

5/29/2024

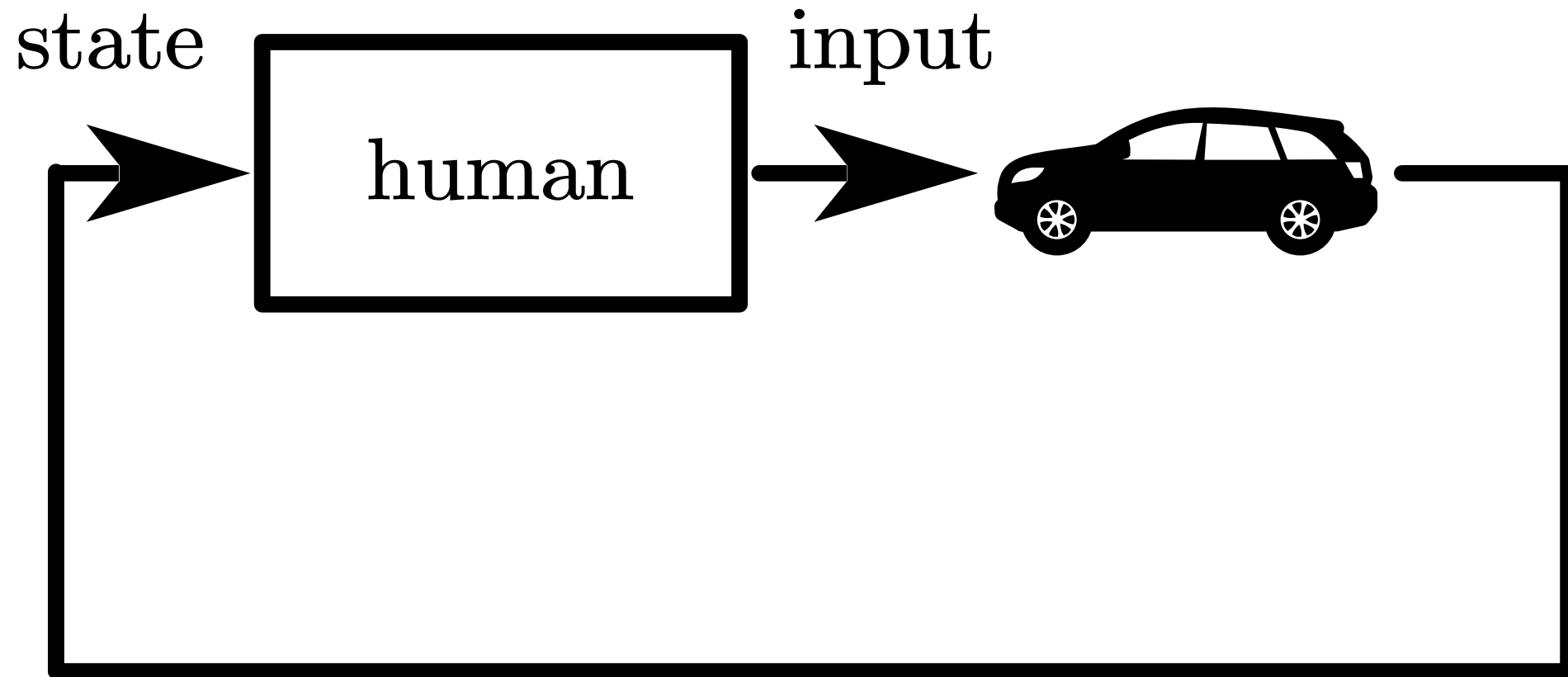
Ph.D. in Aeronautics and Astronautics

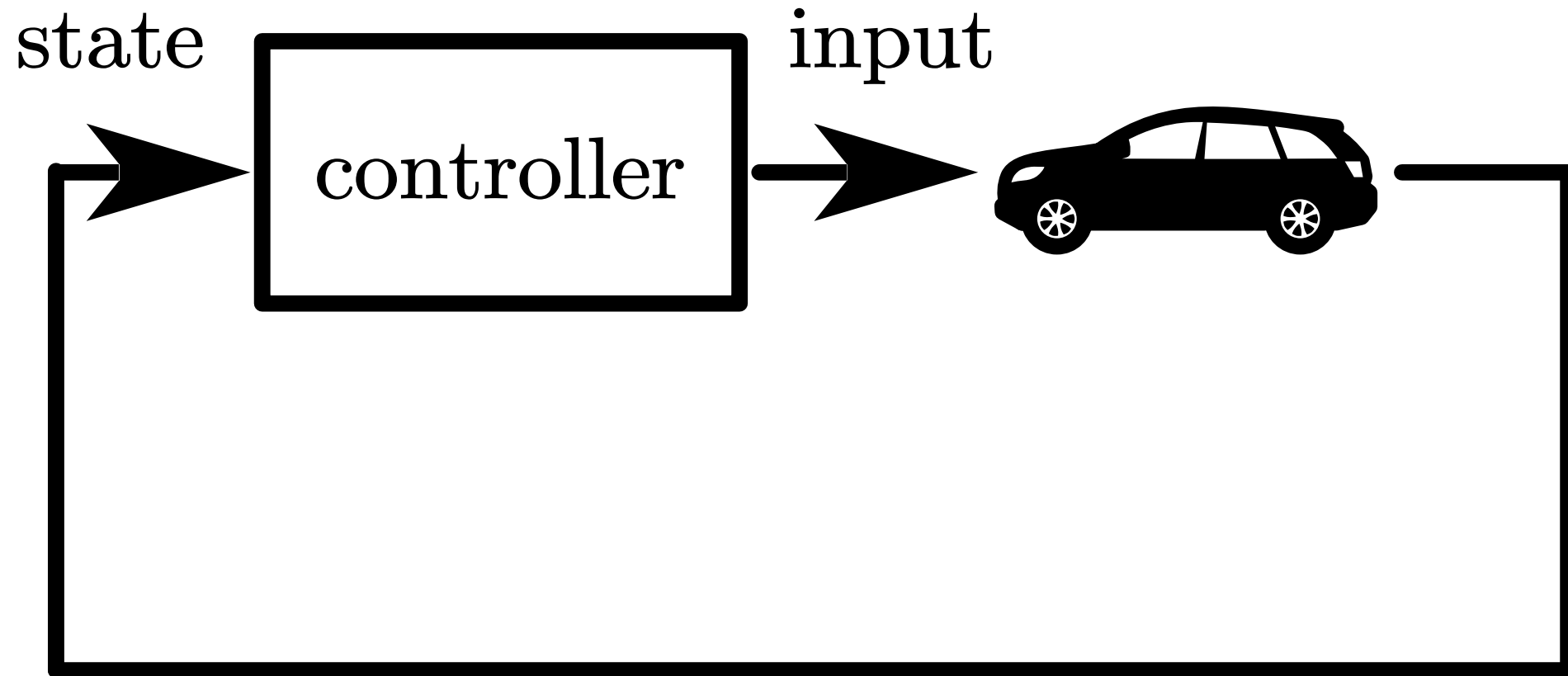
You won't like controllers

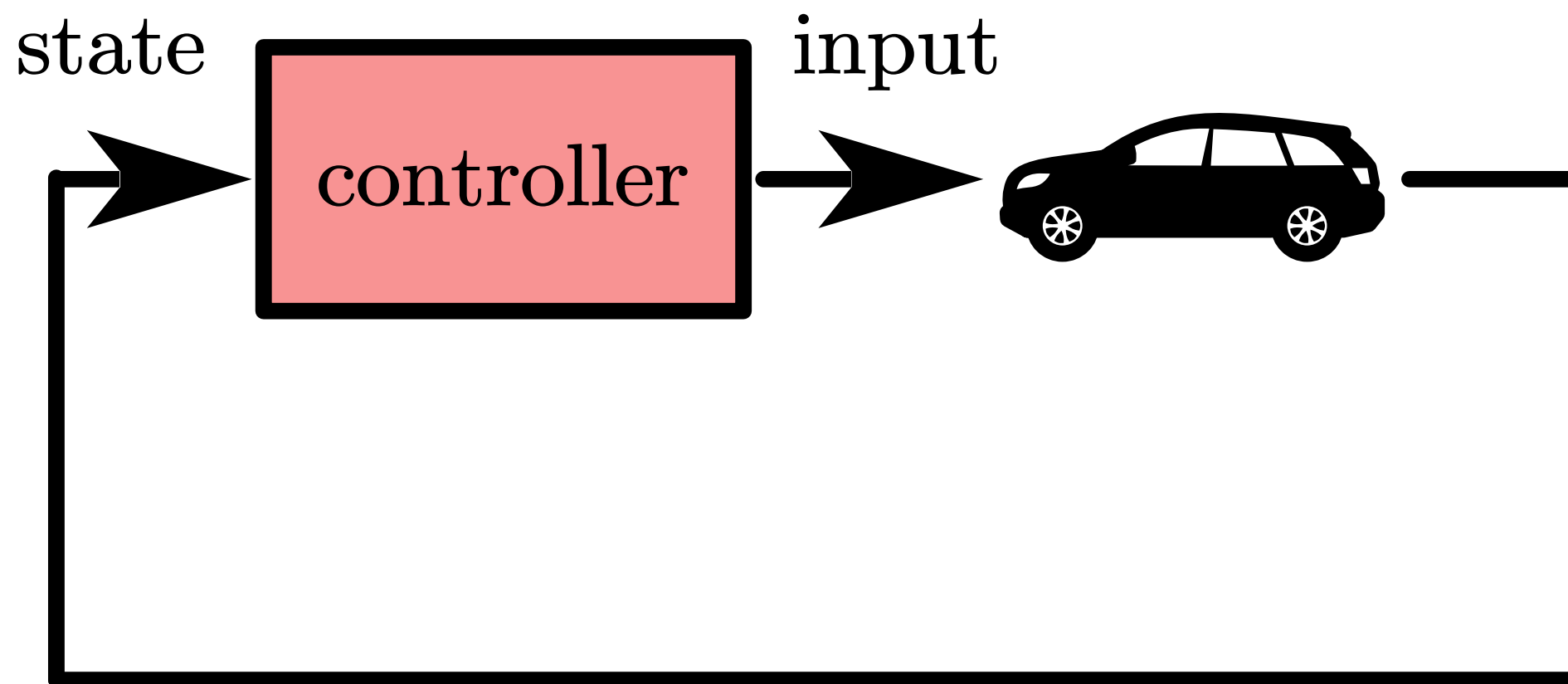
Shen

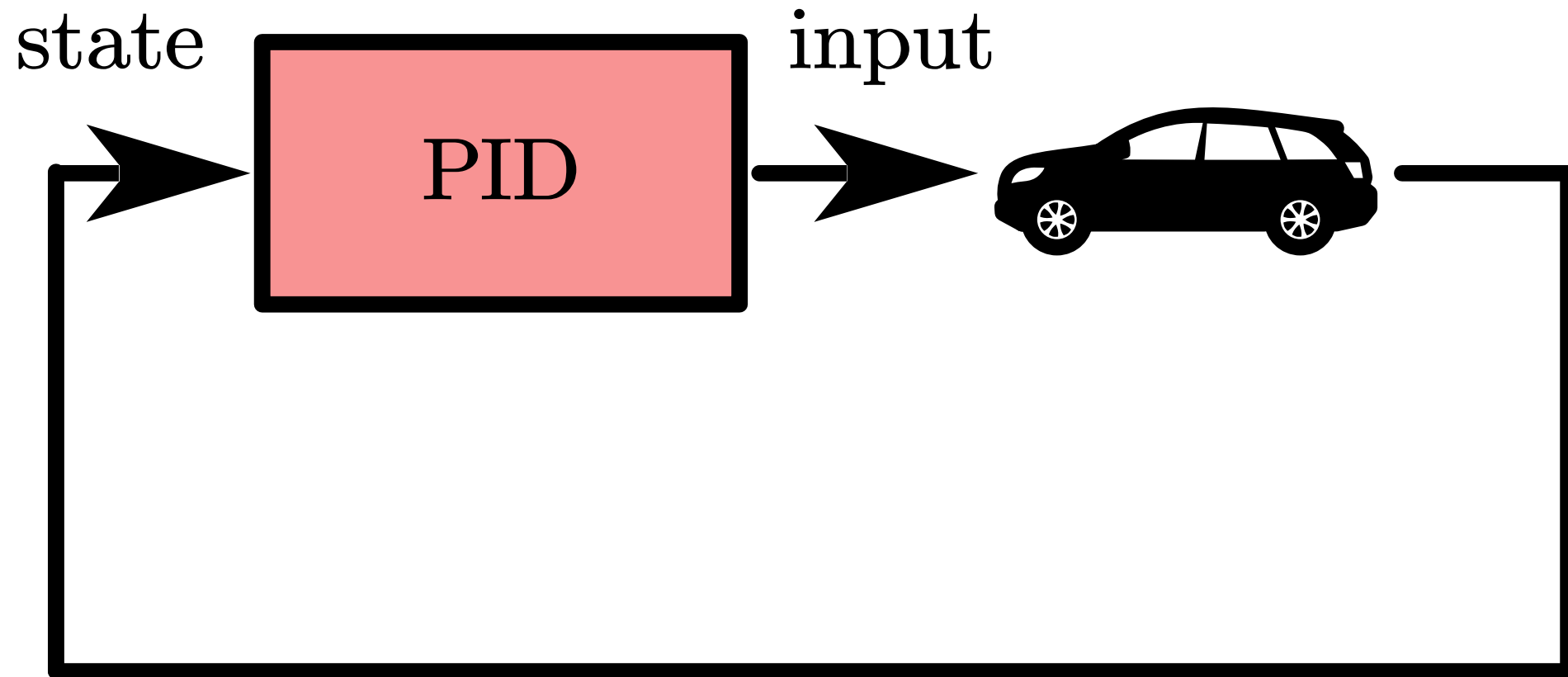
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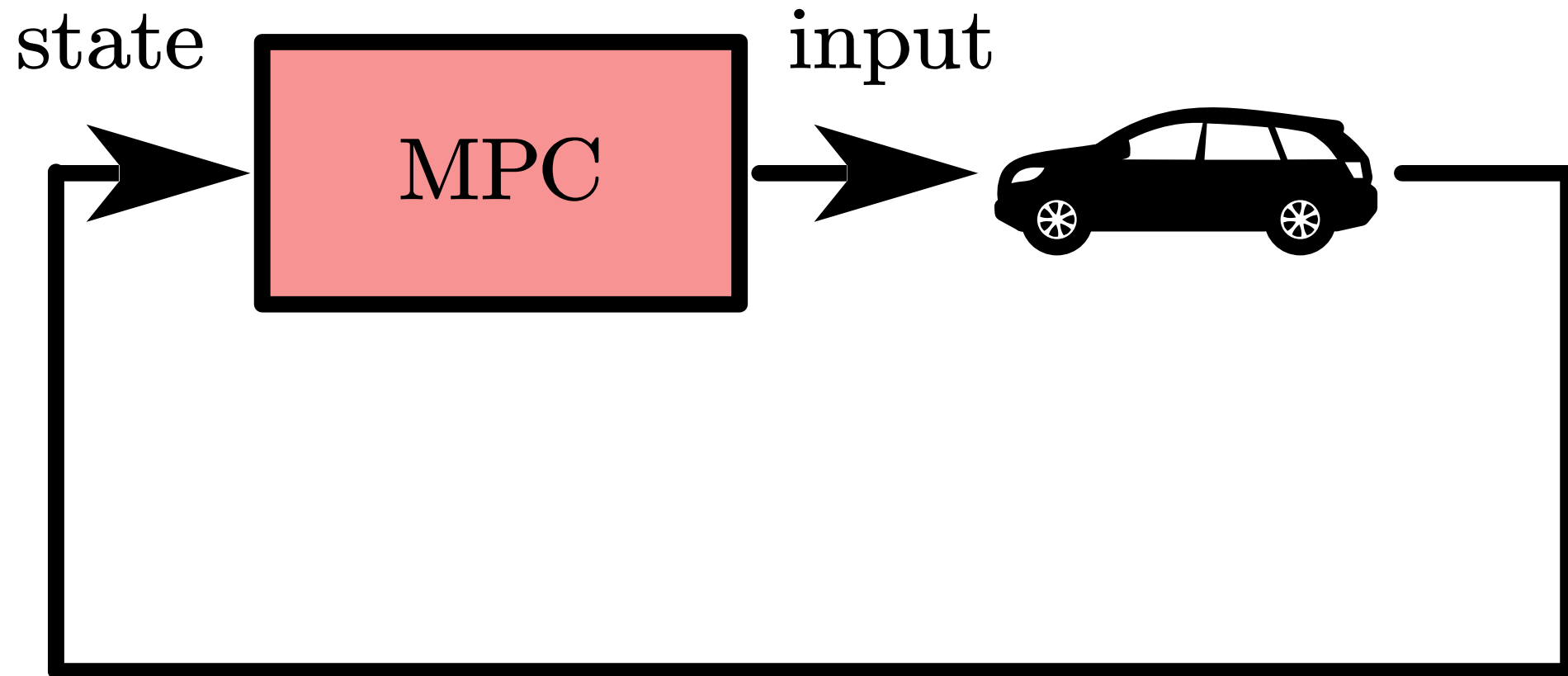
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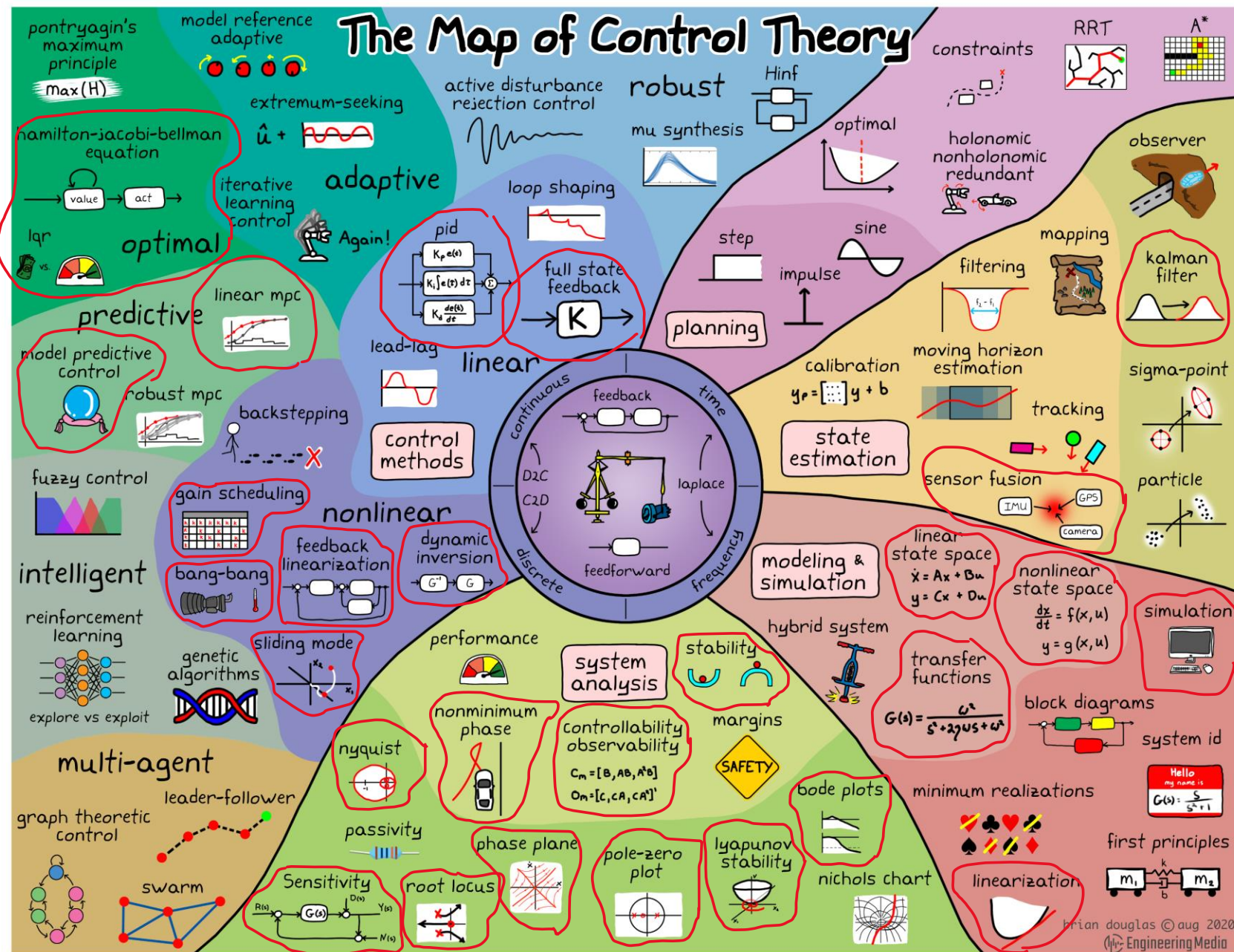


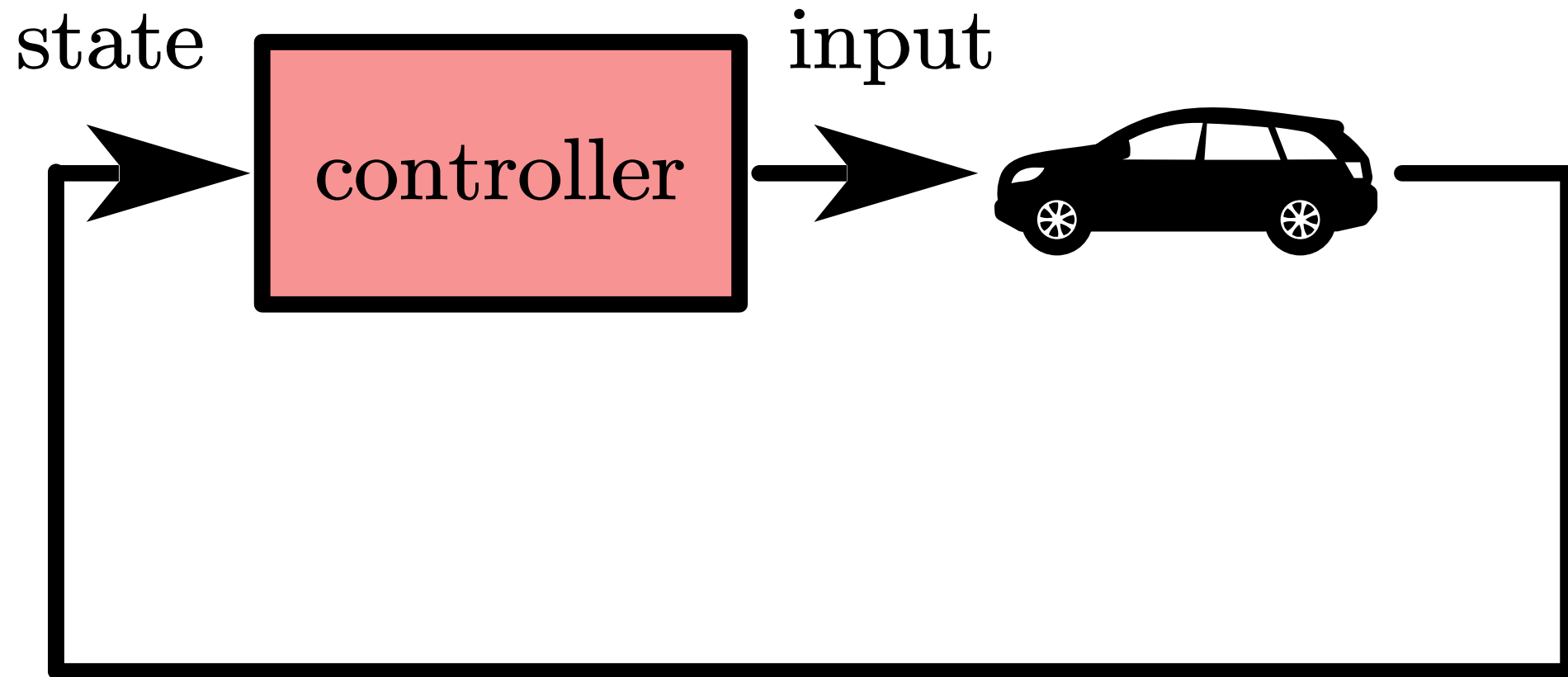










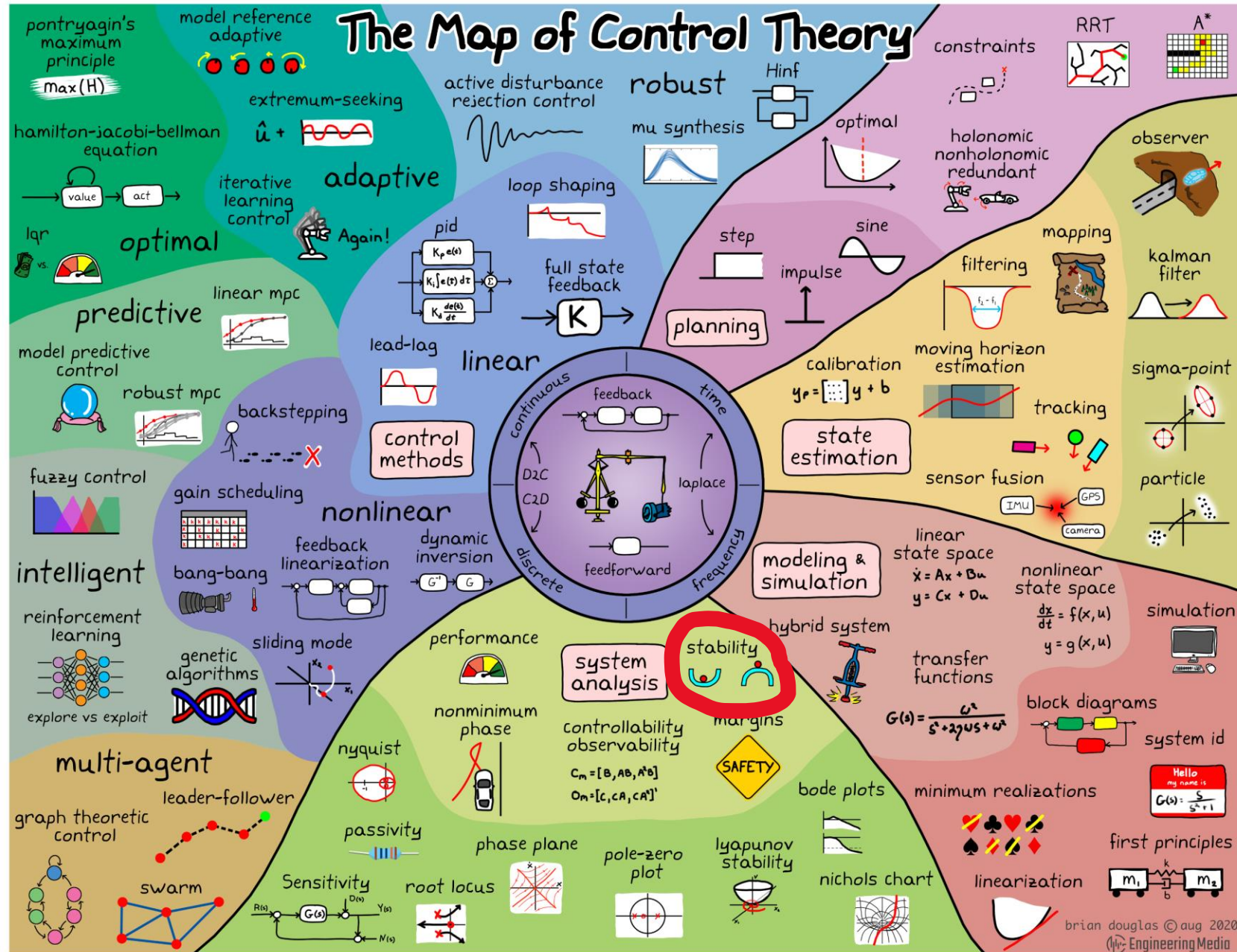


På trods af forskelle i race, farve og ideologi er afstanden fra ethvert land til universet den samme.

Regardless of variations in race, color, or ideology between countries, the distance to the cosmos remains the same.

尽管存在种族、肤色与意识形态的不同，从任何国家出发到宇宙的距离都是相同的。

人種や色、思想の違いはあっても、どの国から宇宙への距離も同じです。



The essence controllers

$$\text{State Error: } e \triangleq r - x$$

Reference: r

State: x

Obviously, we expect a zero State Error after sufficient time.

And that is exactly the purpose of PID controller.

* Stable (asymptotically stable): $\lim_{t \rightarrow \infty} e = 0$

Stable Paradox

State Error: $e \triangleq r - x$ (Stable: $e = 0$)

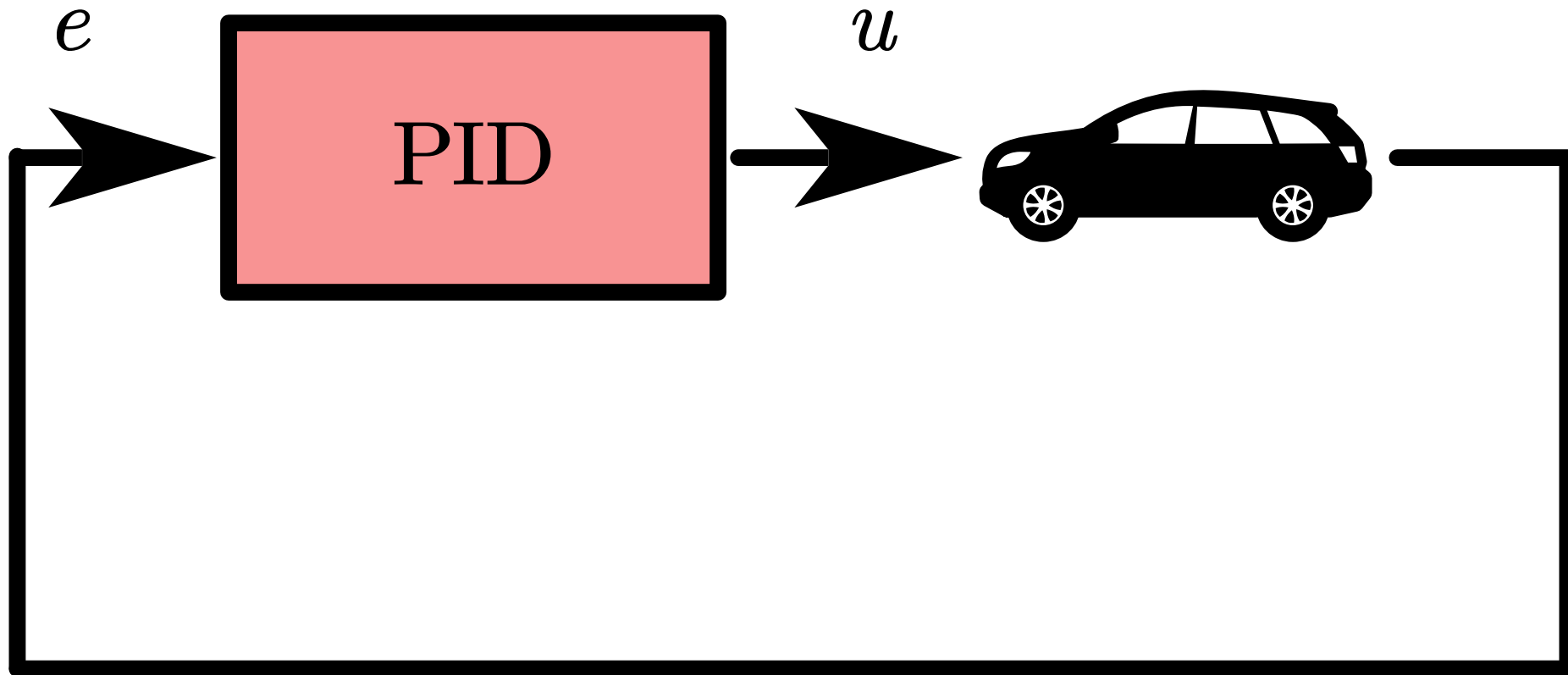
If we proved: $e + 1 = 0$. *If* $e = 0 \Rightarrow 1 = 0$ Paradox!!

Then this system is not stable (asymptotically stable).

This is because that $e \neq 0$

PID Controller in Autoware: find u , which leads to a zero e .

State Error: $e \triangleq r - x$ (Stable: $e = 0$)



PID Controller in Autoware: find u , which leads to a zero e .

State Error: $e \triangleq r - x$ (Stable: $e = 0$)

PID: Proportional + Integral + Derivative

$$u = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e}$$

\ddot{r} is the feedforward part. It is the desired acceleration.

u is the input. It is the acceleration command. $u = \ddot{x}$

PID Controller in Autoware: find u , which leads to a zero e .

State Error: $e \triangleq r - x$ (Stable: $e = 0$)

PID: Proportional + Integral + Derivative

$$u = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e}$$

$$u = \ddot{x} \quad \Rightarrow \quad \ddot{x} = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e}$$

$$e = r - x \quad \Rightarrow \quad 0 = \ddot{e} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e}$$

$$e = \int \dot{e} \cdot dt \quad \Rightarrow \quad 0 = K_P \cdot \dot{e} + K_I \cdot e + (K_D + 1) \cdot \ddot{e} \quad \Rightarrow \quad (K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0$$

PID Controller in Autoware: find u , which leads to a zero e .

State Error: $e \triangleq r - x$ (Stable: $e = 0$)

PID: Proportional + Integral + Derivative

$$u = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} \quad \Rightarrow \quad (K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0$$

Argument 1. Will e be 0 after sufficient time?

$$(K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0 \quad \Rightarrow \quad e = f(t, K_P, K_I, K_D)$$

$$\Rightarrow e = 0, \text{ if and only if } \begin{cases} \frac{K_P}{K_D + 1} > 0 \\ \frac{K_I}{K_D + 1} > 0 \end{cases}$$

PID Controller in Autoware: find u , which leads to a zero e .

Dr. Simon: We received a client's comment ...

The client found that KD in Autoware is always zero.

Now, we may have a better reply to this question.

$$\Rightarrow e = 0, \text{ if and only if } \begin{cases} \frac{K_P}{K_D + 1} > 0 \\ \frac{K_I}{K_D + 1} > 0 \end{cases}$$

PID Controller in Autoware: find u , which leads to a zero e .

Argument 2. Unstable on the ice road.

Dr. Maxime: We received a client's comment ...

The client found that the controller cannot work on the ice road.

$$u = u_{PID} + D \quad D \text{ (**NONZERO** unknown constant): disturbance}$$

$$u = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} + D \quad \Rightarrow \quad (K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e + D = 0$$

Stable Paradox!!

$$\text{If } e = 0 \Rightarrow D = 0 \quad \text{Paradox!!} \quad \Rightarrow e \neq 0 \quad \text{Unstable}$$

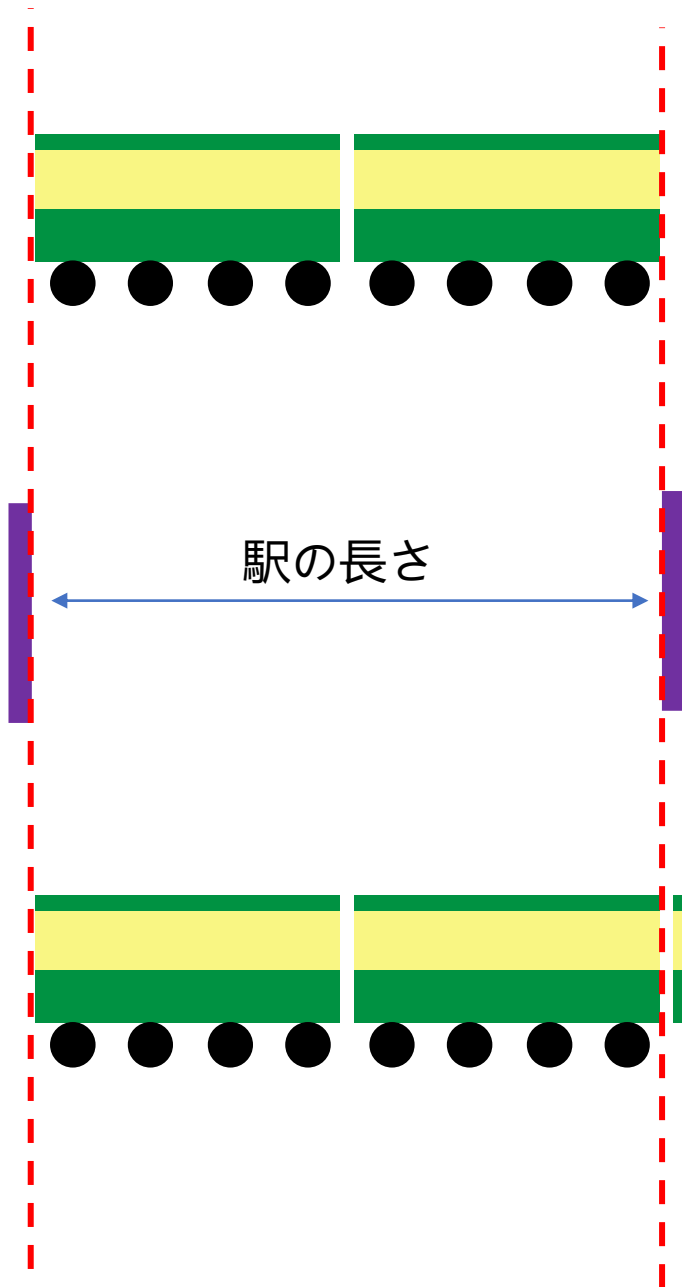
Conclusion: The current PID controller cannot stabilize the car on ice.



[1] 江ノ島電鉄 Wikipedia, Japan

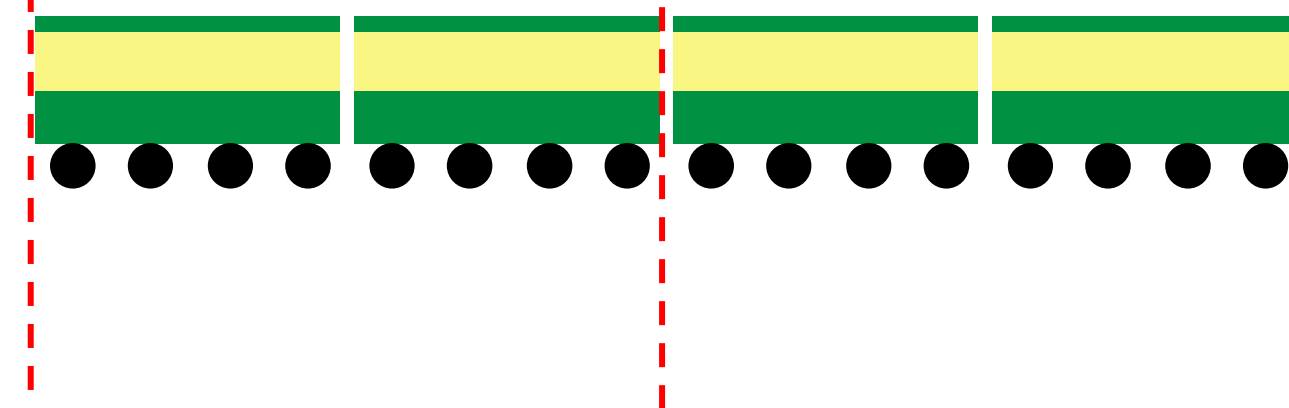
[2] File:Enoden305 01.jpg Wikipedia, Japan

2005:



“電車の長さが駅の長さより長いので、2つの
ドアしか開けることができません”

2021:



Redundancy

PID Controller in Autoware: find u , which leads to a zero e .

Argument 3. Redundancy.

First-order Linear Differential Equation:

$$\dot{e} + a \cdot e = 0 \Rightarrow e \text{ is uniquely determined}$$

Number of orders (1) == Number of coefficients (1, that is a)

$$m \cdot \dot{e} + a \cdot e = 0 \Rightarrow \dot{e} + \frac{a}{m} \cdot e = 0$$

Number of orders (1) < Number of coefficients (2, they are a and m)

Redundancy!!

m is not necessary!

PID Controller in Autoware: find u , which leads to a zero e .

Argument 3. Redundancy.

Second-order Linear Differential Equation:

$$\ddot{e} + a \cdot \dot{e} + b \cdot e = 0 \Rightarrow e \text{ is uniquely determined}$$

Number of orders (2) == Number of coefficients (2, they are a and b)

$$m \cdot \ddot{e} + a \cdot \dot{e} + b \cdot e = 0 \Rightarrow \ddot{e} + \frac{a}{m} \cdot \dot{e} + \frac{b}{m} \cdot e = 0$$

Number of orders (2) < Number of coefficients (3, they are a, b, and m) Redundancy!!

m is not necessary!

PID Controller in Autoware: find u , which leads to a zero e .

Argument 3. Redundancy.

Number of orders == Number of coefficients

$$u = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} \quad \Rightarrow \quad (K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0$$

Number of orders: 2.

Number of coefficients: 3. Redundancy!!

KP
KI
KD

$$\Rightarrow \quad \ddot{e} + \frac{K_P}{K_D + 1} \cdot \dot{e} + \frac{K_I}{K_D + 1} \cdot e = 0$$

It is a PD controller, rather than a PID controller.

You can NOT say "our PID controller". Control researcher can doubt your honesty.

PID Controller in Autoware: find u , which leads to a zero e .

Argument 3. Redundancy.

Number of orders == Number of coefficients

$$u = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} \quad \Rightarrow \quad (K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0$$

Number of orders: 2.

Number of coefficients: 3. Redundancy!!

Method 1. Admit we are using a PD controller and use it CORRECTLY.

$$(K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0$$

$$K_D = 0 \quad \Rightarrow \quad \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0 \quad \text{Number of orders (2) = Number of coefficients (2)}$$

PID Controller in Autoware: find u , which leads to a zero e .

Argument 3. Redundancy.

Method 2. Use PID controller CORRECTLY.

Number of coefficients: 3. \Rightarrow The desired number of orders: ~~2~~ 3

KP
KI
KD

We should raise the order of the system!!

jerk $j = \dot{a} = \ddot{x}$

$$a = \int j \cdot dt$$

PID Controller in Autoware: find u , which leads to a zero e .

$$\text{State Error: } e \stackrel{\Delta}{=} r - x \quad (\text{Stable: } e = 0)$$

PID: Proportional + Integral + Derivative

$$j = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} \quad u = \int j \cdot dt$$

$$\Rightarrow 0 = \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e + K_D \cdot \ddot{e}$$

$$\Rightarrow \ddot{e} + K_D \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0 \quad \text{Number of orders (3) = Number of coefficients (3)}$$

It is a standard beautiful PID controller.

You can now RPOUDLY declare “our PID controller”.

PID Controller in Autoware: find u , which leads to a zero e .

State Error: $e \triangleq r - x$ (Stable: $e = 0$)

PID: Proportional + Integral + Derivative

$$j = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} \qquad u = \int j \cdot dt$$

$$\Rightarrow 0 = \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e + K_D \cdot \ddot{e}$$

$$\Rightarrow \ddot{e} + K_D \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e = 0 \qquad \text{Number of orders (3) = Number of coefficients (3)}$$

It is stable if and only if $\begin{cases} K_P, K_I, K_D > 0 \\ K_P \cdot K_D > K_I \end{cases}$

PID Controller in Autoware: find u , which leads to a zero e .

Argument 2. Unstable on the ice road.

Dr. Maxime: We received a client's comment ...

The client found that the controller cannot work on the ice road.

$$u = u_{PID} + D \quad D \text{ (**NONZERO** unknown constant): disturbance}$$

$$u = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} + D \quad \Rightarrow \quad (K_D + 1) \cdot \ddot{e} + K_P \cdot \dot{e} + K_I \cdot e + D = 0$$

Stable Paradox!!

$$\text{If } e = 0 \Rightarrow D = 0 \quad \text{Paradox!!} \quad \Rightarrow e \neq 0 \quad \text{Unstable}$$

Conclusion: The current PID controller cannot stabilize the car on ice.

PID Controller in Autoware: find u , which leads to a zero e .

Argument 2. Unstable on the ice road.

$$j = \ddot{r} + K_P \cdot \dot{e} + K_I \cdot \int \dot{e} \cdot dt + K_D \cdot \ddot{e} \qquad u_{PID} = \int j \cdot dt$$

$$u = u_{PID} + D \quad D \text{ (**NONZERO** unknown constant): disturbance}$$

$$\Rightarrow u = \ddot{r} + K_P \cdot e + K_I \cdot \int e \cdot dt + K_D \cdot \dot{e} + D$$

$$\Rightarrow 0 = \ddot{e} + K_P \cdot e + K_I \cdot \int e \cdot dt + K_D \cdot \dot{e} + D$$

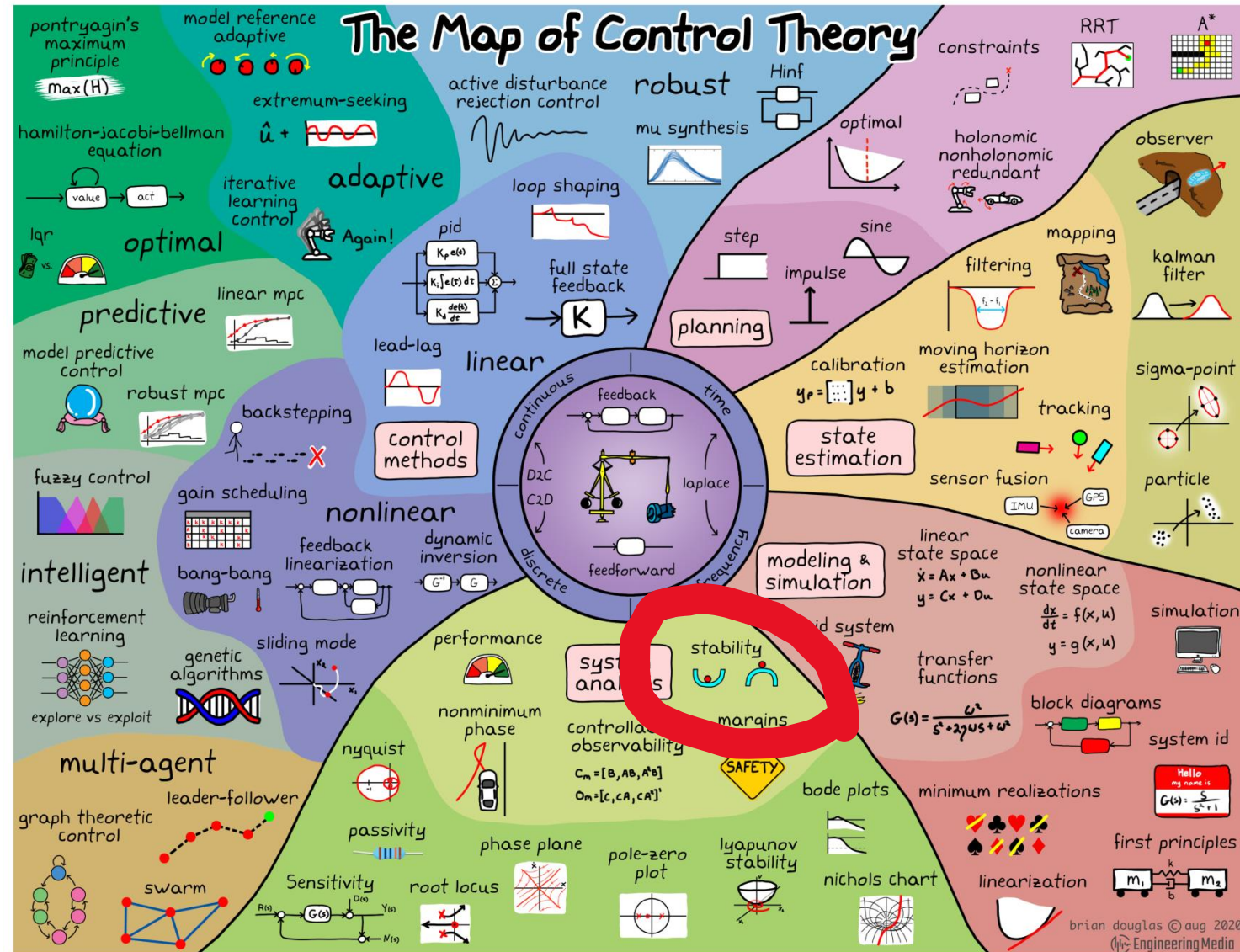
$$\Rightarrow K_I \cdot \int e \cdot dt + K_P \cdot e + K_D \cdot \dot{e} + \ddot{e} + D = 0$$

$$\frac{d}{dt} D = 0 \quad \Rightarrow \quad \ddot{e} + K_D \cdot \dot{e} + K_P \cdot e + K_I \cdot \int e \cdot dt = 0$$

Conclusion: This PID controller stabilizes the car on ice.

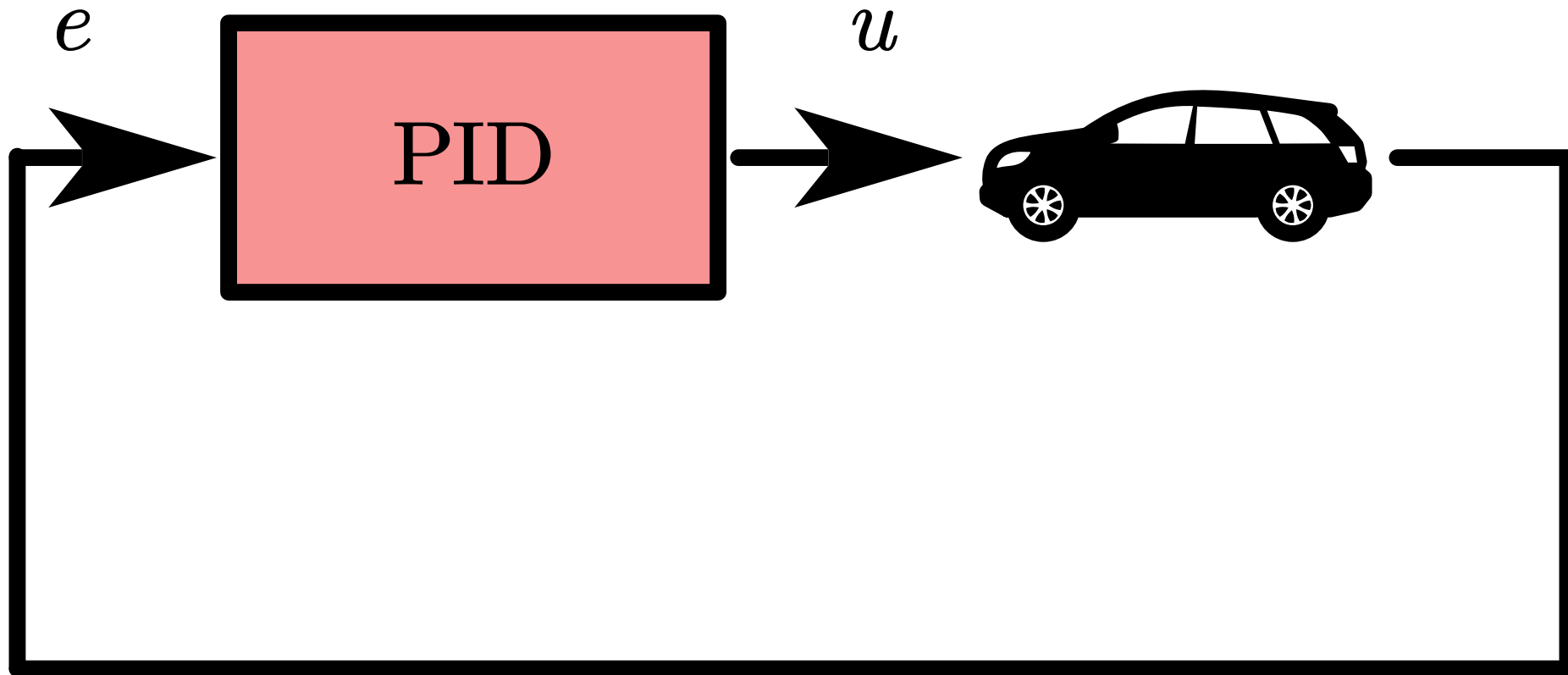
if and only if $\begin{cases} K_P, K_I, K_D > 0 \\ K_P \cdot K_D > K_I \end{cases}$

Check the stability proof while applying the controller.

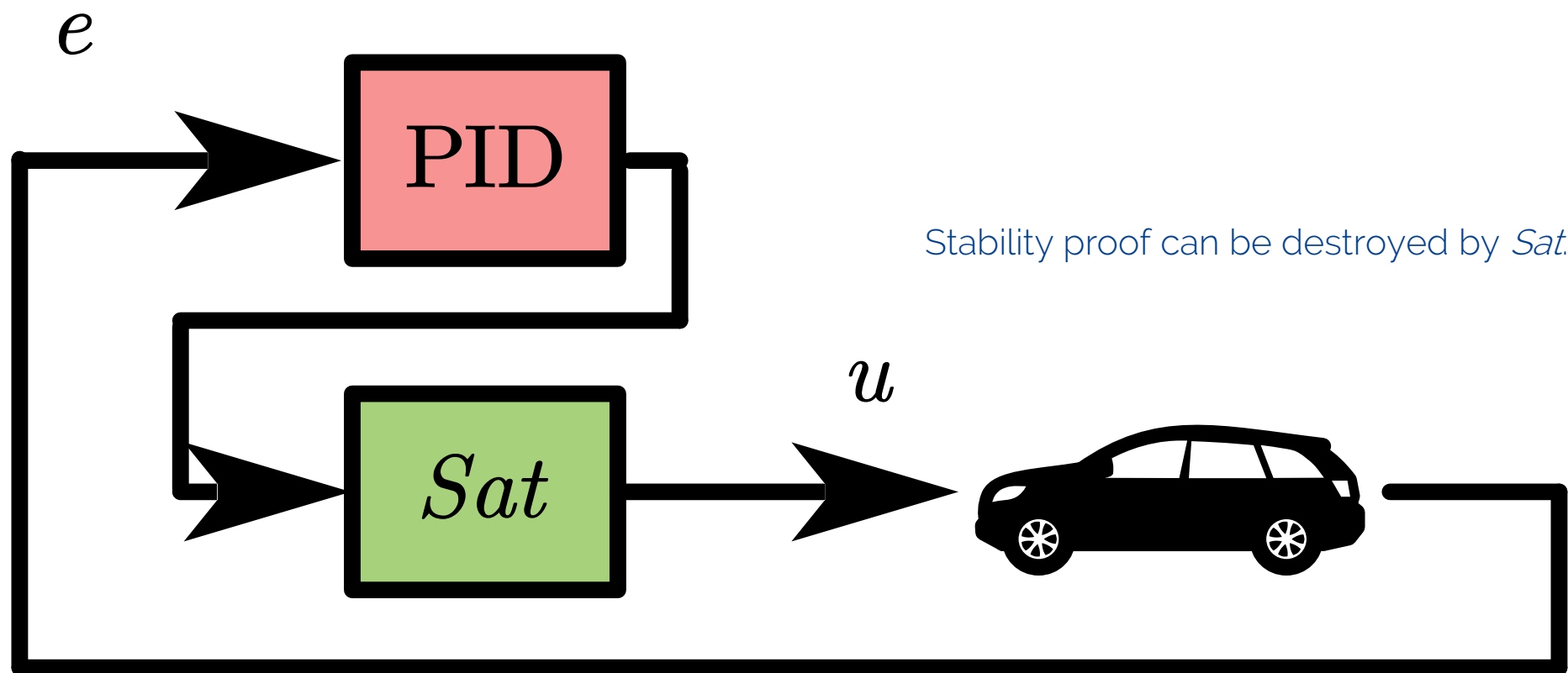


Be cautious about the post process of the controller

We have the stability proof for PID

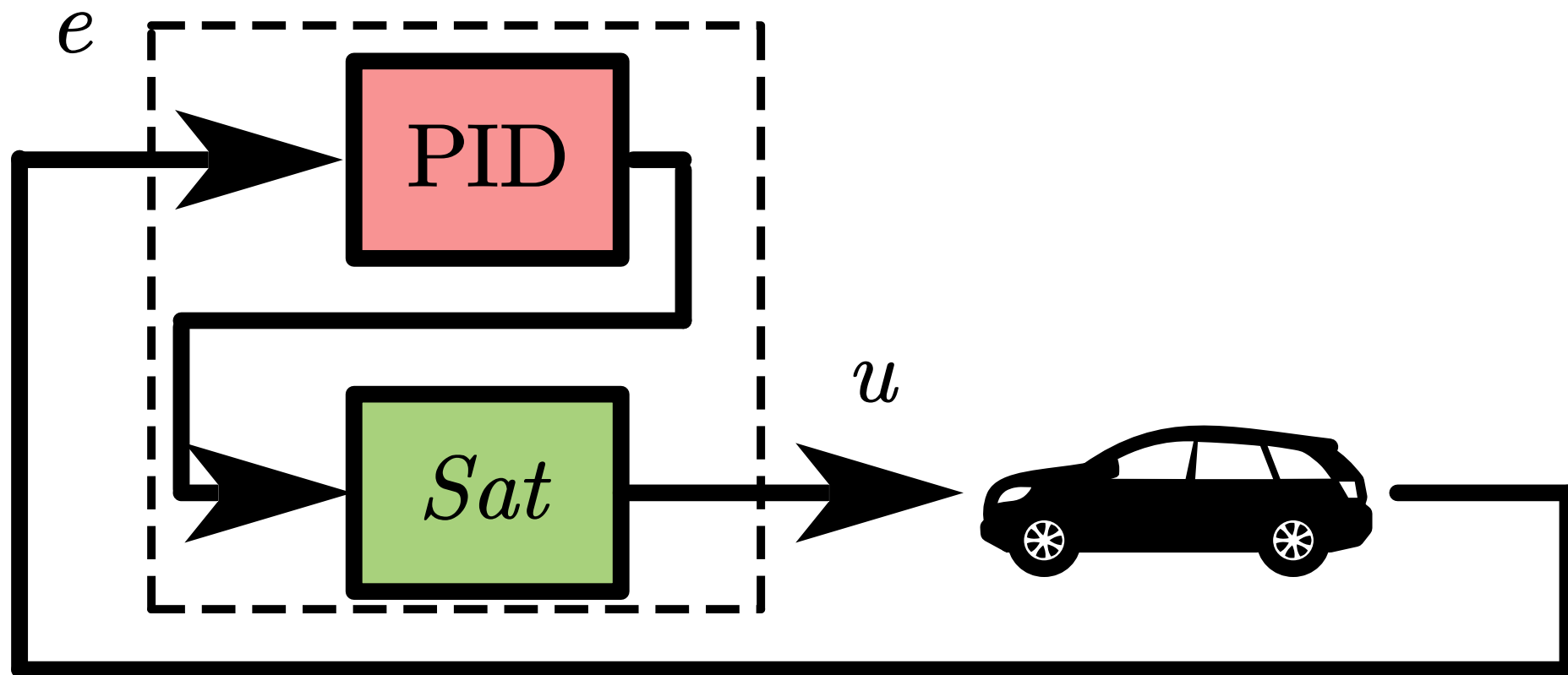


Be cautious about the post process of the controller



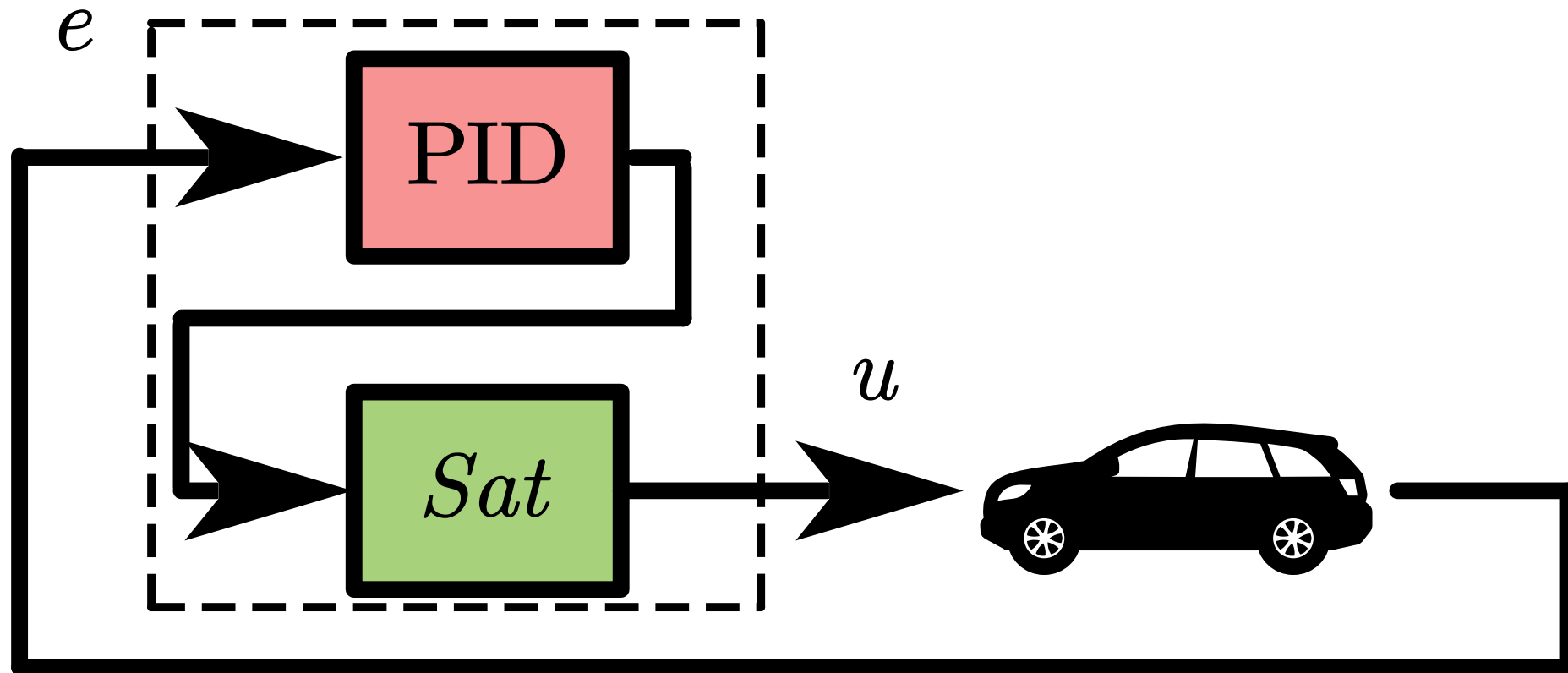
Be cautious about the post process of the controller

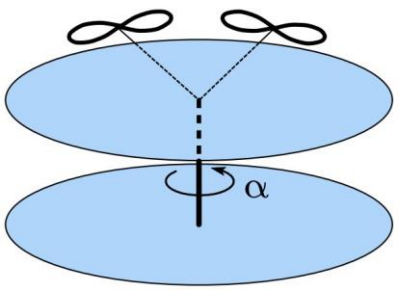
Sat can be regarded as a part of the controller.

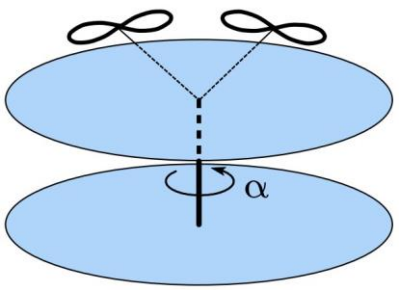


Be cautious about the post process of the controller

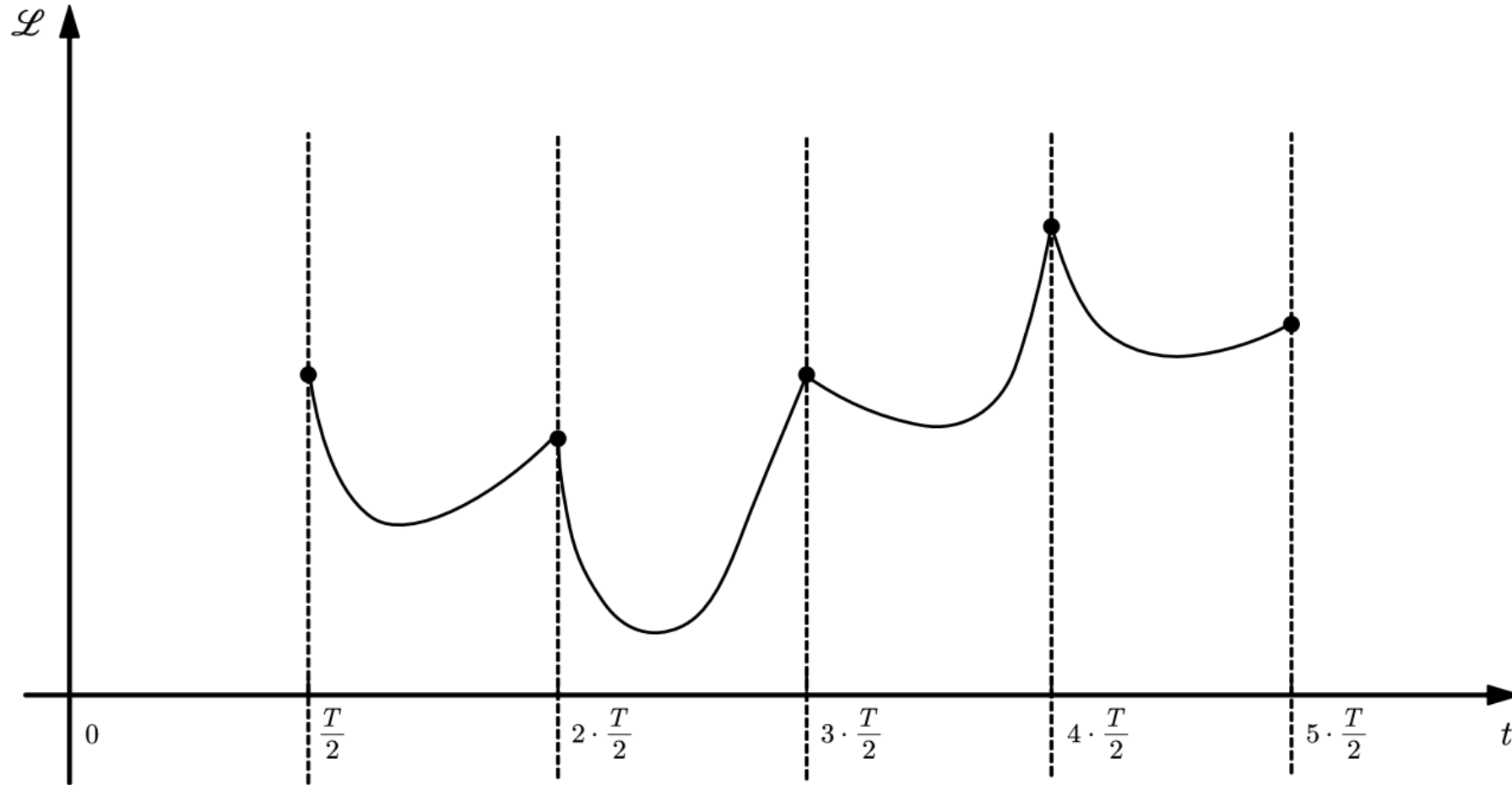
New stability proof can be found for this section.





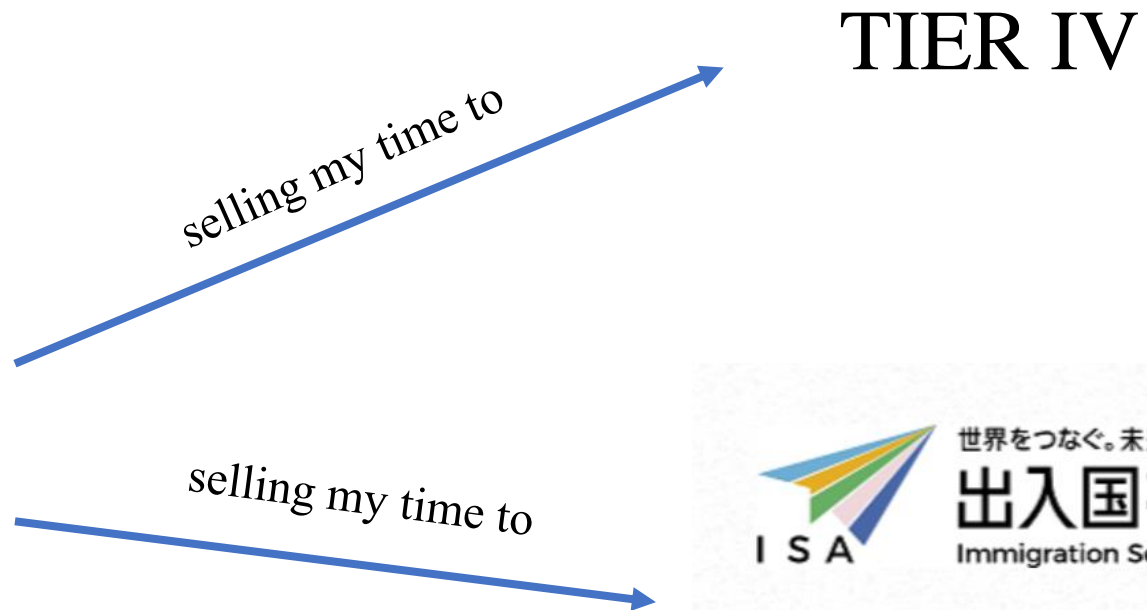


Advanced stability proof may be demanded: stable in the sense of Lyapunov



Lyapunov candidate.

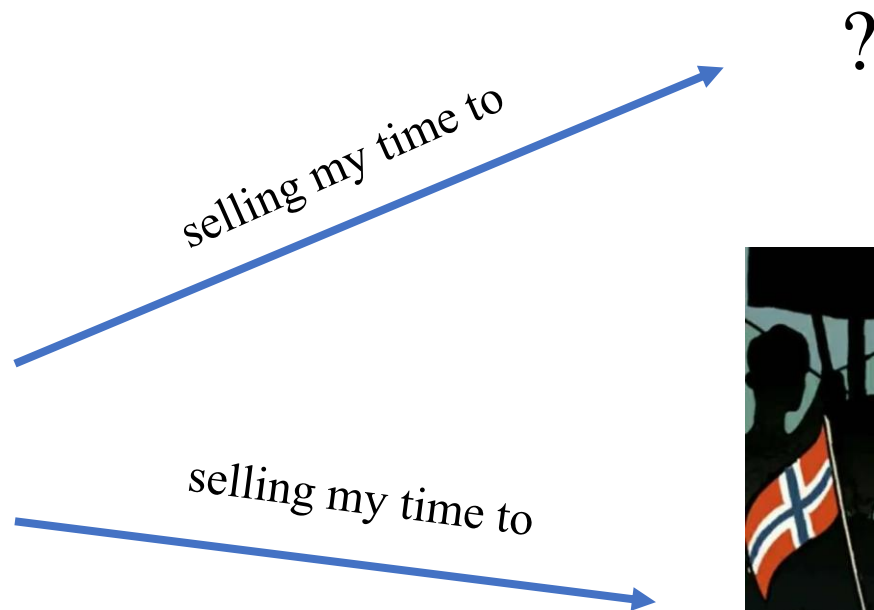
Redundancy



Japan Permanent Residence Permit:
Work continuously for 12 months.

Current me: 8 months

Redundancy



5/29/2024

Ph.D. in Aeronautics and Astronautics

**Thank you for
listening**

Shen