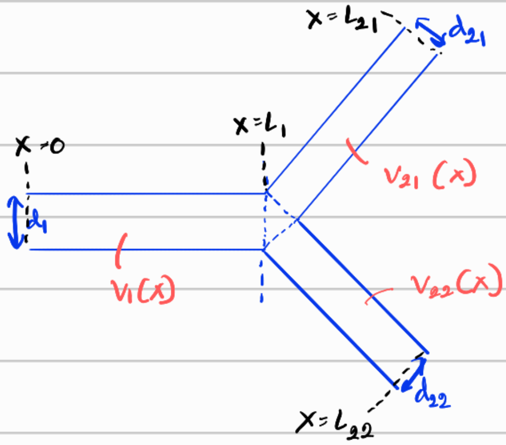
Assignment 2 - Dendritic Tree Aproximation.



$$X = \frac{x}{d_c}$$

$d_c \rightarrow$ not equal for all branches.

$$V = \frac{d^2 V}{dX^2}$$

Question 01

$$V_1(X) = A_1 e^{-X} + B_1 e^{X} \quad ; \quad 0 \leq X \leq L_1 \quad \text{— (I)}$$

$$V_{21}(X) = A_{21} e^{-X} + B_{21} e^{X} \quad ; \quad L_1 \leq X \leq L_{21} \quad \text{— (II)}$$

$$V_{22}(X) = A_{22} e^{-X} + B_{22} e^{X} \quad ; \quad L_1 \leq X \leq L_{22} \quad \text{— (III)}$$

Boundary conditions;

(i) $$\left(\frac{dV_1}{dX}\right)_{X=0} = (-r d_c)_1 I_{app}$$

From (I); $$\frac{dV_1}{dX} = -A_1 e^{-X} + B_1 e^{X} \Rightarrow \left(\frac{dV_1}{dX}\right)_{X=0} = -A_1 + B_1$$

$$(-r d_c)_1 I_{app} = -A_1 + B_1$$

$$A_1 - B_1 = (r d_c)_1 I_{app} \quad \text{— (a)}$$

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(ii) $V_{21}(L_{21}) = 0$

From (II) when $X = L_{21}$;

$$V_{21}(L_{21}) = A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \quad \text{--- (b)}$$

(iii) $V_{22}(L_{22}) = 0$

From (III) when $x = L_{22}$;

$$V_{22}(L_{22}) = A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \quad \text{--- (c)}$$

Nodal Conditions;

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1)$$

From (I) & (II) when $X = L_1$;

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1}$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0 \quad \text{--- (d)}$$

From (II) & (III) when $X = L_1$;

$$A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1}$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0 \quad \text{--- (e)}$$

$$\frac{-1}{(r_i d_c)_1}\left(\frac{dV_1}{dx}\right)_{x=L_1} = \frac{-1}{(r_i d_c)_{21}}\left(\frac{dV_{21}}{dx}\right)_{x=L_1} + \frac{-1}{(r_i d_c)_{22}}\left(\frac{dV_{22}}{dx}\right)_{x=L_1} \quad \text{--- (*)}$$

$$\left(\frac{dV_1}{dx}\right)_{x=L_1} = -A_1 e^{-L_1} + B_1 e^{L_1} \quad \text{--- (P)}$$

$$\frac{dV_{21}}{dx} = -A_{21} e^{-x} + B_{21} e^{x} \Rightarrow \left(\frac{dV_{21}}{dx}\right)_{x=L_1} = -A_{21} e^{-L_1} + B_{21} e^{L_1} \quad \text{--- (q)}$$

$$\frac{dV_{22}}{dx} = -A_{22}e^{-x} + B_{22}e^{x} \Longrightarrow \left(\frac{dV_{22}}{dx}\right)_{x=L_1} = -A_{22}e^{-L_1} + B_{22}e^{L_1} \quad \text{—— (R)}$$

Substituting (P), (Q), (R) to (*)

$$\frac{-1}{(r_i d_c)_1}\left[-A_1 e^{-L_1} + B_1 e^{L_1}\right] = \frac{-1}{(r_i d_c)_{21}}\left[-A_{21}e^{-L_1} + B_{21}e^{L_1}\right] + \frac{-1}{(r_i d_c)_{22}}\left[-A_{22}e^{-L_1} + B_{22}e^{L_1}\right]$$

$$\frac{A_1 e^{-L_1}}{(r_i d_c)_1} - \frac{B_1 e^{L_1}}{(r_i d_c)_1} = \frac{A_{21}e^{-L_1}}{(r_i d_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i d_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i d_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i d_c)_{22}}$$

$$-\frac{A_1 e^{-L_1}}{(r_i d_c)_1} + \frac{B_1 e^{L_1}}{(r_i d_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i d_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i d_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i d_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i d_c)_{22}} = 0 \quad //$$

Question 02

$$Ax = b$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{-L_1} \\ \frac{-e^{-L_1}}{(r_i d_c)_1} & \frac{e^{L_1}}{(r_i d_c)_1} & \frac{e^{-L_1}}{(r_i d_c)_{21}} & \frac{-e^{L_1}}{(r_i d_c)_{21}} & \frac{e^{-L_1}}{(r_i d_c)_{22}} & \frac{e^{-L_1}}{(r_i d_c)_{22}} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{bmatrix} = \begin{bmatrix} (r_i d_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A_1 & -B_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{21}e^{-L_{21}} & B_{21}e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{22}e^{-L_{22}} & B_{22}e^{L_{22}} \\ A_1 e^{-L_1} & B_1 e^{L_1} & -A_{21}e^{-L_1} & -B_{21}e^{L_1} & 0 & 0 \\ 0 & 0 & A_{21}e^{-L_1} & B_{21}e^{L_1} & -A_{22}e^{-L_1} & -B_{22}e^{-L_1} \\ \frac{-A_1 e^{L_1}}{(r_i d_c)_1} & \frac{B_1 e^{L_1}}{(r_i d_c)_1} & \frac{A_{21}e^{-L_1}}{(r_i d_c)_{21}} & \frac{-B_{21}e^{L_1}}{(r_i d_c)_{21}} & \frac{A_{22}e^{-L_1}}{(r_i d_c)_{22}} & \frac{-B_{22}e^{L_1}}{(r_i d_c)_{22}} \end{bmatrix} = \begin{bmatrix} (r_i d_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_1 - B_1 = (r_i d_c)_1 I_{app}$$

$$A_{21} e^{-L_2} + B_{21} e^{L_{21}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0$$

$$-\frac{A_1 e^{-L_1}}{(r_i d_c)_1} + \frac{B_1 e^{L_1}}{(r_i d_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i d_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i d_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i d_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i d_c)_{22}} = 0 \quad //$$