Advanced Topics in Operations Research Notes

Andres Espinosa

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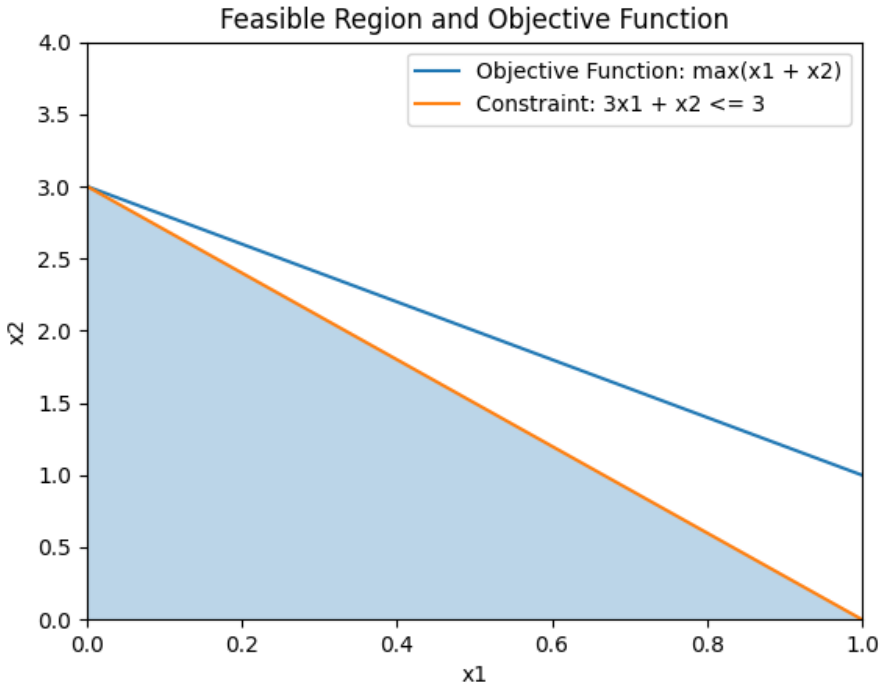
1 Wednesday 08/28/2024

$$max\ x_1 + x_2 \tag{1}$$
$$s.t.\ 3x_1 + x_2 \leq 3 \tag{2}$$
$$x_1 \geq 0, x_2 \geq 0 \tag{3}$$



x^* is an extreme point if we cannot find $x_1, x_2 \in S$, such that $x_1 \neq x_2$ and $x^* = \lambda x_1 + (1-\lambda)x_2$ for some $\lambda \in (0, 1)$
For an n-dimensional problem, there are n-many constraints that are active/binding and their coefficients are linearly independent.
Optimal solutions can be found that are not basic feasible solutions when constraints and objective functions lie on the same plane.

2 Friday 08/30/2024

2.1 Complexity Introduction

Three elements for complexity analysis:

- Model or problem formulation - The known part of a problem. Includes the formulation and problem descriptions.
- Oracle - The smallest computing unit, whose details inside can be ignored.
- Target outcome - When to stop

Two types of complexity:

- Analytical complexity - The number of calls of the oracle which is necessary to solve a given problem formulation up to accuracy ϵ
- Numerical complexity - The number of arithmetic operations which is necessary to solve a given problem formulation up to accracy ϵ

2.2 P, NP, NP-hard

A **problem** is a function $F : I \rightarrow B$, where I is the set of instances encoded as strings of characters and B is the set of problem outputs.

For example, linear programming in $\mathbb{R}^{mxn}$ is a problem where the instance parameters are (m, n, A, c, b) and the output B is the optimal solution found to the objective function.
Another example, making a decision takes in an instance I and outputs a decision of yes or no.

$$I = \textit{Problem-specific information parameters}, B \in (yes, no) \tag{4}$$