Assignment 2: Household Labor Supply and Taxes

Household Behavior over the Life Cycle

Hans Christian Jul Lehmann*

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*gsd758@alumni.ku.dk

Contents

1 Reformulating the Utility Function

As a point of departure, I state the original utility function (the full model setup can be found on GitHub)

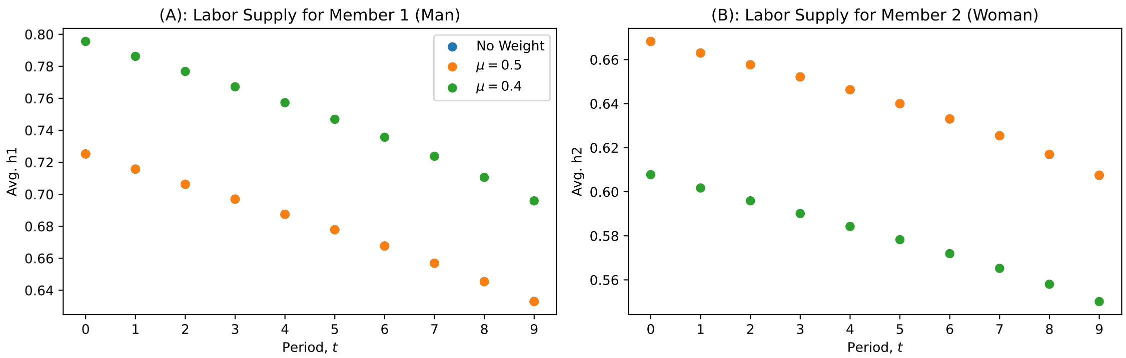
$$U\left(c_t, h_{1,t}, h_{2,t}\right) = 2\frac{(c_t/2)^{1+\eta}}{1+\eta} - \rho_1 \frac{h_{1,t}^{1+\gamma}}{1+\gamma} - \rho_2 \frac{h_{2,t}^{1+\gamma}}{1+\gamma}. \tag{1.1}$$

Now, to change the utility function to be a weighted sum of the individual utilities in the household, I rewrite Eq. (1.1) as

$$U\left(c_t, h_{1,t}, h_{2,t}\right) = \mu\left(\frac{(c_t/2)^{1+\eta}}{1+\eta} - \rho_1 \frac{h_{1,t}^{1+\gamma}}{1+\gamma}\right) + (1-\mu)\left(\frac{(c_t/2)^{1+\eta}}{1+\eta} - \rho_2 \frac{h_{2,t}^{1+\gamma}}{1+\gamma}\right), \tag{1.2}$$

where $\mu = \lambda_1/(\lambda_1 + \lambda_2)$ is a convenient normalization and λ_i for $i \in \{1, 2\}$ governs the weight share on each member of the household. Furthermore, I obtain $\mu = 0.5$, by setting $\lambda_1 = \lambda_2 = 1$ (i.e. putting equal weight on each household member), which gives the same simulated behavior as the original equation.

Figure 1: Simulated Labor Supply for Different (Weighted) Utility Function



I have plotted Figure 1 for $\mu = 0.5$. As expected, the weighted utility curve aligns precisely with the original. As a sanity check, I change the value of μ to 0.4. Since I am effectively decreasing the weight on member 1's utility, the labor supply of member 1 (the man) increases, as shown in Panel A.

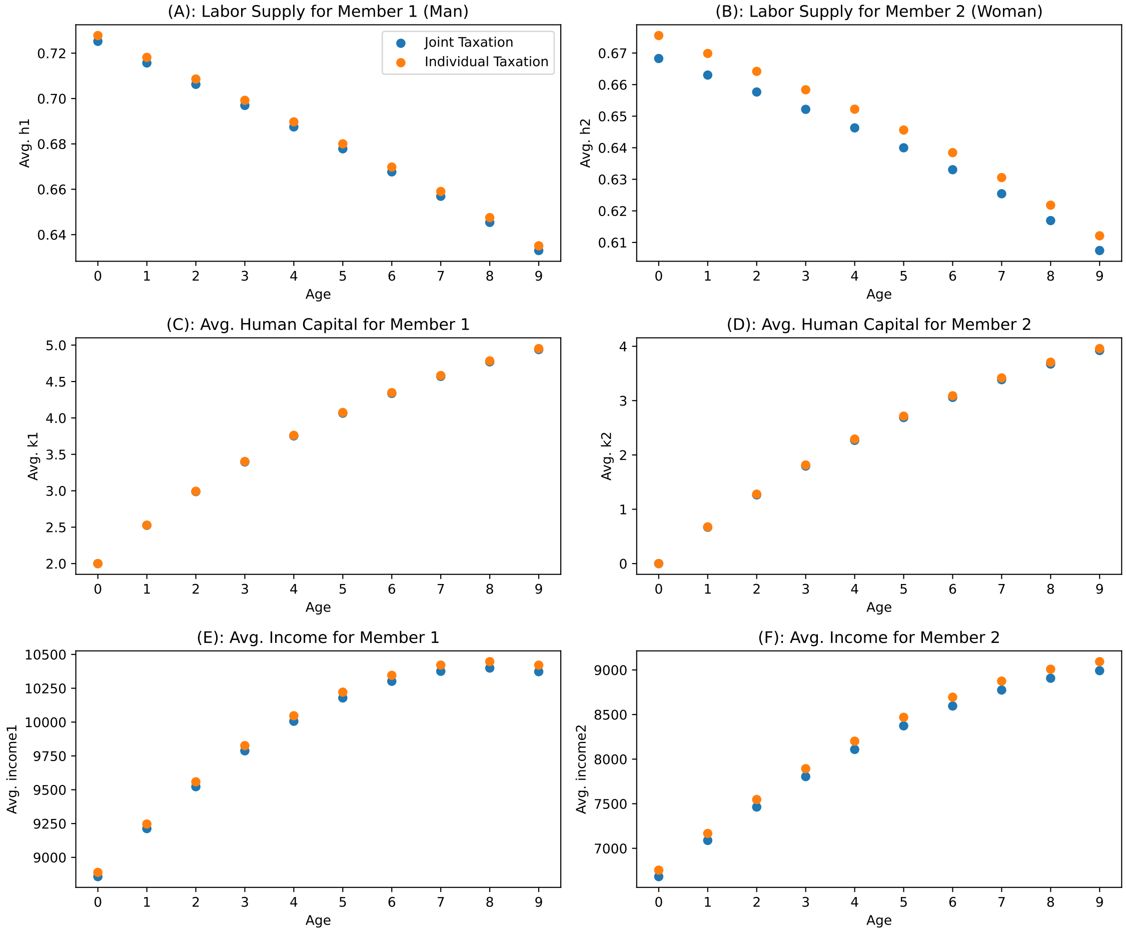
2 Intuition Behind the Weighted Utility

Intuitively, when I weight the household members equally ($\mu = 0.5$) and sum as done in Eq. (1.2), it is the same as summing the individual members' utilities. Although the scaling between the original utility function in Eq. (1.1) and the reformulated utility function in Eq. (1.2) differs, this difference does not affect the household's decision-making. Therefore, the behavior remains unchanged. In conclusion, only the relative weight of utility matters.

3 Labor Supply Responses from Different Tax Schemes

I now explore labor supply responses under different tax systems: *individual taxation* and *joint taxation*. In Figure 2, I have plotted the simulated behavior of the households for both tax systems.

Figure 2: Simulated Behavior from Different Tax Schemes over the Life Cycle



Note: I simulate the model with $1,000$ households.

The simulation shows that when the household is taxed jointly the labor supply expressed along the intensive margin, work hours, reduces. Notably, the labor supply for household member two (the woman) is lowered because the woman faces a higher marginal tax rate captured in the parameter τ reflecting the progression of the tax system. Combining this with a disutility to work captured by the parameter γ, the joint taxation results in a greater disincentive to supply extra work hours for the woman. Furthermore, labor participation is

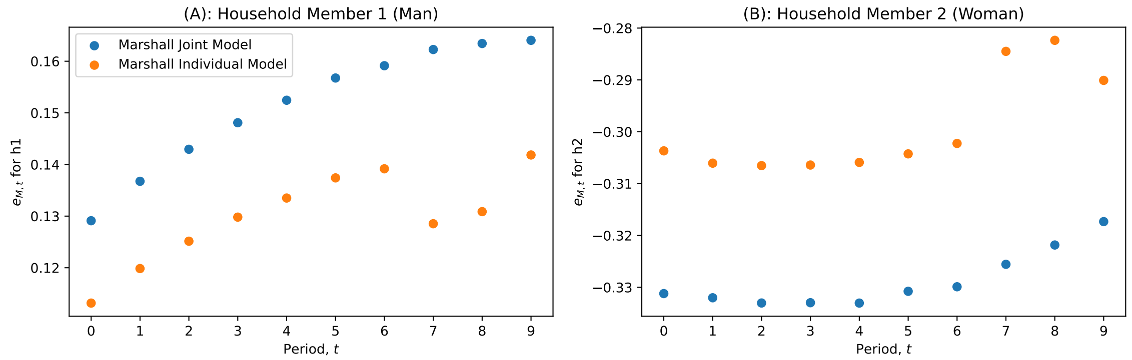
greater for the man than the woman, which aligns with Borella et al.
Lastly, the average income is lower under joint taxation partly due to greater taxation and stemming from the fact that lower labor supply manifests in a reduced accumulation of human capital from which the wage depends through a Mincer-type specification.

Now, looking at the Marshallian elasticity for each household member, defined as *"the percent increase in labor supply that accompanies a 1 percent increase in the after-tax wage rate for the given household member"* cf. Keane, 2011. Formally, I state the Marshallian elasticity as

$$e_{M,t} = \frac{h_t(w(1+0.01), N) - h_t(w, N)}{h_t(w, N)} \cdot 100. \tag{3.1}$$

From Figure 3, I first note that for household member 1 (the man), the substitution effect dominates the income effect, thus increasing labor supply. In contrast, the income effect dominates the substitution effect for the woman. The economic intuition behind this result is that individuals with more human capital are better off, and in this case, the man starts with more human capital than the woman. Consequently, there is a degree of specialization in the household's division of labor in which the man reaps the economic benefits and the woman has a lower labor market participation.
Secondly, there is a clear difference in responsiveness depending on the tax system. The response in labor supply is greater during a joint system due to a more progressive taxation scheme than individual taxation. The larger response implies a greater fostering of specialization in the household's division of labor.

Figure 3: Simulated Marshallian Elasticity from Different Tax Schemes



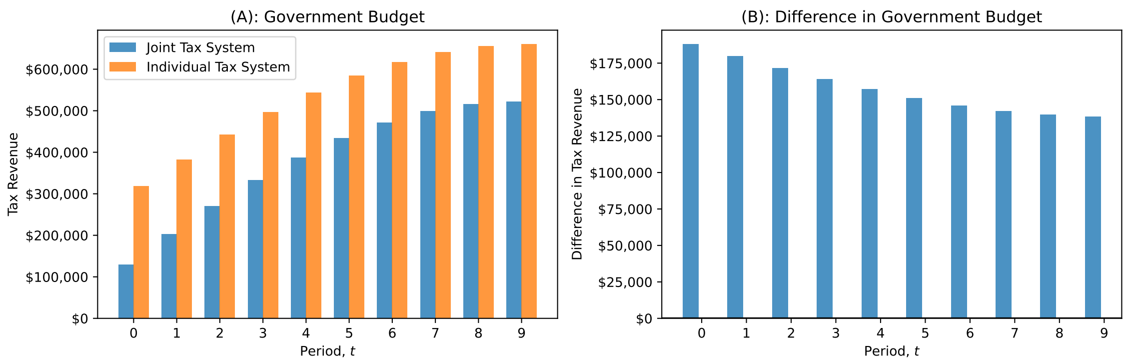
4 Tax Scheme Effect on Government Budget

Next, I calculate the government budget under the two tax systems and explore the transition from a joint taxation to an individual taxation system. To calculate the government budget, I aggregate tax revenues both across time and individual households. Then, I compare the differences in government revenue generated under the different tax regimes. Formally, I define the difference in government revenue as

$$\begin{aligned}\mathrm{T}^{\text{Joint}\rightarrow\text{Indiv}} &= \sum_{i=1}^{N}\sum_{t=1}^{T} T_{it}^{\text{Indiv}} - \sum_{i=1}^{N}\sum_{t=1}^{T} T_{it}^{\text{Joint}} \\ &= \sum_{i=1}^{N}\sum_{t=1}^{T} \left(1-\lambda^{\text{Indiv}} Y_{1,it}^{-\tau^{\text{Indiv}}}\right)\cdot Y_{1,it} + \left(1-\lambda^{\text{Indiv}} Y_{2,it}^{-\tau^{\text{Indiv}}}\right)\cdot Y_{2,it} - \\ &\sum_{i=1}^{N}\sum_{t=1}^{T} \left(1-\lambda^{\text{Joint}}\left(Y_{1,it}+Y_{2,it}\right)^{-\tau^{\text{joint}}}\right)\cdot\left(Y_{1,it}+Y_{2,it}\right).\end{aligned} \tag{4.1}$$

Using Eq. (4.1), I find the difference in government budget to be $\mathrm{T}^{\text{Joint}\rightarrow\text{Indiv}} \approx 5{,}342{,}973.4 - 3{,}765{,}393.4 = 1{,}577{,}580$. Figure 4 below shows how the government budget evolves under the different tax systems. From Panel A, I note that the tax revenue generated increases over time, as a result of wages being endogenously dependent on human capital. Furthermore, looking at Panel B, it is clear that the individual taxation system generates a larger government revenue in all periods. The larger government budget should be seen in the light of the labor responses in Section 3. There I concluded that under joint taxation, the labor participation is reduced (hence lower income), particularly for the woman. Hence, joint taxation results in a reduced tax base and lower government revenue.

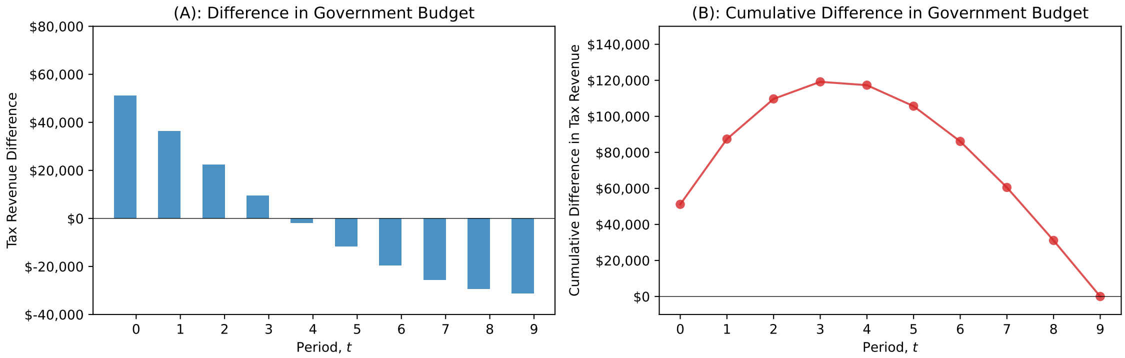
Figure 4: Simulated Government Revenue from Different Tax Schemes



5 Equivalent Government Budget

I wish to find the tax scale parameter, $\lambda^{\text{Private},*}$, that ensures the government revenue over the life cycle is the same regardless of the tax system. I opt for Scipy's root-finding scalar *brentq* to find the $\lambda^{\text{Private},*}$ that yields equivalent revenue under both tax systems. Using the solver, I find that $\lambda^{\text{Private},*} \approx 1.7655$, thus increasing the tax burden from 1.75 to 1.7655 lowers tax revenue from individual taxation such that it corresponds to tax revenue generated from joint taxation. Figure 5 Panel A illustrates a period-specific difference in government revenue. However, over the life cycle, the cumulative revenue difference is approximately zero cf. Panel B.

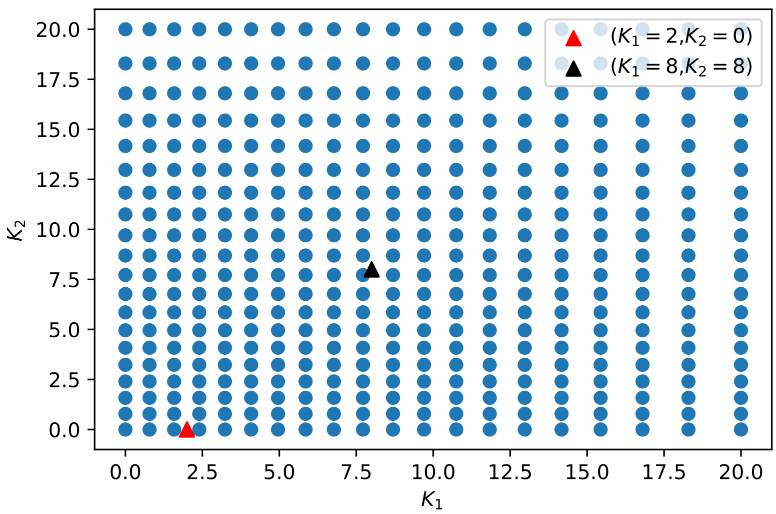
Figure 5: Simulated Government Revenue from Different Tax Schemes, $\lambda^{\text{Private},\star} \approx 1.765469$



6 Initial Tax System Preference

A general approach to solving finite horizon models is to discretize the continuous state variables, secondly, solve by backward recursion on the grid, and thirdly apply a method to interpolate between grid points (Keane et al., 2011). Looking at Figure 6, it is evident that I need to interpolate between grid points to obtain the value function in the two relevant points, respectively.

Figure 6: Tensor Product 2D Grid



Note: The two sets of grids, denoted by $\vec{K}_1$ and $\vec{K}_2$, are used to form the Cartesian product.

I compare the interpolated value functions under two different taxation systems at the beginning of the households' life. From Table 1, it can be seen that a household with initial human capital of two for the man and zero for the woman is indifferent between the individual and joint taxation systems as the interpolated value functions are approximately the same. In the case of high human capital for both members (which I interpret as both members being well-educated), there is a preference for the individual tax system. The intuition behind the results is that if the household possesses a high level of human capital, they derive greater benefit under the individual tax system because the marginal taxation is lower.

Table 1: Comparison of $\check{V}_0$ for Different Taxation Schemes and Initial Human Capital Levels

	$\check{V}_0\left(K_{1,0}, K_{2,0}\right)$		
Scenario	Joint Taxation	Individual Taxation	Difference
$K_{1,0} = 2, K_{2,0} = 0$	-0.2268036560245404	-0.22677299424853892	0.00003
$K_{1,0} = 8, K_{2,0} = 8$	-0.18915572656119886	-0.1884092670012815	0.00075

References

Borella, M., M. De Nardi, and F. Yang (forthcoming). "Are Marriage-Related Taxes and Social Security Benefits Holding Back Female Labor Supply?" In: *Review of Economic Studies*.

Keane, Michael P. (2011). "Labor Supply and Taxes: A Survey". In: *Journal of Economic Literature* 49.4, pp. 961–1075.

Keane, Michael P., Petra E. Todd, and Kenneth I. Wolpin (Jan. 2011). "Chapter 4 - The Structural Estimation of Behavioral Models: Discrete Choice Dynamic Programming Methods and Applications". In: *Handbook of Labor Economics*. Ed. by Orley Ashenfelter and David Card. Vol. 4. Elsevier, pp. 331–461.