Floyd-Warshall Algorithm

The Floyd-warshall algorithm is a method for finding the Shortest Paths between all Pairs of vertices in a weighted graph. It's a type of dynamic Programming algorithm that Systematically find the Shortest Paths by checking every Possible intermediate Vertex.

→ works on both directed and undirected graph.

Why Use it:-

The Floyed-Warshall algorithm is useful when you need to find the Shortest Path between every Single Pair of nodes in a graph.

① In network routing, it can be used to find the most efficient Path for data to travel between any two routers.

② It's also a great tool to detecting negative cycles in a graph.

How It Works

① Initialization:- Start with a distance matrix that represents the graph. If there's a direct edge between two vertices, use its weight.

* If not, use infinity to show there's no direct Path. The distance from a vertex to itself is always 0.

(2) Iterative Improvement:- The algorithm uses three nested loops to go through every Possible Path. (1) The outer loop, controlled by a variable k, represents an intermediate vertex. For each k, the algorithm checks every Pair of vertices (i, j) to see.

* If going from i to j via k is shorter than the currently know path.

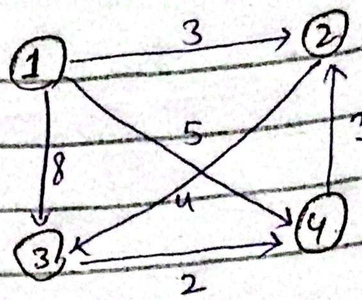
(3) Updated Rule:- The core of the algorithm is a simple formula:

$$\boxed{D[i][j] = \min(D[i][j], (D[i][j] + D[k][j]))}$$

This means the new shortest distance from i to j is either the existing distance or the distance from i to k Plus the distance from k to j, whichever is smaller.

(4) Final Result:- After the loops are complete, the matrix will contain the shortest Path distances between all Pair of vertices.

Example:-



Step 1:- Distance Matrix.

$$d^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 8 & 5 \\ \infty & 0 & 4 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & 3 & \infty & 0 \end{bmatrix} \end{array}$$

Step 2:- Iterations with intermediate vertex.

$k = 1$ (vertex 1).

$$d^{(1)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 8 & 5 \\ \infty & 0 & 4 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & 3 & \infty & 0 \end{bmatrix} \end{array}$$

No any value update

⇒ diagonal remains same and row one and colum 1 also same because 1 — anynumber and anynumber to 1. distance remain same in this iteration.

Step 3:- Iteration $K = 2$ using (vertex 2)

$$d^{(2)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 7 & 5 \\ \infty & 0 & 4 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & 3 & 7 & 0 \end{bmatrix} \end{array}$$

① $1 \to 2 \to 3 = 7$

$7 < 8$ update

for $4 \to 3$

② $4 \to 2 \to 3 = 7$

$7 < \infty$. update

Step 4:- Iteration $k = 3$ using (vertex 3)

$$d^{(3)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 7 & 5 \\ 2 & \infty & 0 & 4 & 6 \\ 3 & \infty & \infty & 0 & 2 \\ 4 & \infty & 3 & 7 & 0 \end{array}$$

① for $2 \rightarrow 4$

$2 \rightarrow 3 \rightarrow 4 = 6$

$6 < \infty$ update

Step 5:- Iteration $k = 4$ using (vertex '4')

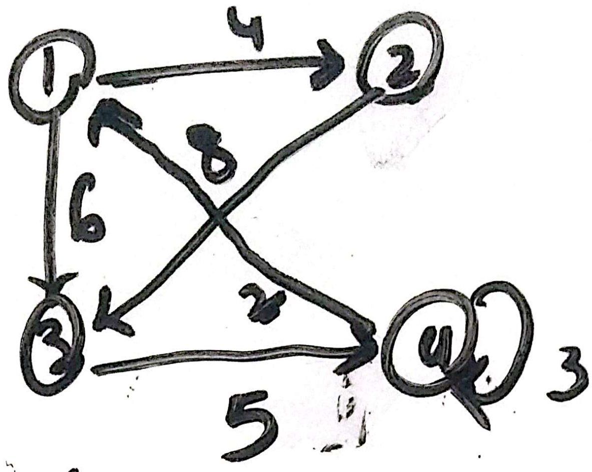
$$d^{(4)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 7 & 5 \\ 2 & \infty & 0 & 4 & 6 \\ 3 & \infty & 5 & 0 & 2 \\ 4 & \infty & 3 & 7 & 0 \end{array}$$

① for $3 \rightarrow 2$

$3 \rightarrow 4 \rightarrow 2 = 5$

$5 < \infty$ update

* $d^{(4)}$ Contains the Shortest Path distances between all Pairs of vertices.



graph having loop.

$$d^{(0)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 6 & \infty \\ 2 & \infty & 0 & 2 & \infty \\ 3 & \infty & \infty & 0 & 5 \\ 4 & 8 & \infty & \infty & 0 \end{array}$$

$4 \to 4 = 3$
first shortest Path vertex to itself is 0

$K = 1$

$$d^{(1)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 6 & \infty \\ 2 & \infty & 0 & 2 & \infty \\ 3 & \infty & \infty & 0 & 5 \\ 4 & 8 & 12 & 14 & 0 \end{array}$$

~~for~~
① $4 \to 2$
$4 \to 1 \to 2 = 12$
$12 < \infty$ update
② for $4 \to 3$
$4 \to 1 \to 3 = 14$
$14 < \infty$ update

$$d^{(2)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 6 & \infty \\ 2 & \infty & 0 & 2 & \infty \\ 3 & \infty & \infty & 0 & 5 \\ 4 & 8 & 12 & 14 & 0 \end{array}$$

No update.

$K = 3$

$$d^{(3)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 6 & 11 \\ 2 & \infty & 0 & 2 & 7 \\ 3 & \infty & \infty & 0 & 5 \\ 4 & 8 & 12 & 14 & 0 \end{array}$$

① for $1 \to 4$

$1 \to 3 \to 4 = 11$

$11 < \infty$ update

② for $2 \to 4$

$2 \to 3 \to 4 = 7$

$7 < \infty$ upadate

$$d^{(4)} \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 6 & 11 \\ 2 & 15 & 0 & 2 & 17 \\ 3 & 13 & 17 & 0 & 5 \\ 4 & 8 & 12 & 14 & 0 \end{array}$$

① for $2 \to 1$

$2 \to 3 \to 4 \to 1 = 15$

$15 < \infty$

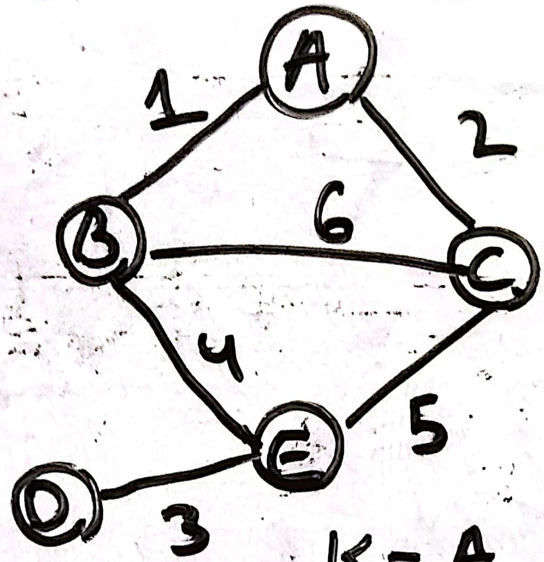
② for $3 \to 1$

$3 \to 4 \to 1 = 13$

$13 < \infty$

③ for $3 \to 2$

$3 \to 4 \to 1 \to 2 = 17 < \infty$ upadate



$$d^{(0)} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 2 & \infty & \infty \\ B & 1 & 0 & 6 & \infty & [illegible] \\ C & 2 & 6 & 0 & \infty & 5 \\ D & \infty & \infty & \infty & 0 & 3 \\ E & \infty & 4 & 5 & 3 & 0 \end{array}$$

$\underline{K = A}$

$$d^{(A)} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 2 & \infty & \infty \\ B & 1 & 0 & 3 & \infty & 4 \\ C & 2 & 3 & 0 & \infty & 5 \\ D & \infty & \infty & \infty & 0 & 3 \\ E & \infty & 4 & 5 & 3 & 0 \end{array}$$

① for $B \to C$

$B \to A \to C = 3 < 6$

② for $C \to B$

$C \to A \to B = 3 < 6$

$\underline{K = B}$

$$d^{(B)} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 2 & \infty & 5 \\ B & 1 & 0 & 3 & \infty & 4 \\ C & 2 & 3 & 0 & \infty & 5 \\ D & \infty & \infty & \infty & 0 & 3 \\ E & 5 & 4 & 5 & 3 & 0 \end{array}$$

① for $A \to E$

$A \to B \to E = 5 < \infty$

② for $E \to A$

$E \to B \to A = 5$

$5 < \infty$

$$d^{(C)} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & \infty & 5 \\ 1 & 0 & 3 & \infty & 4 \\ 2 & 3 & 0 & \infty & 5 \\ \infty & \infty & \infty & 0 & 3 \\ 5 & 4 & 5 & 3 & 0 \end{bmatrix} \end{array}$$

$K = D$

$$d^{(D)} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & \infty & 5 \\ 2 & 0 & 3 & \infty & 4 \\ \infty & 3 & 0 & \infty & 4 \\ \infty & \infty & \infty & 0 & 3 \\ 5 & 4 & 4 & 3 & 0 \end{bmatrix} \end{array}$$

$K = E$

$$d^{(E)} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 8 & 5 \\ 1 & 0 & 3 & 7 & 4 \\ 2 & 3 & 0 & 8 & 4 \\ 8 & 7 & 8 & 0 & 3 \\ 5 & 4 & 4 & 3 & 0 \end{bmatrix} \end{array}$$

① $A \to D$

$A \to B \to E \to D = 8 < \infty$

② $B \to D$

$B \to E \to D = 7$

$7 < \infty$

③ $C \to D$

$C \to E \to D = 8$

$8 < \infty$