Gradient Boosting for Classification

Sunday, January 26, 2025 4:16 PM

Steps:

1. Initialize $f_0(x) = \arg\min_\gamma \sum_{i=1}^{N} L(y, \gamma)$
2. For $m = 1$ to M
   a. For $i = 1, 2, 3, \ldots, N$
      Compute $r_{im} = \left[\frac{\delta L(y,\gamma)}{\delta f(x_i)}\right]$
   b. Fit the regression tree to the targets r_{im} giving terminal regions $R_{jm}, j = 1, 2, 3, \ldots, J_m$
   c. For $j = 1, 2, 3, \ldots, J_m$
      Compute $\gamma_{jm} = \arg\min_\gamma \sum_{x_i \in R_{jm}} L(y_i, f_{m-1}(x_i) + \gamma)$
   d. Update $f_m(x) = f_{m-1}(x) + \sum_{j=1}^{J_m} \gamma_{jm} I(x \in R_{jm})$
3. Update $\hat{f}(x) = f_M(x)$

CGPA	IQ	Placed
6.82	118	0
6.36	125	1
5.39	99	1
5.50	106	1
6.39	148	0
9.13	148	1
7.17	147	1
7.72	72	0

Step 1:

1. Calculate $\log_e odds$

   $$\log_e odds = \log_e \frac{5}{3} = \frac{\log \frac{5}{3}}{\log e} = \frac{\log 1.6667}{1} = 0.51$$

2. Calculate the probability

   $$\frac{1}{1 + e^{-\log odds}} = \frac{1}{1 + e^{-0.51}} = 0.625$$

CGPA	IQ	Placed	Log(odds) 1	Probability 1
6.82	118	0	0.51	0.625
6.36	125	1	0.51	0.625
5.39	99	1	0.51	0.625
5.50	106	1	0.51	0.625
6.39	148	0	0.51	0.625
9.13	148	1	0.51	0.625

7.17	147	1	0.51	0.625
7.72	72	0	0.51	0.625

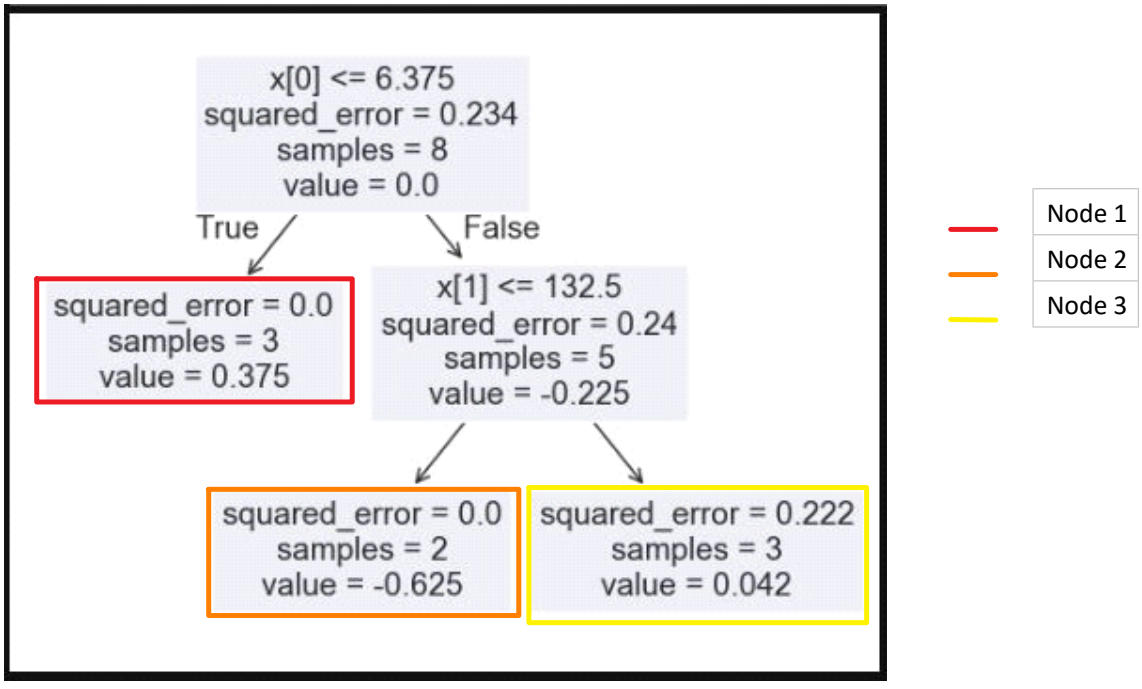
Step 2:

Calculate pseudo-residuals (Target - Probability)

CGPA	IQ	Placed	Log(odds) 1	Probability 1	Residuals 1
6.82	118	0	0.51	0.625	-0.625
6.36	125	1	0.51	0.625	0.375
5.39	99	1	0.51	0.625	0.375
5.50	106	1	0.51	0.625	0.375
6.39	148	0	0.51	0.625	-0.625
9.13	148	1	0.51	0.625	0.375
7.17	147	1	0.51	0.625	0.375
7.72	72	0	0.51	0.625	-0.625

Step 3:

Train a Decision Tree (it has to be a weak regression learner since the outputs are continuous) with features as the previous inputs and targets as residuals



Step 4:

Calculate $\log odds\ (Node) = \frac{\sum Residuals}{\sum(Previous\ Probability*(1-Previous\ Probability))}$ for each end node

CGPA	IQ	Placed	Log(odds) 1	Probability 1	Residuals 1	Leaf entry 1
6.82	118	0	0.51	0.625	-0.625	2
6.36	125	1	0.51	0.625	0.375	1
5.39	99	1	0.51	0.625	0.375	1
5.50	106	1	0.51	0.625	0.375	1
6.39	148	0	0.51	0.625	-0.625	3
9.13	148	1	0.51	0.625	0.375	3
7.17	147	1	0.51	0.625	0.375	3
7.72	72	0	0.51	0.625	-0.625	2

$$\log odds\ (Node\ 1) = \frac{0.375 + 0.375 + 0.375}{0.625 * 0.375 + 0.625 * 0.375 + 0.625 * 0.375} = \frac{3 * 0.375}{3 * 0.625 * 0.375} = 1.6$$

$$\log odds\ (Node\ 2) = \frac{-0.625 - 0.625}{0.625 * 0.375 + 0.625 * 0.375} = \frac{-2 * 0.625}{2 * 0.625 * 0.375} = -2.667$$

$$\log odds\ (Node\ 3) = \frac{-0.625 + 0.375 + 0.375}{0.625 * 0.375 + 0.625 * 0.375 + 0.625 * 0.375} = \frac{0.125}{3 * 0.625 * 0.375} = 0.178$$

Step 5:

Calculate $\log odds = \log odds\ 1 + \log odds\ (respective\ node)$
Calculate Probability

CGPA	IQ	Placed	Log(odds) 1	Probability 1	Residuals 1	Leaf entry 1	Log(odds) 2	Probability 2
6.82	118	0	0.51	0.625	-0.625	2	-2.175	0.102
6.36	125	1	0.51	0.625	0.375	1	2.11	0.8919
5.39	99	1	0.51	0.625	0.375	1	2.11	0.8919
5.50	106	1	0.51	0.625	0.375	1	2.11	0.8919
6.39	148	0	0.51	0.625	-0.625	3	0.688	0.6655
9.13	148	1	0.51	0.625	0.375	3	0.688	0.6655
7.17	147	1	0.51	0.625	0.375	3	0.688	0.6655
7.72	72	0	0.51	0.625	-0.625	2	-2.175	0.102

Step 6:

Calculate residuals

CGPA	IQ	Placed	Log(odds) 1	Probability 1	Residuals 1	Leaf entry 1	Log(odds) 2	Probability 2	Residuals 2
6.82	118	0	0.51	0.625	-0.625	2	-2.175	0.102	-0.102
6.36	125	1	0.51	0.625	0.375	1	2.11	0.8919	0.1081
5.39	99	1	0.51	0.625	0.375	1	2.11	0.8919	0.1081
5.50	106	1	0.51	0.625	0.375	1	2.11	0.8919	0.1081
6.39	148	0	0.51	0.625	-0.625	3	0.688	0.6655	-0.6655
9.13	148	1	0.51	0.625	0.375	3	0.688	0.6655	0.3345
7.17	147	1	0.51	0.625	0.375	3	0.688	0.6655	0.3345
7.72	72	0	0.51	0.625	-0.625	2	-2.175	0.102	-0.102

Step 7:

Repeat the steps until the residuals are zero