The fractal cortex

A multi-scale surface-preserving analysis suggests all cortices are approximations of a single universal shape

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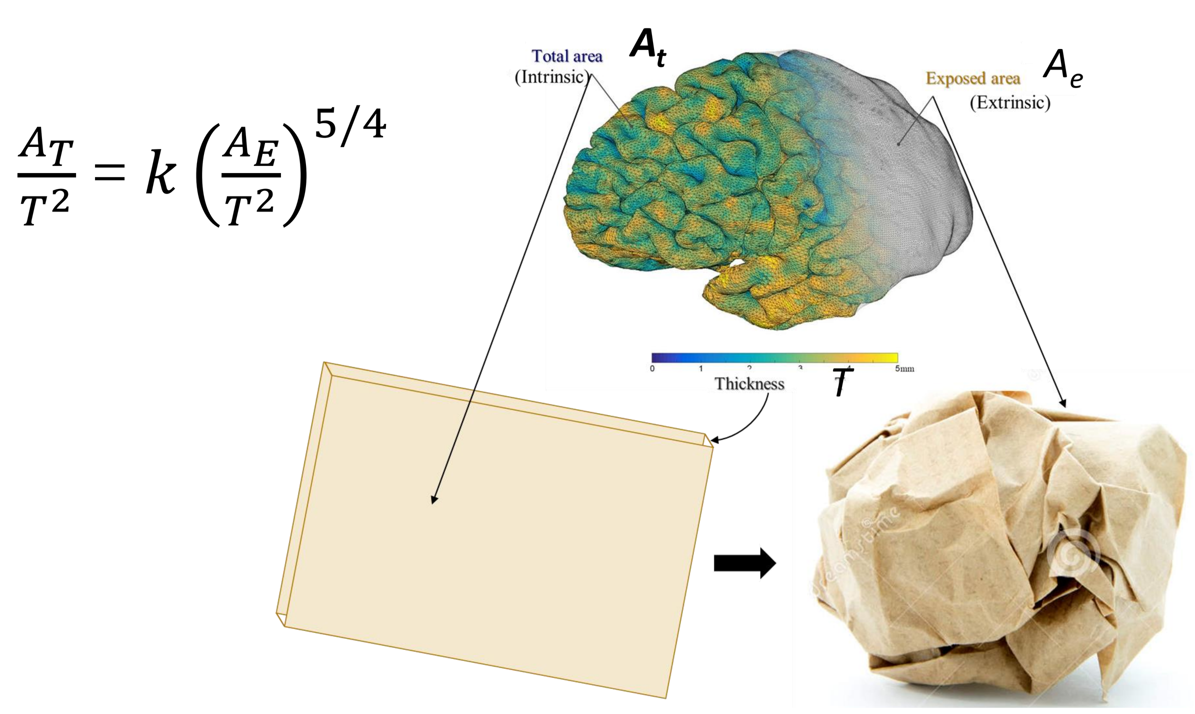
Abstract

The shape of the mammalian cerebral cortex is probably one of the most complex structures ever studied by science. At first glance, any attempt to express it, or significant features thereof, from first principles, would seem doomed to failure. Indeed, there are many ways of comparing the morphological features of cerebral cortices. In what follows, working from first principles, we will express cortical morphology using a more natural set of variables across length scales, rather than across spatial position. Using this new framework, we will show that the morphology of all cerebral cortices analyzed so far, across both mammalian species and individuals, are approximations of a single, universal fractal shape; and that the main distinction between different cortical shapes is the range of scales at which this approximation remains valid. This universal description of the cortical shape is at the same time mechanistically insightful and in full agreement with empirical data across species and individuals. Prospectively, we hope this new framework for expressing and analyzing cortical morphology, besides revealing a hitherto hidden regularity of nature, can become a powerful tool to characterize and compare cortices of different species and individuals, across development and aging, and across health and disease.

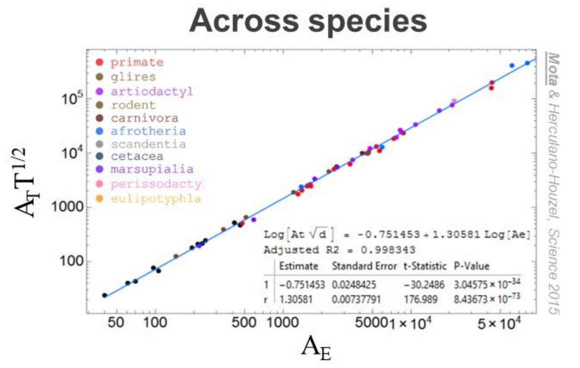
Background

We have previously shown that the cortical folding across mammalian species[1] follows a universal scaling law that can be derived from a simple theoretical model.

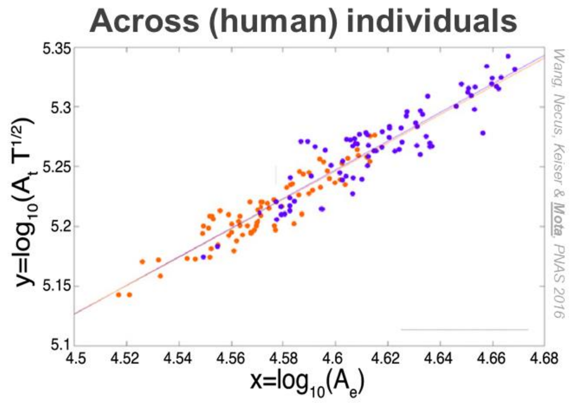
$$\frac{A_T}{T^2} = k\left(\frac{A_E}{T^2}\right)^{5/4}$$



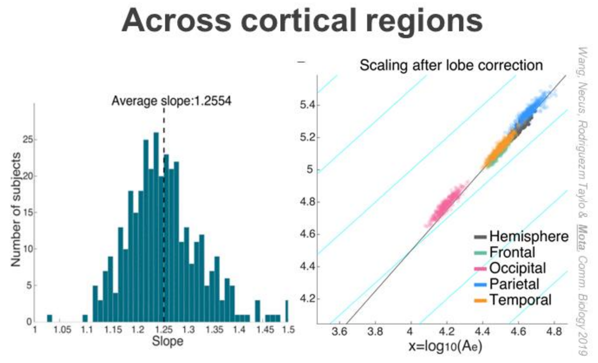
Across species



Across (human) individuals



Across cortical regions

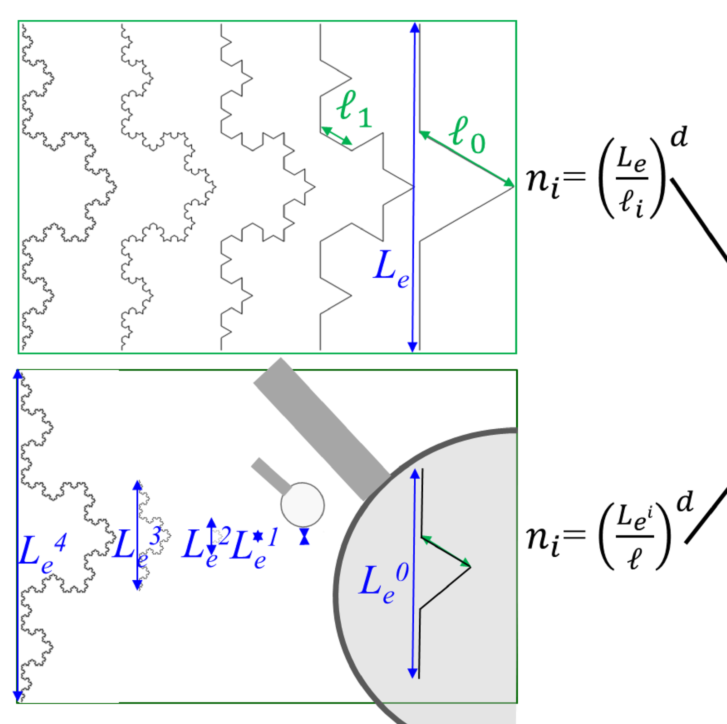


Across length scales?

?

Fractals and scaling

The Koch curve



$L_{intrinsic} = nl$

$$\frac{L_i}{\ell} = \left(\frac{L_e}{\ell}\right)^d$$

d is the fractal dimension
ℓ is the fundamental scale

$d_{line} = 1$

$$d_{Koch} = \frac{\log 4}{\log 3} = 1.262$$

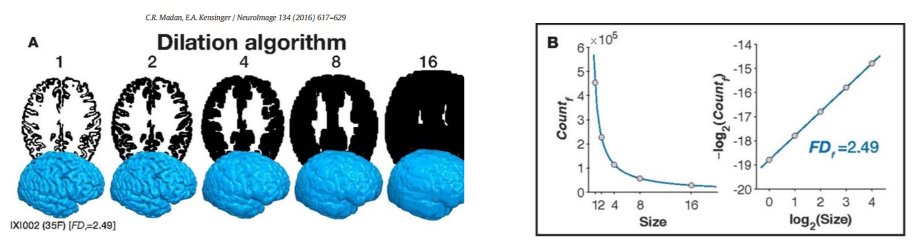
Summary & perspectives

- There is a **natural** and **independent** set of morphometrics to express cortical shape
  - They express size (I), the invariant aspect of shape (K) and shape complexity (S)
- How to express cortical shape?
  - As a **trajectory** in **morphometric space**
  - How I, K and S change over different length scales
- What is the shape of all cortices?
  - Overlapping self-similar trajectories
  - **All cortices are approximations of the same fractal shape**
  - They differ only on the **range of length scales** in which they are **self-similar**
    - The largest scale is the overall cortex size
    - The smallest is proportional to cortical thickness
    - The range is function of the gyrification index $g = A_t/A_e$

Methods

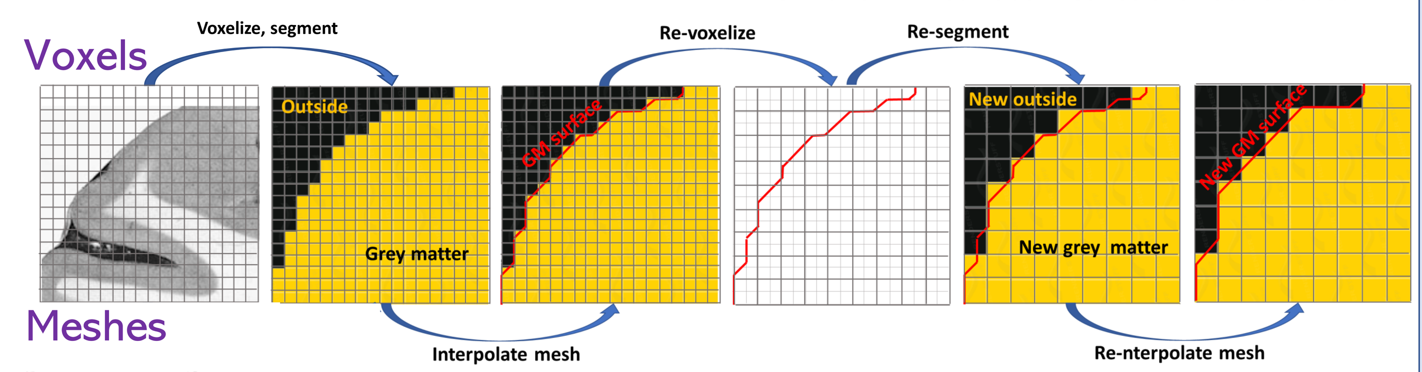
Is the cortex fractal?

- No! Fractals are mathematical abstractions
- But many shapes in nature are well represented by approximations (iterates) of a fractal
- The relation $\frac{A_T}{T^2} = k\left(\frac{A_E}{T^2}\right)^{5/4}$ suggests it may be, with a d_{cortex}=2.5. But this is not a proof
- Indeed, a volume coarse-graining ('lowering resolution') method found d_{cortex}=2.49
- But this method destroys the integrity of cortical surfaces (WM and GM): Iterates are not directly comparable to cortices

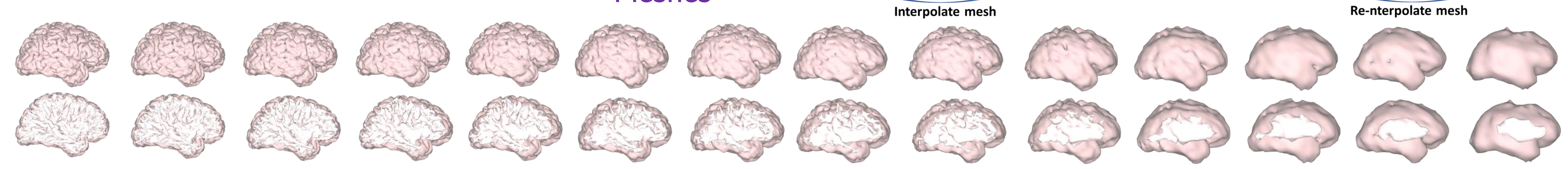


Coarse-graining maintaining surface integrity

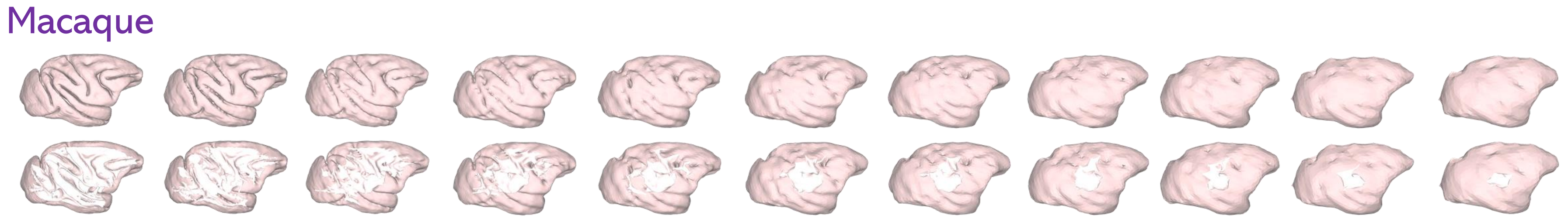
- The GM and WM outer boundaries are nested non-intersecting surfaces
- We need a method of coarse-graining that preserves all these properties
- In this case all iterates would be directly comparable to one another and actual cortices
- We have achieved this with a two-step procedure: Voxelization and surface mesh interpolation
- From MRI images a cortex is separated into voxels, and these assigned to GM or WM
- An algorithm finds the WM and GM surfaces meshes by threshold interpolation
- Increase voxel size, and re-segment
  - For all voxels, if most of the voxel is within GM outer surface and any of it is outside the GM-WM boundary, it is assigned to GM
- GM and WM surface meshes are recalculated



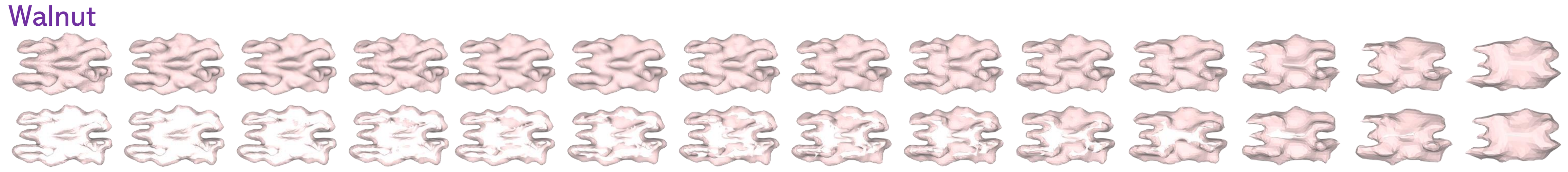
Human



Macaque



Walnut



Voxel size (mm)

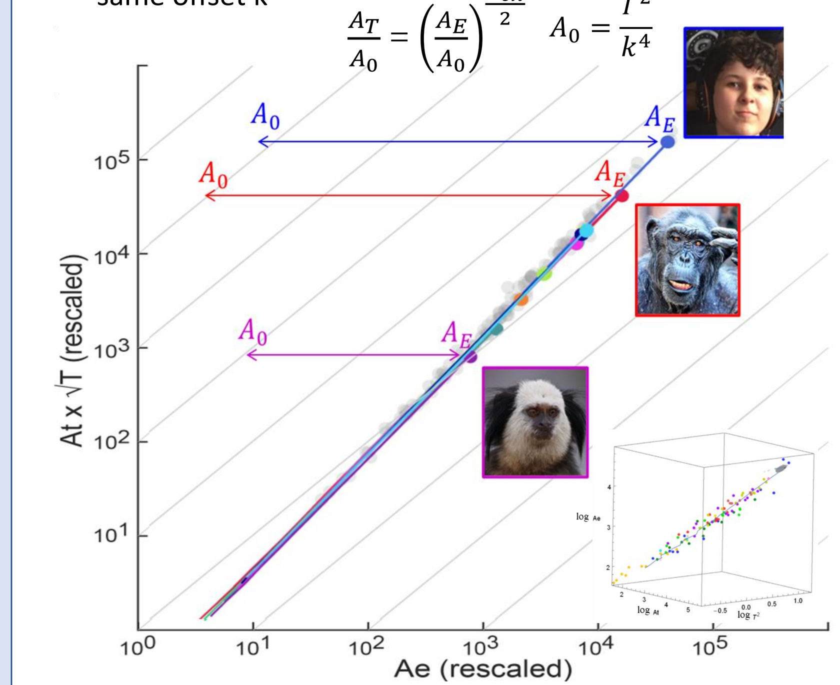
0.50 0.75 1.00 1.25 1.50 1.75 2.00 2.25 2.50 2.75 3.00 4.00 5.00 6.00 7.00 8.00

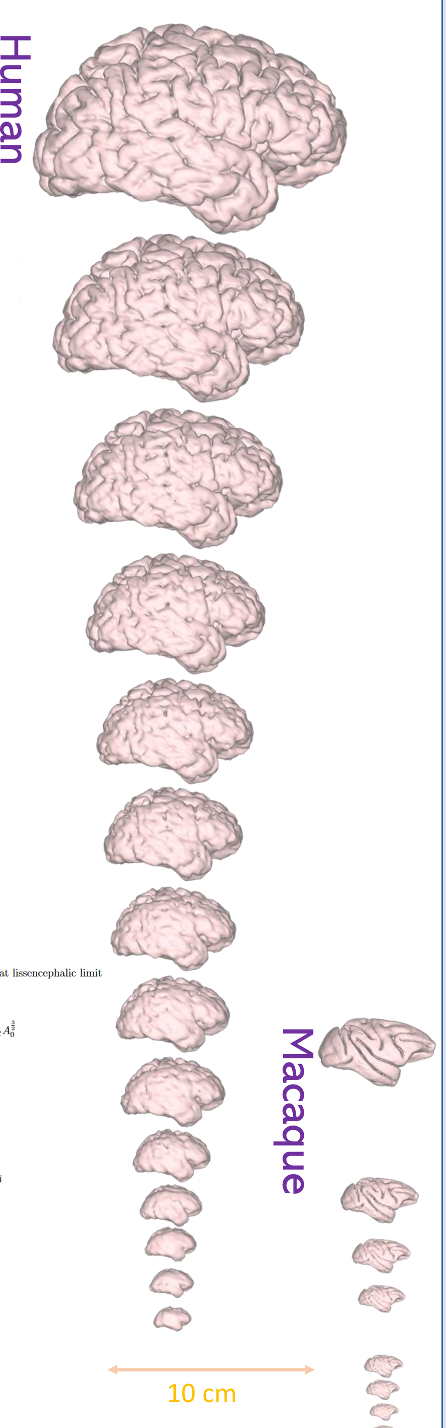
Results

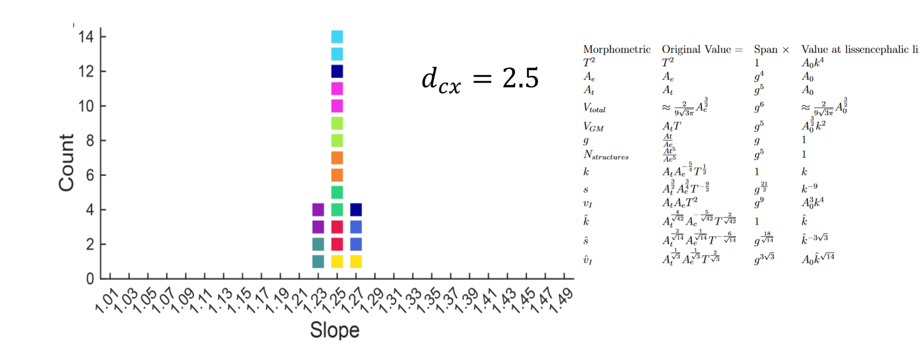
The cortex is fractal!

- Iterates for 11 primate species follow the same morphometric trajectory over coarse-graining
- They are all self-similar with the same slope 1.25 and same offset k

$$\frac{A_T}{A_0} = \left(\frac{A_E}{A_0}\right)^{\frac{d_{cx}}{2}} \quad A_0 = \frac{T^2}{k^4}$$

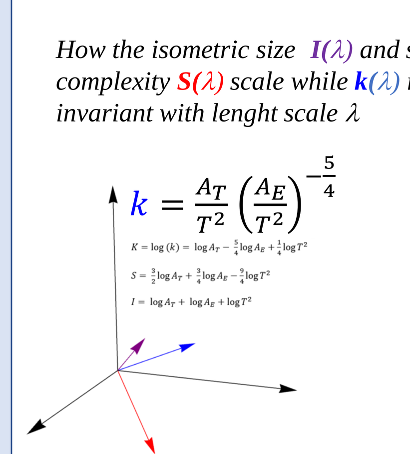




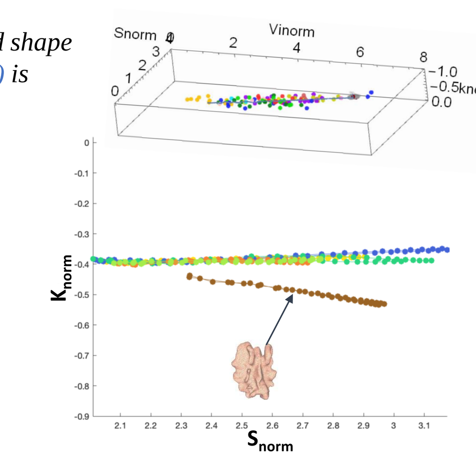


- Less gyrified cortices are similar to coarse-grained versions of more gyrified ones
- The range of length scales for which self-similarity holds accounts for most of morphological diversity

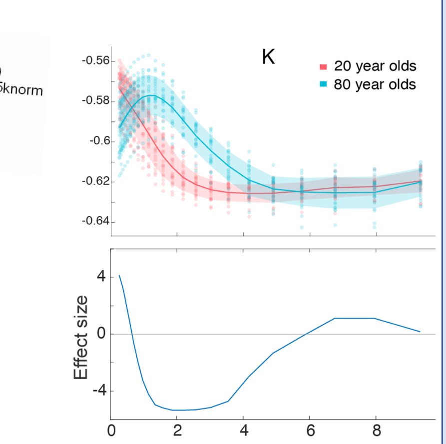
A new syntax for cortical morphology



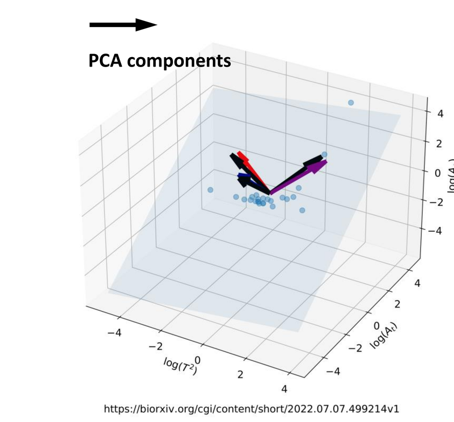
New and old variables:



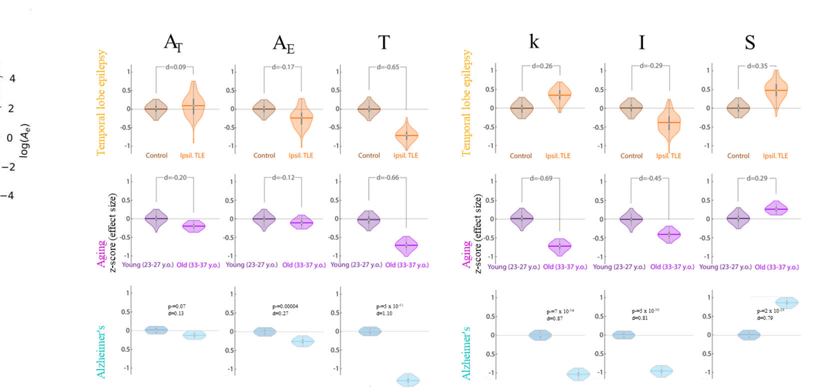
Not in a nutshell



Capturing age-related change



New variables are independent



New vs old variables: telling conditions apart