Big Data in String Theory

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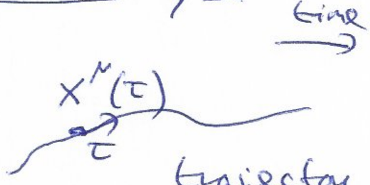
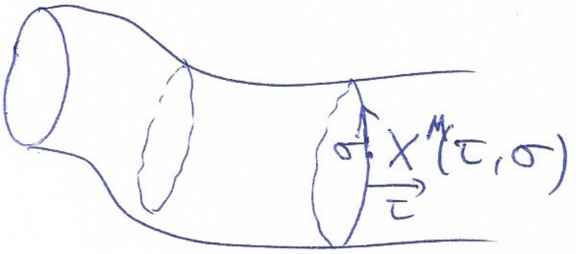
- QFT point-like particle



$\underbrace{\mu = 0,1,2,3}$ Why 4D?

- string theory : extended objects : strings

closed string time $\longrightarrow$



$M = 0,1,2,3,\ldots,9$

$\uparrow$ fixed by string theory

unification of QFT & gravity

$\Rightarrow$ extra dimensions

hide extra dimensions from observation, i.e. make them compact $\hat{=}$ compactification

4D Minkowski space-time

$\underbrace{\hspace{6cm}}$ tiny six-dim. compact space C

- SUPERSYMMETRY in 4D $\hat{=}$ C has special property "Calabi-Yau" (smooth manifold)

$\updownarrow$

"orbifold" (singularities)

Outline

- Motivation
- What happens in compactification?

  Examples: 5D complex scalar on circle $\Rightarrow$ learn: internal momentum $\hat{=}$ mass in 4D

  6D SU(3) gauge theory on torus $\Rightarrow$ learn: gauge theory in extra dim. gives gauge theory + matter in 4D
- Orbifold compactification
- Strings on orbifolds
- The data set

complex scalar in 5D compactified on circle S^1

- 5D space-time with coord. x^M, $M = \underbrace{0,1,2,3}_{\substack{\text{our 4D} \\ \text{space-time} \\ \hat{=}\, \text{index } \mu}}, \underbrace{4}_{\substack{\text{extra} \\ \text{dimension} \\ y := x^4}}$

with Minkowski metric $\eta_{MN} = \mathrm{diag}(1,-1,-1,-1,-1)$

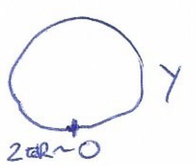
- complex scalar in 5D

action

$$S_{5D}[\varphi] = \int d^5x \, (\partial^M \varphi)^* (\partial_M \varphi)$$

- compactify on circle of radius R

$$y \sim y + 2\pi R$$

$\Rightarrow$ 

on S^1 circle

$\Rightarrow$ boundary condition:

$$\varphi(x^\mu, y + 2\pi R) = \varphi(x^\mu, y)$$

$\Rightarrow$ Fourier expansion of $\varphi(x^\mu, y)$

$$\varphi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \varphi_n(x^\mu) \exp\left(\frac{i n y}{R}\right)$$

like momentum in y direction (but quantized) test boundary condition

$$\varphi(x^\mu, y + 2\pi R) = \sum_{n=-\infty}^{\infty} \varphi_n(x^\mu) \underbrace{\exp\left(\frac{in(y+2\pi R)}{R}\right)}$$

$$= \exp\left(\frac{iny}{R}\right) \underbrace{\exp(2\pi i n)}_{=1}$$

$$= \varphi(x^\mu, y) \checkmark$$

$\Rightarrow$ <u>plug $\varphi(x^\mu, y)$ into $S_{5D}[\varphi]$:</u>

$$\partial^\mu \varphi = \sum_{n=-\infty}^{\infty} (\partial^\mu \varphi_n(x^\mu)) \exp\left(\frac{iny}{R}\right)$$

$$\partial^4 \varphi = \frac{\partial}{\partial x_4}\varphi = -\frac{\partial}{\partial x^4}\varphi = -\frac{\partial \varphi}{\partial y}$$

$$= -\sum_{n=-\infty}^{\infty} \varphi_n(x^\mu)\left(\frac{in}{R}\right) \exp\left(\frac{iny}{R}\right)$$

$\Rightarrow$ $\boxed{\begin{array}{l}\text{internal momentum} \\ \hat{p}^4 = +i\partial^4 \\ \hat{p}^4\varphi \approx \frac{n}{R}\varphi, \ n \in \mathbb{Z}\end{array}}$

$$\partial_4 \varphi = -\partial^4 \varphi$$

then

$$S_{5D}[\varphi] = \int d^5x \left[(\partial^\mu \varphi)^*(\partial_\mu \varphi) + (\partial^4 \varphi)^*(\partial_4 \varphi)\right]$$

$$= \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \int d^5x \Big[(\partial^\mu \varphi_{n_1}(x^\mu))^* \exp\left(\frac{-in_1 y}{R}\right)$$

$$(\partial_\mu \varphi_{n_2}(x^\mu)) \exp\left(\frac{in_2 y}{R}\right)$$

$$- \varphi_{n_1}(x^\mu)^* \varphi_{n_2}(x^\mu) \left(\frac{-in_1}{R}\right)\left(\frac{in_2}{R}\right) \exp\left(\frac{-in_1 y}{R}\right) \exp\left(\frac{in_2 y}{R}\right)\Big]$$

$$= \sum_{n_1,n_2} \int d^5x \Big[(\partial^\mu \varphi_{n_1})^* (\partial_\mu \varphi_{n_2}) \exp\left(\frac{iy}{R}(n_2-n_1)\right)$$

$$- (\varphi_{n_1})^* (\varphi_{n_2}) \frac{n_1 n_2}{R^2} \exp\left(\frac{iy}{R}(n_2-n_1)\right)\Big]$$

$$\int_0^{2\pi R} dy \, \exp\left(\frac{iy}{R}(n_2-n_1)\right) = 2\pi R \, \delta_{n_1 n_2}$$

$$\delta_{n_1 n_2} = \begin{cases} 1 & \text{if } n_1 = n_2 \\ 0 & \text{if } n_1 \neq n_2 \end{cases}$$

$$\Rightarrow = \sum_{n_1,n_2} \int d^4x \Big[(\partial^\mu \varphi_{n_1})^* (\partial_\mu \varphi_{n_2}) \, \delta_{n_1 n_2}$$

$$- \frac{n_1 n_2}{R^2} (\varphi_{n_1})^* (\varphi_{n_2}) \, \delta_{n_1 n_2} \Big] \, 2\pi R$$

$$= 2\pi R \int d^4x \sum_n \Big[(\partial^\mu \varphi_n)^* (\partial_\mu \varphi_n) - \frac{n^2}{R^2} |\varphi_n|^2 \Big]$$

$\Rightarrow$ 5D scalar $\varphi(x^\mu, y)$ on circle (massless) $\hat{=}$ infinite number of 4D scalars $\varphi_n(x^\mu)$

(mass $m_n^2 = \frac{n^2}{R^2}$)

$\pm n \Rightarrow$ same m_n^2

Kaluza-Klein tower of massive particles

for $n = 0$: $\varphi_0(x^\mu)$ is massless so-called <u>zero-mode</u>

internal momentum $\hat{=}$ mass in 4D

6D SU(3) gauge theory on torus

$$S = \int d^6x \left[-\frac{1}{4} F^a_{MN} F^{a\,MN} \right]$$

$M, N = 0,1,2,3,\underbrace{4,5}_{\text{compactify}}$
sum over $a = 1,\dots,8$

where field strength of SU(3):

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + \underbrace{g}_{\text{coupling constant}} \underbrace{f^{abc}}_{\substack{\text{anti-sym. in } a,b,c \\ \text{structure constants} \\ \text{of gauge group}}} A^b_M A^c_N$$

"easy" way to get zero-mode action : set $\partial_i = 0 \Leftrightarrow$ set $A^a_N(x^M)$ to $A^a_N(x^\mu)$

$$\Rightarrow F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$F^a_{ij} = \underbrace{\partial_i A^a_j}_{=0} - \underbrace{\partial_j A^a_i}_{=0} + g f^{abc} A^b_i A^c_j$$

$$= i g A^b_i \underbrace{(-i f^{abc})}_{=:(T^a_{adj})^{bc}} A^c_j = i g A^T_i T^a_{adj} A_j$$

$(T^a_{adj})^{bc}$ generators of adj. repres. of SU(3)

$$F^a_{\mu i} = \partial_\mu A^a_i - \underbrace{\partial_i A^a_\mu}_{=0} + g f^{abc} A^b_\mu A^c_i$$

$$= \partial_\mu A^a_i - i g \underbrace{(i f^{abc})}_{=-i f^{bac} = (T^b_{adj})^{ac}} A^b_\mu A^c_i$$

$$= \left(\delta^{ac} \partial_\mu - i g A^b_\mu (T^b_{adj})^{ac} \right) A^c_i$$

gauge covariant derivative

$$D_\mu = \mathbb{1}\,\partial_\mu - ig A_\mu^b T^b_{adj}$$

$$\Rightarrow F^a_{\mu i} = (D_\mu A_i)^a$$

then

$$-\frac{1}{4} F^a_{MN} F^{aMN} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

$$-\frac{1}{4} F^a_{\mu i} F^{a\mu i} \quad -\frac{1}{4} \underbrace{F^a_{i\mu}}_{=-F^a_{\mu i}} \underbrace{F^{ai\mu}}_{=-F^{a\mu i}}$$

$$-\frac{1}{4} F^a_{ij} F^{aij}$$

$$= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

$$-\frac{1}{2} F^a_{\mu i} \underbrace{F^{a\mu i}}_{=-F^{a\mu}_{\ \ i}}$$

$$-\frac{1}{4} F^a_{ij} F^a_{ij}$$

$$= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} (D_\mu A_i)^a (D^\mu A_i)^a$$

$$+ \frac{g^2}{4} \left(A_i^T T^a_{adj} A_j\right)\left(A_i^T T^a_{adj} A_j\right)$$

next

• show that $A_i^T T^a_{adj} A_i = 0$:

$$\left(A_i^T T^a_{adj} A_i\right)^T = A_i^T \left(T^a_{adj}\right)^T A_i \underset{\uparrow}{=} -A_i^T T^a_{adj} A_i = +A_i^T T^a_{adj} A$$

$$\Rightarrow A_i^T T^a_{adj} A_i = 0$$

using
$$\left(T^a_{adj}\right)^{bc} = -if^{abc} \quad \left(T^a_{adj}\right)^{cb} = -\left(T^a_{adj}\right)^{bc}$$

- complex scalar

$$\chi := \frac{1}{\sqrt{2}}\left(A_4 + i A_5\right)$$ vectors with 8 comp. for $SU(3)$

$$\Rightarrow \bar{\chi} = \frac{1}{\sqrt{2}}\left(A_4 - i A_5\right)$$

and

$$A_4 = \frac{1}{\sqrt{2}}\left(\chi + \bar{\chi}\right)$$

$$A_5 = -\frac{i}{\sqrt{2}}\left(\chi - \bar{\chi}\right)$$

- potential of complex scalar

then

$$A_5^T T^a_{adj} A_6 = -\frac{i}{2}\left(\chi + \bar{\chi}\right)^T T^a_{adj}\left(\chi - \bar{\chi}\right)$$

$$= -\frac{i}{2}\Big[\underbrace{\chi^T T^a_{adj}\chi}_{=0} - \overbrace{\chi^T T^a_{adj}\bar{\chi}}^{=-\bar{\chi}^T T^a_{adj}\chi}$$

$$+ \bar{\chi}^T T^a_{adj}\chi - \underbrace{\bar{\chi}^T T^a_{adj}\bar{\chi}}_{=0}\Big]$$

$$= -i\,\bar{\chi}^T T^a_{adj}\chi$$

and

$$A_6^T T^a_{adj} A_5 = -A_5^T T^a_{adj} A_6 = i\,\bar{\chi}^T T^a_{adj}\chi$$

$$\Rightarrow \frac{g^2}{4}\left(A_i^T T^a_{adj} A_j\right)\left(A_i^T T^a_{adj} A_j\right) = -\frac{g^2}{2}\left(\bar{\chi}^T T^a_{adj}\chi\right)^2$$

$$\Rightarrow \boxed{V(\chi) = \frac{g^2}{2}\sum_{a=1}^{8}\left(\bar{\chi}^T T^a_{adj}\chi\right)^2}$$

see D-term potential in SUSY

- Kinetic energy of complex scalar

$$\frac{1}{2}(D_\mu A_i)^a(D^\mu A_i)^a = \frac{1}{4}\Big[(D_\mu(\chi+\bar{\chi}))^a(D^\mu(\chi+\bar{\chi}))^a$$

$$-(D_\mu(\chi-\bar{\chi}))^a(D^\mu(\chi-\bar{\chi}))^a\Big]$$

$$= \frac{1}{4}\Big[\cancel{(D_\mu\chi)^a(D^\mu\chi)^a} + (D_\mu\chi)^a(D^\mu\bar{\chi})^a + (D_\mu\bar{\chi})^a(D^\mu\chi)^a$$

$$+\cancel{(D_\mu\bar{\chi})^a(D^\mu\bar{\chi})^a} - \cancel{(D_\mu\chi)^a(D^\mu\chi)^a} + (D_\mu\chi)^a(D^\mu\bar{\chi})^a$$

$$+(D_\mu\bar{\chi})^a(D^\mu\chi)^a - \cancel{(D_\mu\bar{\chi})^a(D^\mu\bar{\chi})^a}\Big]$$

$$= \frac{1}{2}\Big[(D_\mu\chi)^a(D^\mu\bar{\chi})^a + (D_\mu\bar{\chi})^a(D^\mu\chi)^a\Big]$$

$$D_\mu = \mathbb{1}\partial_\mu - igA_\mu^a T^a_{adj} \Rightarrow (D_\mu)^* = \mathbb{1}\partial_\mu + igA_\mu^a \underbrace{(T^a_{adj})^*}_{=-T^a_{adj}}$$

$$= D_\mu$$

$$\ldots = \frac{1}{2}\Big[(D_\mu\chi)^a(D^\mu\chi)^{*a} + (D_\mu\chi)^{*a}(D^\mu\chi)^a\Big]$$

$$= (D_\mu\chi)^\dagger(D^\mu\chi)$$

$$\Rightarrow \boxed{\begin{aligned} &\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + (D_\mu\chi)^\dagger(D^\mu\chi) - V(\chi) \\ &\text{where} \\ &V(\chi) = \frac{g^2}{2}\sum_a(\chi^\dagger T^a_{adj}\chi)^2 \\ &D_\mu = \mathbb{1}\partial_\mu - igA_\mu^a T^a_{adj}\end{aligned}}$$

SU(3) gauge theory in 6D compactified $\Longrightarrow$ zero-modes: SU(3) gauge theory in 4D plus complex scalar in adj. representation of SU(3)

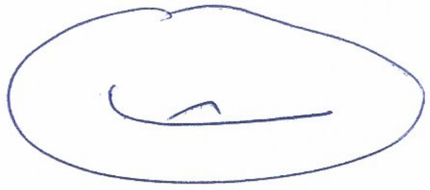
(if SUSY: scalar + fermion)

Orbifold compactification

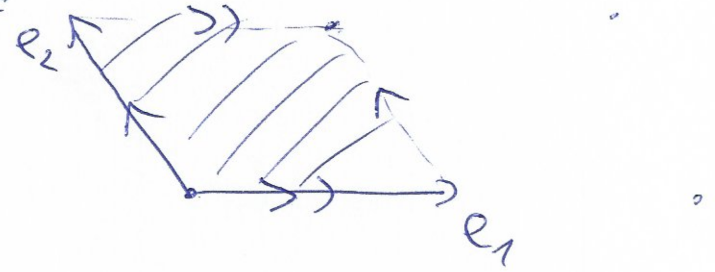
example for two extra dimensions:

torus T^2

$\mathbb{R}^2/\Lambda$ ← two-dim. lattice spanned by e_1 & e_2



$\hat{=}$



for $Y \in \mathbb{R}^2$ identify

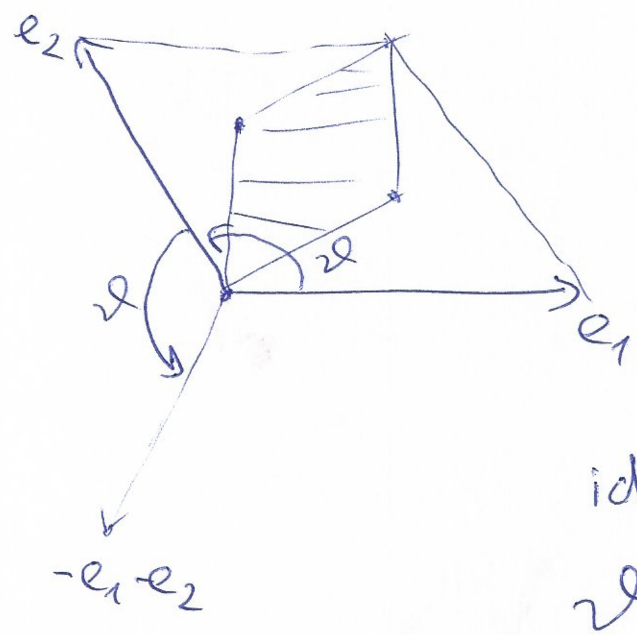
Y and $Y + n_1 e_1 + n_2 e_2$

for all $n_1, n_2 \in \mathbb{Z}$

torus: flat metric

orbifold $T^2/\mathbb{Z}_3$

identify points on T^2 if they differ by 120° rotation



$\vartheta e_1 = e_2$

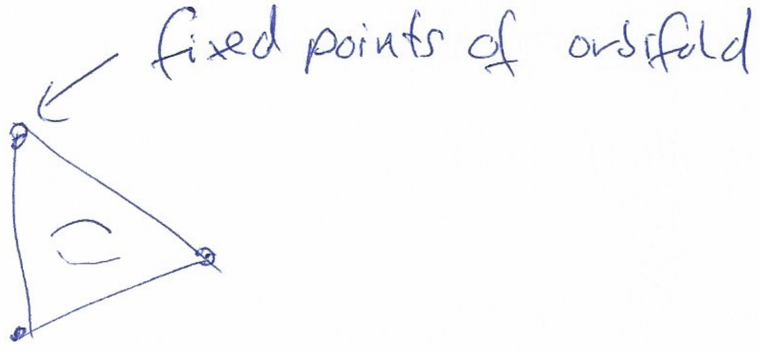
$\vartheta e_2 = -e_1 - e_2$

identify $Y \in \mathbb{R}^2$ and

$\vartheta^k Y + n_1 e_1 + n_2 e_2$

for all $k \in \{0,1,2\}$, $n_1, n_2 \in \mathbb{Z}$

orbifold: flat metric except for:



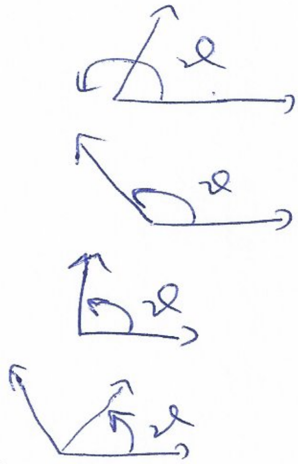
<u>orbifolds in two-dim.</u>

$\vartheta^2 = 1\!\!1 \qquad \mathbb{Z}_2$

or $\vartheta^3 = 1\!\!1 \qquad \mathbb{Z}_3$

or $\vartheta^4 = 1\!\!1 \qquad \mathbb{Z}_4$

or $\vartheta^6 = 1\!\!1 \qquad \mathbb{Z}_6$



- $\mathbb{Z}_N$ orbifolds with $N = 2, 3, 4, 6$
- no other order of rotation in two-dim.

- in total 17 different orbifolds in two-dim. (wall paper groups)

- classification using crystallography in D-dim.

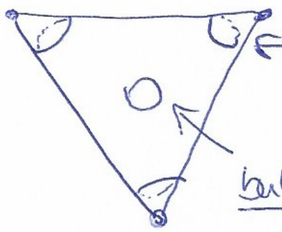
- in $D = 6$ with "Calabi-Yau" condition

$\Rightarrow$ $\boxed{\begin{array}{l}\text{138 orbifolds} \\ \mathbb{Z}_N \text{ or } \mathbb{Z}_N \times \mathbb{Z}_M\end{array}}$ plus 331 orbifolds T^6/P where P is non-Abelian ($a, b \in P$ then $ab \neq ba$ for some a, b)

Strings on orbifolds

heterotic string: 10D $\mathcal{N}=1$ $E_8 \times E_8$

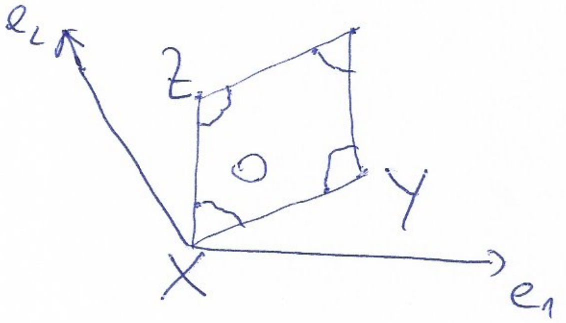
closed strings



← localized at fixed point

($\hat{=}$ no momentum in extra dimension)

bulk: gives gauge bosons + matter



3 "twisted strings" (X, Y, Z)

gives matter

boundary conditions

- $X(\tau, \sigma+1) = X(\tau, \sigma)$ $\quad X(\tau,\sigma)$
- $X(\tau, \sigma+1) = \vartheta X(\tau, \sigma)$ $\Rightarrow$ string X
- $X(\tau, \sigma+1) = \vartheta X(\tau, \sigma) + e_1$ $\Rightarrow$ string Y
- $X(\tau, \sigma+1) = \vartheta X(\tau, \sigma) + e_1 + e_2$ $\Rightarrow$ string Z

Orbifolder

arXiv: 1110.5229 hep-th

- choose orbifold (out of 138 choices)
- choose further compactification parameters

$\Rightarrow$ 4D theory with fixed gauge group, matter spectrum and interactions

The dataset

Ramos-Sanchez et al. 1808.06622 hep-th

$\approx$ 100,000 MSSM-like orbifold models from various $T^6/\mathbb{Z}_N$ and $T^6/\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds

MSSM-like spectrum

$SU(3)_C \times SU(2)_L \times U(1)_Y$

with matter:

$3\,(\underline{3},\underline{2})_{1/6}$ $\quad q_i$ $\qquad i = 1,2,3$ three generations

$3\,(\underline{\bar{3}},\underline{1})_{-2/3}$ $\quad \bar{u}_i$

$3\,(\underline{\bar{3}},\underline{1})_{1/3}$ $\quad \bar{d}_i$

$3\,(\underline{1},\underline{2})_{-1/2}$ $\quad \ell_i$

$3\,(\underline{1},\underline{1})_{1}$ $\quad \bar{e}_i$

and Higgs-pair:

$1\,(\underline{1},\underline{2})_{-1/2}$ $\quad H_d$

$1\,(\underline{1},\underline{2})_{1/2}$ $\quad H_u$

$$W = Y^u_{ij} H_u q_i \bar{u}_j + Y^d_{ij} H_d q_i \bar{d}_j + \dots$$

plus "vector-like exotics" for example

$$1\,(\underline{3},\underline{2})_{1/6} + 1\,(\underline{\bar{3}},\underline{2})_{-1/6} \qquad q_4 + \bar{q}_4$$

or

$$1\,(\underline{1},\underline{1})_{1/2} + 1\,(\underline{1},\underline{1})_{-1/2} \qquad S^+ + S^-$$

then

$$W = M_{Pl}\, q_4\, \bar{q}_4 + M_{Pl}\, S^+ S^- + \dots$$