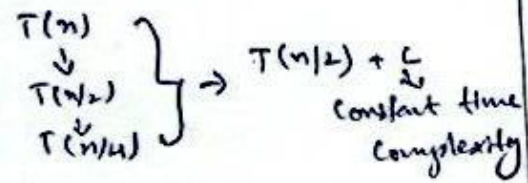
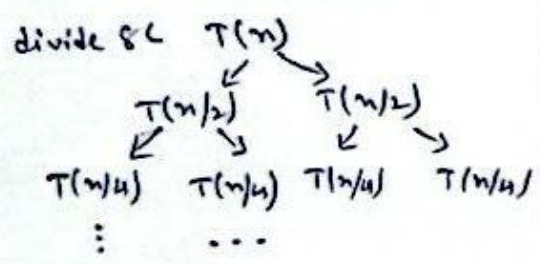


# 1. Substitution Method



Ex:  $T(n/2) + c$ , if  $n > 1$   
 $1$ , if  $n = 1$

$$T(n) = T(n/2) + c \quad \text{--- (1)}$$

$$T(n/2) = T(n/4) + c \quad \text{--- (2)}$$

$$T(n/4) = T(n/8) + c \quad \text{--- (3)}$$

Substitute eq (2) into eq (1)

$$T(n) = T(n/2) + c$$

$$T(n) = [T(n/4) + c] + c$$

$$T(n) = T(n/4) + 2c$$

Substitute eq (3) into

$$T(n) = [T(n/8) + c] + 2c$$

$$T(n) = T(n/8) + 3c$$

$$T(n) = T(n/2^3) + 3c$$

$$= T(n/2^k) + Kc$$

let  $2^k = n$

$$T(n) = T(n/2^k) + Kc$$

$$2^k = n$$

$$T(n) = T(n/n) + Kc$$

$$T(n) = T(1) + Kc$$

$$T(n) = 1 + Kc$$

$$T(n) = Kc, K = ?$$

$$2^k = n \Rightarrow \log_2 2^k = \log_2 n$$

$$k = \log_2 n$$

$T(n) = k = \log n$

$$\hookrightarrow T(n) = k \cdot c$$

$$= c \log(n)$$

# 2. Master Theorem

$$T(n) = aT(n/b) + f(n)$$

$$a \geq 1, b > 1$$

Sol:  $T(n) = n^{\log_b a} \cdot U(n)$   
 where  $U(n)$  depends on  $h(n)$

$$h(n) = \frac{f(n)}{n^{\log_b a}}$$

if $h(n)$	$U(n)$
$n^r, r > 0$	$O(n^r)$
$n^r, r < 0$	$O(1)$
$(\log n)^i, i \geq 0$	$\frac{(\log n)^{i+1}}{i+1}$

$$T(n) = T(n/2) + c$$

$$\frac{1}{a} \geq 1 \checkmark \quad \frac{2}{b} > 1 \checkmark \quad f(n) = c$$

$$T(n) = n^{\log_b a} \cdot U(n)$$

$$= n^{\log_2 1} \cdot U(n)$$

$$= n^0 \cdot U(n)$$

$$h(n) = \frac{f(n)}{n^{\log_b a}} = \frac{c}{1}$$

$$T(n) = n^0 \cdot U(n)$$

$$= 1 \cdot U(n)$$

$$h(n) = c = (\log n)^0 \cdot c$$

$$= \frac{1 \cdot (\log n)^{0+1} \cdot c}{0+1}$$

$T(n) = (\log n) c$

$$T(n) = 8T(n/2) + n^2$$

$$a = 8, b = 2, f(n) = n^2$$

$$T(n) = n^{\log_b a} \cdot U(n)$$

$$= n^{\log_2 8} \cdot U(n)$$

$$= n^{\log_2 2^3} \cdot U(n)$$

$$= n^3 \log^4 \cdot U(n)$$

$$= n^3 \cdot U(n)$$

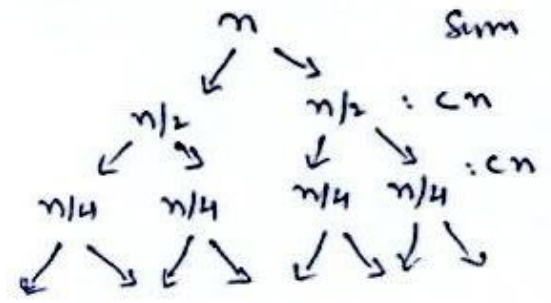
$$h(n) = \frac{f(n)}{n^{\log_b a}} = \frac{n^2}{n^3} = n^{-1}$$

$$h(n) = n^{-1}, n^r, r < 0 \rightarrow O(1) = 1$$

$$T(n) = n^3 \cdot U(n) = n^3 \cdot O(1) = n^3$$

# 3. Recurrence Tree

$$T(n) = 2T(n/2) + cn$$



Level 1	16	Divide by 2
Level 2	8	
Level 3	4	
Level 4	2	

$$\log_2 16 = \log_2 2^4 = 4 \text{ Levels}$$

$$T(n) = 2T(n/2) + cn$$

$$\log n \cdot cn$$

$$O(n \log n) \cdot c$$

$O(n \log n)$