Lecture # 01

SUKKUR IBA UNIVERSITY

Student's Name: ______ Date. ______

Class: ______ CMS ID. ______ Course: ______

Instructor: ______ No. ______

D.E

An equation containing the derivatives of one or more unknown functions, with respect to one or more independent variables, is said to be Differential equation.

Classification by type

ODE	PDE
If DE contains only ordinary derivatives of one or more unknown functions with respect to a single independent variable, it is said to be ODE.	An equation involving partial derivatives of one or more unknown functions of two or more independent variables is called PDE.

$$\frac{dy}{dx} + 5y = e^x$$

ODE can contains more than one unknown function

$$\frac{dx}{dt} + \frac{dy}{dt} = 2x + y$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0$$

$$\frac{d^2\overset{\text{Dependent}}{y}}{d\underset{\text{independent}}{x^2}} + 16y = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t}$$

Order: The order of differential equation (either ODE PDE) is the order of highest derivative in the equation.

$$\cos x \overset{\text{2nd order}}{\frac{d^2y}{dx^2}} + \sin x \overset{\text{1st order}}{\left(\frac{dy}{dx}\right)^2} + 8y = \tan x$$

$$\underset{\text{2nd order}}{\frac{d^2y}{dx^2}} + 5\overset{\text{1st order}}{\left(\frac{dy}{dx}\right)^3} - 4y = e^x$$

Degree:- Degree of the highest derivative after removing the radical sign and fraction.

order 2, degree 1.

$$\left(\frac{d^2y}{dx^2}\right) + 4x = 0$$

order 2, degree 2

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

General form "n^{th}-order DE in one dependent variable"

$$F(x, y, y', y'', \ldots y^{(n)}) = 0$$

x → Independent variable; y → Dependent variable

where F is a real values function of $n+2$ variables

$$\frac{d^ny}{dx^n} = f(x, y, y', y'', \ldots y^{(n-1)})$$

f → real valued continuous function

normal form: $\frac{dy}{dx} = f(x, y)$

$$\frac{d^2y}{dx^2} = f(x, y, y')$$

Solution of D.E

Solution of D.E is a functional relation b/w the variables involved which satisfies the equation.

General Solution	Particular solution	Singular Solution
Solution of D.E in which the number of arbitrary constant is equal to the order of D.E. also called complete solution.	If particular values are assign to arbitrary constants in general solution, then the solution obtained is called particular solution.	Solution does not contain any arbitrary constant and also does not obtained from general solution by giving particular values to arbitrary constant is called Singular Solution.

ex: $\frac{dy}{dx} = x^2 - 3$

$dy = (x^2-3)dx$

$\int dy = \int x^2 dx - 3\int dx$

$$\boxed{y = \frac{x^3}{3} - 3x + C}$$

This is our General Solution

ex: Particular Solution → Assign for particular Solution.

$(x^3+2)y = x(y^4+3)\frac{dy}{dx}$ when $\boxed{x=1 \ \& \ y=1}$

$\left(\frac{x^3+2}{x}\right)dx = \left(\frac{y^4+3}{y}\right)dy$

$\int\left(x^2+\frac{2}{x}\right)dx = \int\left(y^3+\frac{3}{y}\right)dy$

$\int x^2dx + 2\int\frac{1}{x}dx = \int y^3dy + 3\int\frac{1}{y}dy$

$\frac{x^3}{3} + 2\ln x = \frac{y^4}{4} + 3\ln y + C$ → General Solution

Now assign particular values to x and y

$\frac{1}{3} + 2\ln(1) = \frac{1}{4} + 3\ln(1) + C$ $\quad \ln(1) = 0$

$\frac{1}{3} - \frac{1}{4} = C$ $\quad \boxed{C = 1/12}$

So Particular Solution is

$\frac{x^3}{3} + 2\ln x = \frac{y^4}{4} + 3\ln y + 1/12$

$\frac{x^3}{3} - \frac{y^4}{4} + \ln x^2 - \ln y^3 = 1/2$

$\frac{x^3}{3} - \frac{y^4}{4} + \ln\frac{x^2}{y^3} = 1/2$

⇒ For Singular Solution

$9\left(\frac{dy}{dx}\right)^2(y-2)^2 = 4(3-y) \rightarrow i$

let $f(x,y,p)=0$

$9p^2(y-2)^2 = 4(3-y)$

$F = 9p^2(y-2)^2 - 4(3-y) = 0$

$\frac{\partial f}{\partial p} = 18p(y-2)^2$

let $\frac{\partial f}{\partial p} = 0$

$0 = 18p(y-2)^2$

$\boxed{p=0}$

put in eq(i)

$9(0)(y-2)^2 = 4(3-y)$

$0 = 4(3-y)$

$\boxed{y=3}$ is a singular solution

Variable separable

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

Soln $\frac{dy}{dx} = e^y(e^x + x^2)$

$$\frac{1}{e^y}\,dy = (e^x + x^2)\,dx$$

$$\int e^{-y}\,dy = \int e^x\,dx + \int x^2\,dx$$

$$\frac{e^{-y}}{-1} = \frac{e^x}{1} + \frac{x^3}{3} + C$$

$$e^x + e^{-y} + \frac{x^3}{3} + c = 0 \text{ Ans}$$

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$

$$y - x\frac{dy}{dx} = ay^2 + a\frac{dy}{dx}$$

$$y - ay^2 = (a+x)\frac{dy}{dx}$$

$$\left(\frac{1}{a+x}\right)dx = \left(\frac{1}{y-ay^2}\right)dy$$

$$\int \frac{1}{a+x}\,dx = \int \frac{1}{y(1-ay)}\,dy$$

$$\int \frac{1}{a+x}\,dx = \int \frac{1}{y}\,dy + \int \frac{a}{1-ay}\,dy$$

$$\ln(a+x) = \ln y + a\ln(1-ay)\cdot\frac{1}{(-a)} + \ln C$$

$$\ln(a+x) = \ln y - \ln(1-ay) + \ln C$$

$$\ln(a+x) + \ln(1-ay) - \ln y = \ln c$$

$$\cancel{\ln}\left(\frac{(a+x)(1-ay)}{y}\right) = \cancel{\ln} c$$

$$y = (a+x)(1-ay) \text{ Ans}$$

Worksheet #01

Q1- Write the order and degree of the D.E.

i- $\frac{d^2y}{dx^2} + a^2x = 0$

ii- $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$

iii- $x^2\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$

Q2- Obtain the D.E $y^2 = 4a(x+a)$ and $Ax^2 + By^2 = 1$

Q3- By eliminating constant find D.E

$$y = e^x(A\cos x + B\sin x)$$

Q4- Variable Seprable

i- $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ Ans: $y\sin y = x^2\log x + c$

ii- $x^4\frac{dy}{dx} + x^3y = -\sec(xy)$ Ans: $\sin(xy) = \frac{1}{2x^2} + c$

iii- $\cos(x+y)dy = dx$ Ans: $y - \tan\left(\frac{x+y}{2}\right) = c$

iv- $(2x^2+3y^2-7)x\,dx - (3x^2+2y^2-8)y\,dy = 0$ Ans: $x^2+y^2-3 = c(x^2-y^2-1)^5$

v- $\frac{dx}{x} = \tan y\,dy$ Ans: $x\cos y = c$

vi- $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1+x^2}}$ Ans: $\sin^{-1}y = \sin^{-1}x + c$

vii- $y(1+x^2)^{1/2}dy + x\sqrt{1+y^2}\,dx = 0$ Ans: $\sqrt{1+y^2} + \sqrt{1+x^2} = c$

viii- $(e^y+1)\cos x\,dx + e^y\sin x\,dy = 0$ Ans: $(e^y+1)\sin x = c$

ix- $(e^y+2)\sin x\,dx - e^y\cos x\,dy = 0$ Ans: $(e^y+2)\cos x = c$

x- $\frac{dy}{dx} = 1 + \tan(y-x)$ hint [Put $y - x = z$] Ans: $\sin(y-x) = e^{x+c}$