Linear D.E's applications

Ex:- Suppose a family depositing a money into a bank account continuously at the rate of about 10,000 per year, and the account earns interest of 4% annually. The family began their first year with 23,000 in the account. Assuming they don't make any withdrawls, how much money in the account after 4 years?

Sol:

Change in money = Saving + interest

$$\frac{dy}{dt} = 10,000 + 4\%\,y \quad \text{or} \quad \frac{dA}{dt} = 4\%A + 10,000$$

$$\frac{dA}{dt} = 0.04A + 10,000$$

↳ This eq is Linear

initially they deposit $y(0) = 23,000$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$I.F = e^{-\int 0.04\,dt} = e^{0.04x} \text{ or } e^{0.4t}$$

$$\boxed{y.IF = \int Q(x).IF\,dx + C}$$

$$y.e^{0.4t} = \int 10,000\, e^{0.04t}\,dt + C$$

$$= 10,000\, e^{0.04t}.\left(\frac{-1}{0.04}\right) + C$$

$$y.e^{-0.4t} = -250000e^{-0.4t} + C$$

$$y = \frac{-250000e^{-0.4t}}{e^{-0.4t}} + \frac{C}{e^{0.4t}}$$

$$\boxed{y(t) = Ce^{0.4t} - 250000}$$

$$y(0) = Ce^{0.4(0)} - 250000$$

$$23000 + 250000 = C$$

$$C = 273000$$

$$y(t) = 273000e^{0.04t} - 25000$$

$$y(4) \approx 70368.5$$

OK

Investments

Ex: If Rs: 10,000 are invested with annual interest of 10% compounded continuously. What will be the total amount after 5 years?

Solu $\frac{dA}{dt} \propto A \Rightarrow \frac{dA}{dt} = kA \xrightarrow{\text{here } k \text{ is } 10\%} \frac{dA}{dt} = 10\% A$

$\int \frac{1}{A} dA = 0.1 \int dt \Rightarrow \ln A = 0.1t + c$

$A(t) = A_0 e^{0.1t}$ $\quad A_0 = 10{,}000$

$\boxed{A(t) = 10{,}000\, e^{0.1t}}$

$A(5) = 10{,}000\, e^{0.1(5)} \approx 16487$ Ans

Ex:- How long will it take a bank deposite to triple in value if interest is compounded continuously at a rate of $2\frac{1}{4}\%$ per annum?

Solu $\frac{dA}{dt} = \frac{21}{4}\% A \Rightarrow \frac{dA}{dt} = 0.0525A$

$\boxed{A(t) = A_0 e^{0.0525t}}$

when $t=0$, $A = A_0$

Now $A = 3A_0$ then

$3A_0 = A_0 e^{0.0525t}$

$\ln(3) = 0.0525t$

$t = \frac{\ln(3)}{0.0525}$

$\boxed{t \approx 21 \text{ years}}$ Ans