TTK4225 - Systems Theory, Autumn 2020

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Connections with step responses

Roadmap

- definition of step response
- properties for first order systems
- other important examples

Definition: step response

$$Y(s) = H(s)U(s), \qquad U(s) = \frac{1}{s}$$

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Discussion: may we easily implement step responses in real systems? What are the limitations?

Step responses for first order systems

Example: $\dot{x}(t) = ax(t) + bu(t)$, with $u(t) = H(t)$, implies

$$sX(s) - x_0 = aX(s) + bU(s) \qquad \Longrightarrow \qquad X(s) = \frac{1}{s-a}x_0 + \frac{b}{s(s-a)}$$

and thus $x(t) = e^{at}x_0 + \dfrac{b}{a}\left(e^{at} - H(t)\right)$.

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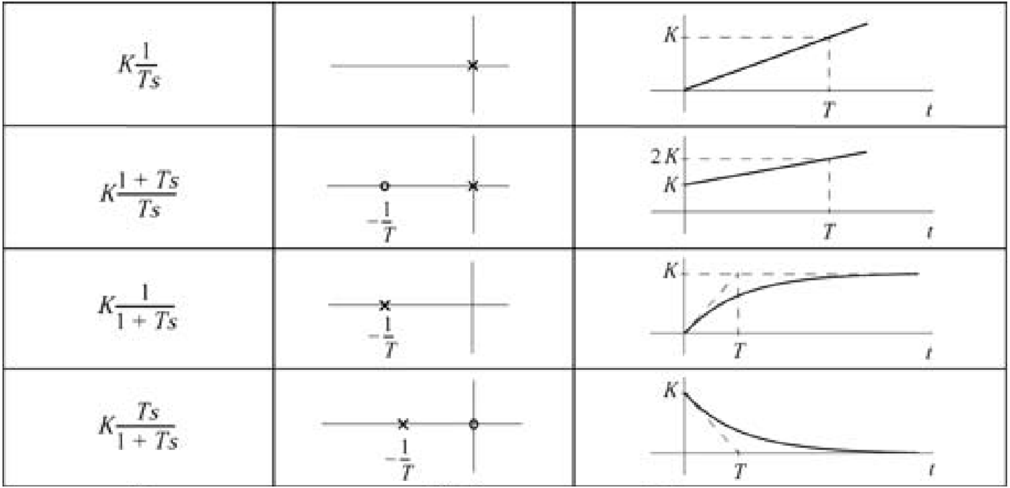
Discussion:

- does the system converge somewhere?
- may the system response pass this value?
- may we compute the time constant of the system in this case?

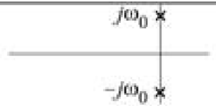
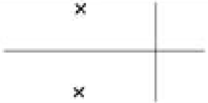
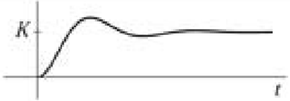
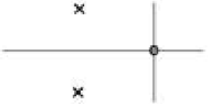
Examples for zeroth order systems

K		

Examples for first order systems



Examples for second order systems

$\dfrac{K}{1+\left(\dfrac{s}{\omega_0}\right)^2}$		
$\dfrac{K}{1+2\zeta\dfrac{s}{\omega_0}+\left(\dfrac{s}{\omega_0}\right)^2}$		
$\dfrac{Ks}{1+2\zeta\dfrac{s}{\omega_0}+\left(\dfrac{s}{\omega_0}\right)^2}$		

Important message

if the input is $\dot{u}$ then the step response gives the impulse response
that we would get if the input were u, instead

Discussion

What about $\dfrac{K(s+a)}{1+2\xi\frac{s}{\omega}+\left(\frac{s}{\omega}\right)^2}$?

?

Block diagrams

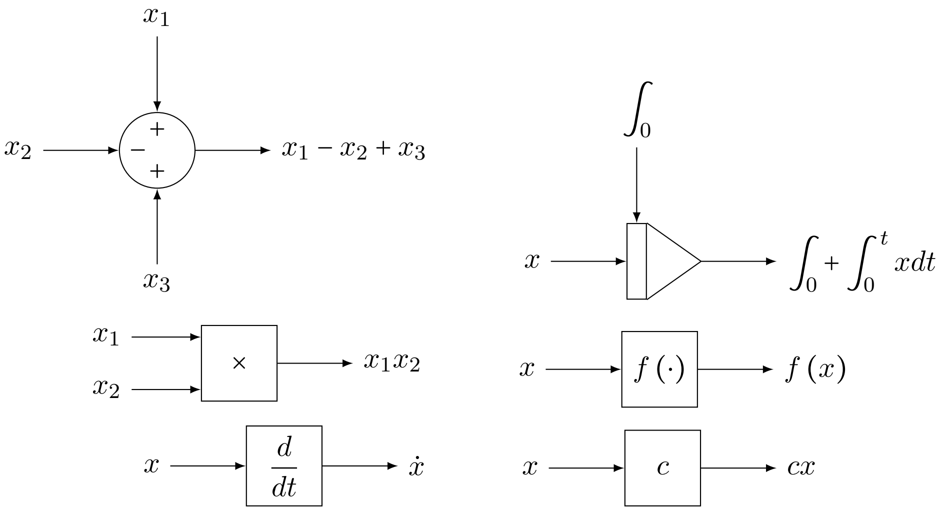
Roadmap

- recap of the diagrams in the time domain
- recap of the diagrams in the frequency domain
- rules for how to transform the diagrams
- examples

Block diagrams - why?

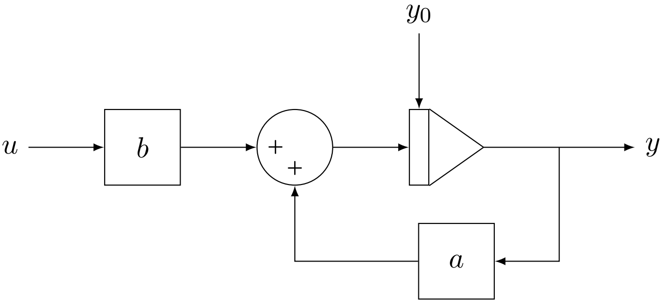
- used very often in companies
- aid visualization *(until a certain complexity is reached...)*
- enable "drag & drop" way of programming
- here primarily used for interpretations

Most common block diagrams in the time domain



Representing a first order DE with a block scheme

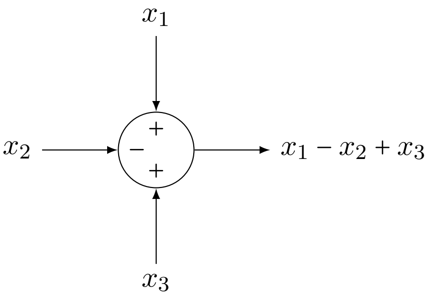
$$\dot{y} = ay + bu$$

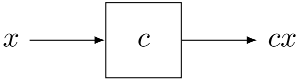


Discussion: how do we represent $\ddot{x} + \frac{f}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}u$?

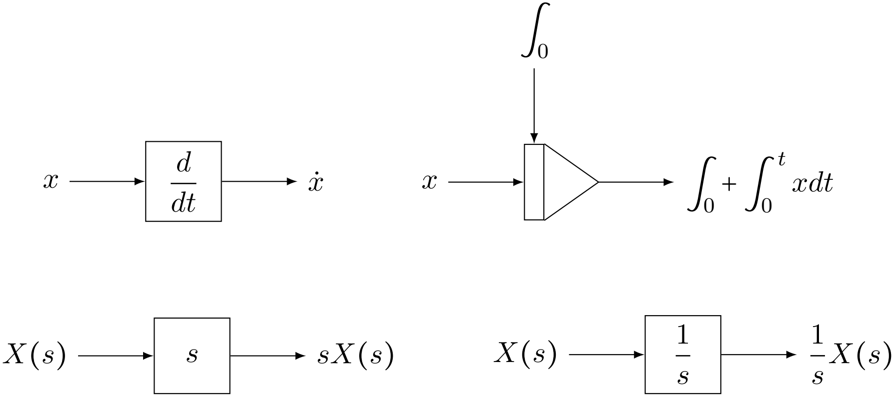
?

Block diagrams that are equal in both time and frequency domains

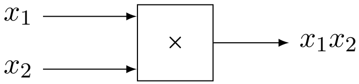


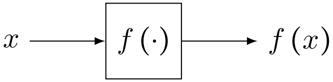


Block diagrams that are logically the same in both time and frequency domains



Block diagrams that do not exist in the frequency domain





Discussion: why?

rules for manipulating block diagrams in the frequency domain

Series of transfer functions

$$U(s) \longrightarrow \boxed{H_a(s)} \longrightarrow \boxed{H_b(s)} \longrightarrow Y(s)$$

is equivalent to

$$U(s) \longrightarrow \boxed{H_a(s)H_b(s)} \longrightarrow Y(s)$$

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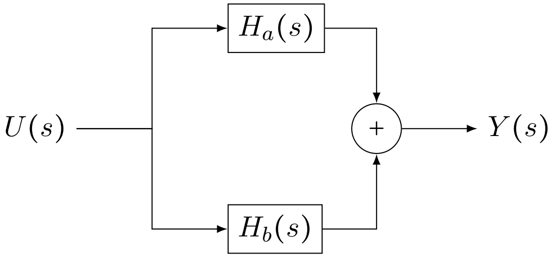
Parallel of transfer functions



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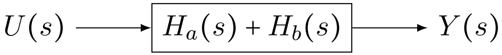

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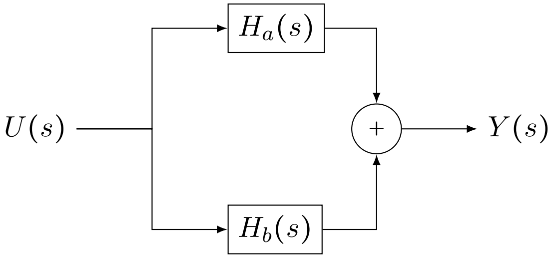

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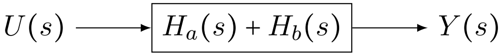
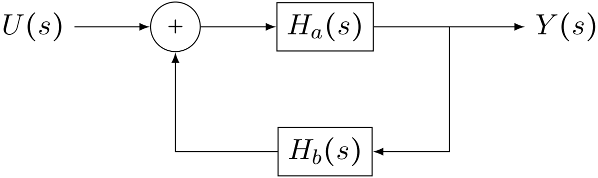


Discussion: why?

Elimination of feedback loops



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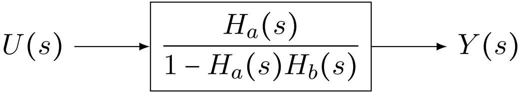
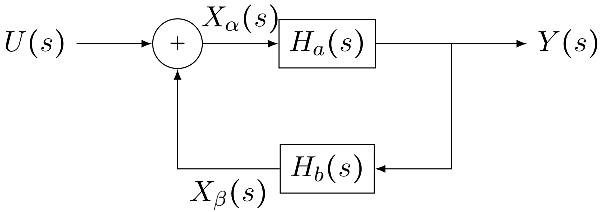


Elimination of feedback loops: how to remember the formula



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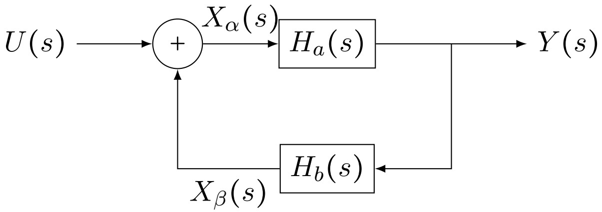

- $Y = H_a X_\alpha$
- $X_\alpha = U + X_\beta$
- $X_\beta = H_b Y$

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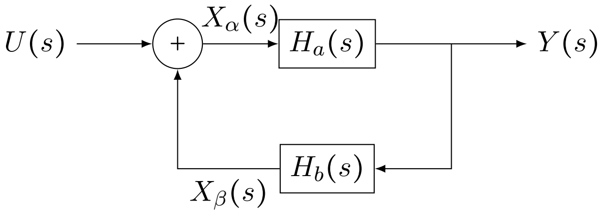

- $Y = H_a X_\alpha$
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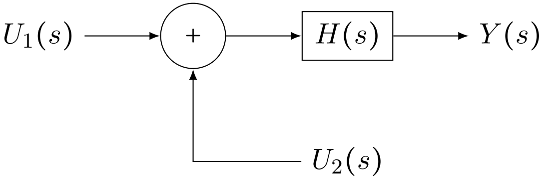
- $X_\beta = H_b Y$
- $\implies Y = H_a\left(U + H_b Y\right)$

Moving blocks around sum operators



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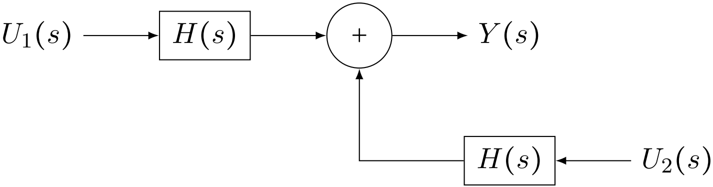

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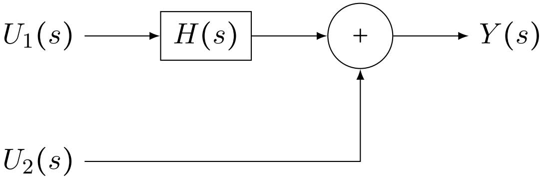

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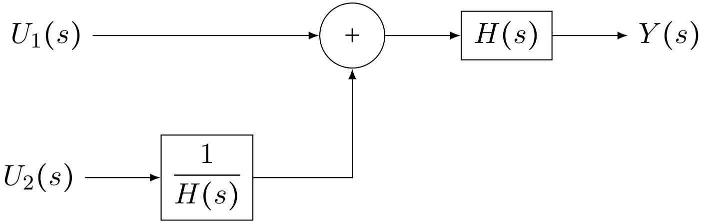
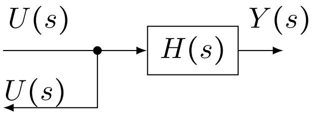


Moving blocks around connections



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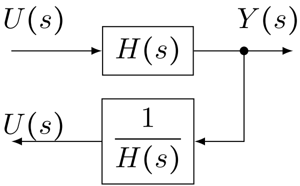

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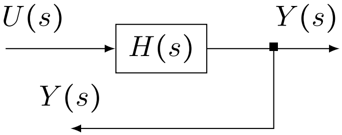

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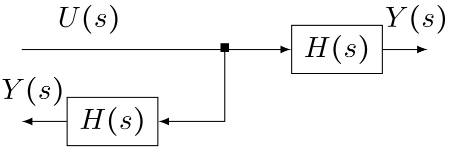
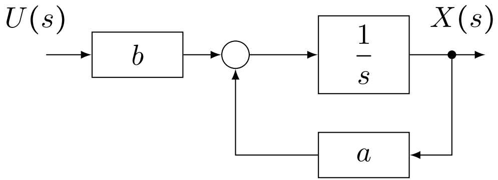


Example: $\dot{x} = ax + bu$



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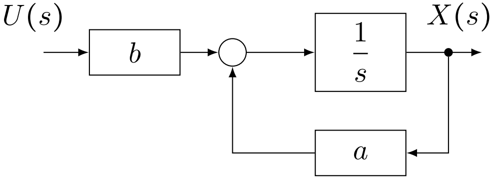

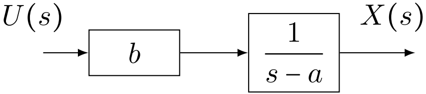

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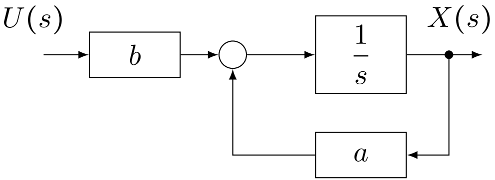

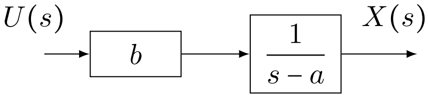

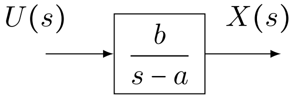

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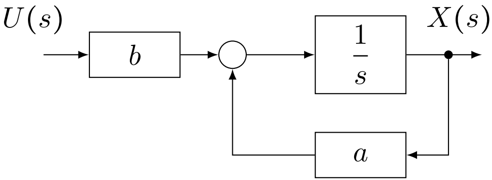

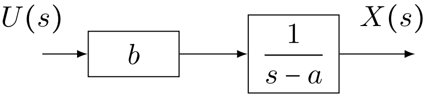

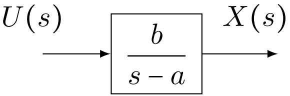

Discussion: what about the stability of the feedback loop?

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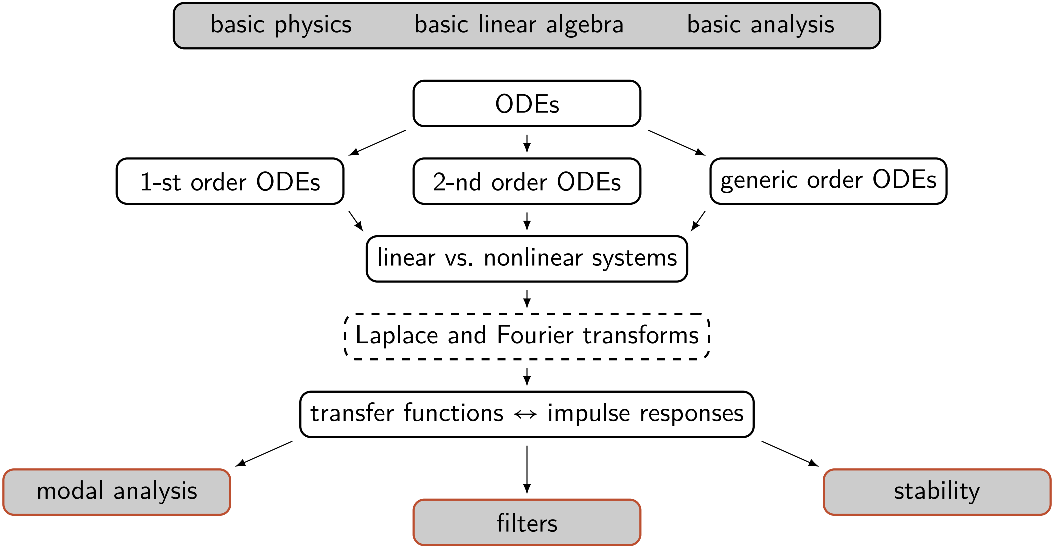
Equilibria

Summary / Roadmap of TTK4225



Roadmap

- what does “equilibrium” mean?
- examples
- equilibria in LTI systems

Equilibrium, what does it mean?

$$\dot{\boldsymbol{y}} = \boldsymbol{f}\left(\boldsymbol{y}, \boldsymbol{u}\right)$$

Example: exponential growth

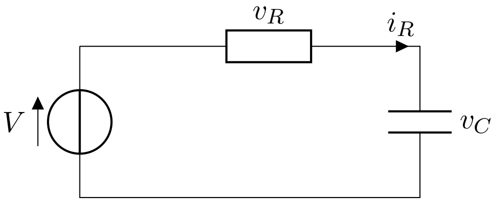
Scalar version:

$$\dot{y} = \alpha y + \beta u \tag{1}$$

Matricial version:

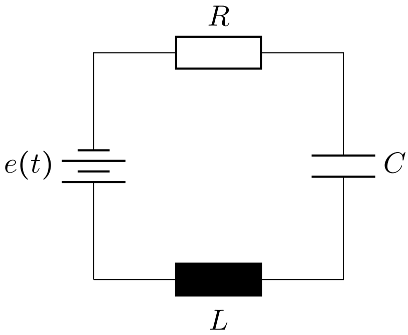
$$\dot{\boldsymbol{y}} = A\boldsymbol{y} + B\boldsymbol{u} \tag{2}$$

Example: RC-circuit



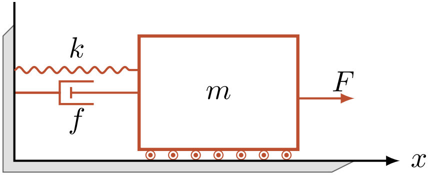
$$\dot{v}_C = -\frac{1}{RC}v_C + \frac{1}{RC}V(t)$$

Example: RCL-circuit



$$e(t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau$$

Example: spring-mass systems



$$m\ddot{x}(t) = -kx(t) - f\dot{x}(t) + F(t)$$

Example: Lotka-Volterra

- y_{prey} := prey
- y_{pred} := predator

$$\begin{cases} \dot{y}_{\text{prey}} & = \alpha y_{\text{prey}} - \beta y_{\text{prey}} y_{\text{pred}} \\ \dot{y}_{\text{pred}} & = -\gamma y_{\text{pred}} + \delta y_{\text{prey}} y_{\text{pred}} \end{cases}$$

Example: Van-der-Pol oscillator

$$\begin{cases} \dot{y}_1 & = \mu \left(y_1 - \dfrac{y_1^3}{3} - y_2 \right) \\ \dot{y}_2 & = \dfrac{y_1}{\mu} \end{cases}$$

Example: balancing robot

$$
\begin{aligned}
\left(I_b + m_b l_b^2\right)\ddot{\theta}_b &= +m_b l_b g \sin\left(\theta_b\right) - m_b l_b \ddot{x}_w \cos\left(\theta_b\right) - \frac{K_t}{R_m} v_m + \left(\frac{K_e K_t}{R_m} + b_f\right)\left(\frac{\dot{x}_w}{l_w} - \dot{\theta}_b\right) \\
\left(\frac{I_w}{l_w} + l_w m_b + l_w m_w\right)\ddot{x}_w &= -m_b l_b l_w \ddot{\theta}_b \cos\left(\theta_b\right) + m_b l_b l_w \dot{\theta}_b^2 \sin\left(\theta_b\right) + \frac{K_t}{R_m} v_m - \left(\frac{K_e K_t}{R_m} + b_f\right)\left(\frac{\dot{x}_w}{l_w} - \dot{\theta}_b\right)
\end{aligned}
$$

Example: insulin concentration

- x_1 := sugar concentration
- x_2 := insulin concentration
- u_1 := food intake
- u_2 := insulin intake
- c := sugar concentration in fasting

$$\begin{cases} \dot{x}_2 = a_{21}\left(x_1 - c\right) - a_{22}x_2 + b_2u_2 & x_1 \geq c \\ \dot{x}_2 = -a_{22}x_2 + b_2u_2 & x_1 < c \end{cases}$$

$$\begin{cases} \dot{x}_1 = -a_{11}x_1x_2 - a_{12}\left(x_1 - c\right) + b_1u_1 & x_1 \geq c \\ \dot{x}_1 = -a_{11}x_1x_2 + b_1u_1 & x_1 < c \end{cases}$$

Very important case: what are the equilibria here?

$$\dot{\boldsymbol{y}} = A\boldsymbol{y}$$

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$$\dot{\boldsymbol{y}} = A\boldsymbol{y}$$

for linear systems $\mathbf{0}$ is always an equilibrium, and if $\overline{\boldsymbol{y}} \neq \mathbf{0}$ is also an equilibrium then every $\alpha\overline{\boldsymbol{y}}$ is an equilibrium too

The different types of stability properties of an equilibrium

Roadmap

- simple stability
- convergence
- asymptotic stability
- examples

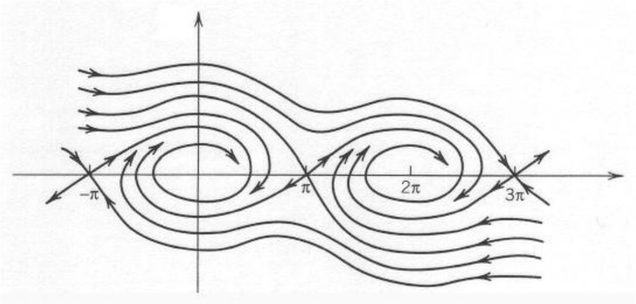
Roadmap

- simple stability
- convergence
- asymptotic stability
- examples

important assumption in this course: $\boldsymbol{u} = \overline{\boldsymbol{u}} = const.$

The different nature of different equilibria

pendulum with friction: $\ddot{\theta} = -\lambda\dot{\theta} - g\sin(\theta)$



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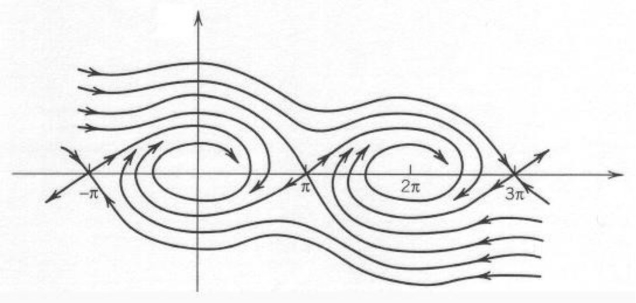
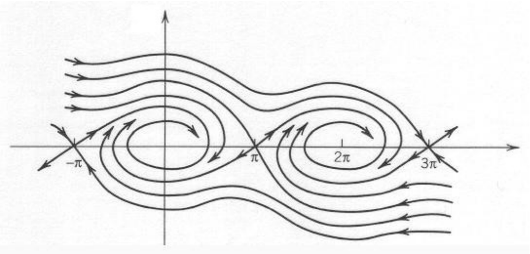


Discussion: why are these equilibria different?

Simply stable equilibrium (continuous time case)

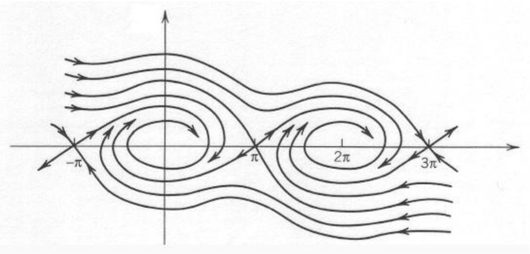


$\overline{\boldsymbol{u}} = \boldsymbol{u}_e = const.$

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}_e) \\ \boldsymbol{y} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}_e) \end{aligned}$$

$(\boldsymbol{x}_e, \boldsymbol{u}_e)$ = equilibrium

Simply stable equilibrium (continuous time case)



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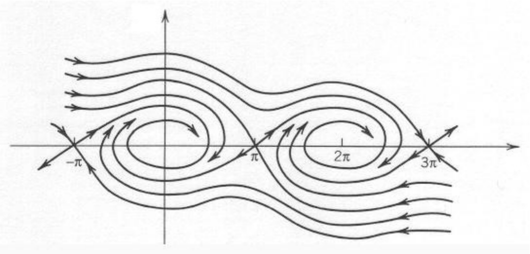
$(\boldsymbol{x}_e, \boldsymbol{u}_e)$ = equilibrium

Definition (simply stable equilibrium)

$\boldsymbol{x}_e$ *is simply stable if* $\forall \varepsilon > 0\ \exists \delta > 0$ *s.t. if* $\|\boldsymbol{x}_0 - \boldsymbol{x}_e\| \leq \delta$ *then* $\|\boldsymbol{x}(t) - \boldsymbol{x}_e\| \leq \varepsilon \quad \forall t \geq 0$

Convergent equilibrium



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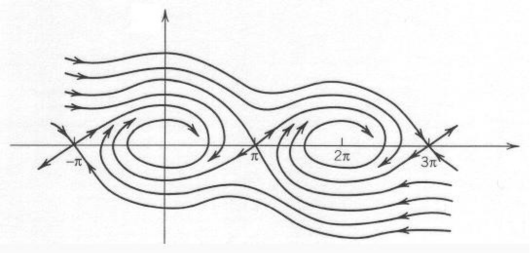

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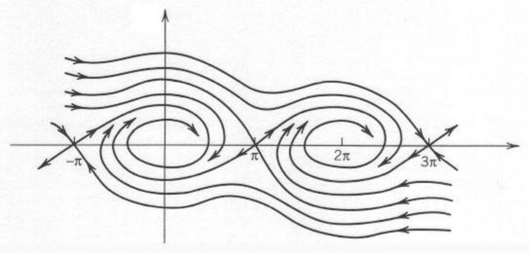
$\left(\boldsymbol{x}_e, \boldsymbol{u}_e\right)$ = equilibrium

Definition (convergent equilibrium)

$\boldsymbol{x}_e$ = *is convergent if* $\exists \delta > 0$ *s.t. if* $\|\boldsymbol{x}_0 - \boldsymbol{x}_e\| \leq \delta$ *then* $\boldsymbol{x}(t) \xrightarrow{t \to +\infty} \boldsymbol{x}_e$

Important differences



simple stability: I can confine arbitrarily the trajectory by reducing ε opportunely

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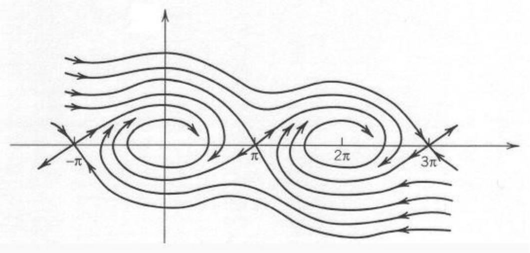


simple stability: I can confine arbitrarily the trajectory by reducing ε opportunely

convergent equilibrium: I *cannot* confine arbitrarily the trajectory, but I know that if I start close enough then *eventually* the distance $\|\boldsymbol{x}(t) - \boldsymbol{x}_e\|$ will go to zero

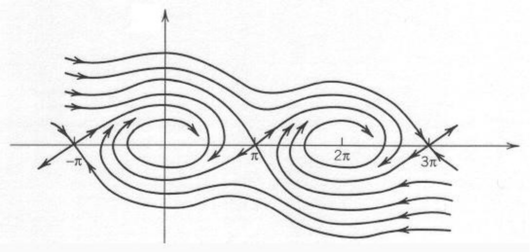
Discussion: Consider the discrete time system

$$x(k+1) = \begin{cases} 2x(k) & \text{if } |x(k)| < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Which type of equilibrium is 0? Possibilities:

1. a simply stable equilibrium
2. a convergent equilibrium
3. nothing special

Asymptotic stability



$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}\left(\boldsymbol{x}, \overline{\boldsymbol{u}}\right) \\ \boldsymbol{y} &= \boldsymbol{g}\left(\boldsymbol{x}, \overline{\boldsymbol{u}}\right) \end{aligned}$$

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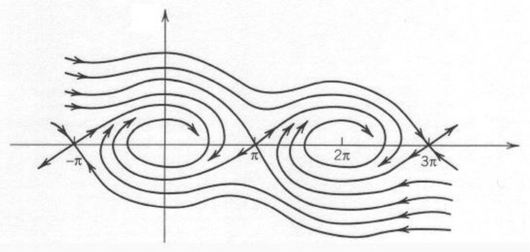

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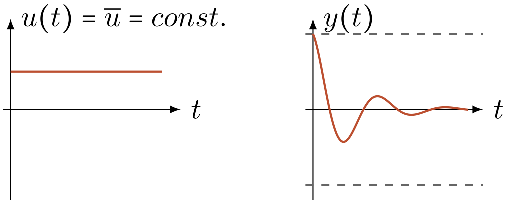
$\boldsymbol{x}_e$ = equilibrium

Definition (asymptotically stable equilibrium)

the equilibrium $\boldsymbol{x}_e$ is said to be asymptotically stable if it is simultaneously simply stable & convergent

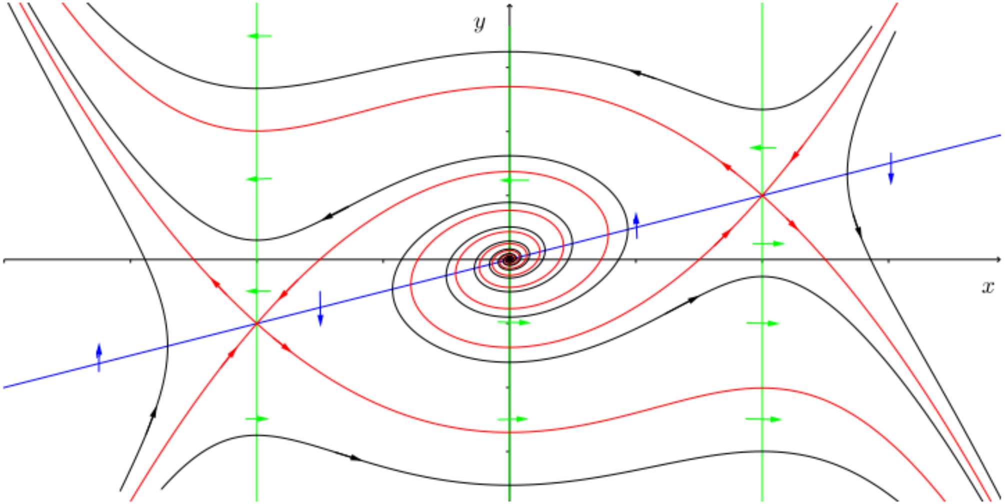
Asymptotic stability, graphically



Very important point

to have instability it is enough to have one trajectory that escapes (example: “constrained” flipped pendulum)

Some examples of phase portraits



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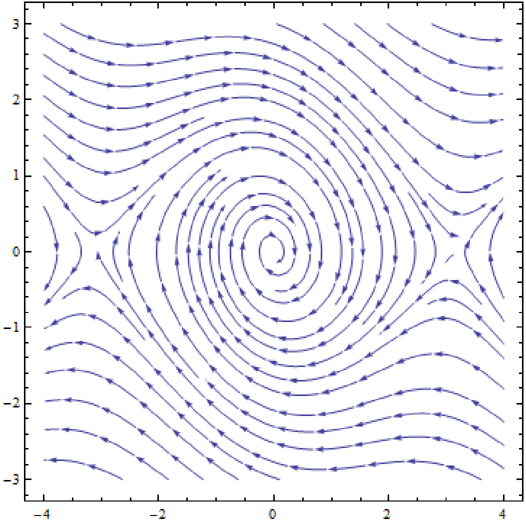

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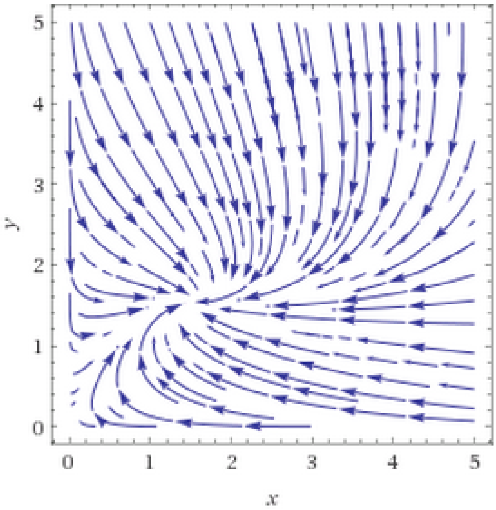

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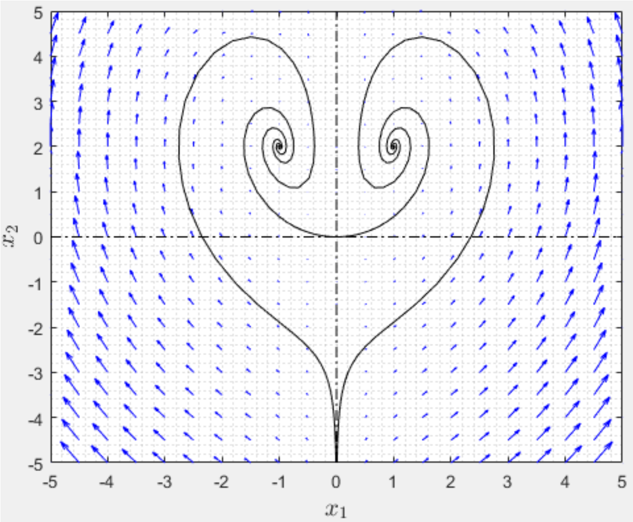

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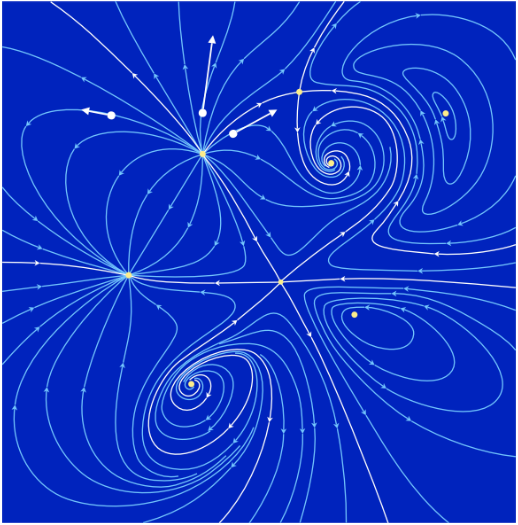

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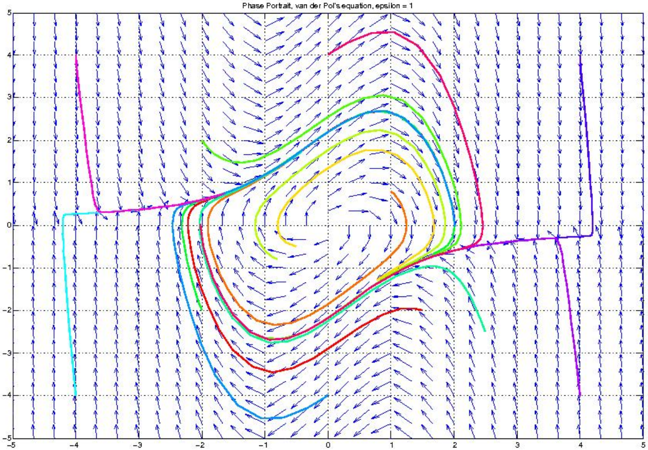

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BIBO stability

Roadmap

- generalizing the concept of stability
- BIBO stability
- connecting BIBO stability with the poles of the transfer functions

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- generalizing the concept of stability
- BIBO stability
- connecting BIBO stability with the poles of the transfer functions

important assumption in this course: $\boldsymbol{u} = \overline{\boldsymbol{u}} = const.$

Important: the term "stability" may refer to specific equilibrium points or specific systems

"Stability" referring to specific equilibria:

1. simply stable equilibrium
2. convergent equilibrium
3. asymptotically stable equilibrium

"Stability" referring to specific systems:

1. Bounded Input Bounded Output (BIBO) stable systems (we will see this now)
2. Input to State Stable (ISS) systems (we will not see this in this course)

BIBO stability

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{y} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u})\end{aligned}$$

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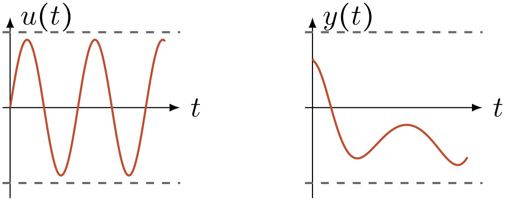
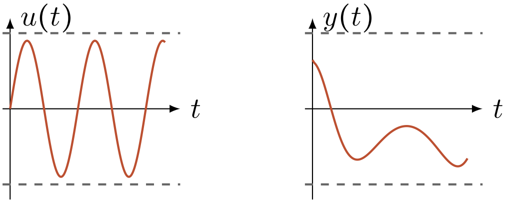
Definition (BIBO stability)

the system $(\boldsymbol{f}, \boldsymbol{g})$ *is said to be Bounded Input Bounded Output (BIBO) stable if*
$\|\boldsymbol{u}\| \leq \gamma_u \implies \|\boldsymbol{y}\| \leq \gamma_y$

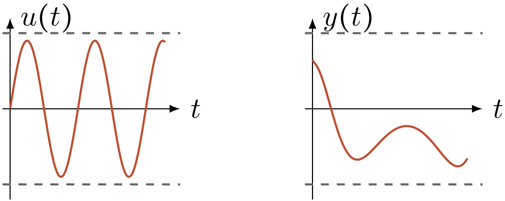


BIBO stability



Discussion: is the system $\dot{x} = x^2 u$ BIBO stable?

BIBO stability



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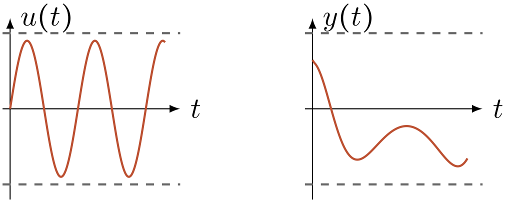
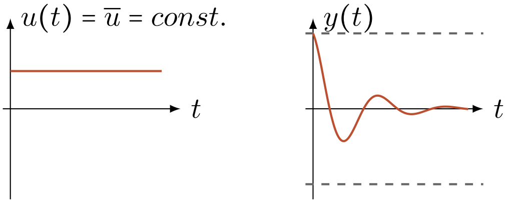
BIBO stability



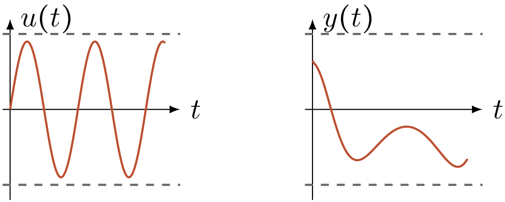
Discussion: is the system $\dot{x} = x^2 u$ BIBO stable? And the system $\dot{x} = -x^2 u$? To check BIBO stability one can use the "small gain theorem": not in this course! Here we will check the BIBO stability checking either the impulse response or the transfer function

Summarizing

Asymptotic stability



BIBO stability



The very important result that we will find now

For general nonlinear systems:

BIBO stable system $\neq$ asymptotically stable equilibria $\neq$ simply stable equilibria

For LTIs:

BIBO stable system $=$ asymptotically stable equilibria $\neq$ simply stable equilibria

Towards coupling the BIBO stability to the system's impulse response or transfer function

$$X(s) = H(s)U(s)$$

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Discussion points:

- $|u(t)| < M_u$ means a "non diverging" $u(t)$. Assuming that $u(t)$ has a rational Laplace transform, where do the poles of $U(s)$ live in this case?

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- what happens if $h(t)$ is converging to 0? Where are the poles of the associated $H(s)$? And may I choose some non-diverging $u(t)$ that makes $y(t)$ diverging?
- what happens if $h(t)$ is non-diverging and non-converging? May I choose some non-diverging $u(t)$ that makes $y(t)$ diverging?

BIBO stability = absolute integrability of the impulse response

BIBO stability:

$$|u(t)| < M_u \quad \Longrightarrow \quad |y(t)| < M_y$$

Impulse response:

$$y(t) = h * u(t) = \int_0^t h(\tau)u(t-\tau)d\tau$$

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$$\text{if } \int_{-\infty}^{+\infty} |h(\tau)|\, d\tau < +\infty \text{ then BIBO stability}$$

Coupling the BIBO stability concept with the poles of a TF

$$\text{if } \int_{-\infty}^{+\infty} |h(\tau)| \, d\tau < +\infty \text{ then BIBO stability}$$

$$H(s) = \frac{N(s)}{D(s)}$$

Discussion: if we want the system be BIBO stable, may $D(s)$ have poles on the imaginary axis?

Important result

BIBO stable LTI system = all the poles have strictly negative real part

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all the poles have strictly negative real part = asymptotically stable equilibria

asymptotically stable equilibria $\neq$ simply stable equilibria

?

Nomenclature

BIBO stable LTI system = LTI with all its equilibria asymptotically stable

marginally stable LTI system = LTI with all its equilibria simply stable

unstable LTI system = LTI with unstable equilibria

Examples: are these systems BIBO stable, marginally stable, or unstable?

$$H(s) = \frac{1}{(s+2)(s+1)} \tag{3}$$

$$H(s) = \tag{4}$$

$$H(s) = \tag{5}$$

$$H(s) = \tag{6}$$

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Examples: are these systems BIBO stable, marginally stable, or unstable?

$$H(s) = \frac{1}{(s+2)(s+1)} \qquad \Longrightarrow \quad Ae^{-2t} + Be^{-t} \tag{3}$$

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And what about nonlinear systems?

more complicated! Will treat this through Lyapunov theory in more advanced courses

The very important result that we found

For general nonlinear systems:

BIBO stable system $\neq$ asymptotically stable equilibria $\neq$ simply stable equilibria

For LTIs:

BIBO stable system $=$ asymptotically stable equilibria $\neq$ simply stable equilibria

Summarizing, once again

Different types of system stability:

asymptotic input-output (system) stability: independently of $u(t)$, $x(t) \to 0$ when $t \to +\infty$

marginal (or simply input-output) (system) stability: as soon as $|u(t)| < M_u$, $|x(t)| < M_x$ when $t \to +\infty$

(system) instability: there exists at least one signal $u(t)$ for which we cannot do the bound $|x(t)| < M_x$ when $t \to +\infty$

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