Robotics

Estimation and Learning
with Dan Lee

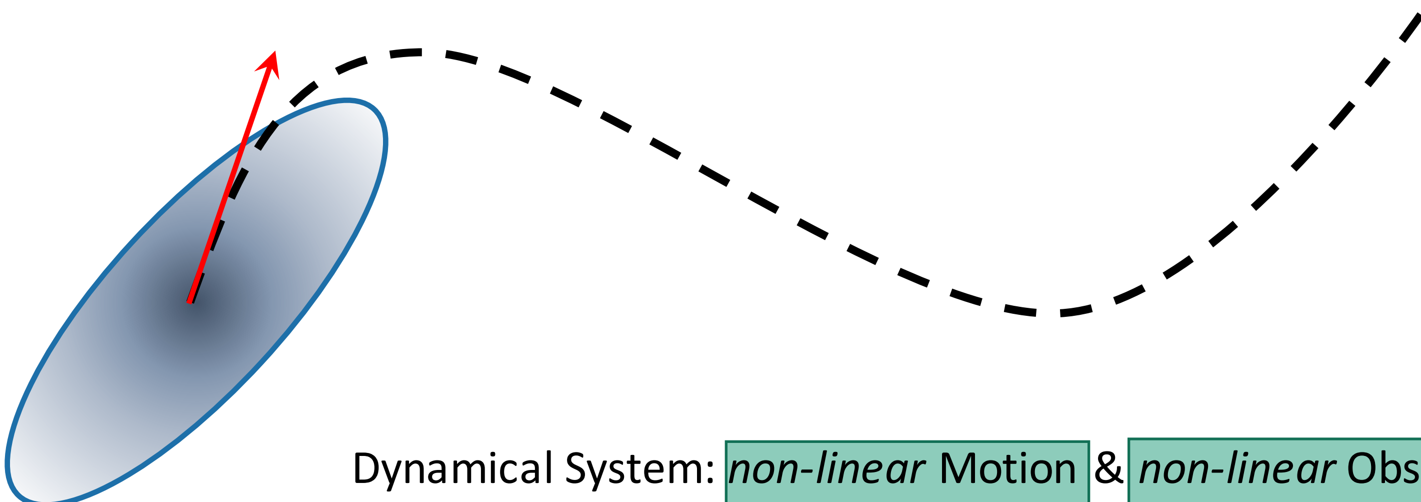
Week 2.
Kalman Filter

2.4 Non-linear Kalman filters

Penn Engineering

Extended Kalman Filter

- Linearize around the transition function
  - Jacobian represents the derivative in matrix form
- Calculate the uncertainty update via linearization



Dynamical System: *non-linear* Motion & *non-linear* Observation

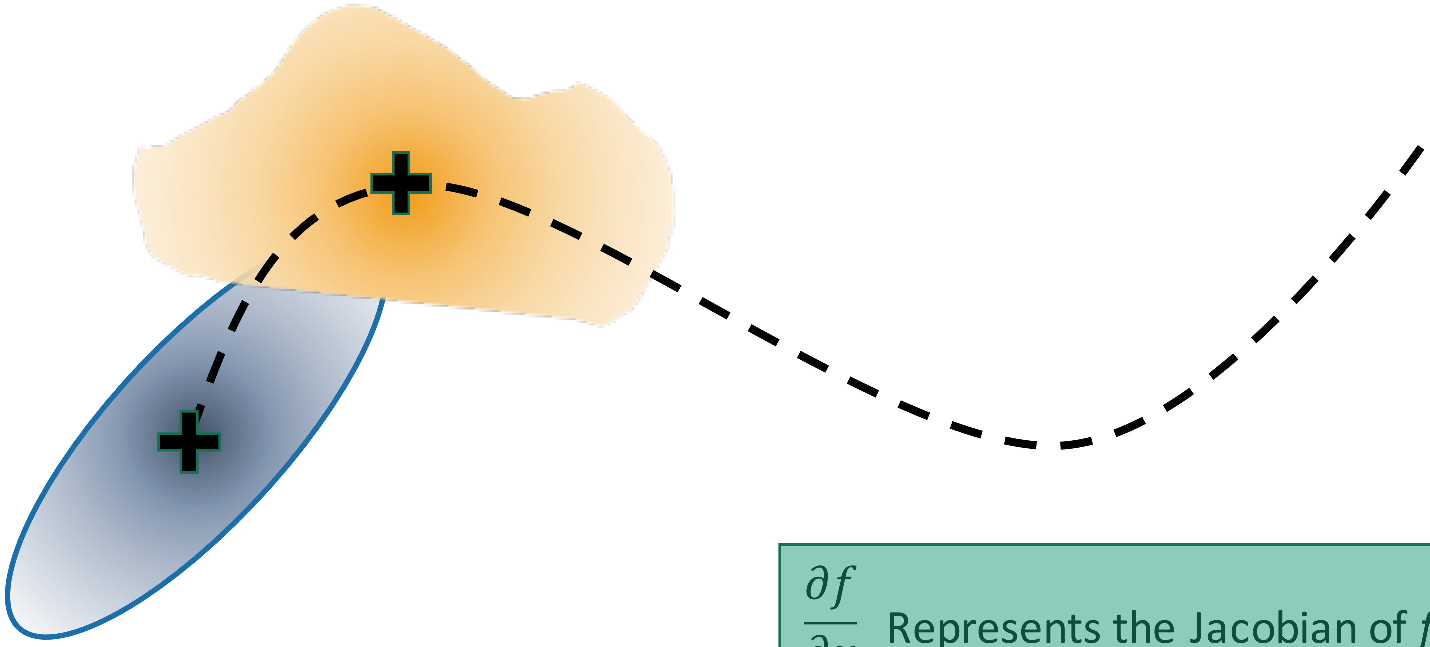
$x_{t+1} = f(x_t)$ $z_{t+1} = h(x_t)$

Extended Kalman Filter

- Covariance prediction

$$p(x_{t+1}|x_t) = \mathcal{N}(Ax_t, AP_tA^T + \Sigma_m)$$

$$p(x_{t+1}|x_t) = \mathcal{N}\left(f(x_t), \frac{\partial f}{\partial x} P_t \frac{\partial f}{\partial x}^T + \Sigma_m\right)$$



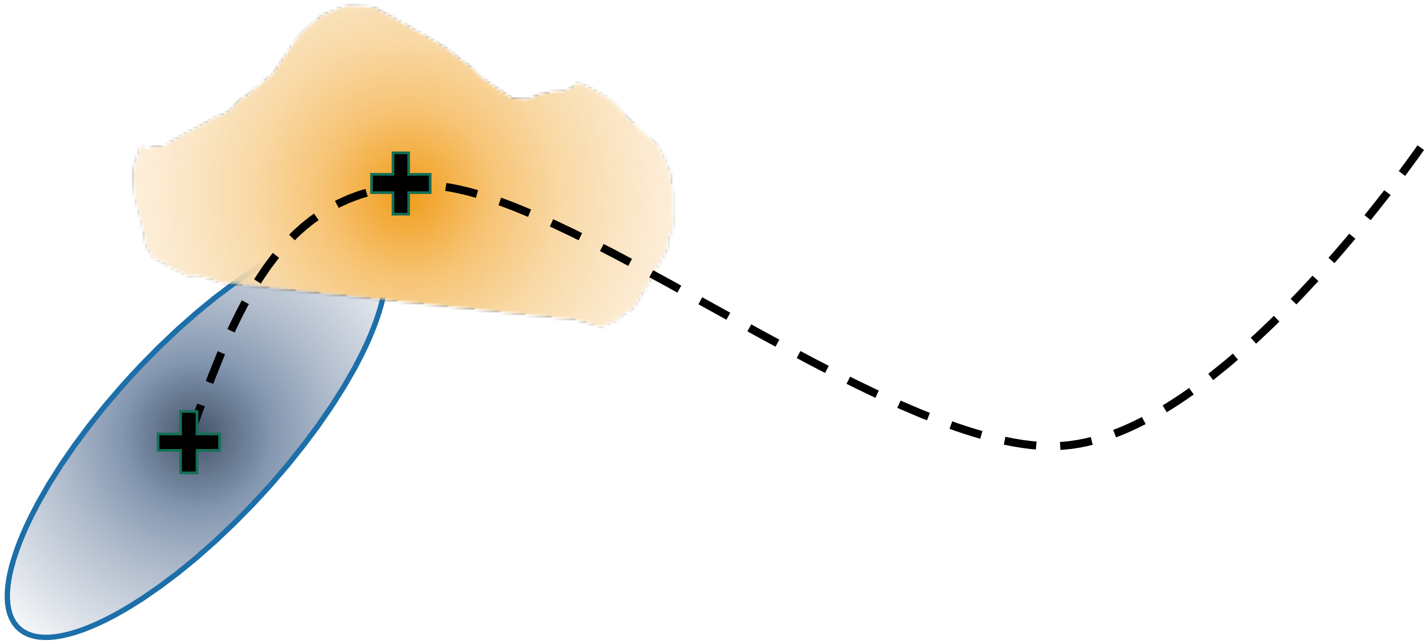
$\frac{\partial f}{\partial x}$ Represents the Jacobian of f

Extended Kalman Filter

- Kalman Gain

$$K = PC^T(\Sigma_o + CPC^T)^{-1}$$

$$K = P\frac{\partial h^T}{\partial x}\left(\Sigma_o + \frac{\partial h}{\partial x}P_t\frac{\partial h^T}{\partial x}\right)^{-1}$$

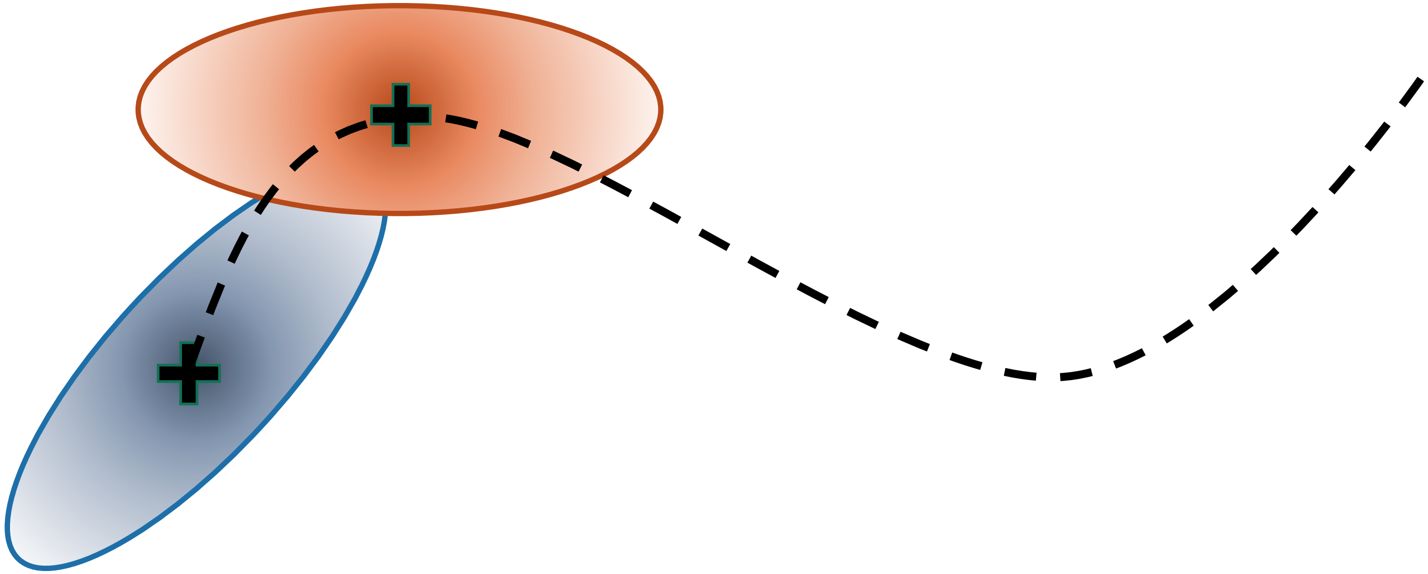


Extended Kalman Filter

- Overall update

$$\hat{x}_t = f(x_{t-1}) + K(z_t - h(f(x_t)))$$

$$\hat{P}_t = P - K\frac{\partial h}{\partial x}P$$

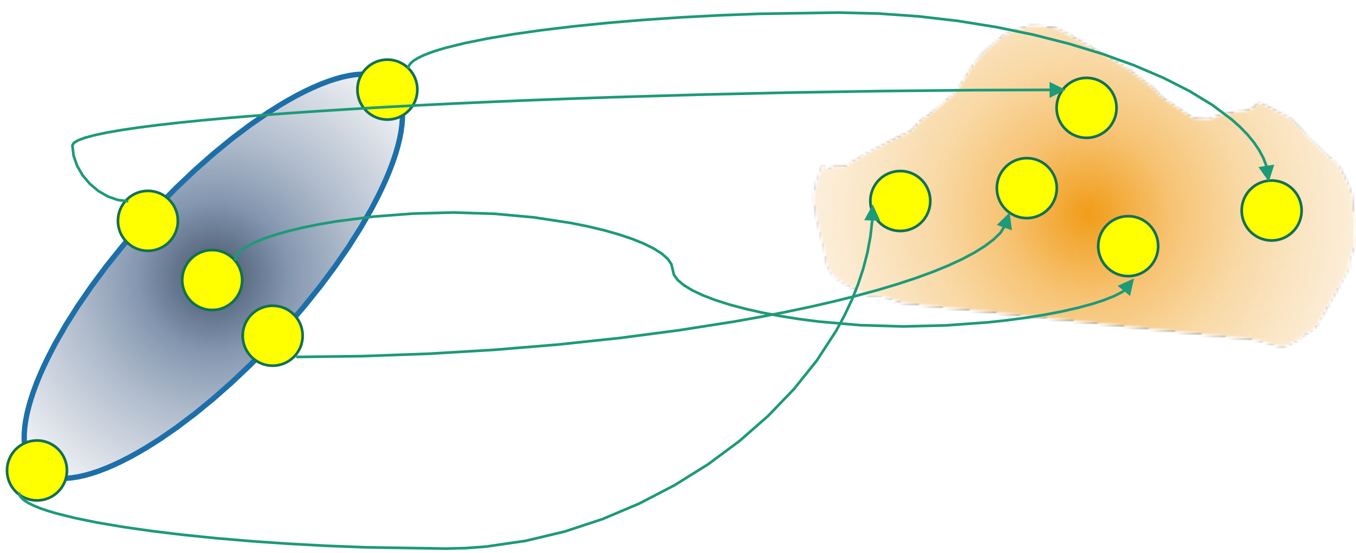


Unscented Kalman Filter

- Noise distribution characterized by a set of points
- Transform these points in non-linear fashion
- Re-estimate mean and covariance for recalculating Gaussian distribution to characterize uncertainty

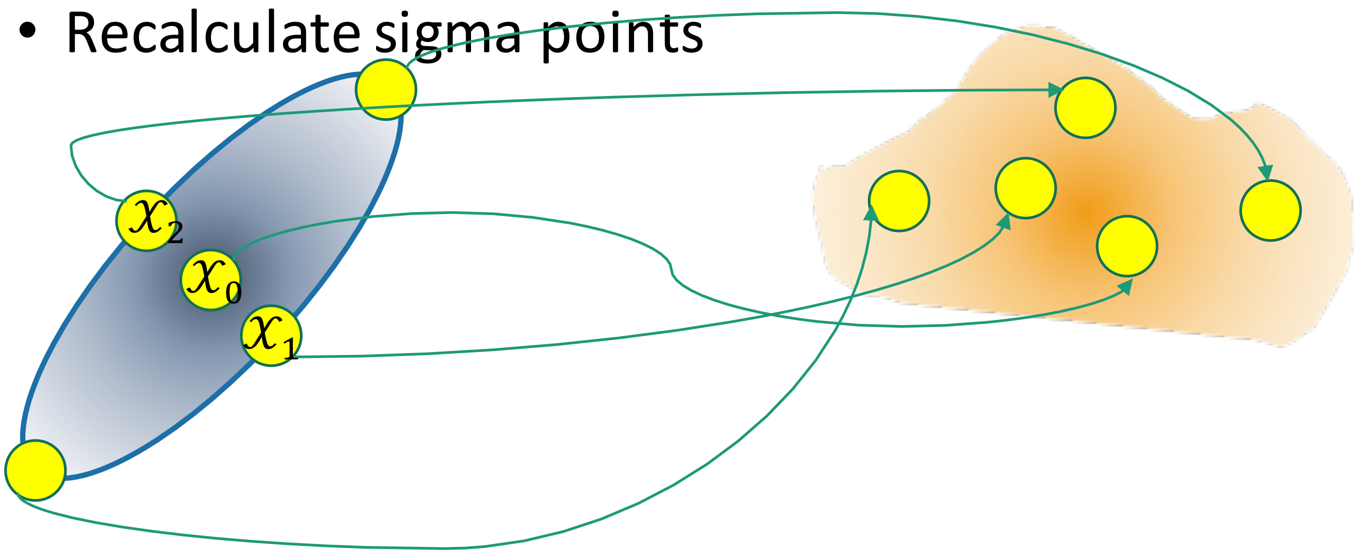
Unscented Kalman Filter

- Model the distribution based on a set of *sigma points*
  - Statistics of these points captures the distribution



Unscented Kalman Filter

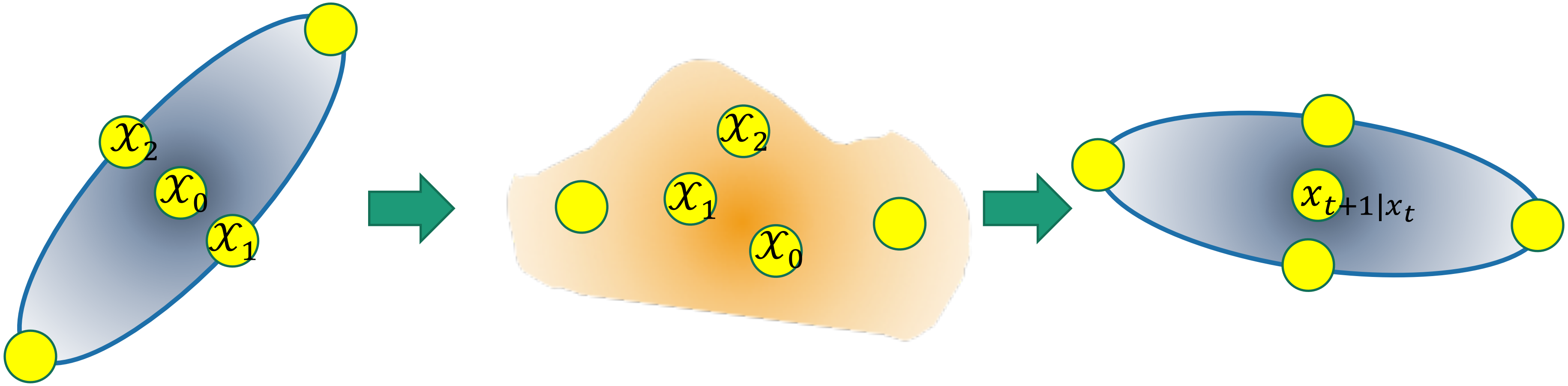
- Approximate new distribution by computing the statistics of the *sigma points* χ_i
- Approximate a Gaussian with first two moments
  - Recalculate sigma points



Unscented Kalman Filter

- State prediction
  - Mean of Sigma points run through dynamic system

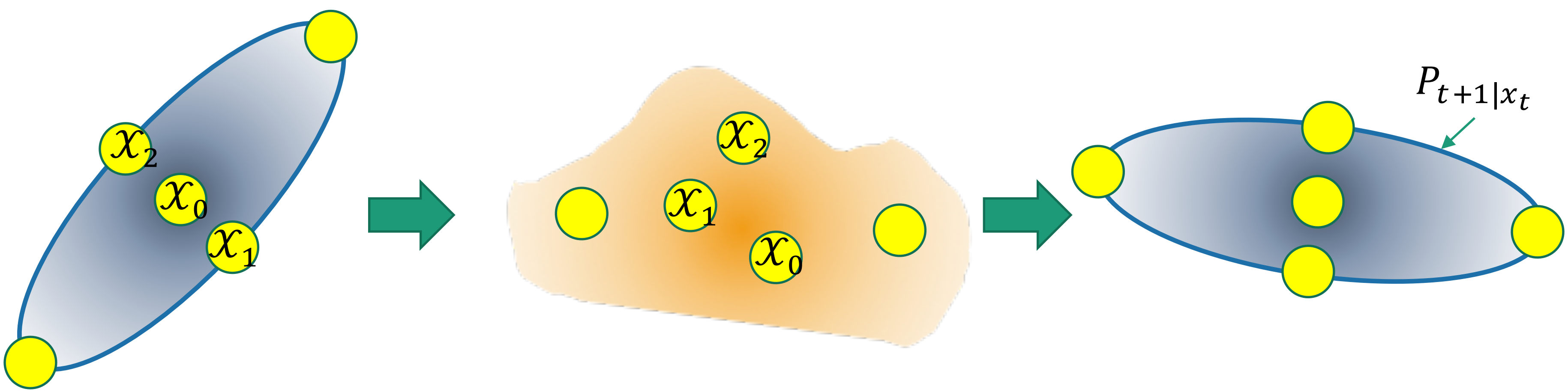
$$x_{t+1|x_t} = \frac{1}{N_{\mathcal{X}}} \sum_i f(\mathcal{X}_i)$$



Unscented Kalman Filter

- Uncertainty prediction
  - Covariance of sigma points run through dynamic system

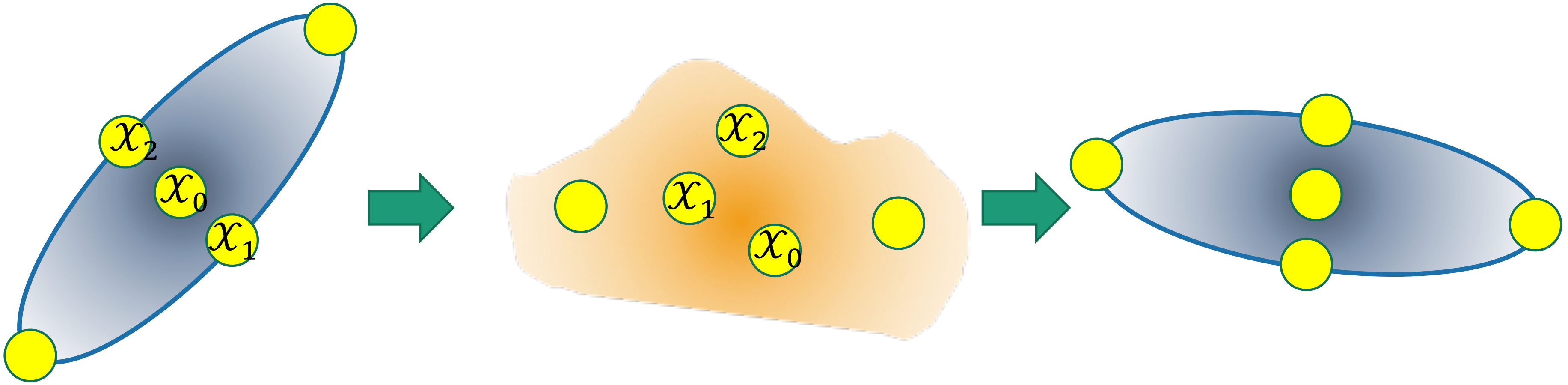
$$P_{t+1|x_t} = \frac{1}{N_{\mathcal{X}}} \sum_i \left(f(\mathcal{X}_i) - x_{t+1|x_t}\right)\left(f(\mathcal{X}_i) - x_{t+1|x_t}\right)^T$$



Unscented Kalman Filter

- Expected Observation
  - Mean of Sigma points' expected obsrvation

$$z_{t+1|x_t} = \frac{1}{N_{\mathcal{X}}} \sum_i h(f(\mathcal{X}_i))$$

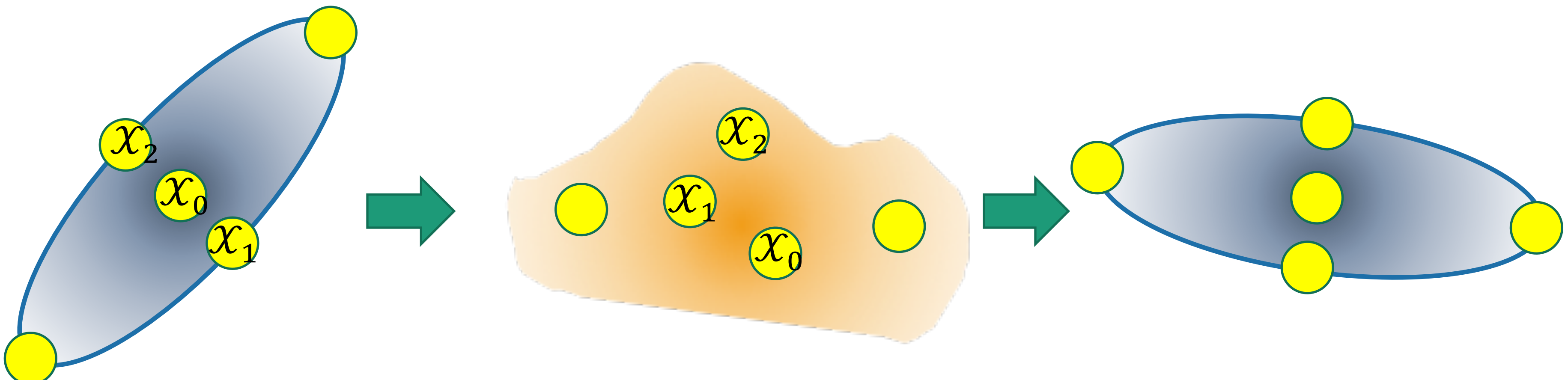


Unscented Kalman Filter

Recall Linear System
$K = PC^T(\Sigma_o + CPC^T)^{-1}$

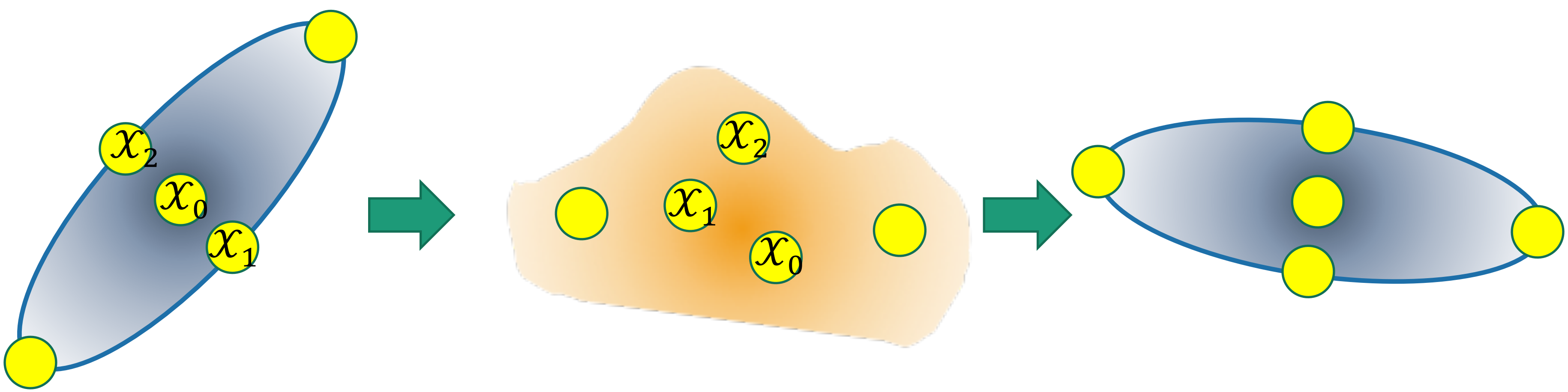
- Kalman Gain
  - Utilize sigma points, not observation model

$$K = \frac{1}{N_\chi}\sum_i \left(f(\mathcal{X}_i) - x_{t+1|x_t}\right)\left(h\left(f(\mathcal{X}_i)\right) - z_{t+1|x_t}\right)^T$$
$$\cdot\left(\frac{1}{N_\chi}\sum_i \left(h\left(f(\mathcal{X}_i)\right) - z_{t+1|x_t}\right)\left(h\left(f(\mathcal{X}_i)\right) - z_{t+1|x_t}\right)^T\right)^{-1}$$



Unscented Kalman Filter

- Update just as the linear filter
  - The covariance update will be slightly different
- See notes for good resources on further details



Particle Filter

- Take the unscented filter to the limit, and use many points to characterize the distribution
- Distribution is not limited to Gaussian
- Discuss in Week 4