Course Code: BTMAT119B/A

Max Time: 3 hours

Course Title: Mathematics-II

Max Marks: 70

Instructions:

Question Number **one** (PART-I) is compulsory and carries total 14 marks (Each sub Question carries two Marks). Question Numbers 2(two) to 5(five) carry fourteen marks each with internal choice.

Note: Normal table is provided with this paper. Use of scientific calculator allowed.

PART -I

Q. No.1

(a) A distribution consists of three components with frequencies 300, 200 and 600, having their mean 16, 8 and 4 respectively. Find the mean of combined distribution. [2]

(b) What are the characteristics of probability mass function of a discrete random variable. [2]

(c) A random variable X has mean 12 and variance 9 and an unknown probability distribution. Using Chebyshev's theorem, estimate $P(6<X<18)$. [2]

(d) Let X be a random variable with density function [2]

$$f(x) = \begin{cases} x^2/3, & -1 < x < 2 \\ 0, & elsewhere \end{cases}$$

Find the variance of the random variable $g(X) = 4X+3$.

(e) What are the different types of Sampling? [2]

(f) State Central Limit theorem with suitable diagram/example. [2]

(g) What is the limiting form of Student's t distribution when $n \to \infty$. [2]

PART –II

Q. No.2

(a) Two airplanes bomb a target in succession. The probability of each correctly scoring a hit is 0.3 and 0.2 respectively. The second will bomb only if the first misses the target. Find the probability that (a) target is hit, (b) both fails to score hits. [7]

(b) The first four moments of distribution about the value 5 of the variable are 2, 20, 40 and 50. Obtain as far as possible the various characteristics of the distribution on the basis of the information given. [7]

OR

(a) A random variable X has the following probability [7]

X	1	2	3	4	5	6	7
P(x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find k, distribution function and minimum value of a such that

$$P(X \le a) > 1/2$$

(b) Assume that a firm has selected a random sample of 100 from its production line and has obtained the data shown in the table below: [7]

Class interval	Frequency	Class interval	Frequency
130-134	3	150-154	19
135-139	12	155-159	12
140-144	21	160-164	5
145-149	28		

Compute Karl Pearson's coefficient of skewness.

Q. No.3

(a) Let X and Y be two random variables each taking three values -1,0, and 1, and having joint probability distribution [7]

Y↓ X→	-1	0	1
-1	0	0.1	0.1
0	0.2	0.2	0.2
1	0	0.1	0.1

Show that X and Y have different expectation and are uncorrelated. Also find variance of Y.

(b) Seven coins are tossed and number of heads noted. The experiment is repeated 128 times and the following distribution is obtained: [7]

No. of Heads	0	1	2	3	4	5	6	7
Frequencies	7	6	19	35	30	23	7	1

Calculate the error when fitted with a Binomial Distribution assuming that the coin is unbiased.

OR

(a) The variable X and Y are connected by the equation $aX + bY + c = 0$. Show that the correlation between them is -1 if the signs of a and b alike and 1 if they are different. [7]

(b) A manufacturer, who produces medicines bottles, find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain: [7]
(i) no defective, and (ii) at least two defectives.

Q. No.4

(a) A continuous random variable X has a probability density function given by [7]
$$f(x) = \begin{cases} kxe^{-\theta x}, & x \geq 0, \ \theta > 0 \\ 0, & otherwise \end{cases}$$
Determine the constant k and obtain the mean and variance of X.

(b) An electrical firm manufacturers light bulb that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. [7]

OR

(a) The joint probability distribution of two random variables X and Y is given by: [7]
$$P(X = 0, Y = 1) = \frac{1}{3}, P(X = 1, Y = -1) = \frac{1}{3}, and \ P(X = 1, Y = 1) = \frac{1}{3}.$$
Find marginal distributions of X and Y and the conditional probability distribution of X given Y = 1.

(b) The pressure of the gas corresponding to various volumes V is measured, given by the following data: [7]

$V\ (cm^3)$	50	60	70	90	100
$P\ (kg/cm^2)$	64.7	51.3	40.5	25.9	78

Fit the data to the equation $PV^Y = c$.

Q. No.5

(a)   A manufacturer of lens is qualifying a new grinding a new grinding machine **[7]**
and will qualify the machine if the percentage of polished lenses that contain
surface defect does not exceed 2%. A random sample of 250 lenses contain
six defective lenses. Formulate and test an appropriate set of hypotheses to
determine if the machine can be qualified.

(b)   The guaranteed average life of a certain type of electric light bulb is 1000 hrs **[7]**
with a standard deviation of 125 hrs. It is decided to sample the output so as
to ensure that 90% of the bulbs do not fall short of the guaranteed average by
more than 2.5%. What must be the minimum size of the sample.

OR

(a)   A sample analysis of examination results of 200 MBA's was made. It was found that **[7]**
46 students had failed, 68 secured a third division, 62 secured a second division and
the rest were placed in first division. Are these figures commensurate with the general
examination result which is in the ratio of 4:3:2:1 for various categories respectively?
[Given $\chi^2_{0.05}\big|_4 = 9.488$ and $\chi^2_{0.05}\big|_3 = 7.815$

(b)   Two random samples gave the following results **[7]**

Sample	Size	Sample Mean	Sum of squares of deviation from mean
1	12	14	108
2	10	15	90

Test whether the same come from the same normal population at 5% level of
significance.
[Given $F_{0.05}(9,11) = 2.90$; $F_{0.05}(11,9) = 3.10$; $t_{0.05}(20) = 2.086$]

STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998