

# Non-interacting fermions on a tight-binding chain

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Here we consider non-interacting spinless fermions (or spin-polarized fermions, equivalently) on a tight-binding chain. Its Hamiltonian is given by

$$\hat{H} = \sum_{\ell=1}^{L-1} (-t_{\ell} \hat{c}_{\ell+1}^{\dagger} \hat{c}_{\ell} - t_{\ell}^* \hat{c}_{\ell}^{\dagger} \hat{c}_{\ell+1}),$$

where the chain has  $L$  sites,  $t_{\ell}$  indicates the hopping amplitude between sites  $\ell$  and  $\ell + 1$ , and  $\hat{c}_{\ell}^{\dagger}$  creates a particle at a site  $\ell \in [1, L]$ .

## Exercise (a): Compute the energy and degeneracy of ground states

Write a script or function that compute the ground-state energy and degeneracy of this non-interacting tight-binding chain. The script or function takes the following input and output:

### < Input >

- $\mathbf{t}$  : [numeric vector] Each element  $\mathbf{t}(\ell)$  indicates a hopping amplitude  $t_{\ell}$ . The length of the vector  $\text{length}(\mathbf{t})$  equals to the number of chain sites minus 1.

### < Output >

- $E_G$  : [numeric scalar] Ground-state energy.
- $d_G$  : [numeric scalar] Ground-state degeneracy.

Once you complete a script or function, compute the ground-state energies and degeneracies for the following three cases:

(1)  $L = 10$ ,  $t_{\ell} = 1$  for all  $\ell$ 's.

(2)  $L = 11$ ,  $t_{\ell} = 1$  for all  $\ell$ 's.

(3)  $L = 11$ ,  $t_{\ell} = e^{i\ell}$ .