My Solutions for Exercises of Deep Learning Fundamentals by Bishop

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Contents

1 The Deep Learning Revolution

1.1 No Exercises

2 Probabilities

Exercise 2.1

Bayes rule

$$P[C=1|T=1] = \frac{P[T=1|C=1]*P[C=1]}{P[T=1|C=1]*P[C=1]+P[T=1|C=0]*P[C=0]} \tag{1}$$

$$P[C=1|T=1] = \frac{0.90*0.001}{0.90*0.001+0.03*0.999} = 0.0292 \tag{2}$$

Given that the test result was positive, there is a 2.92% chance that you have cancer.

2.1 Exercise 2.2

Not attempted

Exercise 2.3

$$p(\mathbf{y}) = \int p_{\mathbf{u},\mathbf{v}}(\mathbf{u},\mathbf{y}-\mathbf{u})d\mathbf{u} \tag{3}$$

$$= \int p_{\mathbf{u}}(\mathbf{u})p_{\mathbf{v}}(\mathbf{y}-\mathbf{u})d\mathbf{u} \tag{4}$$

2.2 Exercise 2.4

Not attempted

Exercise 2.5

Exponential:

$$p(x|\lambda) = \lambda e^{-\lambda x} \tag{5}$$

Laplace:

$$p(x|\mu,\gamma) = \frac{1}{2\gamma}e^{-\frac{|x-\mu|}{\gamma}} \tag{6}$$

Verifying that the exponential distribution is normalized:

$$p(x|\lambda) = \lambda e^{-\lambda x} \tag{7}$$

$$\int_0^{\infty} \lambda e^{-\lambda x} = -e^{-\lambda x}\Big|_0^{\infty} = \frac{1}{e^{\infty}} + \frac{1}{e^0} \tag{8}$$

$$= 1 \tag{9}$$

Verifying the laplace distribution:

$$p(x|\mu,\gamma) = \frac{1}{2\gamma}e^{-\frac{|x-\mu|}{\gamma}} \tag{10}$$

$$\begin{cases} \frac{1}{2\gamma}e^{-\frac{x-\mu}{\gamma}} & \text{if } x \geq \mu \\ \frac{1}{2\gamma}e^{-\frac{-x+\mu}{\gamma}} & \text{if } x < \mu \end{cases} \tag{11}$$

$$\int_{\mu}^{\infty} \frac{1}{2\gamma} e^{-\frac{x-\mu}{\gamma}} = \frac{1}{2} e^{-\frac{x-\mu}{\gamma}} \Big|_{\mu}^{\infty} = -\frac{1}{2}(e^{-\infty} - e^{0}) \tag{12}$$

$$= \frac{1}{2} \tag{13}$$

$$\int_{-\infty}^{\mu} \frac{1}{2\gamma} e^{-\frac{-x+\mu}{\gamma}} = \frac{1}{2} e^{-\frac{-x+\mu}{\gamma}} \Big|_{-\infty}^{\mu} = \frac{1}{2}(e^{0} - e^{-\infty}) \tag{14}$$

$$= \frac{1}{2} \tag{15}$$

$$\frac{1}{2} + \frac{1}{2} = 1 \tag{16}$$

2.3 Exercise 2.6

Not attempted

2.4 Exercise 2.7

$$P(x|D) = \frac{1}{N} \sum_{n=1}^{N} \delta(x - x_n) \tag{17}$$

$$E[f] = \int p(x) f(x) dx \tag{18}$$

$$\text{Substituting:} \tag{19}$$

$$E[f] = \int \frac{1}{N} \sum_{n=1}^{N} \delta(x - x_n) f(x) dx \tag{20}$$

$$E[f] = \frac{1}{N} \sum_{n=1}^{N} \int_{x_n - \varepsilon}^{x_n + \varepsilon} \delta(x - x_n) f(x) dx \tag{21}$$

$$E[f] = \frac{1}{N} \sum_{n=1}^{N} f(x_n) \int_{x_n - \varepsilon}^{x_n + \varepsilon} \delta(x - x_n) dx \tag{22}$$

$$E[f] = \frac{1}{N} \sum_{n=1}^{N} f(x_n) \tag{23}$$

2.5 Exercise 2.8

Not attempted

2.6 Exercise 2.9

$$cov[X, y] = E_{x,y}[xy] - E[x]E[y] \tag{24}$$

If x and y are independent, the joint distribution is equal to the product of the marginals. $p(x, y) = p(x)p(y)$. If $E_{x,y}[xy] = E[x]E[y]$, then the covariance will be zero.

2.7 Exercise 2.10

Not attempted

2.8 Exercise 2.11

Proving $E[x] = E_y[E_x[x|y]]$:

$$E_x[x|y] = \int p(x|y)xdx \tag{25}$$

$$\text{Substituting:} \tag{26}$$

$$E[x] = E_y[\int p(x|y)xdx] \tag{27}$$

$$E[x] = \int E_y[p(x|y)]xdx \tag{28}$$

$$E[x] = \int \int p(x|y)xp(y)dxdy \tag{29}$$

$$E[x] = \int \int \frac{p(x,y)}{p(y)}xp(y)dxdy \tag{30}$$

$$E[x] = \int \int p(x,y)xdxdy \tag{31}$$

$$E[x] = E[x] \tag{32}$$

2.9 Exercise 2.12

Not attempted

2.10 Exercise 2.13

$$E[x] = \int_{-\infty}^{\infty} x\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{1}{2\sigma^2}(x-\mu)^2}dx \tag{33}$$

$$\text{Change of variables } z = \frac{x-\mu}{\sigma}, \sigma dz = dx \tag{34}$$

$$E[x] = \int_{-\infty}^{\infty} \sigma\frac{\sigma z+\mu}{\sqrt{2\pi\sigma^2}}e^{\frac{1}{2}z^2}dz \tag{35}$$

$$E[x] = \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} ze^{\frac{1}{2}z^2} + \frac{\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\frac{1}{2}z^2} \tag{36}$$

$$E[x] = \frac{\sigma}{\sqrt{2\pi}} * 0 + \frac{\mu}{\sqrt{2\pi}} * \sqrt{2\pi} \tag{37}$$

$$E[x] = \mu \tag{38}$$

2.11 Exercise 2.14

Not attempted

2.12 Exercise 2.15

Solving for μ_{ml}:

$$\log p(x|\mu, \sigma^2) = \frac{-1}{2\sigma^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \log \sigma^2 - \frac{N}{2} \log 2\pi \tag{39}$$

$$\frac{d}{d\mu} logp(x|\mu, \sigma^2) = \frac{1}{2\sigma^2} \sum_{n=1}^{N} 2(x_n - \mu) \tag{40}$$

$$0 = \frac{1}{\sigma^2} \sum_{n=1}^{N} (x_n - \mu) \tag{41}$$

$$0 = \sum_{n=1}^{N} x_n - \sum_{n=1}^{N} \mu \tag{42}$$

$$N\mu = \sum_{n=1}^{N} x_n \tag{43}$$

$$\mu_{ml} = \frac{1}{n} \sum_{n=1}^{N} x_n \tag{44}$$

Solving for σ_{ml}:

$$\log p(x|\mu, \sigma^2) = \frac{-1}{2\sigma^2} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2} \log \sigma^2 - \frac{N}{2} \log 2\pi \tag{45}$$

$$\frac{d}{d\sigma^2} logp(x|\mu, \sigma^2) = \frac{1}{2\sigma^4} \sum_{n=1}^{N} (x_n - \mu)^2 - \frac{N}{2\sigma^2} \tag{46}$$

$$\frac{N}{2\sigma^2} = \frac{1}{2\sigma^4} \sum_{n=1}^{N} (x_n - \mu)^2 \tag{47}$$

$$\sigma_{ml}^2 = \frac{1}{N} \sum_{n=1}^{N} (x_n - \mu_{ml})^2 \tag{48}$$

2.13 Exercise 2.16

not attempted

2.14 Exercise 2.17

Finding expectation of $\hat{\sigma}^2$

$$E[\hat{\sigma}^2] = E[\frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2] \tag{49}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E[x_n^2 - 2x_n\mu + \mu^2] \tag{50}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E[x_n^2] - E[2x_n\mu] + E[\mu^2] \tag{51}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E[x_n^2] - 2E[x_n]E[x_n] + E[\mu^2] \tag{52}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E[x_n^2] - E[x_n]^2 \tag{53}$$

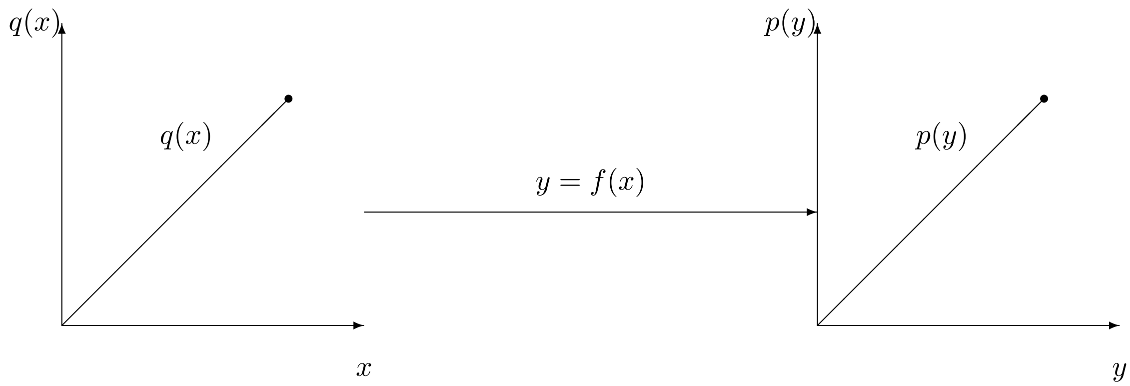
$$= \frac{1}{N}\sum_{n=1}^{N} \mu^2 + \sigma^2 - \mu^2 \tag{54}$$

$$= \sigma^2 \tag{55}$$

2.15 Exercise 2.18

Not attempted

2.16 Exercise 2.19



2.17 Exercise 2.20

Not attempted

2.18 Exercise 2.21

Showing $h(p^2) = 2h(p)$:

$$h(p) = h(p(x_1)) + h(p(x_2)) + \cdots + h(p(x_n)) \tag{56}$$
$$h(p^2) = h(x_1^2) + h(x_2^2) + \cdots + h(x_n^2) \tag{57}$$
$$\because h(x) = -\log_2 p(x), \tag{58}$$
$$h(p^2) = 2h(x_1) + 2h(x_2) + \cdots + 2h(x_n) \tag{59}$$
$$h(p^2) = 2h(p) \tag{60}$$

This can be applied to any exponent which inclues any choice of n integer or $\frac{n}{m}$ positive rational number.

$$h(p^x) = xh(p) \; \forall \; Q^+ \tag{61}$$
$$\therefore \tag{62}$$
$$h(p) \propto lnp \tag{63}$$

2.19 Exercise 2.22

Not attempted

2.20 Exercise 2.23

Not attempted

2.21 Exercise 2.24

Not attempted

2.22 Exercise 2.25

Not attempted

2.23 Exercise 2.26

Not attempted

2.24 Exercise 2.27

Not attempted

2.25 Exercise 2.28

Not attempted

2.26 Exercise 2.29

Not attempted

2.27 Exercise 2.30

Not attempted

2.28 Exercise 2.31

Not attempted

2.29 Exercise 2.32

Not attempted

2.30 Exercise 2.33

Not attempted

2.31 Exercise 2.34

Not attempted

2.32 Exercise 2.35

Not attempted

2.33 Exercise 2.36

Not attempted

2.34 Exercise 2.37

Not attempted

2.35 Exercise 2.38

Not attempted

2.36 Exercise 2.39

Not attempted

2.37 Exercise 2.40

Not attempted

2.38 Exercise 2.41

Not attempted