ECH4905 ChemE Optimization HW 1

Andres Espinosa

February 9, 2025

1   Problem 1

Write the following problem in standard form

$$\text{maximize} \quad 3x_1 + 2x_2 \tag{1}$$
$$\text{subject to} \quad -2x_1 + x_2 \geq 1 \tag{2}$$
$$x_1 + 3x_2 = 2 \tag{3}$$

Solution: To solve this problem we want to turn the above problem into the canonical form

$$\text{minimize} \quad c^\top x \tag{4}$$
$$\text{subject to} \quad Ax \leq b \tag{5}$$
$$x \geq 0 \tag{6}$$

Flipping the maximize to minimize

$$\text{minimize} \quad -3x_1 - 2x_2 \tag{7}$$
$$\text{subject to} \quad -2x_1 + x_2 \geq 1 \tag{8}$$
$$x_1 + 3x_2 = 2 \tag{9}$$

Expanding the equality

$$\text{minimize} \quad -3x_1 - 2x_2 \tag{10}$$
$$\text{subject to} \quad -2x_1 + x_2 \geq 1 \tag{11}$$
$$x_1 + 3x_2 \geq 2 \tag{12}$$
$$x_1 + 3x_2 \leq 2 \tag{13}$$

Flipping the $\geq$ inequalities

$$\text{minimize} \quad -3x_1 - 2x_2 \tag{14}$$
$$\text{subject to} \quad 2x_1 - x_2 \leq -1 \tag{15}$$
$$-x_1 + -3x_2 \leq -2 \tag{16}$$
$$x_1 + 3x_2 \leq 2 \tag{17}$$

Bounding the variables to be ≥ 0

$$\text{minimize} \quad -3(x_1^+ - x_1^-) - 2(x_2^+ - x_2^-) \tag{18}$$
$$\text{subject to} \quad 2(x_1^+ - x_1^-) - (x_2^+ - x_2^-) \leq -1 \tag{19}$$
$$-(x_1^+ - x_1^-) + -3(x_2^+ - x_2^-) \leq -2 \tag{20}$$
$$(x_1^+ - x_1^-) + 3(x_2^+ - x_2^-) \leq 2 \tag{21}$$
$$x_1^+, x_1^-, x_2^+, x_2^- \geq 0 \tag{22}$$

To solve this problem in standard form, we turn it into

$$\text{minimize} \quad c^\top x \tag{23}$$
$$\text{subject to} \quad Ax = b \tag{24}$$
$$x \geq 0 \tag{25}$$

We can alter the canonical form easily by adding slack variables

$$\text{minimize} \quad -3(x_1^+ - x_1^-) - 2(x_2^+ - x_2^-) \tag{26}$$
$$\text{subject to} \quad 2(x_1^+ - x_1^-) - (x_2^+ - x_2^-) + s_1 = -1 \tag{27}$$
$$(x_1^+ - x_1^-) + 3(x_2^+ - x_2^-) = 2 \tag{28}$$
$$x_1^+, x_1^-, x_2^+, x_2^-, s_1 \geq 0 \tag{29}$$

2 Problem 2

An engineering factory makes seven products on the following machines: four grinders, two vertical drills, three horizontal drills, one borer and one planer. Each product yields a certain contribution to profit (defined in dollars per unit). These quantities together with the unit production times required on each process are given below.

	P1	P2	P3	P4	P5	P6	P7
Contribution to profit	10	6	8	4	11	9	3
Grinding (hours)	0.5	0.7	-	-	0.3	0.2	0.5
Vertical drilling (hours)	0.1	0.2	-	0.3	-	0.6	-
Horizontal drilling (hours)	0.2	-	0.8	-	-	-	0.6
Boring (hours)	0.05	0.03	-	0.07	0.1	-	0.08
Planning (hours)	-	-	0.01	-	0.05	-	0.05
Demand (units)	500	1000	300	300	800	200	100

Table 1: Production data for the engineering factory

There are some marketing limitations to the production (demand), these are given as the bottom row in table 1. We can assume that the factory works 24 days, and that each day each machine works for 16 hours. Formulate an LP to find the optimal product distribution.
Solution:

3 Problem 3

Solve problem 2 but consider that the demand of each product is a function of the month, i.e., that the demand for product i in month m is given by parameter $d_{i,m}$. In this case, assume that you

can store a maximum amount of 100 units of product each month at a cost of 0.5/unit-month. At the beginning of the planning period there is no inventory, but at the end we want to have 50 units of each product in storage.

When and what should the factory produce to maximize profit?

Solution:

4 Problem 4

Formulate a LP model for the following reaction network (image 1), assuming that you want to maximize the production rate of 3-methyltetrahydrofuran, and that you have an incoming flux of itaconic acid equal to 1. For the sake of simplicity you can ignore the water and hydrogen that would be required to balance the equations.
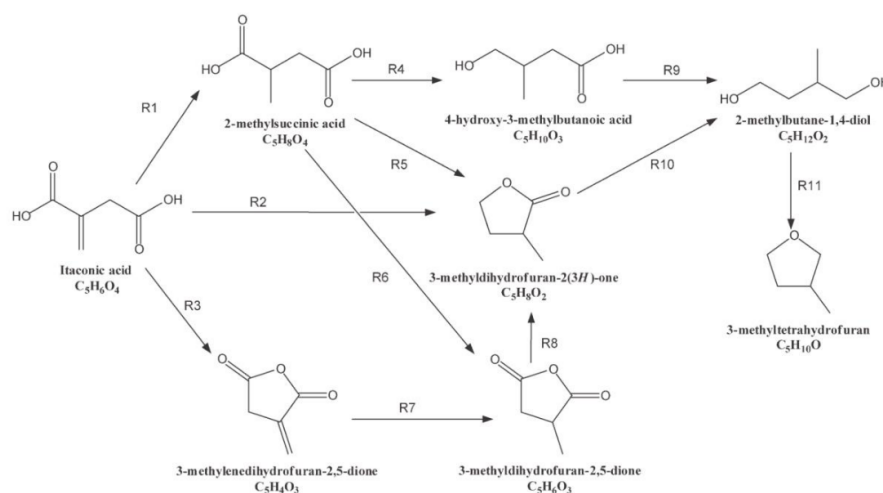

Figure 1: Problem 4 Reaction Network

Solution: