Diagonalize many-body Hamiltonians

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In this tutorial, we will construct many-body Hamiltonians by using tensor networks and diagonalize them. We introduce two new functions that perform the tasks needed in this tutorial.

- `Tensor/getIdentity.m`: It creates a rank-2 identity tensor that spans the space of a specified leg of an input tensor, or a rank-3 identity tensor that merge two specified legs of input tensors.
- `Tensor/getLocalSpace.m`: It generates local operators for spins, spinless fermions, and spinful fermions.

Check out the documentation of these functions for details.

Exercise (a): Spin-1/2 Heisenberg triangle (pen-and-paper)

In Exercises (a) and (b), we study the system of three spin-1/2's, where every pair of spins interact via the Heisenberg exchange of equal strength. The system's Hamiltonian is specified as

$$\hat{H} = J(\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2 + \hat{\vec{S}}_1 \cdot \hat{\vec{S}}_3 + \hat{\vec{S}}_2 \cdot \hat{\vec{S}}_3).$$

Identify the eigenvalues and their degeneracies of the Hamiltonian by hand.

(*Hint*: By exploting the SU(2) spin symmetry, the Hamiltonian can be much simplified.)

Exercise (b): Spin-1/2 Heisenberg triangle (coding)

In this Exercise, we solve the Hamiltonian introduced in Exercise (a) above, with $J = 1$. Obtain the matrix elements of the Hamiltonian in the many-body state basis, by constructing the identity tensors (without trunctating them) and contracting them with spin operators. Then diagonalize the Hamiltonian to obtain the energy eigenvalues. Check whether your numerical result is consistent with pen-and-paper calculations done for Exercise (a) above.

Exercise (c): Non-interacting tight-binding chain

In this Exercise, we consider non-interacting spinless fermions (or spin-polarized fermions, equivalently) on a tight-binding chain. Its Hamiltonian is given by

$$\hat{H} = \sum_{\ell=1}^{N-1} (-t_\ell \hat{c}_{\ell+1}^\dagger \hat{c}_\ell - t_\ell^* \hat{c}_\ell^\dagger \hat{c}_{\ell+1}),$$

where the chain has L sites, t_ℓ indicates the hopping amplitute between sites ℓ and $\ell+1$, and $\hat{c}_\ell^\dagger$ creates a particle at a site $\ell \in [1, N]$. Consider the case of $N = 11$ and $t_\ell = e^{i\ell}$. Obtain the matrix elements of the Hamiltonian in the many-body state basis (without truncation), and diagonalize it to identify the ground-state and

the lowest-excited-state energies and their degeneracies. Compare your "many-body calculation" result with one from `Util/nonIntTB.m`.