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On Linearity in OptiFlow

PRELIMINARY - DO NOT QUOTE OR FORWARD

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Summary

Here is the hypothesis: ”A problem that has a network topology [to be defined], and that may be formulated as an LP problem may also be formulated as a generalized network model.”

Argumentation

Originally OptiFlow was developed in the context of solid waste handling, and was then called OptiWaste. Due to the generality of the formulation of that model it may have applications outside the waste handling sector, therefore it was renamed to OptiFlow.

OptiFlow is formulated as a generalized network model. As it is formulated fairly general it is of value to investigate the range of applications of the model, this is the aim of the present document. More specifically, possible limitations to the modelling will be discussed in relation to a traditional linear programming modelling.

This approach is motivated by two observations. First, because linear programming (LP in the following) is well understood and widely applied, so it serves as a good framework of reference. Second, because it has been questioned whether a generalized network model may handle the same range of problems as an LP model can (James W. Levis, Morton A. Barliz, Joseph F. DeCarolis, S. Ranji Ranjithan: A generalized multistage modeling framework for life cycle assessment-based integrated solid waste management, Environmental Modelling & Software 50 (2013) 51-65).

As an illustration, consider the constraint matrix A of an LP problem formulation, associated with a general formulation of a LP problem

$$\max c'x \tag{1}$$
$$Ax = b \tag{2}$$

Lower and/or upper bounds on individual variables are added according to conventions and needs.

The matrix A may for instance be as illustrated in Table 1. The matrix may represent ....

Note that although the matrix in Table 1 has only coefficients 1, -1 and 0, this is not the case in general.

TODO: Comments on the objective function - it is not needed for the discussion...

Now consider a reformulation of the LP in the direction of a generalized network model, the constraint matrix of this is illustrated in Table 2. The correspondence between the two formulations should be somewhat observable.

In particular note that the formulation in Table 2 does not apply all of the same variables as shown in Table 1, for instance, variable $v2$. Thus, variable $v1$ in Table 2 represents two variable, $v1$ and $v2$. This is possible because values of $v1$ and $v2$ in Table 1 are in proportions $b1/b2$, and values for the pairs $v3$ and $v4$, $v5$ and $v6$, $v7$ and $v8$, $v3$ and $v10$ are also proportional with the same value $b1/b2$, this follows from equations $q1 - q6$ in Table 1. The same holds true for other pairs of variables in Table 1.

In the other hand, the formulation in Table 2 applies some variables and equations that are not present in Table 1. They are introduced in order to maintain linearity. The main construction is application of a Transform process (in the OptiFlow terminology), shown as equations $q20-q23$ in Table 2. Some of the variables related to those processes are new, viz. $v51 - v52$ and $v131-v132$. [Note that this is discussed on the OptiWaste documentation, version 0.95, Section 2.3]

Another difference is that some of the coefficients in corresponding places in the two matrixes differ. This is explained by the additional Transform processes in Table 2. The values of the coefficients may be derived from the corresponding values related to the formulation in Table 1, taking into account also the units applied in the two formulations.

As an example consider Source2. The LP formulation ... $b11 - b12$ ... [Here I derive a_{22}^{131} and a_{23}^{132}]

So as seen [after further argumentation .... to come], a generalized network model may be formulated to represent the same problem as modelled as LP in Table 1.

Here is the statement we want to arrive at: "A problem that has a network topology [to be defined], and that may be formulated as an LP problem may also be formulated as a generalized network model."

	v 1	v 2	v 3	v 4	v 5	v 6	v 7	v 8	v 9	v 10	v 11	v 12	v 13	v 14	v 15	v 16	v 17	v 18	v 19	v 20	v 21	v 22	v 23	v 24	v 25	v 26	v 27	v 28	
q1	1		-1																										Source1
q2		1		-1																									Source1
q3			1		-1		-1																						VSplit
q4				1		-1		-1																					VSplit
q5							1		-1																				Sink1
q6								1		-1																			Sink1
q7											1		-1																Source2
q8												1		-1															Source2
q9															1			-1											Source3
q10																1			-1										Source3
q11																	1			-1									Source3
q12																		1			-1								Transf4
q13																			1			-1							Transf4
q14																				1			-1						Transf4
q15																							1	-1					Sink4
q16					1								1								1				-1				VJoin1
q17						1								1								1				-1			VJoin2
q18																									1		-1		Sink2
q19																										1		-1	Sink3

Table 1: LP formulation of the situation illustrated in Figure ... The table shown is the constraint matrix. Variables $v1 - v28$ are shown as columns, equations $q1 - q19$ are shown as rows. Variables are assumed positive (i.e., non-negative) (NB: SINKS?), equations are all of the equality type, and all right hand sides are 0. Variables $v1 - v2$, $v11 - v12$ and $v16 - v18$ are fixed at non-negative values $b1 - b2$, $b11 - b12$ and $b16 - b18$ respectively (UPS: Notation?). By convention flow entering (NB: men vi har jo ikke 'FLOW' ...

	v1	v3	v5	v7	v9	v11	v13	v15	v18	v21	v22	v23	v24	v25	v26	v27	v28	v51	v52	v131	v132	
q1	1	-1																				Source1
q3		1	-1	-1																		VSplit
q5				1	-1																	Sink1
q7						1	-1															Source2
q9								1	-1													Source3
q12								1		-a^{21}_{12}												Transf4
q13								1	1		-a^{22}_{13}											Transf4
q14								1	1			-a^{23}_{14}										Transf4
q15												1	-1									Sink4
q16										1				-1				1		1		VJoin1
q17											1				-1				1		1	VJoin2
q18														1		-1						Sink2
q19															1		-1					Sink3
q20			1															-a^{51}_{20}				Transf1
q21			1																-a^{52}_{21}			Transf1
q22							1													-a^{131}_{22}		Transf2
q23							1														-a^{132}_{23}	Transf2

Table 2: Network formulation of the LP from Table 1. The network is illustrated in Figure ... To highlight the correspondence between the two formulations, as many as possible of the names of variables and equations are reused here.