Gradient Boosting for Regression

Sunday, January 26, 2025 3:17 PM

Steps:

1. Initialize $f_0(x) = \arg\min_\gamma \sum_{i=1}^{N} L(y, \gamma)$
2. For $m = 1$ to M
   a. For $i = 1, 2, 3, \dots, N$
      Compute $r_{im} = \left[\frac{\delta L(y,\gamma)}{\delta f(x_i)}\right]$
   b. Fit the regression tree to the targets r_{im} giving terminal regions $R_{jm}, j = 1, 2, 3, \dots, J_m$
   c. For $j = 1, 2, 3, \dots, J_m$
      Compute $\gamma_{jm} = \arg\min\limits_{\gamma} \sum_{x_i \in R_{jm}} L(y_i, f_{m-1}(x_i) + \gamma)$
   d. Update $f_m(x) = f_{m-1}(x) + \sum_{j=1}^{J_m} \gamma_{jm} I(x \in R_{jm})$
3. Update $\hat{f}(x) = f_M(x)$

Step 1:

It states that we should have a loss function that should be differentiable

$$f_0(x) = \arg\min_\gamma \sum_{i=1}^{N} L(y, \gamma)$$

$$f_0(x) = \arg\min_\gamma \frac{1}{2} \sum_{i=1}^{N} L(y_i, \gamma)$$

$$\frac{\delta f_0(x)}{\delta \gamma} = \frac{1}{2} \sum_{i=1}^{n} \frac{\delta (y_i - \gamma)^2}{\delta \gamma}$$

$$\frac{\delta f_0(x)}{\delta \gamma} = \sum_{i=1}^{n} (y_i - \gamma)^2 \frac{\delta (y_i - \gamma)}{\delta \gamma}$$

$$\frac{\delta f_0(x)}{\delta \gamma} = -\sum_{i=1}^{n} (y_i - \gamma) = \sum_{i=1}^{n} (\gamma - y_i) = 0$$

R&D Spend	Administration	Marketing Spend	Profit
165	137	472	192
101	92	250	144
29	127	201	91

$$\frac{\delta f_0(x)}{\delta \gamma} = \sum_{i=1}^{3} (\gamma - y_i) = (\gamma - 192) + (\gamma - 144) + (\gamma - 91) = 0$$

$$\frac{\delta f_0(x)}{\delta \gamma} = (\gamma - 192) + (\gamma - 144) + (\gamma - 91) = 0$$
$$3\gamma = 192 + 144 + 91$$
$$\gamma = 142.33$$

R&D Spend	Administration	Marketing Spend	Profit	$f_0(x)$
165	137	472	192	142.33
101	92	250	144	142.33
29	127	201	91	142.33

Step 2:

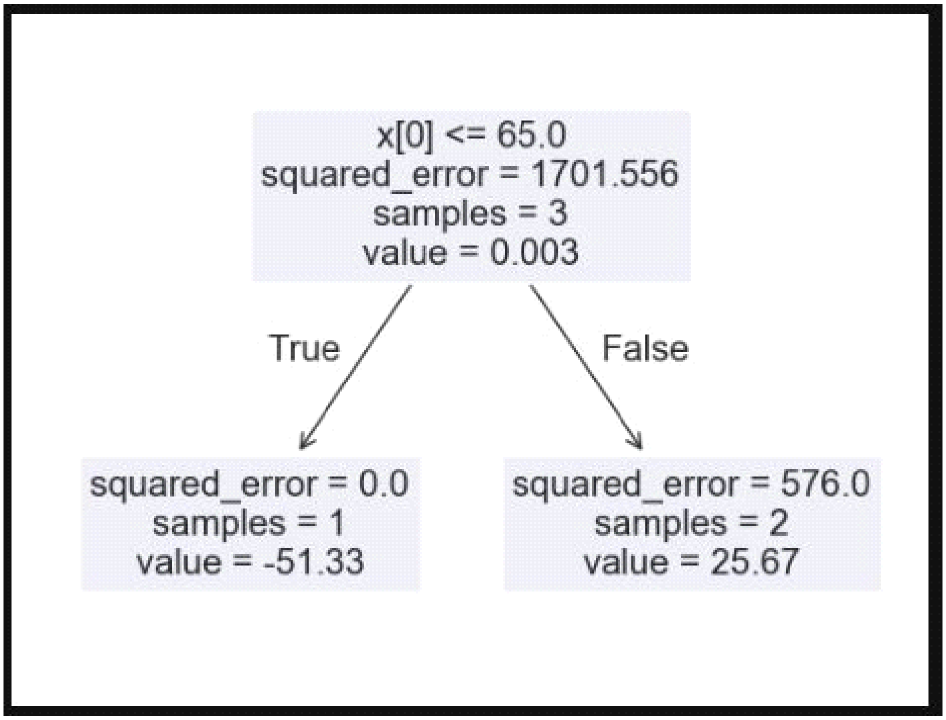
For $i = 1, 2, 3, \ldots, N$

Compute $r_{im} = -\left[\frac{\delta L(y, f(x_i))}{\delta f(x_i)}\right]_{f=f_{m-1}}$ $; i = row, m = decision\ tree$

For $m = 1$

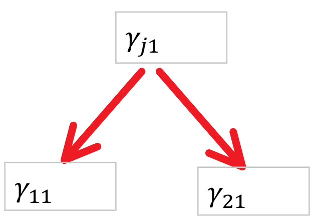
$$r_{i1} = -\left[\frac{\delta L(y, f(x_i))}{\delta f(x_i)}\right]_{f=f_0}$$
$$r_{i1} = (y_i - \gamma)_{f=f_0}$$
$$r_{11} = 192 - 142.33 = 49.67$$
$$r_{21} = 144 - 142.33 = 1.67$$
$$r_{31} = 91 - 142.33 = -51.33$$

R&D Spend	Administration	Marketing Spend	Profit	$f_0(x)$	r_{im}
165	137	472	192	142.33	49.67
101	92	250	144	142.33	1.67
29	127	201	91	142.33	-51.33



Calculate Output values

$$\gamma_{jm} = \arg\min_{\gamma} \sum_{x_i \in R_{jm}} L(y_i, f_{m-1}(x_i) + \gamma)$$



$$\gamma_{j1} = \arg\min_{\gamma} \sum_{x_i \in R_{j1}} L(y_i, f_0(x_i) + \gamma)$$

For $j = 1$

$$\gamma_{11} = \arg\min_{\gamma} \sum_{x_i \in R_{11}} L(y_i, f_0(x_i) + \gamma)$$

$$\gamma_{11} = \frac{1}{2}(y_i - (f_0(x_i) + \gamma))^2 = 0$$

$$\frac{\delta\gamma_{11}}{\delta\gamma} = (y_i - f_0(x_i) - \gamma)\frac{\delta(y_i - f_0(x_i) - \gamma)}{\delta\gamma} = 0$$

$$(y_i - f_0(x_i) - \gamma) = 0$$

$$91 - 142.33 - \gamma = 0$$

$$\gamma_{11} = -51.33$$

$$\gamma_{21} = \arg\min_{\gamma} \sum_{x_i \in R_{11}} L(y_i, f_0(x_i) + \gamma)$$

$$\gamma_{21} = \frac{1}{2}(y_i - (f_0(x_i) + \gamma))^2 = 0$$

$$\frac{\delta\gamma_{21}}{\delta\gamma} = \sum_{1}^{2}(y_i - f_0(x_i) - \gamma)\frac{\delta(y_i - f_0(x_i) - \gamma)}{\delta\gamma} = 0$$

$$(y_1 - f_0(x_1) - \gamma) + (y_2 - f_0(x_2) - \gamma) = 0$$

$$(192 - 142.33 - \gamma) + (144 - 142.33 - \gamma) = 0$$

$$(49.67 - \gamma) + (1.67 - \gamma) = 51.34 - 2\gamma = 0$$

$$\gamma_{21} = 25.67$$

$$f_1(x) = f_0(x) + \boxed{\sum_{j=1}^{J_m} \gamma_{jm} I(x \in R_{jm})} \rightarrow \text{Output from Decision Tree}$$