2D Dipole Model of Maggy

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1 Introduction

This document presents a 2D of Maggy, illustrated in Fig. 1, where all magnets are assumed to be infinitesimal dipoles (we only consider the far fields) and the solenoids simply augment the magnetization of the permanent magnets in the base.

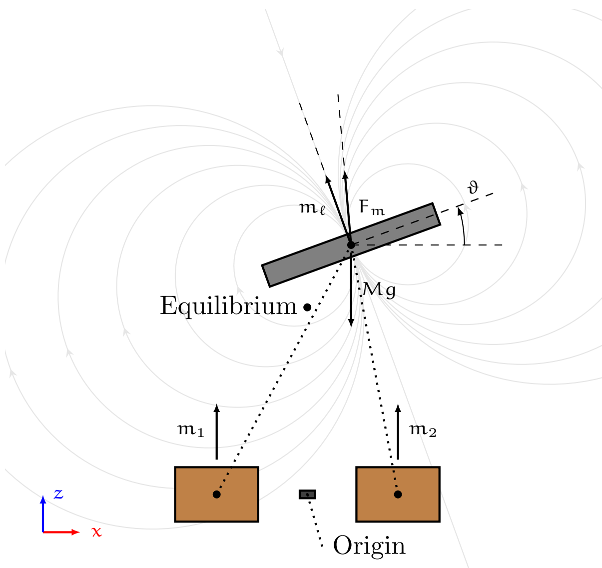

Figure 1: An illustration of the magnetic levitation system.

The levitating magnet, with mass M and moment of inertia J, is characterized by the dipole moment

$$\mathbf{m}_\ell = m_\ell \begin{bmatrix} -\sin\vartheta \\ \cos\vartheta \end{bmatrix},$$

where $m_\ell := \|\mathbf{m}_\ell\|$. Each solenoid, located at $(x_{s,i}, z_{s,i})$ for $i = 1, 2$, has an effective dipole moment

$$m_i = m_{i,\text{nom}} + kI_i, \quad k = n_w \pi r_{s,i}^2,$$

with I_i representing the current in solenoid i. The total magnetic scalar potential, obtained by summing the contributions from both solenoids, governs the magnetic field that exerts force and torque on the levitating magnet. The state of the system is described by

$$\mathbf{x} = \begin{bmatrix} x & z & \vartheta & \dot{x} & \dot{z} & \dot{\vartheta} \end{bmatrix}^\mathsf{T},$$

with x and z the position and ϑ the orientation.

2 Magnetic Field and Scalar Potential

For solenoid i, the magnetic scalar potential is

$$\psi_i = \frac{m_i(z - z_{s,i})}{4\pi\left[(x - x_{s,i})^2 + (z - z_{s,i})^2\right]^{3/2}}, \tag{1}$$

and the total potential is

$$\psi_{tot} = \psi_1 + \psi_2. \tag{2}$$

The resulting magnetic field is obtained by

$$\mathbf{B} = -\mu_0 \nabla \psi_{tot} = -\mu_0 \begin{bmatrix} \frac{\partial \psi_{tot}}{\partial x} \\ \frac{\partial \psi_{tot}}{\partial z} \end{bmatrix}. \tag{3}$$

3 Force, Torque, and System Dynamics

The magnetic force acting on the levitating magnet, derived from the gradient of the dot product $\mathbf{m}_\ell \cdot \mathbf{B}$, is

$$\begin{aligned} \mathbf{F}_m &= m_\ell \mu_0 \nabla \left(\frac{\partial \psi_{tot}}{\partial x} \sin\vartheta - \frac{\partial \psi_{tot}}{\partial z} \cos\vartheta \right) \\ &= m_\ell \mu_0 \begin{bmatrix} \frac{\partial^2 \psi_{tot}}{\partial x^2} \sin\vartheta - \frac{\partial^2 \psi_{tot}}{\partial x \partial z} \cos\vartheta \\ \frac{\partial^2 \psi_{tot}}{\partial x \partial z} \sin\vartheta - \frac{\partial^2 \psi_{tot}}{\partial z^2} \cos\vartheta \end{bmatrix}. \end{aligned} \tag{4}$$

The corresponding torque is

$$\tau_m = m_\ell \mu_0 \left(\sin\vartheta \frac{\partial \psi_{tot}}{\partial z} + \cos\vartheta \frac{\partial \psi_{tot}}{\partial x} \right). \tag{5}$$

Using Newton-Euler's equations of motion, the dynamics of the levitating magnet are described by

$$\ddot{x} = \frac{m_\ell \mu_0}{M} \left(\frac{\partial^2 \psi_{tot}}{\partial x^2} \sin\vartheta - \frac{\partial^2 \psi_{tot}}{\partial x \partial z} \cos\vartheta \right), \tag{6a}$$

$$\ddot{z} = \frac{m_\ell \mu_0}{M} \left(\frac{\partial^2 \psi_{tot}}{\partial x \partial z} \sin\vartheta - \frac{\partial^2 \psi_{tot}}{\partial z^2} \cos\vartheta \right) - g, \tag{6b}$$

$$\ddot{\vartheta} = \frac{m_\ell \mu_0}{J} \left(\sin\vartheta \frac{\partial \psi_{tot}}{\partial z} + \cos\vartheta \frac{\partial \psi_{tot}}{\partial x} \right). \tag{6c}$$

4 Scalar Potential Derivatives

For solenoid i, the first derivatives of the scalar potential are

$$\frac{\partial \psi_i}{\partial x} = -\frac{3\, m_i (x - x_{s,i})(z - z_{s,i})}{4\pi\left[(x - x_{s,i})^2 + (z - z_{s,i})^2\right]^{5/2}}, \tag{7}$$

$$\frac{\partial \psi_i}{\partial z} = \frac{m_i}{4\pi} \left[\frac{1}{\left[(x - x_{s,i})^2 + (z - z_{s,i})^2\right]^{3/2}} - 3 \frac{(z - z_{s,i})^2}{\left[(x - x_{s,i})^2 + (z - z_{s,i})^2\right]^{5/2}} \right]. \tag{8}$$

The mixed and second derivatives are

$$\frac{\partial^2\psi_i}{\partial x\partial z} = -\frac{3\,m_i(x-x_{s,i})}{4\pi}\left[\frac{1}{[(x-x_{s,i})^2+(z-z_{s,i})^2]^{5/2}} - 5\,\frac{(z-z_{s,i})^2}{[(x-x_{s,i})^2+(z-z_{s,i})^2]^{7/2}}\right], \tag{9}$$

$$\frac{\partial^2\psi_i}{\partial x^2} = -\frac{3\,m_i(z-z_{s,i})}{4\pi}\left[\frac{1}{[(x-x_{s,i})^2+(z-z_{s,i})^2]^{5/2}} - 5\,\frac{(x-x_{s,i})^2}{[(x-x_{s,i})^2+(z-z_{s,i})^2]^{7/2}}\right], \tag{10}$$

$$\frac{\partial^2\psi_i}{\partial z^2} = -\frac{3\,m_i(z-z_{s,i})}{4\pi}\left[\frac{3}{[(x-x_{s,i})^2+(z-z_{s,i})^2]^{5/2}} - 5\,\frac{(z-z_{s,i})^2}{[(x-x_{s,i})^2+(z-z_{s,i})^2]^{7/2}}\right]. \tag{11}$$

Since $\psi_{\mathrm{tot}} = \psi_1 + \psi_2$, its derivatives are the sums of the corresponding expressions for $i = 1$ and 2.