Quizzes of TTK4225 - Systems Theory, Autumn 2020

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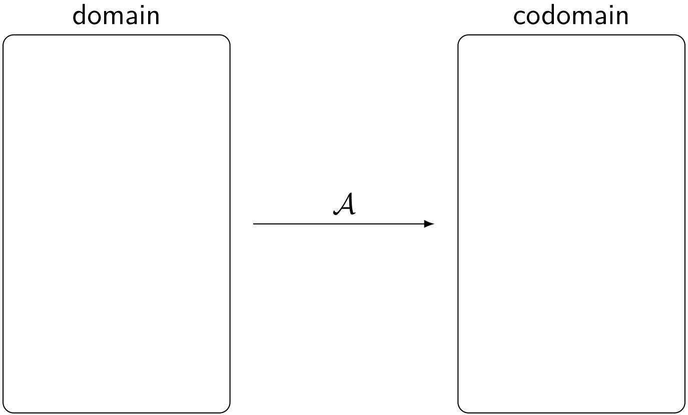
Kernel (or null-space) of a matrix $A \in \mathbb{R}^{n \times m}$

$$\ker(A) = \{\boldsymbol{x} \in \mathbb{R}^m \text{ s.t. } A\boldsymbol{x} = \mathbf{0}\}$$

importance: it defines the space of the equilibria of the autonomous system

How do we characterize $\ker\left(A\right)$ geometrically?



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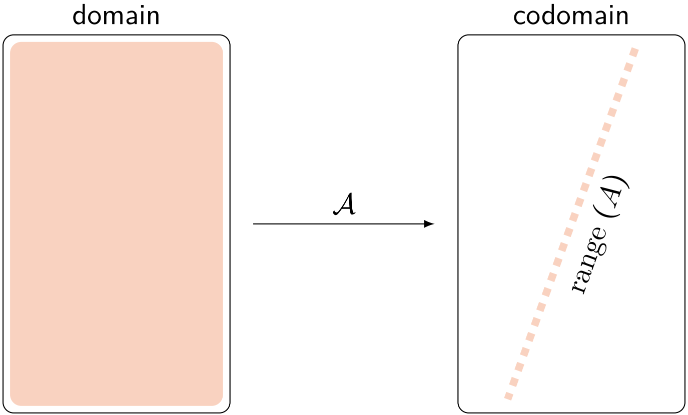

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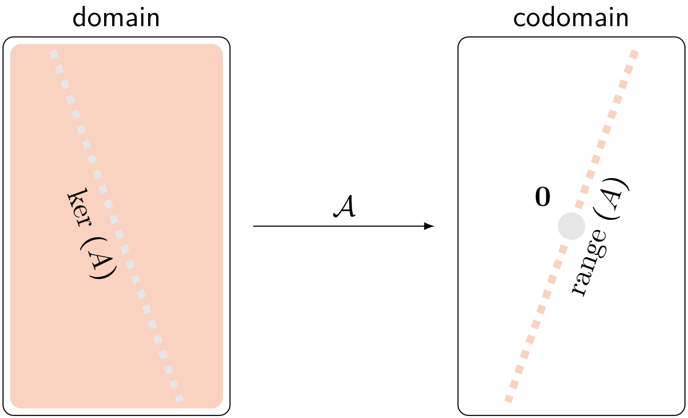
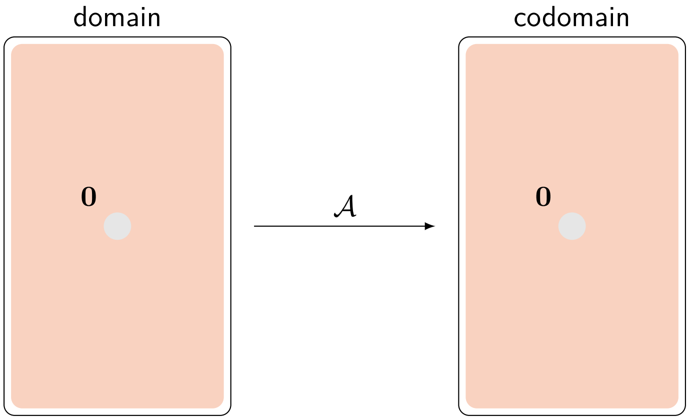
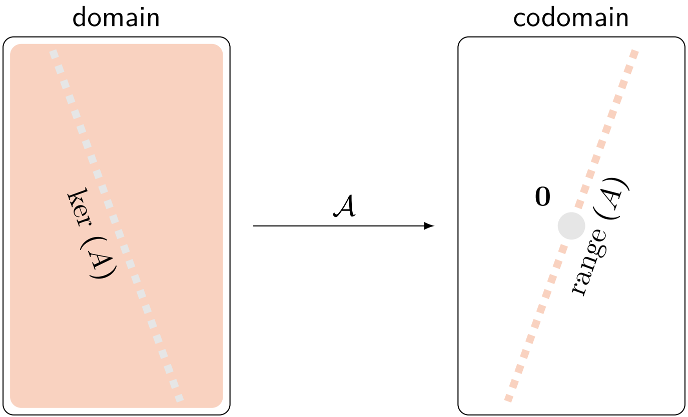


All the possible situations, geometrically



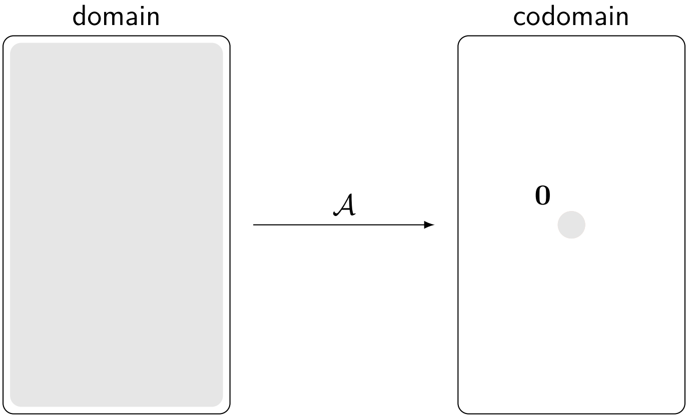
case 1: the kernel is only the $\mathbf{0}$ in the domain

All the possible situations, geometrically



case 2: the kernel is a *subspace* in the domain

All the possible situations, geometrically

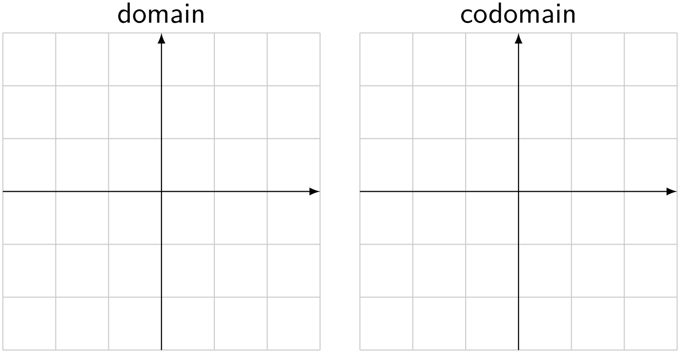


case 3: the kernel is the whole domain

(and this means that $\mathrm{range}\,(A) = \{\mathbf{0}\}$)

Important result: $\ker(A)$ is orthogonal to the rows of A

$$A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}$$



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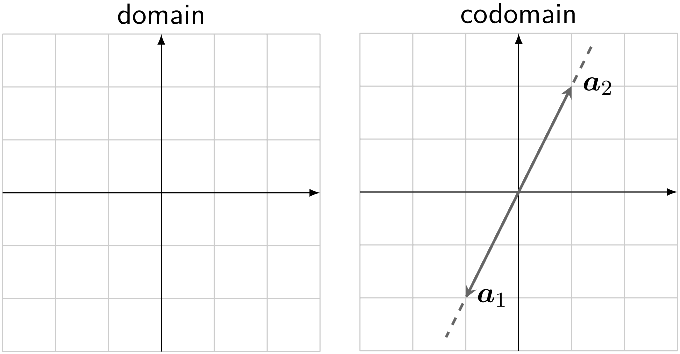

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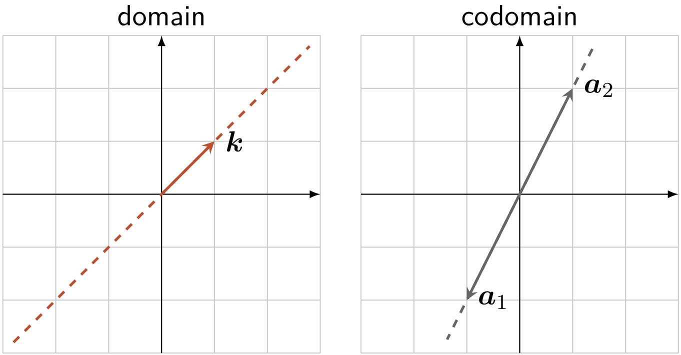

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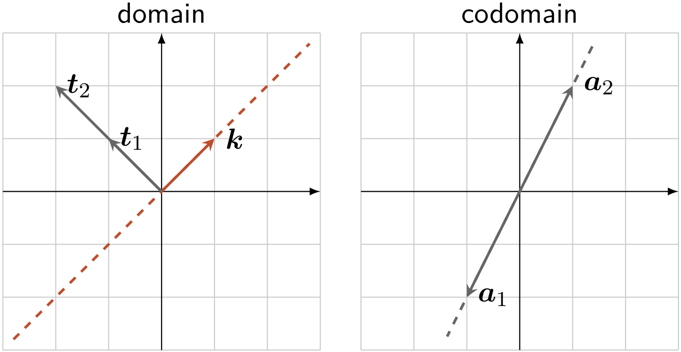
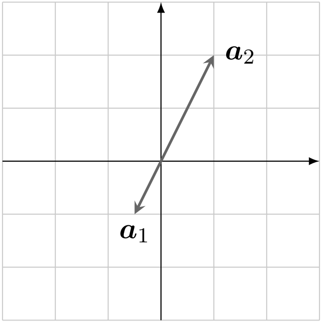


How to visualize the spaces associated to a matrix $A \in \mathbb{R}^{2\times 2}$

$$A = \begin{bmatrix} -0.5 & 1 \\ -1 & 2 \end{bmatrix}$$



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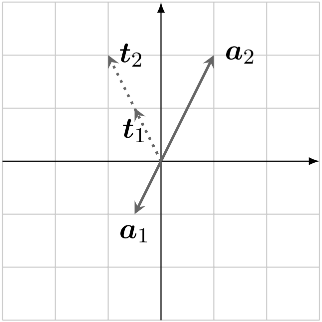
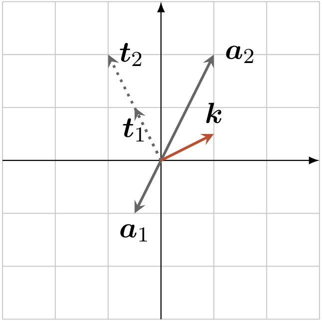


note that in general the fact that $ker\,(A) \neq \{\mathbf{0}\}$ *does not imply* A *to be a projection matrix!*

How to visualize the spaces associated to a matrix $A \in \mathbb{R}^{2\times 2}$

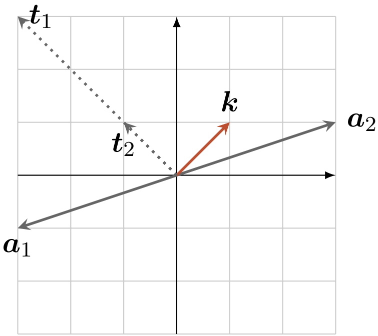
$$A = \begin{bmatrix} -0.5 & 1 \\ -1 & 2 \end{bmatrix}$$



note that in general the fact that $ker\,(A) \neq \{\mathbf{0}\}$ *does not imply* A *to be a projection matrix!*

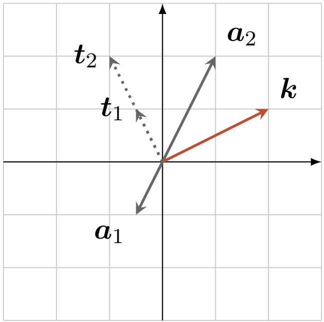
Kernel (or null-space) of a matrix $A \in \mathbb{R}^{n\times m}$: examples

$$A = \begin{bmatrix} -3 & 3 \\ -1 & 1 \end{bmatrix}$$



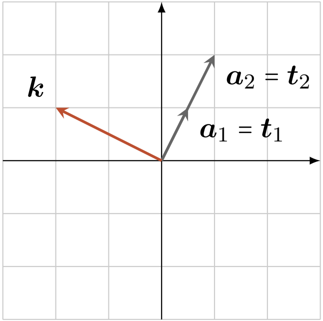
Kernel (or null-space) of a matrix $A \in \mathbb{R}^{n \times m}$: examples

$$A = \begin{bmatrix} -0.5 & 1 \\ -1 & 2 \end{bmatrix}$$



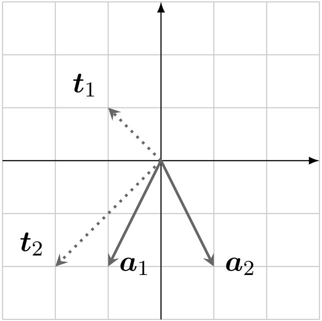
Kernel (or null-space) of a matrix $A \in \mathbb{R}^{n \times m}$: examples

$$A = \begin{bmatrix} 0.5 & 1 \\ 1 & 2 \end{bmatrix}$$ *(symmetric!)*



Kernel (or null-space) of a matrix $A \in \mathbb{R}^{n \times m}$: examples

$$A = \begin{bmatrix} -1 & +1 \\ -2 & -2 \end{bmatrix}$$



Summarizing

$\ker(A)$ orthogonal to the rows of A

(that implies also $\ker(A) \perp \mathrm{range}\left(A^T\right)$)

Summarizing

$$\mathrm{rank}\,(A) + \dim\left(\ker\,(A)\right) = \text{number of columns of} A$$

Question 117

Rewrite the dynamics of a linearized spring mass system, i.e., $m\ddot{y}(t) = -ky(t) - f\dot{y}(t) + F(t)$, as a state space system

Question 118

Graphically visualize the state update matrix of a linearized spring mass system, i.e., $m\ddot{y}(t) = -ky(t) - f\dot{y}(t) + F(t)$ but represented as a state space system, as a function of the parameters of the system. Define what is the range of the matrix, its kernel, and its determinant, again as functions of the parameters of the system.

Question 119

Rewrite the dynamics of a 3rd order ARMA model, i.e., $\dddot{y} + \alpha_2\ddot{y} + \alpha_1\dot{y} + \alpha_0 y = bu$, as a state space system

Question 120

Graphically visualize the state update matrix of a 3rd order ARMA model, i.e., $\dddot{y} + \alpha_2 \ddot{y} + \alpha_1 \dot{y} + \alpha_0 y = bu$ but represented as a state space system, as a function of the parameters of the system. Define what is the range of the matrix, its kernel, and its determinant, again as functions of the parameters of the system.

Question 121

Graphically visualize the state update matrix of a system whose dynamics is defined by

$$\dot{\boldsymbol{x}} = J_{\lambda}^{(n)} \boldsymbol{x} + \boldsymbol{b} u$$

as a function of λ and the dimension of the system. Define what is the range of the matrix, its kernel, and its determinant, again as functions of the parameters of the system.

Question 122

Graphically visualize the state update matrix of a system whose dynamics is defined by

$$\dot{\boldsymbol{x}} = \operatorname{diag}\left(J_{\lambda_1}^{(n_1)}, J_{\lambda_2}^{(n_2)}\right)\boldsymbol{x} + \boldsymbol{b}u$$

as a function of λ and the dimension of the system. Define what is the range of the matrix, its kernel, and its determinant, again as functions of the parameters of the system.