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Branched Cylinders: Dendritic Tree Approximations

by:

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BM2102 Modelling and Analysis of Physiological Systems

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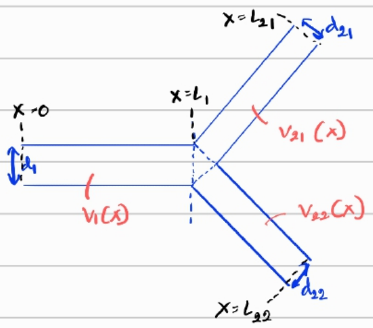
Contents

1 Question (1) and (2)

Assignment 2 – Dendritic Tree Aproximation.



$X = \frac{x}{d_c}$

$d_c \rightarrow$ not equal for all branches.

$V = \frac{d^2V}{dX^2}$

Question 01

$V_1(X) = A_1 e^{-X} + B_1 e^{X}$; $0 \leq X \leq L_1$ — ①

$V_{21}(X) = A_{21} e^{-X} + B_{21} e^{X}$; $L_1 \leq X \leq L_{21}$ — ②

$V_{22}(X) = A_{22} e^{-X} + B_{22} e^{X}$; $L_1 \leq X \leq L_{22}$ — ③

Boundary conditions;

(i) $\left(\frac{dV_1}{dX}\right)_{X=0} = (-r\, d_c)_1 I_{app}$

From ①; $\frac{dV_1}{dX} = -A_1 e^{-X} + B_1 e^{X} \Rightarrow \left(\frac{dV_1}{dX}\right)_{X=0} = -A_1 + B_1$

$(-r d_c)_1 I_{app} = -A_1 + B_1$

$A_1 - B_1 = (r d_c)_1 I_{app}$ — ⓐ

//

(ii) $V_{21}(L_{21}) = 0$

From ② when $X = L_{21}$;

$V_{21}(L_{21}) = A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \quad \text{—— (b)}$$

(iii) $V_{22}(L_{22}) = 0$

From (III) when $x = L_{22}$;

$$V_{22}(L_{22}) = A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \quad \text{—— (c)}$$

Nodal Conditions;

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1)$$

From (I) & (II) when $X = L_1$;

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1}$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0 \quad \text{—— (d)}$$

From (II) & (III) when $X = L_1$;

$$A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1}$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0 \quad \text{—— (e)}$$

$$\frac{-1}{(r_i d_c)_1}\left(\frac{dV_1}{dx}\right)_{x=L_1} = \frac{-1}{(r_i d_c)_{21}}\left(\frac{dV_{21}}{dx}\right)_{x=L_1} + \frac{-1}{(r_i d_c)_{22}}\left(\frac{dV_{22}}{dx}\right)_{x=L_1} \quad \text{—— (*)}$$

$$\left(\frac{dV_1}{dx}\right)_{x=L_1} = -A_1 e^{-L_1} + B_1 e^{L_1} \quad \text{—— (P)}$$

$$\frac{dV_{21}}{dx} = -A_{21} e^{-x} + B_{21} e^{x} \Rightarrow \left(\frac{dV_{21}}{dx}\right)_{x=L_1} = -A_{21} e^{-L_1} + B_{21} e^{L_1} \quad \text{—— (Q)}$$

$$\frac{dV_{22}}{dx} = -A_{22}e^{-x} + B_{22}e^{x} \Rightarrow \left(\frac{dV_{22}}{dx}\right)_{x=L_1} = -A_{22}e^{-L_1} + B_{22}e^{L_1} \quad \text{—— (R)}$$

Substituting (P), (Q), (R) to (*)

$$\frac{-1}{(r_i d_c)_1}\left[-A_1 e^{-L_1} + B_1 e^{L_1}\right] = \frac{-1}{(r_i d_c)_{21}}\left[-A_{21}e^{-L_1} + B_{21}e^{L_1}\right] + \frac{-1}{(r_i d_c)_{22}}\left[-A_{22}e^{-L_1} + B_{22}e^{L_1}\right]$$

$$\frac{A_1 e^{-L_1}}{(r_i d_c)_1} - \frac{B_1 e^{L_1}}{(r_i d_c)_1} = \frac{A_{21}e^{-L_1}}{(r_i d_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i d_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i d_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i d_c)_{22}}$$

$$-\frac{A_1 e^{-L_1}}{(r_i d_c)_1} + \frac{B_1 e^{L_1}}{(r_i d_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i d_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i d_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i d_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i d_c)_{22}} = 0 \quad //$$

<u>Question 02</u>

$$Ax = b$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{-L_1} \\ \frac{-e^{-L_1}}{(r_i d_c)_1} & \frac{e^{L_1}}{(r_i d_c)_1} & \frac{e^{-L_1}}{(r_i d_c)_{21}} & \frac{-e^{L_1}}{(r_i d_c)_{21}} & \frac{e^{-L_1}}{(r_i d_c)_{22}} & \frac{e^{-L_1}}{(r_i d_c)_{22}} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{bmatrix} = \begin{bmatrix} (r_i d_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A_1 & -B_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{21}e^{-L_{21}} & B_{21}e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{22}e^{-L_{22}} & B_{22}e^{L_{22}} \\ A_1 e^{-L_1} & B_1 e^{L_1} & -A_{21}e^{-L_1} & -B_{21}e^{L_1} & 0 & 0 \\ 0 & 0 & A_{21}e^{-L_1} & B_{21}e^{L_1} & -A_{22}e^{-L_1} & -B_{22}e^{-L_1} \\ \frac{-A_1 e^{L_1}}{(r_i d_c)_1} & \frac{B_1 e^{L_1}}{(r_i d_c)_1} & \frac{A_{21}e^{-L_1}}{(r_i d_c)_{21}} & \frac{-B_{21}e^{L_1}}{(r_i d_c)_{21}} & \frac{A_{22}e^{-L_1}}{(r_i d_c)_{22}} & \frac{-B_{22}e^{L_1}}{(r_i d_c)_{22}} \end{bmatrix} = \begin{bmatrix} (r_i d_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_1 - B_1 = (r_i d_c)_1 I_{app}$$

$$A_{21} e^{-L_2} + B_{21} e^{L_{21}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0$$

$$\frac{-A_1 e^{-L_1}}{(r_i d_c)_1} + \frac{B_1 e^{L_1}}{(r_i d_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i d_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i d_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i d_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i d_c)_{22}} = 0 \quad //$$

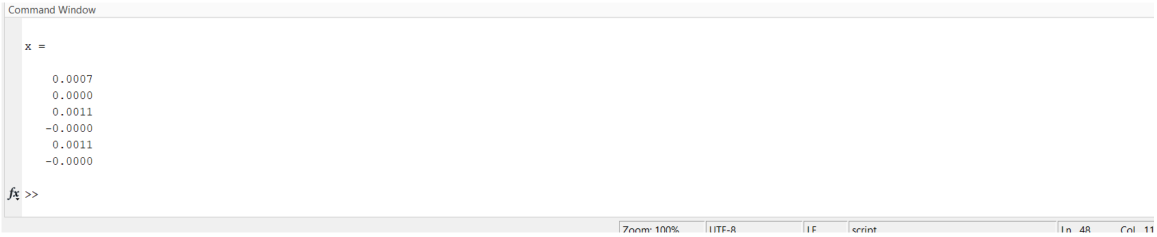
2 Question (3)

Here is the MATLAB code used to calculate the coefficients.

```
% electrical constants and derived quantities for typical
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;              % cm
d21 = 30e-4;           % cm
d22 = 15e-4;           % cm
% d21 = 47.2470e-4;        % E9 cm
% d22 = d21;               % E9 cm

l1 = 1.5;                % dimensionless
l21 = 3.0;               % dimensionless
l22 = 3.0;               % dimensionless

% Electrical properties of compartments

Rm = 6e3;                % Ohms cm^2
Rc = 90;                 % Ohms cm
Rs = 1e6;                % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);      % Ohms
rl21 = c1*d21^(-3/2);    % Ohms
rl22 = c1*d22^(-3/2);    % Ohms

% Applied current

iapp = 1e-9;             % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22 -rl1*exp
         (l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';

```

```
% Question 3
x=A\b;
display(x)
```

Following are the calculated values of the coefficients.



3 Question (4)

Here is the MATLAB code snippet used to plot the steady-state voltage profile in each branch.

```

%Question 4
y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);

v1=x(1)*exp(-y1)+x(2)*exp(y1);
v21=x(3)*exp(-y21)+x(4)*exp(y21);
v22=x(5)*exp(-y22)+x(6)*exp(y22);
plot(y1,v1,'-y',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady Stage Voltage - E5');
```

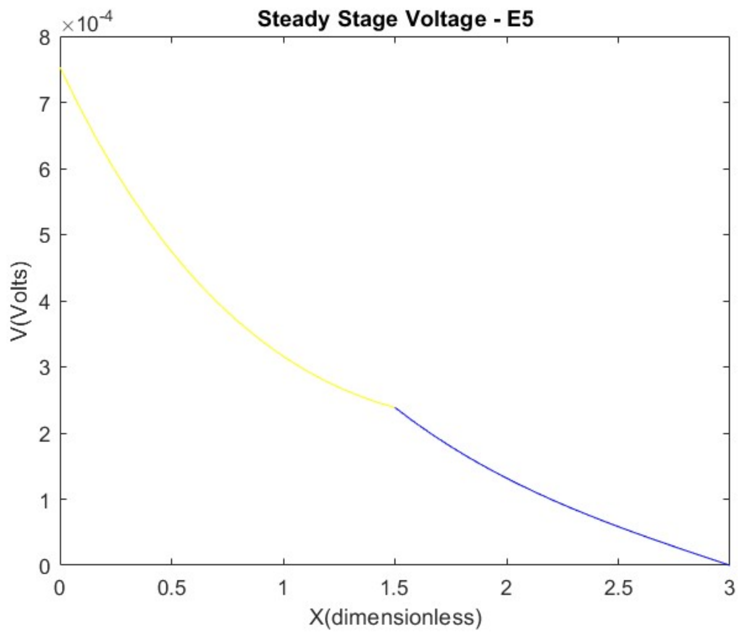


Figure 1: Steady-State Voltage Profile in Each Branch

3.1 Explanation

In the diagram, the absence of a visible red line suggests that it either completely overlaps with the blue line or is so close to it that the difference cannot be distinguished visually. Since the yellow line represents the membrane potential along the parent branch and does not influence the daughter branches, it is reasonable to infer that the red and blue lines represent the same voltage profile. This implies that both daughter branches exhibit identical steady-state voltage distributions.
This interpretation is also supported by the code, where the red and blue lines are used to plot the steady-state voltages of the two daughter branches. The fact that only the blue line is visible suggests it was plotted directly over the red one, confirming that the two profiles are effectively the same.
Therefore, based on both the graphical output and the code structure, it is evident that **the steady-state voltage profiles of the daughter branches are equal**.

4 Question (5)

4.1 Part (a)

```
%Question 5- (a)

A1=A;
A1(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
x=A1\b;


y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);

v1=x(1)*exp(-y1)+x(2)*exp(y1);
v21=x(3)*exp(-y21)+x(4)*exp(y21);
v22=x(5)*exp(-y22)+x(6)*exp(y22);
plot(y1,v1,'-y',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady Stage Voltage - E5');
```

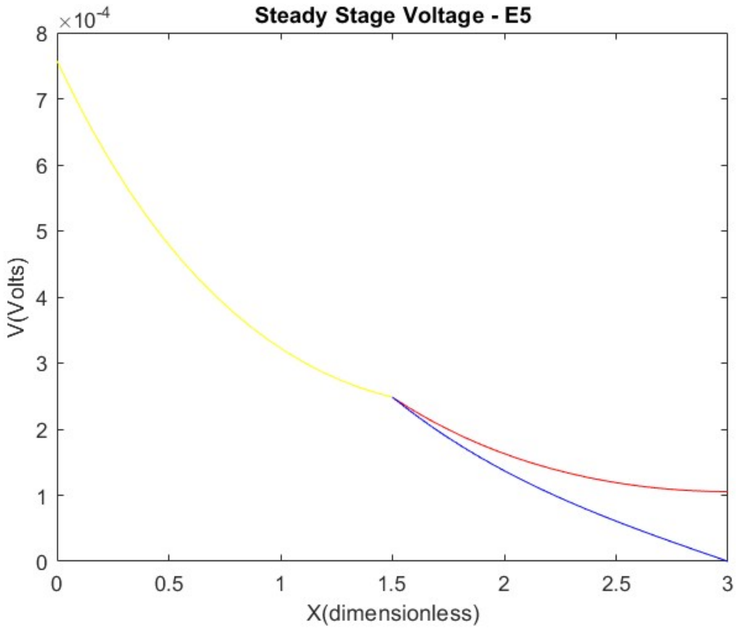


Figure 2: Steady-State Voltage Profile in Each Branch

4.2 Part (b)

```

%Question 5- (b)


A1(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
x=A1\b;


y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);

v1=x(1)*exp(-y1)+x(2)*exp(y1);
v21=x(3)*exp(-y21)+x(4)*exp(y21);
v22=x(5)*exp(-y22)+x(6)*exp(y22);
plot(y1,v1,'-y',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady Stage Voltage - E5');
```

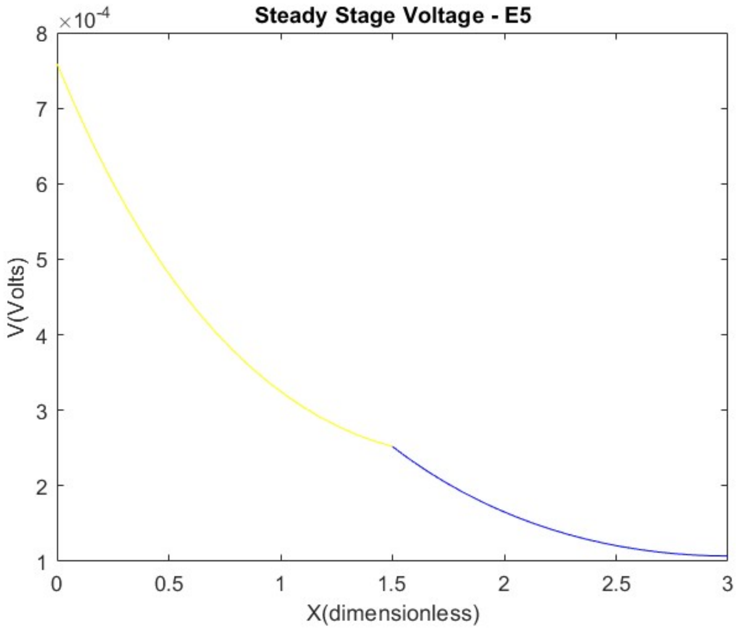


Figure 3: Steady-State Voltage Profile in Each Branch

4.3 Part (c)

```
%Question 5- (c)

A2=A;
A2(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];


b2=b;
b2(1) = 0;
b2(2) = rl21*iapp;

x=A2\b2;


y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);

v1=x(1)*exp(-y1)+x(2)*exp(y1);
v21=x(3)*exp(-y21)+x(4)*exp(y21);
v22=x(5)*exp(-y22)+x(6)*exp(y22);
plot(y1,v1,'-y',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady Stage Voltage - E5');
```

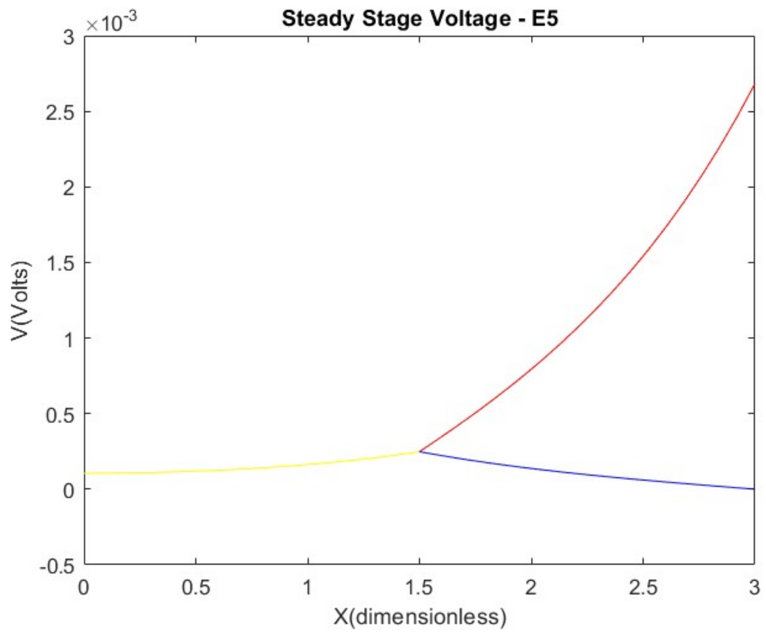


Figure 4: Steady-State Voltage Profile in Each Branch

4.4 Part (d)

```
%Question 5- (d)

b2(3) = rl22*iapp;

x=A1\b2;

y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);

v1=x(1)*exp(-y1)+x(2)*exp(y1);
v21=x(3)*exp(-y21)+x(4)*exp(y21);
v22=x(5)*exp(-y22)+x(6)*exp(y22);
plot(y1,v1,'-y',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady Stage Voltage - E5');
```

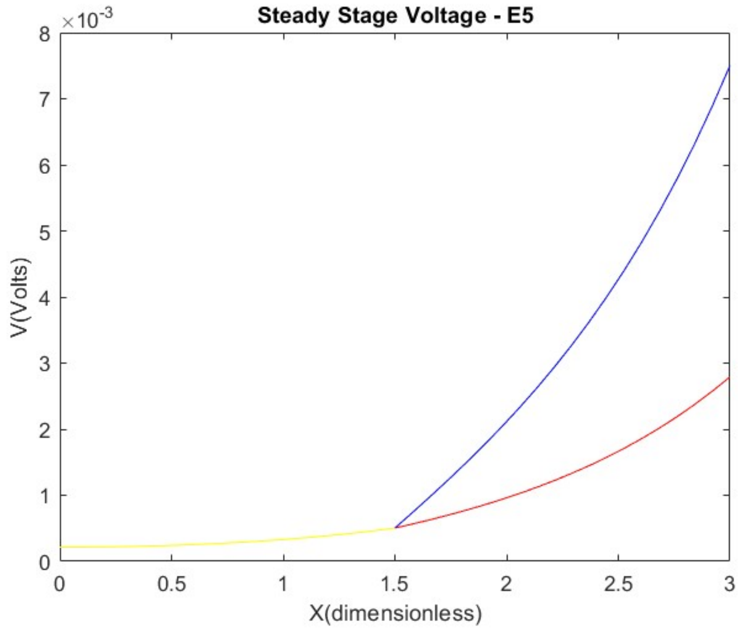


Figure 5: Steady-State Voltage Profile in Each Branch

The membrane voltage gradient refers to how the membrane voltage changes over distance. In the context of daughter branches in a neuron, this gradient is positive at the farthest nodes because the membrane voltage rises as you move away from the node. This happens because these terminal nodes are actively passing an electrical signal to another neuron or branching path.

This electrical signal, known as a depolarization wave, travels along the neuron's axon. When it arrives at the end of a daughter branch, it causes an increase in membrane voltage as the distance from the node increases.

This rise in membrane voltage at the ends of daughter branches leads to a positive voltage gradient at those points. Such a gradient plays an important role in promoting the smooth and effective transmission of the electrical signal from the main (parent) branch to the branching pathways.

5 Question (6)

```
%Question 6


%electrical constants and derived quantities for typical
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;                       % cm
%d21 = 30e-4;                     % cm
%d22 = 15e-4;                     % cm
d21 = 47.2470e-4;        % E9 cm
d22 = d21;                % E9 cm

l1 = 1.5;                         % dimensionless
l21 = 3.0;                        % dimensionless
l22 = 3.0;                        % dimensionless

% Electrical properties of compartments

Rm = 6e3;                         % Ohms cm^2
```

```matlab
Rc = 90;                        % Ohms cm
Rs = 1e6;                       % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);             % Ohms
rl21 = c1*d21^(-3/2);           % Ohms
rl22 = c1*d22^(-3/2);           % Ohms


% Applied current

iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22 -rl1*exp(l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';
```

5.1 Part (b)

```matlab
%Question 6- (b)

A3=A;
A3(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A3(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
x=A3\b;


y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);

v1=x(1)*exp(-y1)+x(2)*exp(y1);
v21=x(3)*exp(-y21)+x(4)*exp(y21);
v22=x(5)*exp(-y22)+x(6)*exp(y22);
plot(y1,v1,'-y',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady Stage Voltage - E5');
```

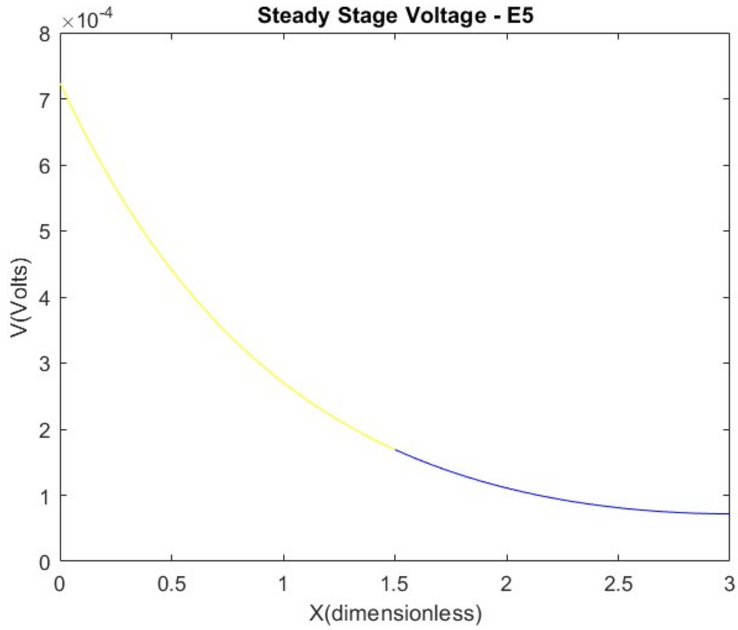


Figure 6: Steady-State Voltage Profile in Each Branch

5.2 Part (d)

```
%Question 6- (d)

A3=A;
A3(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A3(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

b3=b;
b3(1) = 0;
b3(2) = rl21*iapp;
b3(3) = rl22*iapp;

x=A3\b3;


y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);

v1=x(1)*exp(-y1)+x(2)*exp(y1);
v21=x(3)*exp(-y21)+x(4)*exp(y21);
v22=x(5)*exp(-y22)+x(6)*exp(y22);
plot(y1,v1,'-y',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady Stage Voltage - E5');
```

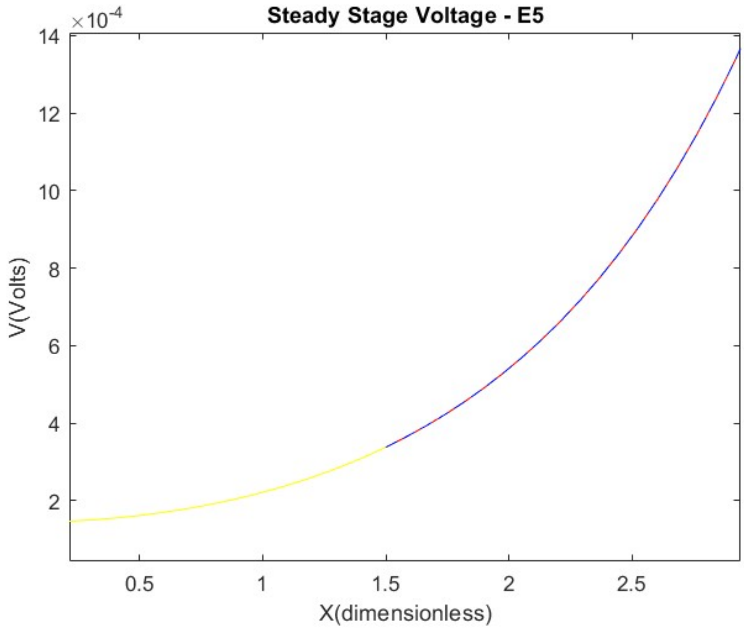


Figure 7: Steady-State Voltage Profile in Each Branch

The continuous differentiability of the graphs in Figures 2(c) and 2(d) indicates that they are smooth, with no abrupt changes or discontinuities. This smoothness is crucial for the uninterrupted transmission of electrical signals from the parent branch to the daughter branches.
Additionally, the minimal voltage difference between the two daughter branches in Figure 2(d) implies that both branches are receiving the electrical signal equally, resulting in nearly identical membrane voltages.
In summary, both interpretations are valid and complement one another. Together, they offer a clear and thorough understanding of how the graphs in Figures 2(b), 2(c), and 2(d) differ and what those differences imply about signal propagation.