Written Exam Economics Summer 2023
Household Behavior over the Life Cycle

21 June 2023 from 9 AM to 23 June at 9 AM

This exam question consists of 6 pages in total.

Answers only in English.

You should hand in a single zip-file with all assignments and the exam. The zip-file should be named after your KU username (e.g. abs123) and have the following folder and file structure:

Assignment_1
Assignment_1.pdf - with text and all results
files for reproducing the results

Assignment_2
Assignment_2.pdf - with text and all results
files for reproducing the results

Assignment_3
Assignment_3.pdf - with text and all results
files for reproducing the results

Exam
Exam.pdf - with text and all results
files for reproducing the results

Setup and Model Economy

We will analyze how couples allocate time to labor market work and home production. This exam text is self-contained but some of the analysis is motivated by Siminski and Yetsenga (2022), who analyze Australian time-use data for couples.

The model framework is a unitary one, in which couples jointly choose how much each member should work in the labor marker, $l_{j,t}$, $j \in \{f,m\}$, and how much each member should work on home production, $h_{j,t}$, $j \in \{f,m\}$, in each year $t = 1,\dots,T$. For example, $l_{f,t} \in [0,24]$ denotes how many hours the female works in the labor market per day during year t. Likewise, for men and home production. Total time spent on working per day in a year is $T_{j,t} = l_{j,t} + h_{j,t}$ which cannot exceed 24 hours.

In each period, household members accumulate human capital, $K_{j,t}$, if they work and wages are a function of the level of human capital. Couples care about market goods, C_t, and home produced goods, H_t, through the composite good $Q_t = C_t^{\omega} H_t^{1-\omega}$ where $\omega \in (0,1)$ is the relative weight on market goods. Home production is a (CES) function of home production of both members,

$$H_t = \left(\alpha h_{f,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) h_{m,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma \in (0,1)$ is the elasticity of substitution and $\frac{\alpha}{1-\alpha} \in (0,1)$ is the productivity in home production of females relative to males (her absolute advantage in home production, AAH). Household preferences over work and consumption are

$$U(T_{f,t}, T_{m,t}, Q_t) = \frac{Q_t^{1-\rho}}{1-\rho} - \nu \left(\frac{T_{f,t}^{1+\frac{1}{\epsilon_f}}}{1+\frac{1}{\epsilon_f}} + \frac{T_{m,t}^{1+\frac{1}{\epsilon_m}}}{1+\frac{1}{\epsilon_m}} \right) \tag{1}$$

in which the last term is the dis-utility from total work hours ($\nu > 0$) and $\epsilon_j > 0$ controls the curvature of the dis-utility. $\rho > 1$ is the constant relative risk aversion coefficient.

Couples cannot save and thus spend all available income (earned income plus non-earned income, X_t) on market goods. The recursive formulation of the model is for $t \leq T$:

$$V_t(K_{f,t}, K_{m,t}) = \max_{l_{f,t}, h_{f,t}, l_{m,t}, h_{m,t}} U(T_{f,t}, T_{m,t}, Q_t) + \beta V_{t+1}(K_{f,t+1}, K_{m,t+1}) \tag{2}$$

$$\text{s.t.} \tag{3}$$

$$C_t = w_{f,t} l_{f,t} + w_{m,t} l_{m,t} + X_t \tag{4}$$

$$H_t = \left(\alpha h_{f,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) h_{m,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{5}$$

$$Q_t = C_t^{\omega} H_t^{1-\omega} \tag{6}$$

$$\log w_{j,t} = \gamma_{j,0} + \gamma_{j,1} K_{j,t},\ j \in \{f, m\} \tag{7}$$

$$K_{j,t+1} = (1-\delta) K_{j,t} + l_{j,t}/24,\ j \in \{f, m\} \tag{8}$$

$$T_{j,t} = l_{j,t} + h_{j,t},\ j \in \{f, m\} \tag{9}$$

$$l_{j,t}, h_{j,t} \geq 0,\ j \in \{f, m\} \tag{10}$$

$$T_{j,t} \leq 24,\ j \in \{f, m\} \tag{11}$$

where β is the discount factor, $\gamma_{j,0}$ is the intercept of log-wages and $\gamma_{j,1}$ is the return to human capital. Human capital depreciates in each period with a rate of $\delta \in [0, 1]$. There are no bequest motive and

$$V_{T+1}(K_{f,T+1}, K_{m,T+1}) = 0. \tag{12}$$

The baseline parameters are $T = 8$, $\beta = 0.98$, $\rho = 2$, $\nu = 0.001$, $\epsilon_f = \epsilon_m = 1$, $\omega = 0.5$, along with $\alpha = 0.7$, $\sigma = 0.3$ and $X_t = 0$, $\gamma_{f,0} = \gamma_{m,0} = 0$, $\gamma_{f,1} = \gamma_{m,1} = 0.1$ and $\delta = 0.1$. When simulating data from the model, each couple $i = 1, \ldots, N$ is initialized with a random tuple of human capital, drawn from independent uniform distributions.

Questions

1. Explain which main trade-offs a couple faces over their life cycle in this model.

2. Explain in your own words how the model is solved numerically.

3. Plot the average simulated age-profiles of all four choice-variables.

4. Siminski and Yetsenga (2022) estimate a regression relating the relative hours in home production to the relative wage rates of men and women:

$$\log(h_{i,f,t}/h_{i,m,t}) = \beta_0 + \beta_1 \log(w_{i,f,t}/w_{i,m,t}) + \varepsilon_{i,t} \tag{13}$$

   and estimate roughly $\hat{\beta}_0 = 0.4$ and $\hat{\beta}_1 = -0.1$.
   Using the simulated data, plot the log of relative hours of home production, $\log(h_{i,f,t}/h_{i,m,t})$ against the log of relative wages of females, $\log(w_{i,f,t}/w_{i,m,t})$. Discuss the resulting plot and compare with the implied relationship from the estimated coefficients in Siminski and Yetsenga (2022). What does a weak link (β_1 close to zero) between relative wages and relative time spend in home production indicate about the allocation of time in the household?
   [*Hint:* The accompanying code includes a method called `model.plot()`. Furthermore, `model.regress()` returns $\hat{\beta}_0$ and $\hat{\beta}_1$ estimated using simulated data.]

5. We now want to add the possibility of a child arriving. Let $n_t \in \{0, 1\}$ denote the presence of a child. Let the probability that a child arrives in period $t + 1$ be

$$p_{t+1}(n_t) = \begin{cases} 0.1 & \text{if } n_t = 0 \\ 0.0 & \text{else.} \end{cases}$$

   Children affect the weight on market purchased and home produced consumption goods through

$$\omega(n_t) = \omega + \omega_n n_t$$

   where baseline parameter values are $\omega = 0.5$ and $\omega_n = -0.2$.
   Write out the new recursive formulation of the model.

6. What is the interpretation of a negative ω_n?

7. Implement the extended version of the model. Solve the model and simulate data from the model. Plot the average simulated age-profiles of all four choice-variables along with the average share of couples with children. Compare with the baseline model and discuss potential differences.

8. Plot the average of the specialization index

$$SI_{i,t} = \frac{h_{i,f,t}}{h_{i,f,t} + h_{i,m,t}} - \frac{l_{i,f,t}}{l_{i,f,t} + l_{i,m,t}}$$

   around childbirth (an event-study graph). Discuss the interpretation of the plot.
   Siminski and Yetsenga (2022) roughly find a 100% increase in average $SI_{i,t}$ from the period just before childbirth to that at childbirth ($t = 0$ vs. $t = -1$). Compare that to your simulations.
   [*Hint:* The index is stored in `sim.specialization`.]

9. Estimate, using simulated method of moments (SMM/SMD), the three parameters $\theta = (\alpha, \sigma, \omega_n) \in (0,1) \times (0,1) \times (-\omega_0, 1 - \omega_0)$ that produce approximately the same results as those discussed above from Siminski and Yetsenga (2022). Concretely match the three moments

$$\Lambda^{data} = \begin{pmatrix} 0.4 \\ -0.1 \\ 1.0 \end{pmatrix} \begin{array}{l} \text{the constant } (\hat{\beta}_0) \text{ in the regression in eq. (13)} \\ \text{the slope } (\hat{\beta}_1) \text{ in the regression in eq. (13)} \\ \text{change in avg. } SI_{i,t} \text{ index at childbirth } (t=0)\text{, relative to } t=-1 \text{ (in pct.)} \end{array}$$

   where the last element, 1.0, refers to the 100% increase in the specialization index at time of birth relative to the period just prior.
   What does the estimates suggest that females' absolute advantage in home production, AAH, $\frac{\alpha}{1-\alpha}$, is (in order to fit the data)?
   [*Hint:* Preferably a numerical solver is used but alternatively a grid-search can be used. Good starting values could be $\theta_0 = (0.98, 0.1, -0.25)$.]

10. Plot and comment on the model fit, i.e. plot the event-study graph and the log of relative hours of home production, $\log(h_{i,f,t}/h_{i,m,t})$ against the log of relative wages of females, $\log(w_{i,f,t}/w_{i,m,t})$.

11. Plot average male working hours, $l_{i,m,t}$, around childbirth (event-study graph). Does the simulated behavior align with your expectations (and existing literature)? If not, what/which parameters could we let depend on children to improve on the model predictions in this dimension?

12. We now want to investigate the role of human capital accumulation for the intra-household allocation of time. We will do so by changing the return to human capital, $\gamma_{j,1}$. Let $\gamma_{f,1} = \gamma_{m,1} = \gamma_1$ and produce a plot with five values of $\gamma_1 \in [0.1, 1]$ on the x-axis and the slope-coefficient $\hat{\beta}_1$ from simulated data on the y-axis. Discuss your findings.

13. Measurement error in wages is a potential driver of the weak relationship between relative wages and the relative time spend in home production. Let observed wages be $w^{obs}_{i,j,t}$ and "true" wages be $w_{ij,,t}$. Assuming that measurement error is additive *iid* Normally distributed with mean zero and variance σ^2_w, we have that observed wages are

$$w^{obs}_{i,j,t} = w_{i,j,t} + \sigma_w \varepsilon_{i,j,t},\ \varepsilon_{i,j,t} \sim iid\mathcal{N}(0,1).$$

Simulate data for five values of $\sigma_w \in [0,1]$ and produce a plot with σ_w on the x-axis and the slope-coefficient $\hat{\beta}_1$ from simulated data on the y-axis. Discuss your findings. [*Hint:* Recall that agents in the economy do not care about the measurement error that we as economists observe in the data. All that matters for agents in the model is therefore $w_{i,j,t}$ as before.]

References

Siminski, P. and R. Yetsenga (2022): "Specialization, ComparativeAdvantage, and the Sexual Divisionof Labor," *Journal of Labor Economics*, 40(4), 851–887.