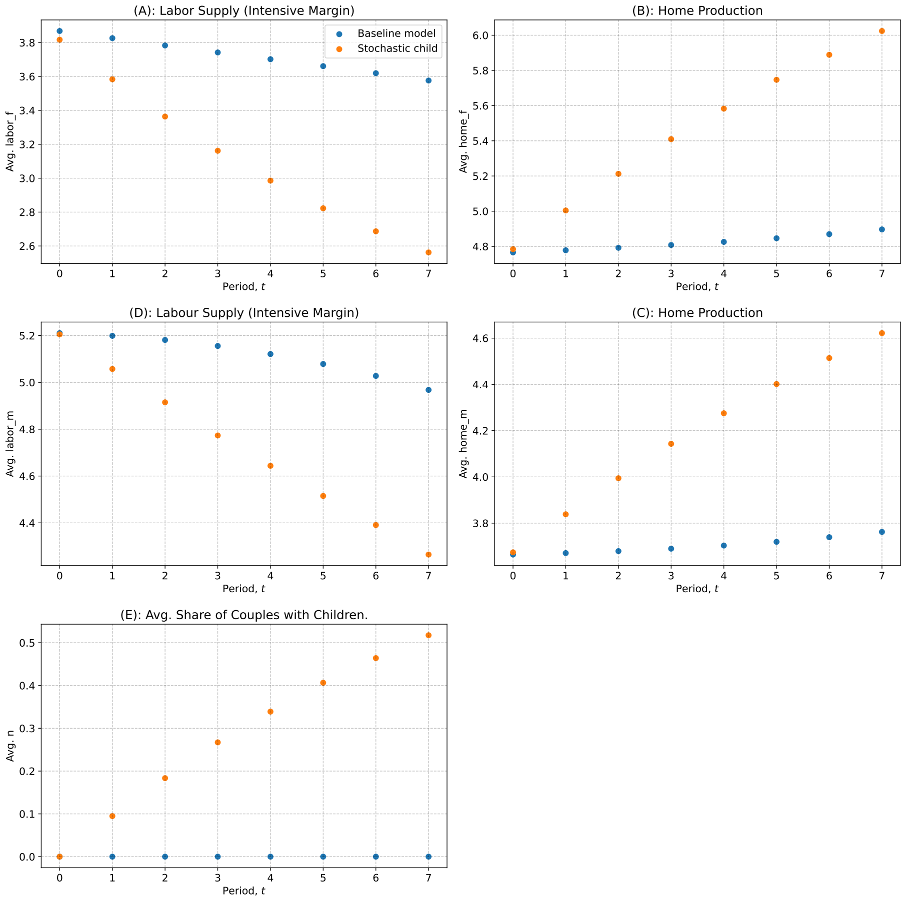
Having implemented the extension (see the associated Python code), I plot the simulated behavior of the extended model against the baseline model in Figure 3. Looking at Panel A and B, the female works fewer hours in the labor market and more hours in home production. Likewise, the male supplies fewer hours in the labor market and more in home production. It is interesting that the male decreases labor supply considering the literature. I will delve into this in Question 11. However, the life cycle pattern makes sense given the interpretation of a negative ω_n in Question 6 due to a child results in an additional requirement of input hours in home production. Although the pattern for both the male and female are similar, a child does foster a greater specialization since the female increases home production relatively more than the male and decreases hours supplied in the labor market relatively more. Finally, as expected, the share of couples with a child increases by about 10 percent each period following the initial period where no couples have a child.
To conclude, raising children requires time, particularly the female's time.

8 Specialization Index Around Childbirth

The within couple specialization index stated in Eq. (8.1), measures the degree to which a female partner specializes in home production work and the male partner in labor market

8 Specialization Index Around Childbirth

Figure 3: Simulated Age-Profiles



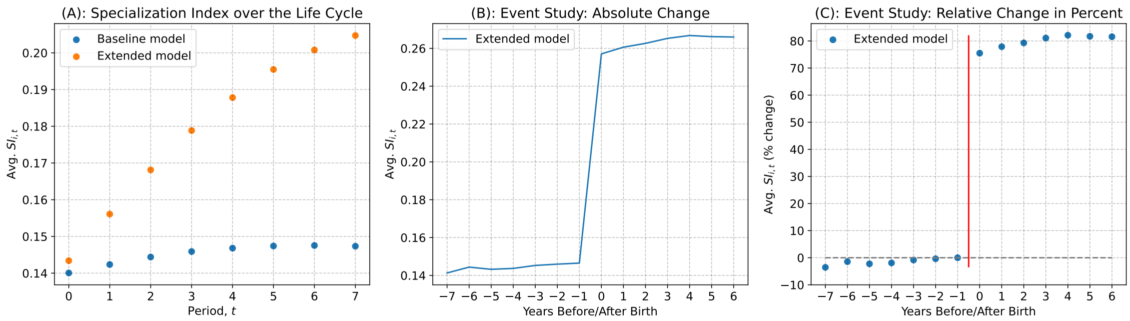
work

$$SI_{i,t} = \frac{h_{i,f,t}}{h_{i,f,t} + h_{i,m,t}} - \frac{l_{i,f,t}}{l_{i,f,t} + l_{i,m,t}}. \tag{8.1}$$

The index corresponds to the specialization index SI_2 described in Siminski and Yetsenga, 2022.

In Figure 4 Panel A, the average specialization index is plotted for both the baseline and extended models. It becomes evident that the female predominantly specializes in home production during all periods - especially when a child arrives. This labor division is particularly noticeable in the event study in Panel B. Sex-based specialization is already significant in the years leading up to birth, which can be attributed to $\alpha = 0.7$ as previously discussed. Moreover, there is a notable surge in the specialization index from the period just before childbirth to that at childbirth. Panel C demonstrates that the relative change in the average specialization index following a birth event is approximately 75.5 percent, somewhat lower than the roughly 100 percent increase reported by Siminski and Yetsenga, 2022. Notably, the increased specialization following childbirth persists; the female continues to specialize in home production at a heightened level. In conclusion, the arrival of a child substantially impacts the labor division within a couple.

Figure 4: Specialization Before and After Children



9 Simulated Method of Moments

I utilize the Simulated Method of Moments (SMM) to perform a structural estimation. The data moments of interest are $\hat{\beta}_0 = 0.4$, $\hat{\beta}_1 = -0.1$, and 1.0 change in average $SI_{i,t}$ at childbirth relative to the period before. I stack the data moments in a vector Λ^d. Next, let $\theta = (\alpha, \sigma, \omega_n) \in (0,1) \times (0,1) \times (-\omega_0, 1-\omega_0)$ be a vector of parameters I wish to estimate. I can identify the parameters since I have an equal number of parameters and moments. Using SMM where the simulated moments are denoted by $\Lambda^m(\theta)$, I minimize the squared distance between the data and simulated moments

$$\hat{\theta} = \arg\min_{\theta} \left(\Lambda^d - \Lambda^m(\theta)\right)' W \left(\Lambda^d - \Lambda^m(\theta)\right), \tag{9.1}$$

where W denotes a weighting matrix that needs to be symmetric and positive semi-definite, I choose the identity matrix because it allows me to assign equal importance to all three target moments.

I implement SMM using the gradient-free method Nelder-Mead with a tolerance ϵ=1e-8 and starting values $\theta_0 = (0.98, 0.1, -0.25)$. In Table 1, I provide the estimated moments for comparison with the data moments. I find that $\hat{\theta} = (\hat{\alpha}, \hat{\sigma}, \hat{\omega}_n) \approx (0.97850413, 0.10417467, -0.24682659)$ yields simulated moments that provide a good fit. However, the accuracy of the slope parameter, $\hat{\beta}_1$, is less than that of the other two target moments.

Table 1: Structural Estimation

Parameter	Data Moment	Simulated Moment
$\hat{\beta}_0$	0.4	0.4003
$\hat{\beta}_1$	-0.1	-0.1039
$\Delta\overline{SI}_{i,0}$	1.0	1.00086

Note: $\hat{\theta} = (\hat{\alpha}, \hat{\sigma}, \hat{\omega}_n) \approx (0.97850413, 0.10417467, -0.24682659)$

Consequently, the estimated absolute advantage in home production is

$$\text{AAH} \equiv \frac{\alpha}{1-\alpha} \Rightarrow \widehat{\text{AAH}} = \frac{0.97850413}{1-0.97850413} \approx 45.52. \tag{9.2}$$

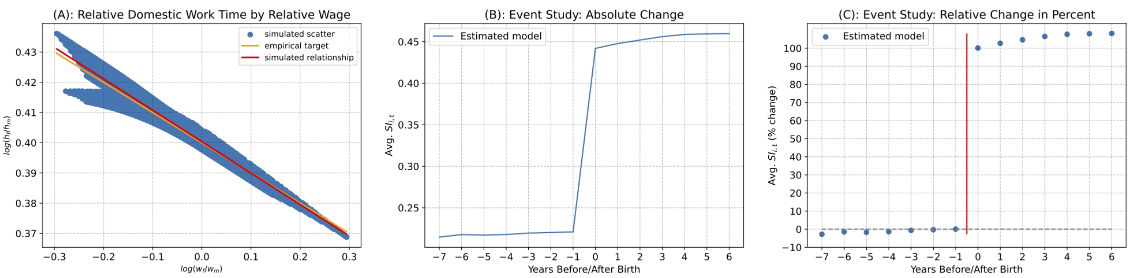
Thus, to fit the data, there must be a significant AHH for the female compared to the baseline model where AAH ≈ 2.33 for $\alpha = 0.7$.

10 Model Fit

In Figure 5, Panels A, B and C illustrate the model's fit by plotting the empirical moments with those derived from the simulated data based on the estimated model. Generally, the model fit is reasonably good considering the few parameters estimated.

The model provides a good fit for both the constant and slope coefficients, even though the slope is somewhat steeper than the data suggests. Furthermore, the model captures the surge in the average change in $SI_{i,t}$ at childbirth relative to the preceding period very well. However, there is no *anticipation effect* evident in the years leading up to the birth, as seen in Panel B. This discrepancy compared to Siminski and Yetsenga, 2022 reflects the fact that in my model, couples cannot fully prepare for the arrival of a child, as a birth event occurs randomly.

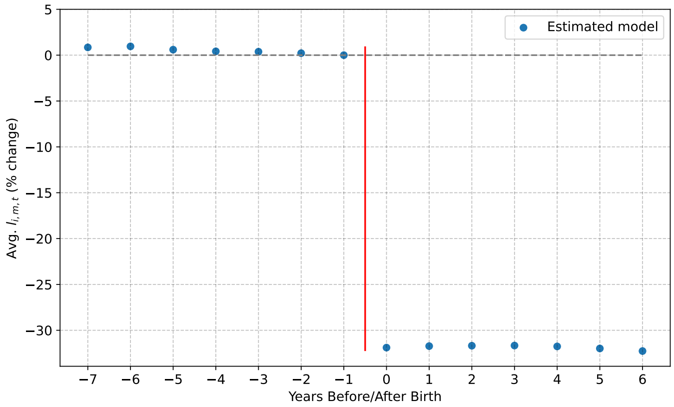
Figure 5: Model Fit



11 Male Work Hours Around Childbirth

A priori, I expect the arrival of a child would lead to a drop in the male's work hours in the labor market, given that the presence of a child would increase the demands of home production cf. Eq. (5.7). Indeed, Figure 6 demonstrates a decrease in work hours of approximately 31.9 percent following childbirth. Looking at the literature, e.g. Kleven et al., 2019 find that following childbirth, the work patterns of males and females diverge such that females reduce labor supply dramatically. In contrast, males maintain or even slightly increase labor market participation. Inspired by Jakobsen et al., 2022, I propose modifying the couple's preferences to account for the arrival of a child influencing the disutility of market work via $\nu(n_t)$. Importantly, the parameter should

Figure 6: Male Work Hours Before and After Children



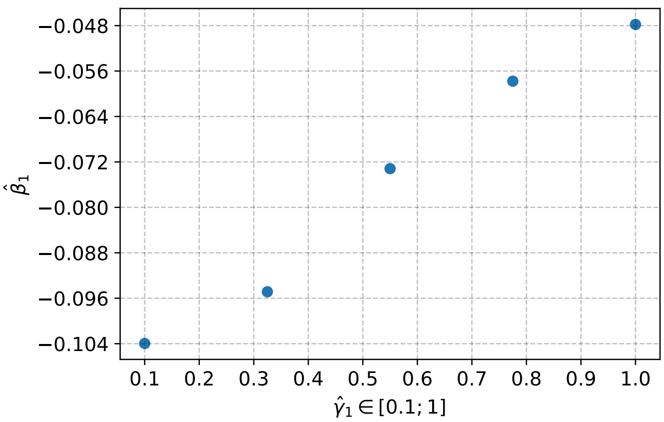
differ for females and males, in line with the notion that gender norms are a central driver of sex-based specialization after childbirth, as per Siminski and Yetsenga, 2022. This modification could help ensure that the male labor supply does not drop following childbirth.

Furthermore, the elasticity parameter $\epsilon_j > 0$ for $j \in \{f, m\}$, which control the curvature of the disutility of work for females and males respectively, could also be adjusted with the arrival of a child to reflect changes in the marginal disutility of work. For instance, assuming that the marginal disutility of work increases more for females than for males following the birth of a child, I could model this by implementing a larger increase in the value of $\epsilon_f(n_t)$ compared to $\epsilon_m(n_t)$.

12 Human Capital Accumulation and Time Allocation

In Figure 7, I plot the estimated slope parameter $\hat{\beta}_1$ for different return values to human capital. The figure shows that the slope parameter value is reduced for a larger return to human capital (i.e. a higher value of γ_1). The findings suggest that a higher level of

Figure 7: $\hat{\beta}_1$ Values for Various Levels of Return to Human Capital

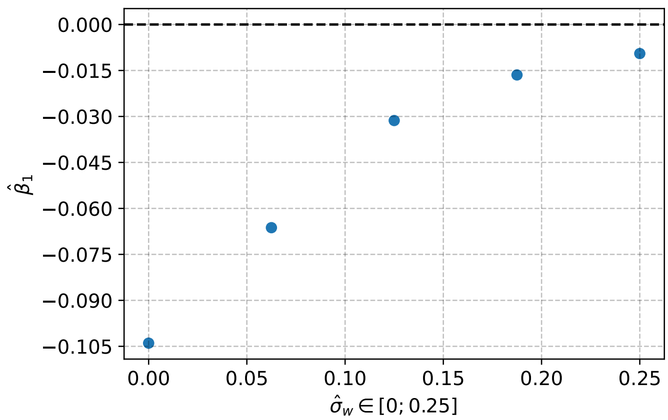


human capital reduces the extent to which comparative advantages in the labor market can explain household allocation. Thus, relative wages have less predictive power on the intra-household time allocation to domestic work hours for couples with higher levels of human capital.

13 Measurement Errors

To investigate measurement errors, it is not necessary to solve the model for different levels of measurement errors since this does not influence the agents' expectations. Hence, I can simply simulate data and run a regression as done in Eq. (4.1) for different levels of σ_w. The results are plotted in Figure 8. From the figure, increasing the measurement

Figure 8: $\hat{\beta}$ Values for Various Levels of Variance in Measurement Error Term



Note: I was unable to solve for values greater than $\sigma_w = 0.25$.

errors causes the relative wages to reflect household productivity less accurately. This subsequently diminishes the ability of relative wages to predict relative domestic work time, as also discussed in Question 4 (i.e. $\hat{\beta}_1$ values closer to zero). The findings align with the observations made by Siminski and Yetsenga, 2022, who note that measurement errors introduce a bias toward zero in the slope parameter. Thus, the bias toward zero is consistent with the estimated $\hat{\beta}_1$ values depicted in Figure 8.

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