Assignment 1: Labor Supply and Children

Household Behavior over the Life Cycle

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Contents

1 Manual Calibration

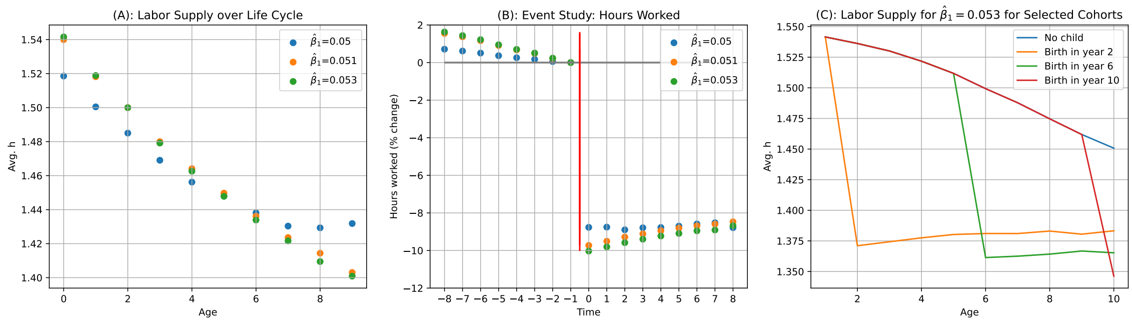
In the paper Kleven et al. (2019), the authors show that gender inequality to a large extent can be attributed to children, the so-called child penalty which they define as *"the percentage by which women fall behind men due to children"*. The child penalty can manifest in both extensive and intensive margins. Regarding the intensive margin, work hours, the authors find a 10 percent decrease in hours in period (or year) 1 due to a birth in period 0 cf. Figure 1B in Kleven et al.

I wish to reproduce this finding from Kleven et al. in the context of my baseline model (the full baseline model setup can be found on GitHub). I restate the equation that relates the disutility of work to the presence of a child

$$\beta(n_t) = \beta_0 + \beta_1 \cdot n_t, \tag{1.1}$$

where β_1 governs the strength of the disutility of working when having a child and n_t takes the value 1 in the presence of a child. I will vary the parameter β_1 in Eq. (1.1) to get a 10 percent drop in work hours in the year following birth. Since β_1 enters the Bellman equation negatively, I will search among values just above 0.05. Using a "handheld" calibration of the model, I find that a $\hat{\beta}_1$ value of 0.053 yields a 10 percent drop in work hours the year following a birth event (see Figure 1 Panel B).[1] In Figure 1 Panel A, I show

Figure 1: Labor Supply for Different $\hat{\beta}_1$ Values



Note: The model was simulated with 1,000 individuals.

that individuals supply the most labor hours at the beginning of the life cycle. This is motivated by a precautionary motive to accumulate human capital before the occurrence of a birth event. Human capital accumulation drives the dynamics of the model rather than, for example, savings since the model includes a Mincer-type wage. The behavior aligns with what is seen in Panel B, where labor supply decreases up to the birth event due to the reduction in uncertainty as the birth event approaches. Finally, Panel C further illustrates the decrease in labor supply for selected cohorts as they approach the birth event.

[1] Alternatively, I could have used structural estimation and let the 10 percent target drop be a *moment* I want to capture, and then use a solver to find the associated β_1 value.

In the remainder of the paper, I set the value of β_1 to 0.053. A complete list of parameter values for the baseline model are shown in Table 1.

Table 1: Overview of Parameter Values in the Baseline Model

Parameter	Description	Value
β_0	Weight on labor disutility	0.10
β_1	Additional weight on labor disutility	0.05
η	CRRA coefficient	-2.00
γ	Curvature on labor hours	2.50
ρ	Discount factor	1/1.02
α	Human capital accumulation	0.30
w	Wage base level	1.00
τ	Fixed income tax rate	0.10
p_n	Probability of child arrival	0.10
r	Interest rate on savings	0.02
T	Time horizon	10

2 Labor Supply and Children

I consider labor supply elasticity, in order to explore the effects of a marginal tax increase on the labor participation of the individual over the life cycle. A particular elasticity is the Marshallian elasticity, known as the *uncompensated* or *total elasticity*. To implement the Marshallian elasticity, I introduce an unanticipated permanent tax increase of 1 percent and solve the model under the new tax regime. I impose the permanent unanticipated tax from period 0 because the individuals have perfect foresight allowing me to interpret the Marshallian elasticity as a long-run elasticity as the tax hike lasts the remaining of the individual's life. The long-run Marshallian elasticity across age is calculated as

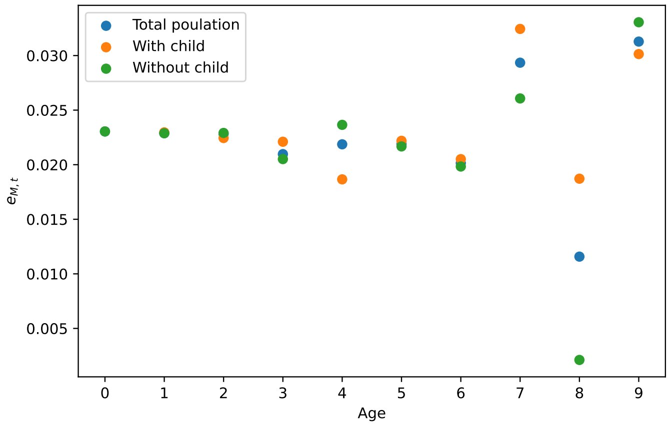
$$e_{M,t} = \frac{h_t(\tau = 0.101, N) - h_t(\tau = 0.1, N)}{h_t(\tau = 0.1, N)} \cdot 100, \tag{2.1}$$

and the average long-run Marshallian elasticity is

$$e_M \equiv \frac{1}{T} \sum_{s=t}^{T} e_{M,s} = \frac{1}{T} \sum_{s=t}^{T} \frac{h_t(\tau = 0.101, N) - h_t(\tau = 0.1, N)}{h_t(\tau = 0.1, N)} \cdot 100. \tag{2.2}$$

I find that the average long-run Marshallian elasticity is $e_M \approx 0.023$. The positive elasticity reflects the income effect is dominating the substitution effect. Looking at Figure 2 illustrating the Marshallian elasticity across age the income effect is dominating at all ages. To investigate the impact of having a child on labor supply, I compare the simulated elasticities across age for the total population against two the sub-populations: people with a child and people who remain childless through out life. There appears to be a pattern

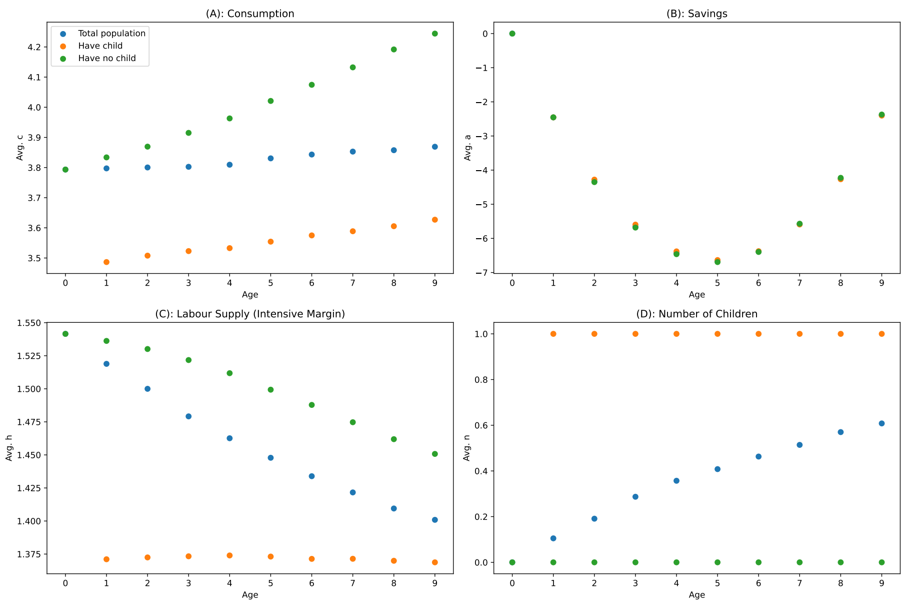
Figure 2: Marshallian Elasticity Across Age for Different Subpopulations



where the labor supply of people with one child is more responsive in the latter periods of the life cycle. This responsiveness may reflect their relatively low income, implying a more substantial income effect. Moreover, there seem to be some spikes that I cannot account for, but I suspect they are due to a numerical approximation error. The conclusions drawn must therefore be interpreted with caution.

Lastly, I plot the simulated behavior of the individuals over the life cycle in Figure 3. From Panel C it is evident people have a substantially lower labor supply than people

Figure 3: Simulated Life Cycle Behavior for Various Sub-Populations in the Baseline Model



who remain childless over the life cycle.

3 Introducing a Spouse

Next, I introduce a spouse who contributes an age-dependent exogenous wage in each period. The exogenous wage of the spouse is

$$y_t = 0.1 + 0.01 \cdot t. \tag{3.1}$$

Note, I do *not* let the spouse be subject to any income tax. Subsequently, I reformulate the affected equations of the model. A complete reformulation of the model, which includes all extensions, can be found in Section 6. The original member of the household, referred to as the "primary earner", has an after-tax Mincer-type wage process defined as

$$w_t = (1 - \tau) \cdot w \left(1 + \alpha k_t\right), \quad \tau \in (0, 1), \tag{3.2}$$

where w is a baseline wage, τ is a fixed income tax rate, and α maps work experience to human capital. Combining Equation (3.1) and (3.2), and taking into account that the primary earner's wage depends endogenously on hours supplied, the total income process of the household is given by

$$\text{income}_t^{hh} = w_t \cdot h + y_t. \tag{3.3}$$

Consequently, the budget constraint is

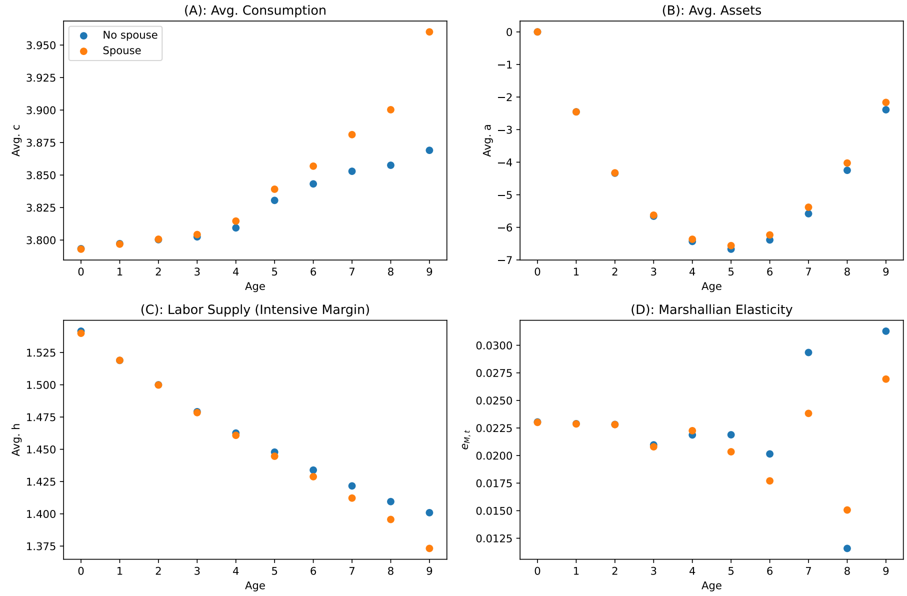
$$a_{t+1} = (1 + r)(a_t + \text{income}_t^{hh} - c_t), \tag{3.4}$$

where a_t refers to assets at time t and c_t refers to consumption at time t. The budget constraint in the final period is

$$a_T = \text{income}_T^{hh} - c_T. \tag{3.5}$$

In Figure 4 I plot the the model with a spouse present against the baseline model. Looking at Panel A, consumption is higher, particularly in the last periods, due to the spouse's wage being age-dependent cf. Eq. (3.1). Furthermore, in Panel C, the hours supplied by the primary earner are reduced since the household's total income has increased. Keeping in mind that both consumption and leisure are normal goods, the increase in income implies that the primary earner works fewer hours and consumes more consumption goods. Finally, in Panel D, the Marshallian elasticity is lower when introducing a spouse. I attribute the lower responsiveness of the labor supply to the increased income resulting in a less incentive to work extra hours following a tax hike.

Figure 4: Simulated Behavior when Introducing a Spouse



4 Introducing Childcare Costs

I introduce childcare costs in the presence of a child. In the model, I let childcare costs defined by the parameter θ depend on the number of children. I define the childcare costs as follows

$$\text{childcarecosts}_t = \theta \cdot n_t, \tag{4.1}$$

enabling me to disable childcare costs by setting $\theta = 0$. I include the childcare costs in the income equation of the household[2]

$$\text{income}_t^{hh} = w_t \cdot h + y_t - \text{childcarecosts}_t, \tag{4.2}$$

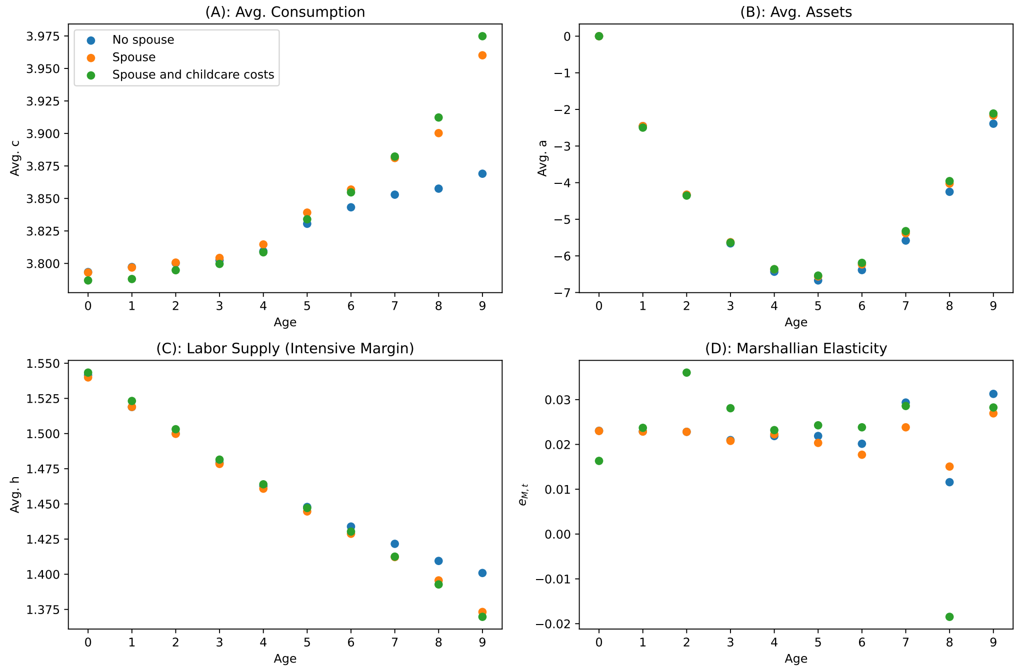
thereby affecting the budget constraint in all periods, including the final period.[3] Figure 5 displays the simulated behavior for the baseline model, the model with the introduction of a spouse, and an alternative model that includes both a spouse and childcare costs.

Ex ante, I expect to see an increase in the Marshallian elasticity because childcare costs negatively affect the household's budget. From Figure 5 Panel D, the labor supply of the primary earner now seems more responsive as the elasticity of the model with spouse and childcare costs lies above the model with a spouse and no childcare costs. Furthermore, from Panel C, the primary earner accumulates more human capital because childcare costs impose a greater child penalty associated with having a child.

[2] I could have incorporated the childcare costs directly into the budget constraint. However, this modification does not alter any of the results.

[3] In order to maintain a positive consumption in the final period while considering childcare costs and spouse income, the minimum hours for the final period should be calculated as $h^{\min} = -(a_T + y_T - \text{childcarecosts}_T)/w_T$.

Figure 5: Simulation: Baseline Model versus Augmented Model with Spouse and Childcare Costs



Note: In the baseline model: $y_t = 0 \; \forall \; t$ and $\theta = 0$, in the model with spouse: $y_t = 0.1 + 0.01 \cdot t$ and $\theta = 0$, and in the model with spouse and childcare costs: $y_t = 0.1 + 0.01 \cdot t$ and $\theta = 0.05$.

5 A Discussion of Endogenous Fertility Adjustments

In parts of the literature, fertility is often treated as exogenous, as discussed by Adda et al., 2017. However, it is crucial to note that not only might a (mechanical) income effect be at play, but a substitution effect could also influence the labor supply decisions of the household. Given the importance of measuring fertility adjustments in response to policy reforms, such as reducing childcare subsidies, it would be inadequate to model fertility exogenously, as explored in Jakobsen et al., 2022. This stance arises from the fact that changes in the opportunity cost of having a child directly influence fertility and, consequently, the labor supply - a consideration that should be incorporated into the model, as demonstrated in Adda et al., 2017.

I would expect that introducing endogenous fertility results in a trade-off between human capital accumulation and having children. As a result, a reform reducing childcare subsidies (effectively increasing childcare costs) would, all things being equal, incentivize households to accumulate more human capital (due to a greater opportunity cost) and reduce fertility. However, the effects may be heterogeneous, as "tastes for" children can vary across households.

6 Extending the Model with a Stochastic Spouse

We need information about choices and states today and transition rules to solve a dynamic model. In the case of the presence of a spouse being stochastic, this alters the transition rules of the model, thus altering the expectations formed by the individuals in the model. Therefore, I need to introduce a new state variable $s_t \in \{0,1\}$ for which I solve the model for each possible state of spouse presence.[4] In the following, I state the complete recursive formulation of the full model. Letting $n_t \in \{0,1\}$ denote the presence of a child, and $s_t \in \{0,1\}$ denote the presence of a spouse, the Bellman equation and the recursive formulation of the model is

$$
\begin{aligned}
V_t(s_t, n_t, a_t, k_t) =& \max_{c_t, h_t} \frac{c_t^{1+\eta}}{1+\eta} - \beta(n_t)\frac{h_t^{1+\gamma}}{1+\gamma} + \rho \mathbb{E}_t[V_{t+1}(s_{t+1}, n_{t+1}, a_{t+1}, k_{t+1})] \\
& \text{s.t.} \\
n_{t+1} =& \begin{cases} n_t + 1 & \text{with probability } p(n_t, s_{t+1}) \\ n_t & \text{with probability } 1 - p(n_t, s_{t+1}) \end{cases}, \\
p(n_t, s_{t+1}) =& \begin{cases} p_n & \text{if } n_t = 0 \text{ and } s_{t+1} = 1 \\ 0 & \text{else} \end{cases}, \\
s_t =& \begin{cases} 1 & \text{with probability } p_s \\ 0 & \text{with probability } 1 - p_s \end{cases}, \\
a_{t+1} =& (1+r)(a_t + \text{income}_t^{hh} - c_t), \\
k_{t+1} =& k_t + h_t,
\end{aligned}
$$

where h_t denotes the hours worked at time t, and k_t is the (dynamic) human capital at time t. Children can only arrive in the presence of a spouse, and the likelihood of spousal presence is $p_s = 0.8$. Furthermore, we have

$$
\begin{aligned}
w_t =& (1-\tau) \cdot w\left(1 + \alpha k_t\right), \\
y_t =& 0.1 + 0.01 \cdot t, \\
\text{childcarecosts}_t =& \theta \cdot n_t, \\
\text{income}_t^{hh} =& w_t \cdot h + y_t \cdot s_t - \text{childcarecosts}_t,
\end{aligned}
$$

where the wage process is endogenous, spouse's income age dependent given spousal presence. The probability of child arrival depends on whether a child is already present such that a household can only have 1 child. Let the disutility of work depend on the presence of children,

$$\beta(n_t) = \beta_0 + \beta_1 \cdot n_t.$$

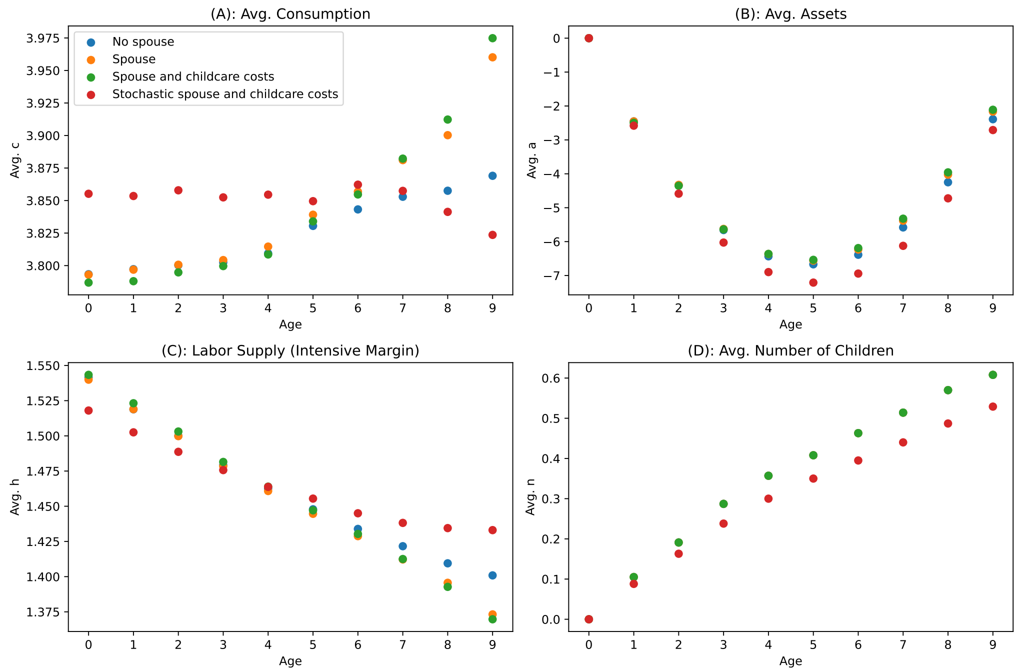
[4]Admittedly, I take a somewhat naive approach.

We obtain the baseline model by setting $\theta = 0$, $y_t = 0 \; \forall \; t$, and $p_s = 1$. I recursively solve the model from the last period of life using Value Function Iteration (VFI), where it is optimal to consume all resources. Thus, the terminal condition is

$$V_T(s_T, n_T, a_T, k_T) = \max_{h_T} \frac{c_T^{1+\eta}}{1+\eta} - \beta(n_T)\frac{h_T^{1+\gamma}}{1+\gamma}$$
$$\text{s.t.}$$
$$c_T = a_T + \text{income}_T^{hh}.$$

Having solved the model, I initialize 80 percent of the households with a spouse and simulate forwards. When comparing the fertility of the extended model against the previous models, the fertility is now lower in all periods following year 0 (see Panel D in Figure 6).

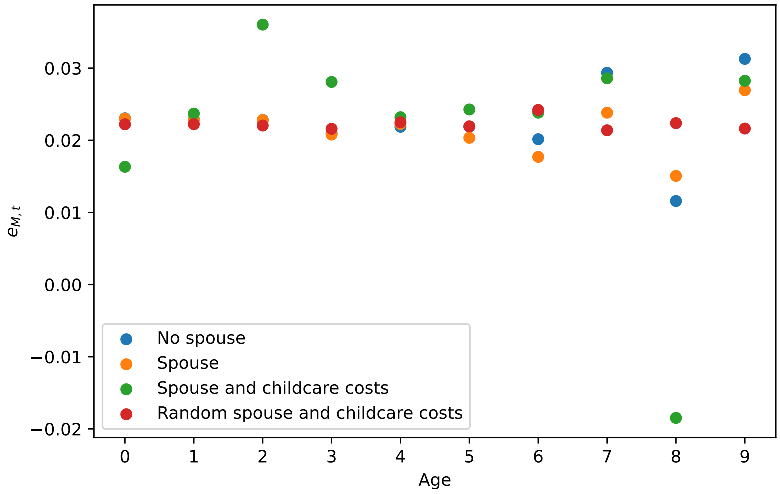
Figure 6: Simulated Behavior over the Life Cycle with Stochastic Spouse



Introducing a "stochastic" spouse has two opposite effects. On the one hand, it creates uncertainty about future income, which tends to lower income. On the other hand, a lower likelihood of having children results in a higher household income. Panel C shows that hours are higher in the new model in the last part of the life cycle since they have fewer children on average.

Finally, the Marshallian elasticity is plotted in Figure 7. Upon examination of these plotted elasticities, it appears that the elasticity is lowered. However, due to the aforementioned opposing effects and potential numerical errors, any conclusions drawn may be invalid.

Figure 7: Simulated Marshallian Elasticity in Model Augmented with Stochastic Spouse



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