2024 23rd IEEE Intersociety Conference on Thermal and Thermomechanical Phenomena in Electronic Systems (ITherm) | 979-8-3503-6433-0/24/$31.00 ©2024 IEEE | DOI: 10.1109/ITherm55375.2024.10709371

Analytical prediction of thermal performance and flow non-uniformity of manifold microchannel heat sinks

Young Jin Lee, Hansol Lee, Sung Jin Kim
Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Daejeon 305-701, Republic of Korea
Corresponding author: sungjinkim@kaist.ac.kr

***Abstract*—An analytical model has been proposed and validated to accurately predict the thermal performance and the flow non-uniformity of the manifold microchannel (MMC) heat sinks. To this end, one-dimensional governing equations are derived from the integral relations of momentum and energy over appropriately-defined two separate control volumes. Specifically, a control volume for the momentum balance is defined in the fluid region, including a dividing flow junction where the main fluid stream branches off from the manifold to the microchannels. A control volume for the energy balance in the solid region is defined so that it incorporates an individual microchannel and its solid substrate. The derived one-dimensional governing equations are solved without introducing any fitting parameters. By solving these equations, the flow distribution among the microchannels and the temperature distribution within the solid substrate are estimated. In order to verify the proposed model, a series of 3-D numerical simulation is conducted over a wide range of geometric parameters and operating conditions. This range includes the channel aspect ratio (AR) from 3 to 15, the Reynolds number (Re_{man}) at the manifold inlet from 560 to 3190, the dimensionless hydraulic flow length (x^+) from 0.012 to 0.123, and the dimensionless thermal flow length (x^*) from 0.002 to 0.023. It is shown that the model provides an accurate prediction of the thermal performance and the flow non-uniformity of MMC heat sinks: the total thermal resistance and the coefficient of variation (CV) of the flow distribution among the microchannels are estimated within the root mean square percentage error (RMSPE) of 6% and 23% for 51 data points. The results emphasize that the non-uniform flow distribution among the microchannels should be taken into account for accurately predicting the thermal performance of MMC heat sinks. By accounting for the non-uniformity in the flow distribution, the significant improvement of the prediction accuracy is made over the earlier model with an error reduction of 81%.**

***Index Terms*—embedded cooling, high heat flux cooling, manifold microchannels, thermal management**

I. INTRODUCTION

With a rapid development of high-performance electronics including advanced processors for machine learning applications and power electronics for the next-generation grid systems, the power density of the electronics has been significantly increasing [1, 2]. Hence, the die-level heat flux of the electronics has been projected to reach over 1 kW/cm^2 [3, 4]. Earlier studies have shown that embedded microchannel liquid cooling can be an effective cooling scheme for such high heat-flux electronics. However, conventional microchannel cooling suffers from a high pressure drop and a significant temperature difference across a chip surface [5–7]. To overcome these major drawbacks, the concept of manifold microchannel (MMC) heat sink has been proposed and experimentally investigated [8–15]. An MMC heat sink distinguishes itself from a conventional microchannel heat sink by having an additional fluid-distributing manifold bonded on top of the microchannels. Since the overlying manifold distributes the fluid through a slot jet array, a flow length within the microchannels becomes very short compared to the overall size of a chip — typically one-tenth of the chip length. Therefore, an MMC heat sink can yield relatively low pressure drop and mitigated temperature difference across the chip surface, making them a promising solution for thermal management of high heat-flux electronics.

To implement an MMC heat sink for thermal management of various industrial applications, it is essential to predict the thermal performance with respect to its geometric parameters and operating conditions. For this, theoretical models have been developed based on the assumption of the uniform flow distribution among the microchannels for simplicity of analysis [9–11, 15, 16]. However, Copeland *et al.* [9] have shown that their model exhibits a large discrepancy of over 100% in predicting the thermal resistance of an MMC heat sink compared to the experimental data, which raises doubts about the validity of the assumption. Results from the recent numerical and experimental studies support that the main fluid stream within the manifold is unevenly distributed to the microchannels, and the thermal-hydraulic performance of an MMC heat sink is significantly affected by the uneven flow distribution [16, 17]. Boteler *et al.* [16] reported that the flow rate in each microchannel deviates by a factor of six depending on the manifold geometry and the total flow rate, which implies that the assumption of the uniform flow distribution among the microchannels may not be justified in general. Jung *et al.* [17] experimentally showed that a difference of over 10K between the maximum and the average temperatures of 5 × 5 mm^2 chip exists at the heat flux of 250 W/cm^2, due mainly to the non-uniform flow distribution. These results suggest that the non-uniform flow distribution and its effect on the temperature distribution should be taken

Identify applicable funding agency here. If none, delete this.

into account to accurately predict the thermal performance of an MMC heat sink.

The objective of this study is to develop a theoretical model to accurately predict both the thermal performance and the flow non-uniformity of an MMC heat sink. The proposed model consists of one-dimensional momentum and energy equations. These two one-dimensional governing equations are derived from integral relations of momentum and energy over appropriately-defined two separate control volumes. After deriving the one-dimensional governing equations, the flow distribution among the microchannels and the resultant temperature distribution within the solid substrate are estimated by solving the one-dimensional governing equations. The total thermal resistance and the coefficient of variation (CV) of the flow distribution predicted by the present model are compared with the 3-D numerical dataset. The accuracy of the present model is compared to that of the earlier theoretical model.

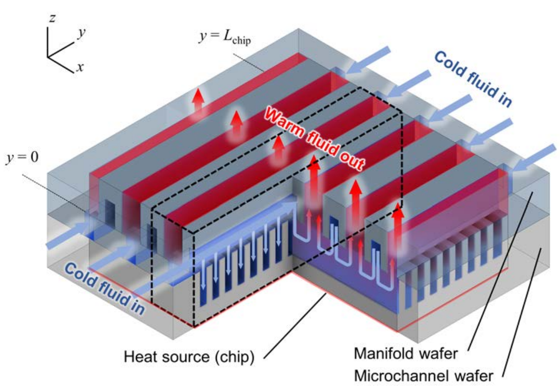


Fig. 1 A schematic of the manifold microchannel (MMC) heat sink with quarter-symmetry section removed.

II. Results and discussion

A. Modeling

The problem considered in this paper concerns the single-phase forced convective flow within an MMC heat sink with plate-fins, as shown in Fig. 1. The MMC heat sink comprises two parts: a fluid-distributing manifold and a substrate with microchannels. The two manifold inlets are located on the opposite sides of the heat sink. The main fluid streams enter these two manifold inlets and flow along the y (or $-y$) direction, distributing the fluid into the microchannels. The flow initially makes a 90-degree turn from the y (or $-y$) direction to the $-z$ direction to enter the top of the microchannels. After impinging on the base plate, it takes another 90-degree turn from the $-z$ direction to the x (or $-x$) direction. The liquid takes heat away from a heat dissipating chip at the backside of the microchannel layer, and finally, it exits through an outlet that is oriented perpendicular to the chip surface in the z direction. The following assumptions are made in analyzing the problem:

1) Steady-state, laminar, and incompressible flow with negligible gravitational effects and viscous dissipation is considered.
2) Heat transfer at the fluid-solid interface occurs at the fin and the primary surfaces of the microchannels, while heat transfer at the manifold wall is neglected.
3) The hydrodynamically and thermally developing flow is assumed for the fluid flow within the microchannels, while the hydrodynamically fully-developed flow is assumed for the main fluid stream within the manifold.

To establish the governing equations describing the flow and temperature distributions within MMC heat sinks, the integral analysis is performed for the appropriate control volumes in the fluid and the solid regions within the MMC heat sinks. Fig. 2(a) describes two different control volumes for estimating the flow distribution among the microchannels in the fluid region and the temperature distribution in the solid region, respectively. The 1st control volume in the fluid region, depicted in Fig. 2(b), includes the dividing flow junction where the main fluid stream branches off from the manifold to the microchannels. The 2nd control volume in the solid region, depicted in Fig. 2(c), incorporates an individual microchannel and its substrate.

The integral relations of the continuity and the momentum equations over the fluid control volume are given as

$$\int_{CS} \mathbf{u}\cdot\mathbf{n}\, dA = 0 \tag{1}$$

$$\int_{CS} \rho(\mathbf{u}\cdot\mathbf{n})\mathbf{u}\, dA = -\int_{CS} P\mathbf{n}\, dA + \int_{CS} \tau\cdot\mathbf{n}\, dA \tag{2}$$

where $\mathbf{u}$ and $\mathbf{n}$ indicate the velocity vector and the surface normal vector, respectively. P and τ are the static pressure and the shear stress acting on the control volume surfaces, and ρ is the fluid density. From the integral relation of the momentum, Eq. (2), the momentum balance across the control volume surfaces is expressed as follows:

$$\rho\left(U+\frac{dU}{dy}\Delta y\right)^2 A_{man,in} - \rho U^2 A_{man,in} + \rho U_{ch,in} V_{ch,in} A_{ch,in} = P A_{man,in} - \left(P+\frac{dP}{dy}\Delta y\right) A_{man,in} - \tau_w L_{p,man,in}\Delta y \tag{3}$$

where P, τ_w, $L_{p,man,in}$ are the static pressure, the wall shear stress acting on the control volume surfaces, and the perimeter of the manifold inlet, respectively. $U_{ch,in}$ is the lateral velocity at the microchannel inlet. $U_{ch,in}$ can be expressed as $U_{ch,in} = \gamma U$, which is the combination of the momentum correction factor, γ, and the average velocity of the main fluid stream at the dividing flow junction [18, 19]. In this study, γ is fixed at 1.2, based on the value suggested in the previous experimental study on the fluid-distributing manifold [18]. The shear stress acting on the control volume surfaces can be estimated by using the friction factor through a duct as follows:

$$\tau_w = f_{man}\left(\frac{1}{2}\rho U^2\right) \tag{4}$$

where f_{man} is the friction factor for the fully-developed flow within a duct, suggested by Muzychka and Yovanovich [20].

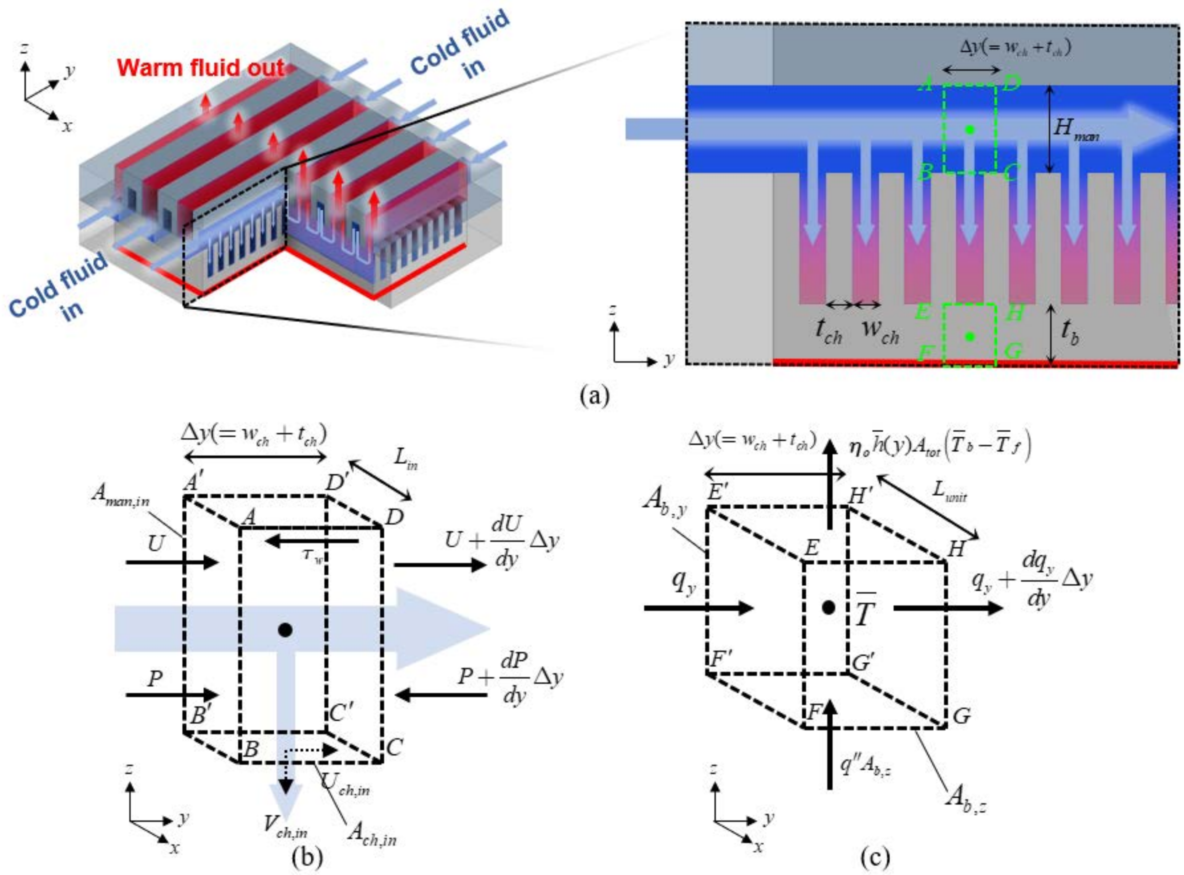


Fig. 2 Control volume definitions to derive one-dimensional governing equations. (a) y-z cross-sectional view of MMC heat sink. (b) 3-D schematic of the control volume in the fluid region. (c) 3-D schematic of the control volume in the solid region.

Finally, the one-dimensional momentum equation is derived as follows:

$$\frac{dP}{dy} = -f_{man}\left(\frac{1}{2}\rho\frac{L_{p,man,in}}{A_{man,in}}\right)U^2 + \rho\left(2-\gamma\right)U\left(-\frac{dU}{dy}\right) \tag{5}$$

The term on the left hand side represents the static pressure gradient along the direction of the main fluid stream within the manifold. The first term on the right hand side indicates the frictional pressure drop due to the wall shear stress. The remaining second term represents the static pressure recovery or static pressure rise due to the branching flow.

The integral relation of energy over the solid control volume is given in Eq. (6).

$$\int_{CS}\left(k_s\nabla T\right)\cdot\mathbf{n}dA = 0 \tag{6}$$

where k_s indicates the thermal conductivity of the solid substrate. The energy balance across the control volume surfaces is estimated by considering y directional and z directional heat transfer components. For the y directional heat transfer component, the conduction heat transfer rate exists. On the other hand, the heat input from the heat flux boundary and the convective heat transfer rate at the fluid-solid interface are present for the z direction. Given the symmetry condition of the control volume surfaces, no heat transfer component exists in the x direction. To quantify the convective heat transfer at the fluid-solid interface through the base and the fin surfaces of the microchannels, the fin model is used. The average heat transfer coefficient and the bulk fluid temperature within the individual microchannel are determined depending on the channel flow rate at each channel. The overall energy balance across the control volume surfaces is expressed as follows:

$$q_y - \left(q_y + \frac{dq_y}{dy}\Delta y\right) - \eta_o\overline{h}A_{tot}\left(\overline{T}_b - \overline{T}_f\right) + q''A_{b,z} = 0 \tag{7}$$

where q_y is the conduction heat transfer rate in the y direction. η_o, $\overline{h}$, and A_{tot} indicate the overall fin efficiency, the average heat transfer coefficient, and the wetted surface area including the fin and primary surfaces, respectively. Utilizing geometric parameters $\alpha = (w_{ch}+t_{ch}+2H_{ch})/(w_{ch}+t_{ch})$ and $\Delta y = w_{ch}+t_{ch}$ simplifies the equation to

$$\frac{d^2\overline{T}}{dy^2} - \frac{\eta_o}{k_s t_b}\alpha\overline{h}\left(\overline{T}_b - \overline{T}_f\right) + \frac{q''}{k_s t_b} = 0 \tag{8}$$

where t_b is the thickness of the substrate. Eq. (8) is the one-dimensional energy equation as a form of second-order linear ordinary differential equation, for which the boundary

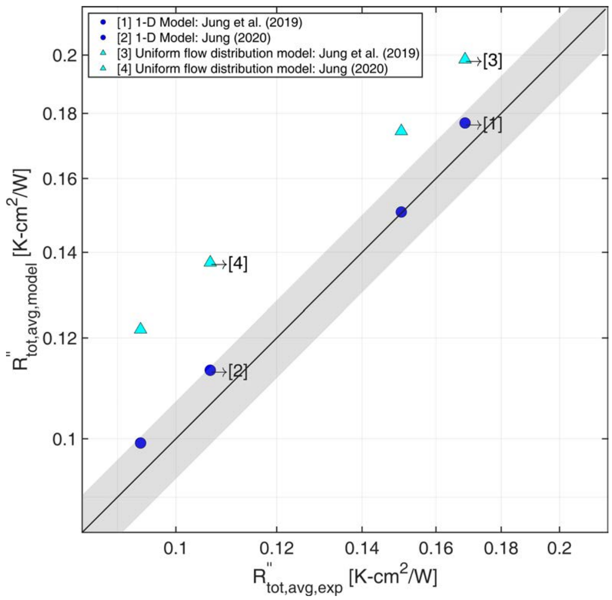


Fig. 3 Experimental validation result. The present one-dimensional thermal-hydraulic model of MMC heat sinks predicts the experimental thermal performance of MMC heat sinks within the maximum error of 7%. This shows a substantial error reduction of 77% compared to the prediction result based on the uniform flow distribution.

conditions related to the temperature are $\left.\frac{d\overline{T}}{dy}\right|_{y=0} = 0$ and $\left.\frac{d\overline{T}}{dy}\right|_{y=L_{chip}/2} = 0$. The unknown average heat transfer coefficient, $\overline{h}$, is estimated by the correlation of the Nusselt number for simultaneously developing flow suggested by Muzychka and Yovanovich [21], which is

$$\overline{\mathrm{Nu}} = \frac{\overline{h}D_{ch}}{k_f} = \frac{1.772}{[1+(1.909\mathrm{Pr}^{1/6})^{9/2}]^{2/9}}\frac{1}{\sqrt{x^*}} \tag{9}$$

Finally, the derived one-dimensional momentum and energy equations are converted into the set of algebraic equations in terms of U and $\overline{T}$ using the finite difference method, respectively. The algebraic equations are solved simultaneously by $fsolve$ solver in MATLAB with Levenberg-Marquardt algorithm; therefore, the flow distribution among the microchannels and the resultant temperature distribution within the substrate of the microchannels are obtained.

B. CFD analysis for validation

The 3-D numerical simulation aims to verify the accuracy of the one-dimensional model in predicting the flow non-uniformity among the microchannels and the resultant temperature distribution across the base substrate. The computational domain consists of the fluid region within a single manifold and the corresponding microchannels, and the solid region of the microchannels including its substrate. This computational domain has been demonstrated to successfully predict the thermal performance and the flow distribution of the MMC heat sinks [16, 22]. At the manifold inlet, the fully-developed flow velocity profile is assigned as the velocity boundary condition. At the manifold outlet, the gauge pressure of 0 Pa is imposed as the pressure boundary condition. The adiabatic condition is imposed at the symmetry planes, and the uniform heat flux is assigned at the bottom of the solid substrate. De-ionized water, with the inlet temperature of 23°C, is used for the fluid domain, and silicon is used for the solid domain. A grid independence test is performed to ensure that the results of the 3-D numerical simulation are invariant to the grid resolution. The MMC heat sink with the channel aspect ratio (AR) of 5 and with the unit cell length (L_{unit}) of 500 μm is chosen for the grid independence study. The grid independence test is conducted with the total flow rate of 400 mL/min, which is the highest flow rate in the current parametric study. The generated mesh consists of conformal polyhedral elements near the fluid-solid interfaces and hexahedral elements in the bulk regions of both the solid and the fluid domains. This ensures a high-quality mesh with a fewer number of elements. The length of one side of the element is approximately 2 μm so that it can fully resolve the conjugated heat transfer characteristics within the boundary layer regions. Successive grid refinements are conducted so that the variations in the base average temperature and the pressure drop between Grid 2 (9.2M elements) and Grid 3 (14.4M elements) are well below 1%. Therefore, the mesh generation setting from Grid 2 is selected as the standard for the parametric study. The computational time of Grid 2 required for the convergence is approximately 20 h using Intel Core i9 24-core 3.0 GHz processors.

C. Experimental validation of CFD analysis

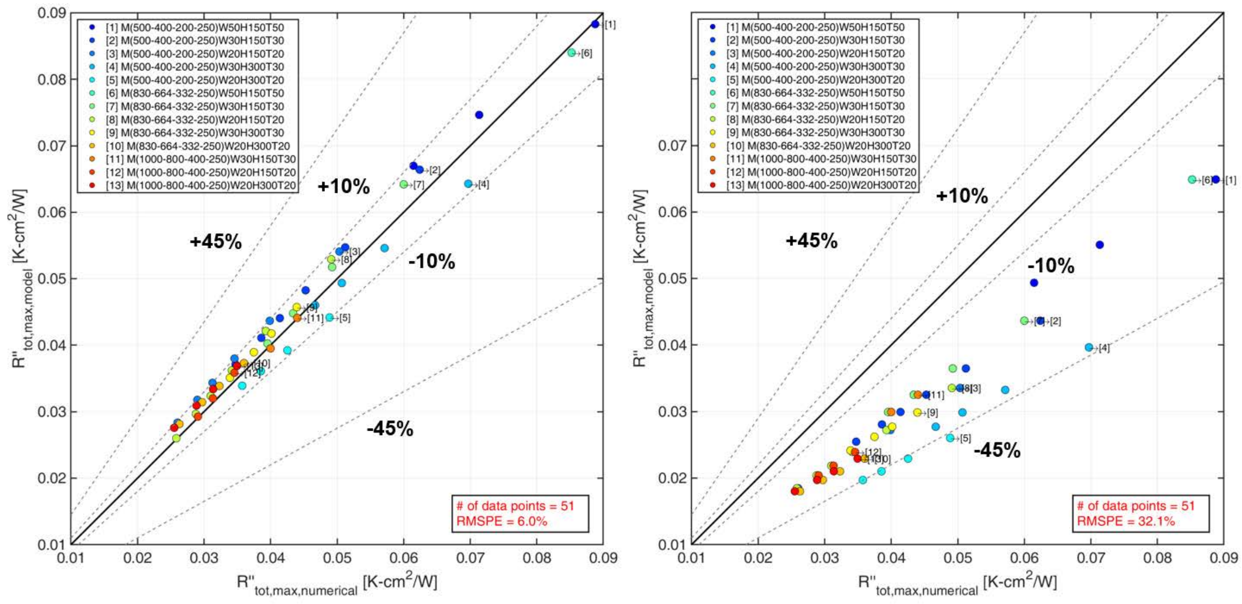
The experimental validation of the proposed one-dimensional model is conducted, and the prediction accuracy is compared with the model based on the uniform flow distribution assumption. Fig. 3 indicates that the present model is capable of predicting the experimental thermal performance of MMC heat sinks within the maximum error of 7%. The predicted thermal performances outside the grey region indicates the prediction result based on the uniform flow distribution assumption. Compared to the prediction based on the uniform distribution assumption, the current model shows the error reduction of 77% by considering the non-uniform flow distribution.

D. Prediction of thermal performance and flow non-uniformity

The thermal performance of the MMC heat sinks is defined by the total thermal resistance based on the maximum temperature at the base substrate.

$$R''_{tot,max} = \frac{T_{b,max} - T_{in}}{q''} \tag{10}$$

Fig. 4 shows the predictions of the total thermal resistance of the MMC heat sinks based on the one-dimensional model and the previous model proposed by Copeland *et al.* [10] which assumes the uniform flow distribution. The prediction using the one-dimensional model shows good agreement with the



(a) (b)

Fig. 4 The prediction results of the total theremal resistance of MMC heat sinks with various manifold and microchannel geometries based on the models. (a) The prediction result using the one-dimensional model. The present model predicts the total thermal resistance of MMC heat sinks within the RMSPE of 6%. (b) The prediction result using the uniform flow distribution model suggested by Copeland *et al.* . The uniform flow distribution model underpredicts the total thermal resistance of MMC heat sinks up to the error of 47%. The nomenclature indicates the geometric parameters of the manifold and the microchannels, which are M(L_{unit}-L_{in}-L_{out}-H_{man})W(w_{ch})H(H_{ch})T(t_{ch}).

total thermal resistance of MMC heat sinks from the 3-D numerical simulation within the RMSPE of 6%. On the other hand, the previous model underestimates the total thermal resistance up to 47% since it does not take into account the non-uniform flow distribution among the microchannels and the resultant temperature distribution across the base substrate. Therefore, the proposed one-dimensional model shows a significant error reduction of 81% compared to the model based on the uniform flow distribution.

To quantify the flow non-uniformity among the microchannels, the interquartile-based coefficient of variation (CV) is utilized, which is defined as

$$\mathrm{CV} = \frac{\mathrm{IQR}}{\mathrm{med}(\dot{m}_{ch})} \tag{11}$$

where IQR and $\mathrm{med}(\dot{m}_{ch})$ indicate the interquartile range and the median of the flow distribution, respectively. The interquartile range is defined by the difference between 75-th and 25-th percentile of the channel flow rates. The interquartile-based CV is widely used in statistical analysis as a measure of the non-uniformity of the given data owing to its superior robustness compared to the conventional definition of CV, which is the standard deviation divided by the mean value. The prediction result of the flow non-uniformity of MMC heat sinks using the one-dimensional model is shown in Fig. 5. The result shows that the one-dimensional model predicts the flow non-uniformity of MMC heat sinks estimated by 3-D numerical simulation within the RSMPE of 23%. On the contrary, the previous model cannot predict the flow non-uniformity since it has assumed the uniform flow distribution among the channels. Thus, the CV value estimated by the previous model turns out to be always zero, which does not account for the flow characteristics of the MMC heat sinks.

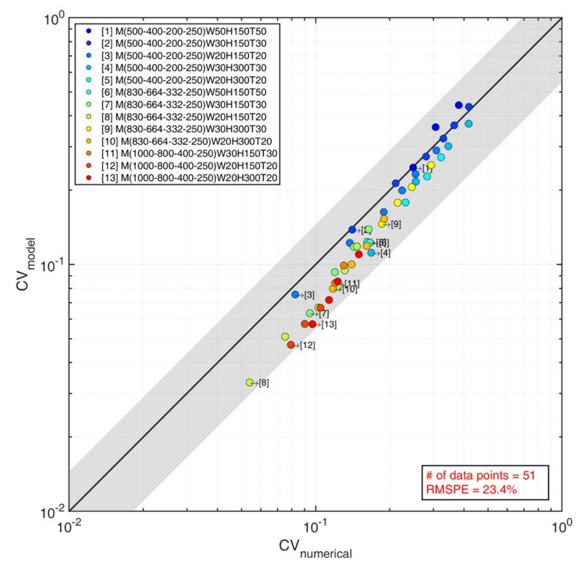


Fig. 5 The prediction result of the flow non-uniformity (CV) of MMC heat sinks with various manifold and microchannel geometries. The present model predicts the CV within the RMSPE of 23%. The grey region indicates the range of error up to 45%.

III. Conclusion

Throughout this study, a one-dimensional model for predicting the thermal performance and the flow non-uniformity of an embedded manifold microchannel (MMC) cooling system has been proposed and validated. For this, one-dimensional governing equations are derived from integral relations of momentum and energy over appropriately-defined two separate control volumes. By solving the one-dimensional governing equations, the flow distribution among the microchannels and the resultant temperature distribution within the solid substrate are estimated. The proposed model is validated over a wide range of geometric parameters and operating conditions by using the dataset of 3-D numerical simulation. Fifty-four data points regarding the total thermal resistance and the coefficient of variation (CV) of the flow distribution among the microchannels have been gathered for comparison with the predictions. The applicable range of the proposed model includes the channel aspect ratio (AR) from 3 to 15, the Reynolds number ($\mathrm{Re}_{m,in}$) at the manifold inlet from 560 to 3190, the dimensionless hydraulic flow length (x^{+}) from 0.012 to 0.123, and the dimensionless thermal flow length (x^{*}) from 0.002 to 0.023. The present model enables the accurate prediction of the thermal performance and flow non-uniformity of an MMC heat sink: The total thermal resistance and the CV are estimated within the root mean square percentage error (RMSPE) of 6% and 23% for 54 data points, respectively. Compared to the earlier model based on the uniform flow distribution assumption, the prediction error for the thermal performance has been significantly reduced by 82%.

Acknowledgment

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2021R1A2C3011944), and by Korea Research Institute for defense Technology planning and advancement (KRIT) grant funded by the Korea government (DAPA(Defense Acquisition Program Administration)) (No. KRIT-CT-22-022).

References

[1] N. Jouppi, C. Young, N. Patil, D. Patterson, Motivation for and evaluation of the first tensor processing unit, IEEE Micro 38 (3) (2018) 10–19. doi:10.1109/MM.2018.032271057.

[2] B. K. Bose, Power electronics, smart grid, and renewable energy systems, Proceedings of the IEEE 105 (11) (2017) 2011–2018. doi:10.1109/JPROC.2017.2745621.

[3] A. Bar-Cohen, P. Wang, E. Rahim, Thermal management of high heat flux nanoelectronic chips, Microgravity Science and Technology 19 (2007) 48–52.

[4] A. Bar-Cohen[1], J. J. Maurer, J. G. Felbinger, Darpa's intra/interchip enhanced cooling (icecool) program, in: CS MANTECH Conference, May 13th-16th, 2013.

[5] D. Tuckerman, R. Pease, High-performance heat sinking for vlsi, IEEE Electron Device Letters 2 (5) (1981) 126–129. doi:10.1109/EDL.1981.25367.

[6] R. Pijnenburg, R. Dekker, C. Nicole, A. Aubry, E. Eummelen, Integrated micro-channel cooling in silicon, in: Proceedings of the 30th European Solid-State Circuits Conference (IEEE Cat. No. 04EX850), IEEE, 2004, pp. 129–132.

[7] R. v. Erp, G. Kampitsis, L. Nela, R. S. Ardebili, E. Matioli, Embedded microchannel cooling for high power-density gan-on-si power integrated circuits, in: 2020 19th IEEE Intersociety Conference on Thermal and Thermomechanical Phenomena in Electronic Systems (ITherm), 2020, pp. 53–59. doi:10.1109/ITherm45881.2020.9190356.

[8] G. Harpole, J. Eninger, Micro-channel heat exchanger optimization, in: Proceedings - IEEE Semiconductor Thermal and Temperature Measurement Symposium, 1991, pp. 59–63.

[9] D. Copeland, H. Takahira, N. Wataru, B. C. Pak, Manifold microchannel heat sinks: Theory and experiment, in: Advances in Electronic Packaging Proc. Int. Intersociety Electronic Packaging Conference (INTERpack '95), 1995, p. 829–835.

[10] D. Copeland, M. Behnia, W. Nakayama, Manifold microchannel heat sinks: isothermal analysis, in: InterSociety Conference on Thermal Phenomena in Electronic Systems, I-THERM V, 1996, pp. 251–257. doi:10.1109/ITHERM.1996.534570.

[11] K. W. Jung, Investigation thermo-fluidic performance of si-based embedded microchannels-3d manifold cooling system for high power density electronic applications, Ph.D. thesis, Stanford University (2020).

[12] E. Cetegen, Forced fed microchannel high heat flux cooling utilizing microgrooved surfaces, Ph.D. thesis, University of Maryland, Colleage Park (2010).

[13] E. Kermani, Manifold micro-channel cooling of photovoltaic cells for high efficiency solar energy conversion systems, Master's thesis, University of Maryland, Colleage Park (2008).

[14] R. van Erp, R. Soleimanzadeh, L. Nela, G. Kampitsis, E. Matioli, Co-designing electronics with microfluidics for more sustainable cooling, Nature 585 (7824) (2020) 211–216. doi:10.1038/s41586-020-2666-1.

[15] W. Escher, T. Brunschwiler, B. Michel, D. Poulikakos, Experimental Investigation of an Ultrathin Manifold Microchannel Heat Sink for Liquid-Cooled Chips, Journal of Heat Transfer 132 (8) (2010) 081402. doi:10.1115/1.4001306.

[16] L. Boteler, N. Jankowski, P. McCluskey, B. Morgan, Numerical investigation and sensitivity analysis of manifold microchannel coolers, International Journal of Heat and Mass Transfer 55 (2012) 7698–7708.

[17] K. W. Jung, C. R. Kharangate, H. Lee, J. Palko, F. Zhou, M. Asheghi, E. M. Dede, K. E. Goodson, Embedded cooling with 3d manifold for vehicle power electronics application: Single-phase thermal-fluid performance, International Journal of Heat and Mass Transfer 130

(2019) 1108–1119.

[18] R. A. Bajura, A model for flow distribution in manifolds, Journal of Engineering for Power 93 (1971) 7–12. doi:10.1115/1.3445410.

[19] J. Wang, Theory of flow distribution in manifolds, Chemical Engineering Journal 168 (2011) 1331–1345. doi:https://doi.org/10.1016/j.cej.2011.02.050.

[20] Y. S. Muzychka, M. M. Yovanovich, Pressure drop in laminar developing flow in noncircular ducts: A scaling and modeling approach, ASME. J. Fluids Eng. 131(11) (2009) 111105.

[21] Y. S. Muzychka, M. M. Yovanovich, Laminar forced convection heat transfer in the combined entry region of non-circular ducts, Journal of Heat Transfer 126 (1) (2004) 54–61. doi:10.1115/1.1643752.

[22] S. Hazra, T. Wei, Y. Lin, M. Asheghi, K. Goodson, M. P. Gupta, M. Degner, Parametric design analysis of a multi-level 3d manifolded microchannel cooler via reduced order numerical modeling, International Journal of Heat and Mass Transfer 197 (2022) 123356. doi:https://doi.org/10.1016/j.ijheatmasstransfer.2022.123356.