Homework: Constrained Dynamics

Refer to data structures in Siggraph course notes “particle dynamics” as template, so that you build your own modularized particle system for your future extension. Use Lagrange Multiplier method in notes “constrained dynamics”.

Problem A: A Bead on a Wire

Simulate the motion of a particle and visualize the result. The particle is defined in a Cartesian coordinate system p=(x,y), and it is constrained by a unit circle centered at the origin. Use the initial position (0, 1), the initial velocity (0, 0), and gravitational force (0,-mg). Constants are given as m=1Kg, and g=9.8m/sec^2.

Problem B: “Double Pendulum” Problem

A double pendulum is a system consisting of two masses, which are particles. The first mass is constrained to lie on the parabola $y = ax^2$, where α is a constant of your choice (not zero!). The second mass must be at a constant distance r from the first mass.

Simulate the double pendulum system with a constant gravity force acting on both particles. You should be able to turn a mouse spring on and off, and pull on either of the two particles. You will want to add some damping to the system. The damping force in R^2 on each particle should be of the form $F_{damp} = -kv$ where v is the particle’s velocity. You should add “feedback” terms onto the right-hand-side of the equation as described in the handout so that small errors in the constraint or the constraints’ first derivative are cancelled. Draw a vector from each particle to indicate the constraint force acting on the particle. (Choose an appropriate scaling factor so that the constraint force vectors are neither too big nor too small to view during the simulation.) Draw the parabola (whatever part fits on the screen), and draw a line between the two particles.

This assignment will be graded based upon:

- Data structures; Correctness of the implementations (60%)
- UI, other SE factors & additional features (20%)
- Other features (derive and analysis, etc.) Write an experiment report (20%)
  - write out the constraint equations; derive the matrices $\mathbf{J} = \partial \mathbf{C} / \partial \mathbf{p}$ and $\dot{\mathbf{J}}$.

Helpful Hint

You'll probably make a mistake somewhere in your derivation, or in coding the double pendulum up. To easily test if your system is working correctly, start the double pendulum so that the first mass has position (0, 0) and the second has position (r, 0), i.e. the pendulum is stretched out horizontally. Turn gravity and damping off. Choose an arbitrary initial velocity for the system that is consistent with the constraints. If you've gotten everything right, then the kinetic energy of the system $\frac{1}{2}(m_1 v_1 \cdot v_1 + m_2 v_2 \cdot v_2)$ should remain constant as the system moves around. If the kinetic energy varies (or your objects fly apart!) you've made a mistake.