Central University of Haryana
Odd Semester Term End Examination Mar 2023
B. Tech. Programme
Branch: Computer science & Engineering

Course Code: BT MAT 111B
Course Title: Mathematics-I

Max Time: 3 Hours
Max Marks: 70

Instructions:
Question Number **one (PART-I)** is compulsory and carries a total of 14 marks (Each sub-Question carries two Marks).
Question Numbers 2(two) to 5(five) carry fourteen marks each with internal choice.

PART -I

Q 1.

a) Find the value of $\Gamma(4.5)$?

b) If λ is an eigenvalue of a matrix A, then an eigenvalue of A^t *and* A^4 is ?

c) If $x = (x_1, x_2, x_3)$ & $y = (y_1, y_2, y_3) \in \mathbb{R}^3$, then determine whether $\langle x, y\rangle$ is a real inner product for $\mathbb{R}^3$ if $\langle x, y\rangle$ defined by $\langle x, y\rangle = |x_1y_1 + x_2y_2 + x_3y_3|$.

d) If the rank of a matrix $A = \begin{bmatrix} a & -1 & 0 \\ 0 & a & -1 \\ -1 & 0 & a \end{bmatrix}$ is 2, then value of a is equal to?

e) Show that $\{(x, y, z) \in \mathbb{R}^3 : x + y - 2z = 5\}$ is not a subspace of $\mathbb{R}^3$.

f) Evaluate the integral $\int_0^\infty e^{-x}\,dx$.

g) The set of vectors $\{(1,2,2), (2,1,2), (2,2,1)\}$ is linearly dependent or independent in $\mathbb{R}^3$?

PART –II

Q 2.

a) Reduce the matrix A to the row-reduced echelon form and hence find its rank.

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ -1 & 2 & 0 & 3 \end{bmatrix}$$

b) If $A = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & -6i \\ -4 & -6i & 3 \end{bmatrix}$, then prove that A is Hermitian and iA is skew-Hermitian.

OR

Q 2.

a) Find the condition for which the system of equations has (i) unique solution (ii) no solution, and (iii) many solutions.

$$3x - 2y + z = b$$
$$5x - 8y + 9z = 3$$
$$2x + y + \lambda z = -1$$

b) Use Gauss Jordan method to find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Q3.

a) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}.$$

b) Use Gram-Schmidt process to obtain an orthonormal basis of the subspace of the Euclidean space $\mathbb{R}^4$ with standard inner product space generated by the linearly independent set $\{(1,1,,0,1),(1,1,0,0),(0,1,0,1)\}$.

OR

Q 3.

a) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix where $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.

b) Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and find A^{-1} & A^9.

Q 4.

a) Obtain the Taylor's series expansion of $f(x) = x^5 + 2x^4 - x^2 + x + 1$ about the point $x = -1$.

b) Determine the area between the curve $y = x^3$ and the parabola $y = 4x^2$.

OR

Q 4.

a) Determine the volume of the solid generated by revolving the plane area bounded by $y^2 = 4x$ and $x = 4$ about the line $x = 4$.

$\pi\left(4 - \frac{y^2}{4}\right)^2$ $\quad \frac{\pi}{16}\left(16 - y^2\right)^2$ $\quad \left(4 - \frac{y}{4}\right)$

b) Evaluate the integral $\int_0^\infty x^4 e^{-x^4}\,dx$.

$2 \int_0^4 \frac{\pi}{16} \times \frac{1}{3} \frac{(16-y^2)^3}{-2y}$

$\left(4 - \frac{y}{4}\right)^2$ $\quad 2 \frac{\pi}{16} \cdot \frac{1}{3} \frac{(16-y)^3}{-1}$ $\quad \frac{\pi}{24} \times 16 \times 16 \times 12$ $\quad 72\pi$

Q 5.

a) For a linear map $f: \mathbb{R}^3 \to \mathbb{R}^3$ define by $f(x_1, x_2, x_3) = (2x_1 + x_2 - x_3, x_2 + 4x_3, x_1 - x_2 + 3x_3)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$. Find the matrix of f relative to the ordered bases $(0,1,1), (1,0,1), (1,1,0)$ of $\mathbb{R}^3$

b) Show that the set $S = \{(1,2,-1,-2),(2,3,0,-1),(1,2,1,4),(1,3,-1,0)\}$ is a basis of $\mathbb{R}^4$.

$256 + y^4 - 32y^2$

$256y + \frac{y^5}{5} - \frac{32}{3}y^3$

OR

$1024 + \frac{1024}{5} - \frac{2048}{3}$

Q 5.

a) For a linear map $f: \mathbb{R}^3 \to \mathbb{R}^3$ define by $f(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$. Show that f is a linear mapping. Find $Ker(f), Im(f), rank(f)$ & $nullity(f)$.

b) Let $S = \{(x, y, z) \in \mathbb{R}^3 : 3x - y + z = 0\}$. Show that S is a sub-space of $\mathbb{R}^3$. Find a basis of S.