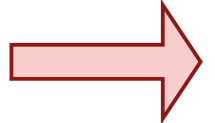
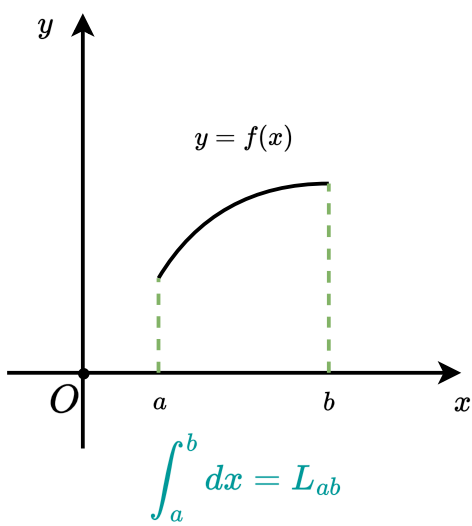


定积分

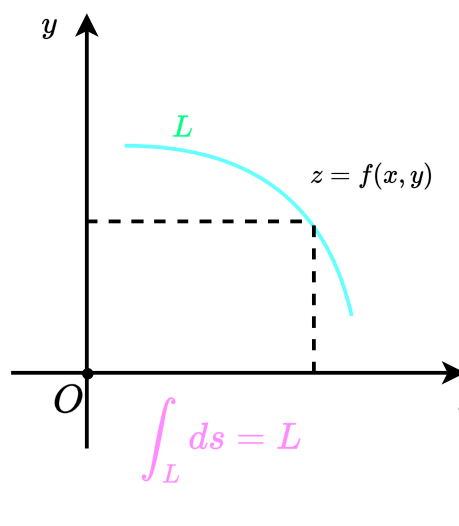
$$\int_a^b f(x)dx = S = \lim_{\lambda \rightarrow 0} \sum_{i=1}^n f(x_i)\Delta x_i$$



第一型曲线积分

$$\int_L f(x,y)ds = \lim_{\lambda \rightarrow 0} \sum_{i=1}^n f(x_i,y_i)\Delta s_i$$

二维形式

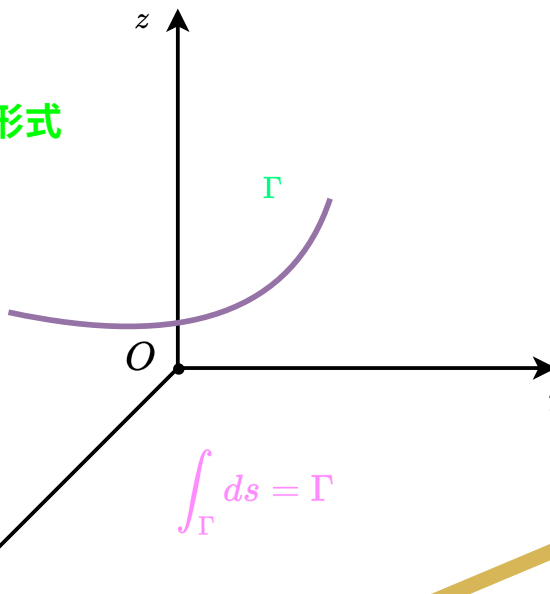


$$ds = \sqrt{1 + (\frac{dy}{dx})^2}dx$$

1. 直角坐标: $\int_L f(x,y)ds = \int_a^b f(x,y(x))\sqrt{1 + [y'(x)]^2}dx$
2. 参数方程: $\int_L f(x,y)ds = \int_\alpha^\beta f(x(t),y(t))\sqrt{[x'(t)]^2 + [y'(t)]^2}dt$
3. 极坐标: $\int_L f(x,y)ds = \int_\alpha^\beta f(r(\theta)\cos\theta, r(\theta)\sin\theta)\sqrt{[r(\theta)]^2 + [r'(\theta)]^2}d\theta$

$$\int_\Gamma f(x,y,z)ds = \lim_{\lambda \rightarrow 0} \sum_{i=1}^n f(x_i,y_i,z_i)\Delta s_i$$

三维形式



一阶偏导数连续

封闭曲线

坐标积分

第二型曲线积分

曲线积分与路径无关

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

格林公式(二维形式斯托克斯)

$$\oint_L Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) d\sigma$$

第二型曲面积分

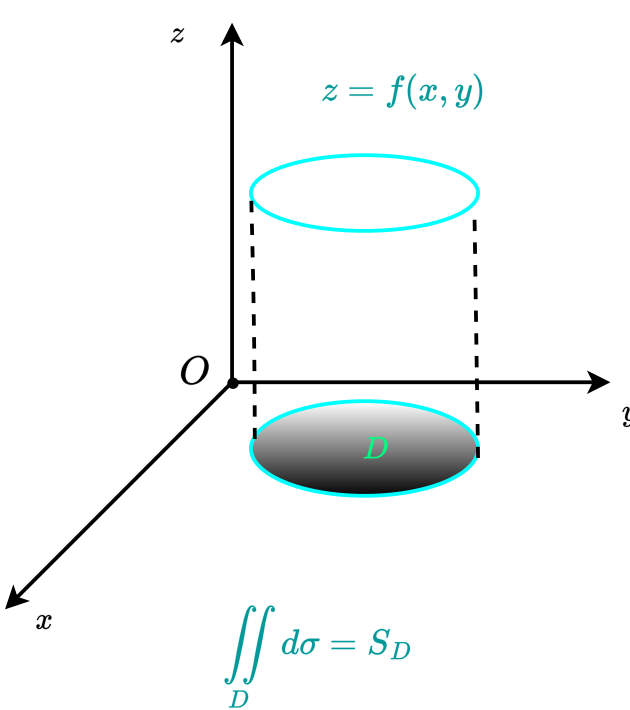
$$\int_\Sigma \mathbf{F}d\mathbf{S} = \iint_\Sigma P(x,y,z)dydz + Q(x,y,z)dx dz + R(x,y,z)dx dy$$

高斯公式

$$\iint_\Sigma Pdydz + Qdx dz + Rdx dy = \iiint_\Omega \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dv$$

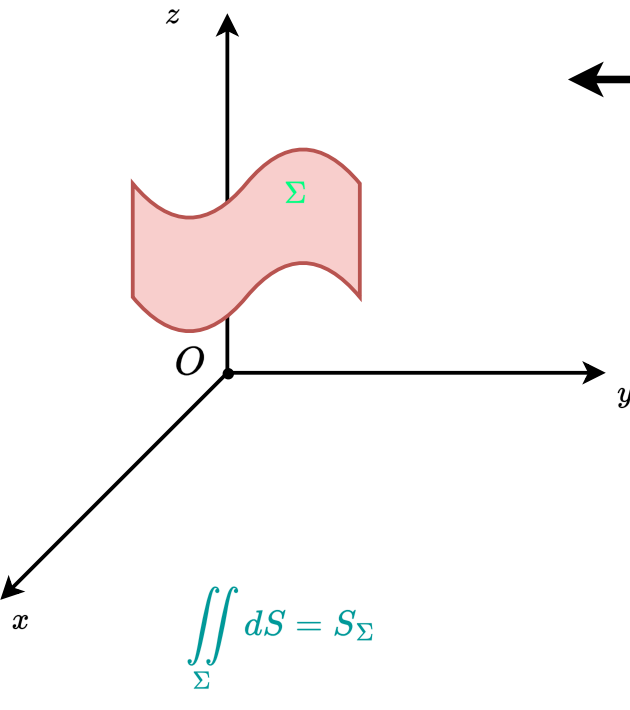
二重积分

$$\iint_D f(x,y)d\sigma = \lim_{\lambda \rightarrow 0} \sum_{i=1}^n f(x_i,y_i)\Delta D_i$$



第一型曲面积分

$$\iint_\Sigma f(x,y,z)dS = \lim_{\lambda \rightarrow 0} \sum_{i=1}^n f(x_i,y_i,z_i)\Delta \Sigma_i$$

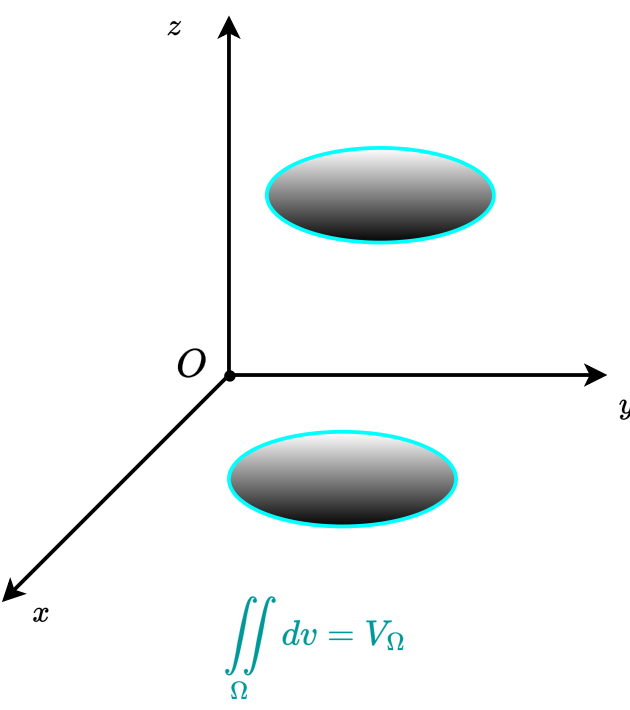


$$\iint_\Sigma f(x,y,z)dS = \iint_{D_{xy}} f(x,y,z(x,y))\sqrt{1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2}dxdy$$

$$\iint_\Sigma f(x,y,z)dS = \iint_{D_{yz}} f(x(y,z),y,z)\sqrt{1 + (\frac{\partial x}{\partial y})^2 + (\frac{\partial x}{\partial z})^2}dydz$$

$$\iint_\Sigma f(x,y,z)dS = \iint_{D_{xz}} f(x,y(x,z),z)\sqrt{1 + (\frac{\partial y}{\partial x})^2 + (\frac{\partial y}{\partial z})^2}dxdz$$

三重积分



$$\iiint_\Omega f(x,y,z)dv = \iiint_{D'} r f(r\cos\theta, r\sin\theta, z)drd\theta dz$$

$$\iiint_\Omega f(x,y,z)dv = \iiint_{D'} r^2 \sin\varphi f(r\sin\varphi\cos\theta, r\sin\varphi\sin\theta, r\cos\varphi)drd\theta d\varphi$$

二维形式

$$\int_L \mathbf{F}ds = \int_L P(x,y)dx + Q(x,y)dy$$

三维形式

$$\int_\Gamma \mathbf{F}ds = \int_\Gamma P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz$$

斯托克斯公式

$$\oint_\Gamma Pdx + Qdy + Rdz = \iint_\Sigma \begin{vmatrix} dydz & dzdx & dxdy \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

第二型曲面积分

$$\oint_\Gamma Pdx + Qdy + Rdz = \iint_\Sigma \begin{vmatrix} \cos\alpha & \cos\beta & \cos\gamma \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} dS$$

第一型曲面积分

$$\mathbf{n} = (\cos\alpha, \cos\beta, \cos\gamma)$$

