

Here is a polynomial for which I need a mathematical proof that it is non-negative:

$$\mathbf{C}_2 = (\alpha\psi + \beta\theta + \gamma(\theta + \psi))^2 - 4(\alpha + \beta + \gamma)\gamma\psi\theta \quad (1)$$

Through some tedious math I can show that \mathbf{C}_2 satisfies the identity

$$(\alpha + \gamma)^2 \frac{\mathbf{C}_2}{\theta^2} = 4\alpha\beta\gamma(\alpha + \beta + \gamma) + \left((\alpha + \gamma)^2 \frac{\psi}{\theta} + \alpha\beta - (\alpha + \beta + \gamma)\gamma \right)^2 \quad (2)$$

The proof introduces the new parameter $\tau = \psi/\theta$ and shows that \mathbf{C}_2/θ^2 can be written in the form

$$\frac{\mathbf{C}_2}{\theta^2} = A_0 + A_1\tau + A_2\tau^2$$

with

$$A_0 = (\beta + \gamma)^2 \quad (3)$$

$$A_1 = 2(\alpha\beta - (\alpha + \beta + \gamma)\gamma) \quad (4)$$

$$A_2 = (\alpha + \gamma)^2 \quad (5)$$

and re-writes it in the form

$$\frac{\mathbf{C}_2}{\theta^2} = E + F(\tau - G)^2$$

where all three polynomials E , F , and G do not contain τ . This gives (2).

I tried to use SymPy for this proof with the python3 code 20200910_sympy.py submitted at the same time.

```
import sympy
sympy.init_printing(use_unicode=True)

alpha, beta, gamma, psi, theta, tau, CCC_2 = \
    sympy.symbols('alpha_beta_gamma_psi_theta_tau_CCC_2')

psi=theta*tau

CCC_2 = (alpha*psi+beta*theta+gamma*(psi+theta))**2 \
        -4*(alpha+beta+gamma)*gamma*psi*theta

sympy.pprint(sympy.collect(CCC_2, tau))
```

The output of the `sympy.pprint` command is

$$\mathbf{C}_2 = -\gamma\tau\theta^2(4\alpha + 4\beta + 4\gamma) + (\alpha\tau\theta + \beta\theta + \gamma(\tau\theta + \theta))^2$$

which can be re-written as

$$\mathbf{C}_2 = -4\gamma\tau\theta^2(\alpha + \beta + \gamma) + \theta^2((\alpha + \gamma)\tau + \beta + \gamma)^2$$

This is similar to (2) but has one important difference: It can be re-written in the form

$$\mathbf{C}_2 = E + F(\tau - G)^2$$

but whereas the decomposition in my proof required that all three polynomials E , F , and G do not contain τ , the above $E = -4\gamma\tau\theta^2(\alpha + \beta + \gamma)$ does contain τ . This is an error in the `sympy.collect` command.