Homework 1

1. Consider the following matrix and perform the following calculations showing all your steps (no credit will be given by presenting just the responses).

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

   a. Determinant of A
   b. Eigenvalues and eigenvectors of A
2. Check if set of all polynomials with real coefficients form a vector space
3. Consider the following function and perform the following calculations

$$f(x_1, x_2) = x_1^3 x_2 - x_1 x_2^3$$

   a. Gradient of the function
   b. Hessian of the function
   c. Write the second order Taylor expansion around a point (x_1^*, x_2^*)
4. Check if the following function is convex

$$f(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2$$

5. Check if the following function is convex

$$g(x_1, x_2) = 5x_1^2 - 4x_1 x_2$$

6. Consider a set of linear equalities $Ax = b$ as well as a set of convex nonlinear inequalities $g(x) \leq 0$. Consider the feasible region constrained by these linear and nonlinear inequalities. Assuming that this region is non-empty, show that this feasible region is convex.
7. Consider the following optimization problem

$$\begin{gathered} \min(x_1) \\ x_1 + x_2 \leq 10 \\ x_1 - 2x_2 \geq 1 \\ x_1 \geq 0, x_2 \geq 0 \\ x_1, x_2 \in R \end{gathered}$$

   a. What type of problem is this (MILP, MINLP…) justify
   b. Draw the feasible region of the problem
   c. Is this region convex or non-convex, justify
8. Consider the following optimization problem

$$\begin{gathered} \min(x_1) \\ x_1 + x_2 \leq 10 \\ x_1 - 2x_2 \geq 1 \\ x_1 \geq 0, x_2 \geq 0 \\ x_1, x_2 \in \{0,1\} \end{gathered}$$

   d. What type of problem is this (MILP, MINLP…) justify
   e. Draw the feasible region of the problem
   f. Is this region convex or non-convex, justify