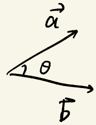
cross product.

① $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta$.



② famula.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$ ← 行列式的表达形式.

⇒ 行列式的展开是怎么样的?

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

对于三阶的行列式:

选这一行

选出一行1列, 乘上对应去掉行列的剩余行列式.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(-1)^{1+1} \begin{vmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ \cancel{a_{21}} & a_{22} & a_{23} \\ \cancel{a_{31}} & a_{32} & a_{33} \end{vmatrix} +$$

$$a_{12}(-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{33} \end{vmatrix}$$

$$\therefore \vec{a} \times \vec{b} = \vec{i}(-1)^{1+1} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + \vec{j}(-1)^{1+2} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \vec{k}(-1)^{1+3} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2)\vec{i} - (a_1b_3 - a_3b_1)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$$

⇒ 可以用矩阵的形式来展开.

$\vec{a} = (a_x \ a_y \ a_z)$

$\vec{b} = (b_x \ b_y \ b_z)$.

$$\vec{a} \times \vec{b} = [(a_yb_z - b_ya_z), -(a_xb_z - b_xa_z), (a_xb_y - b_xa_y)]$$

$$\Updownarrow$$

$$\begin{bmatrix} 0, & -a_z, & a_y \\ a_z, & 0, & -a_x \\ -a_y, & a_x, & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [(a_yb_z - a_zb_y), (a_zb_x - a_xb_z), (a_xb_y - a_yb_x)]$$