

Here is a polynomial for which i need a mathematical proof that it is non-negative:

$$\mathbf{C}_2 = (\alpha\psi + \beta\theta + \gamma(\theta + \psi))^2 - 4(\alpha + \beta + \gamma)\gamma\psi\theta \quad (??)$$

Through some tedious math I can show that \mathbf{C}_2 satisfies the identity

$$(\alpha + \gamma)^2 \frac{\mathbf{C}_2}{\theta^2} = 4\alpha\beta\gamma(\alpha + \beta + \gamma) + \left((\alpha + \gamma)^2 \frac{\psi}{\theta} + \alpha\beta - (\alpha + \beta + \gamma)\gamma \right)^2 \quad (1)$$

The proof consists in writing \mathbf{C}_2 in the form

$$\frac{\mathbf{C}_2}{\theta^2} = E_0 + E_1\tau + E_2\tau^2$$

with

$$E_0 = (\beta + \gamma)^2 \quad (2)$$

$$E_1 = 2(\alpha\beta - \alpha\gamma - \beta\gamma - \gamma^2) = 2(\alpha\beta - (\alpha + \beta + \gamma)\gamma) \quad (3)$$

$$E_2 = (\alpha + \gamma)^2 \quad (4)$$

and then re-write it in the form

$$\mathbf{C}_2 = E + F(\tau - G)^2$$

where all three polynomials E , F , and G do not contain τ . Then I tried to use SymPy for this proof with the following code included in the same zip file: The output of the `sympy.pprint` command is

$$\mathbf{C}_2 = -\gamma\tau\theta^2(4\alpha + 4\beta + 4\gamma) + (\alpha\tau\theta + \beta\theta + \gamma(\tau\theta + \theta))^2$$

which can be re-written as

$$\mathbf{C}_2 = -4\gamma\tau\theta^2(\alpha + \beta + \gamma) + \theta^2((\alpha + \gamma)\tau + \beta + \gamma)^2$$

This has the form

$$\mathbf{C}_2 = E + F(\tau - G)^2$$

but whereas decomposition I used for my proof required that all three polynomials E , F , and G do not contain τ , the above $E = -4\gamma\tau\theta^2(\alpha + \beta + \gamma)$ does contain τ . I think this is a design error in the `sympy.collect` command.