My Solutions for Exercises of Deep Learning Fundamentals by Bishop

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Contents

1 The Deep Learning Revolution

1.1 No Exercises

2 Probabilities

Exercise 2.1

Bayes rule

$$P[C=1|T=1] = \frac{P[T=1|C=1]*P[C=1]}{P[T=1|C=1]*P[C=1]+P[T=1|C=0]*P[C=0]} \tag{1}$$

$$P[C=1|T=1] = \frac{0.90*0.001}{0.90*0.001+0.03*0.999} = 0.0292 \tag{2}$$

Given that the test result was positive, there is a 2.92% chance that you have cancer.

2.1 Exercise 2.2

Not attempted

Exercise 2.3

$$p(\mathbf{y}) = \int p_{\mathbf{u},\mathbf{v}}(\mathbf{u}, \mathbf{y}-\mathbf{u})d\mathbf{u} \tag{3}$$

$$= \int p_{\mathbf{u}}(\mathbf{u})p_{\mathbf{v}}(\mathbf{y}-\mathbf{u})d\mathbf{u} \tag{4}$$

2.2 Exercise 2.4

Not attempted

Exercise 2.5

Exponential:

$$p(x|\lambda) = \lambda e^{-\lambda x} \tag{5}$$

Laplace:

$$p(x|\mu,\gamma) = \frac{1}{2\gamma}e^{-\frac{|x-\mu|}{\gamma}} \tag{6}$$

Verifying that the exponential distribution is normalized:

$$p(x|\lambda) = \lambda e^{-\lambda x} \tag{7}$$

$$\int_0^\infty \lambda e^{-\lambda x} = -e^{-\lambda x}\Big|_0^\infty = \frac{1}{e^\infty} + \frac{1}{e^0} \tag{8}$$

$$= 1 \tag{9}$$

Verifying the laplace distribution:

$$p(x|\mu,\gamma) = \frac{1}{2\gamma}e^{-\frac{|x-\mu|}{\gamma}} \tag{10}$$

$$\begin{cases} \frac{1}{2\gamma}e^{-\frac{x-\mu}{\gamma}} & \text{if } x \geq \mu \\ \frac{1}{2\gamma}e^{-\frac{-x+\mu}{\gamma}} & \text{if } x < \mu \end{cases} \tag{11}$$

$$\int_{\mu}^{\infty} \frac{1}{2\gamma} e^{-\frac{x-\mu}{\gamma}} = \frac{1}{2} e^{-\frac{x-\mu}{\gamma}} \Big|_{\mu}^{\infty} = -\frac{1}{2}(e^{-\infty} - e^{0}) \tag{12}$$

$$= \frac{1}{2} \tag{13}$$

$$\int_{-\infty}^{\mu} \frac{1}{2\gamma} e^{-\frac{-x+\mu}{\gamma}} = \frac{1}{2} e^{-\frac{-x+\mu}{\gamma}} \Big|_{-\infty}^{\mu} = \frac{1}{2}(e^{0} - e^{-\infty}) \tag{14}$$

$$= \frac{1}{2} \tag{15}$$

$$\frac{1}{2} + \frac{1}{2} = 1 \tag{16}$$

2.3 Exercise 2.6

Not attempted

2.4 Exercise 2.7

$$P(x|D) = \frac{1}{N} \sum_{n=1}^{N} \delta(x - x_n) \tag{17}$$

$$E[f] = \int p(x) f(x) dx \tag{18}$$

$$\text{Substituting:} \tag{19}$$

$$E[f] = \int \frac{1}{N} \sum_{n=1}^{N} \delta(x - x_n) f(x) dx \tag{20}$$

$$E[f] = \frac{1}{N} \sum_{n=1}^{N} \int_{x_n-\varepsilon}^{x_n+\varepsilon} \delta(x - x_n) f(x) dx \tag{21}$$

$$E[f] = \frac{1}{N} \sum_{n=1}^{N} f(x_n) \int_{x_n-\varepsilon}^{x_n+\varepsilon} \delta(x - x_n) dx \tag{22}$$

$$E[f] = \frac{1}{N} \sum_{n=1}^{N} f(x_n) \tag{23}$$

2.5 Exercise 2.8

Not attempted

2.6 Exercise 2.9

$$cov[X, y] = E_{x,y}[xy] - E[x]E[y] \tag{24}$$

If x and y are independent, the joint distribution is equal to the product of the marginals. $p(x, y) = p(x)p(y)$. If $E_{x,y}[xy] = E[x]E[y]$, then the covariance will be zero.

2.7 Exercise 2.10

Not attempted

2.8 Exercise 2.11

Proving $E[x] = E_y[E_x[x|y]]$:

$$E_x[x|y] = \int p(x|y)xdx \tag{25}$$

$$\text{Substituting:} \tag{26}$$

$$E[x] = E_y[\int p(x|y)xdx] \tag{27}$$

$$E[x] = \int E_y[p(x|y)]xdx \tag{28}$$

$$E[x] = \int\int p(x|y)xp(y)dxdy \tag{29}$$

$$E[x] = \int\int \frac{p(x,y)}{p(y)}xp(y)dxdy \tag{30}$$

$$E[x] = \int\int p(x,y)xdxdy \tag{31}$$

$$E[x] = E[x] \tag{32}$$

2.9 Exercise 2.12

Not attempted

2.10 Exercise 2.13