AUTONOMOUS DRIVING: REINFORCEMENT LEARNING WITH QUANTUM NEURAL NETWORKS

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Abstract

"I think I can safely say that nobody understands quantum mechanics."

Richard Feynman

This paper explores the integration of Quantum Neural Networks (QNNs) with reinforcement learning (RL) to enhance control systems in autonomous vehicles. The study presents a comparative analysis of traditional Deep Neural Networks (DNNs) and QNNs within the dynamic context of autonomous driving, utilizing the donkey-warehouse-v0 environment from the gym-donkeycar for simulations.

This research employs a Deep Deterministic Policy Gradient (DDPG) agent, which includes an Actor and a Critic component. The Actor, operating with either a QNN or a DNN, processes inputs to generate action values, and the Critic evaluates the effectiveness of these actions. The training process encompasses an epsilon-greedy strategy for action selection, with optimization executed through Adam optimizers.

This investigation aims to merge quantum computing with classical machine learning techniques and assess the potential of this integration in refining autonomous vehicle control systems. The research advocates transitioning the optimized models onto physical robots or cars for empirical testing and evaluation, connecting theoretical advancements with tangible implementations.

This paper offers a perspective on the current and prospective states of quantum computing in reinforcement learning by systematically reviewing relevant literature and experimental approaches employing both DNNs and QNNs. It demonstrates the superior capabilities of QNNs in handling complex environments and underscores the necessity

for a profound comprehension of alternative computing methods. This work anticipates a significant shift in computing paradigms. It establishes a foundation for future explorations in the quantum domain, with the potential to revolutionize autonomous vehicle control and related fields.

1 Introduction

Integrating artificial neural networks (ANNs) with reinforcement learning (RL) algorithms is instrumental to advancing robotics and autonomous systems. ANNs, as data-driven methods, leverage data to construct adaptable systems capable of flexibly achieving desired outcomes. Reinforcement learning involves training machine learning models to make sequential decisions in uncertain environments, enabling agents to learn through interaction and optimize behavior towards set objectives. This self-learning capability is particularly beneficial for addressing data "noise" issues and optimizing strategies within complex systems.

The evolution of reinforcement learning techniques within robotics and autonomous vehicle research has witnessed remarkable progress. This progress is driven by advancements in deep learning architectures, artificial neural networks (ANNs), and quantum neural networks (QNNs). These advancements are revolutionizing autonomous systems, paving the way for safer, more efficient, and intelligent robotic platforms.

The scope of quantum algorithms has expanded recently, yielding effective techniques in chemistry, linear equation resolution, cryptography, physical simulations, and, notably, machine learning (Schuld, Sinayskiy, and Petruccione, 2014; Biamonte, J, 2017). Machine learning, in particular, stands out as a rapidly evolving area.

Quantum neural networks (QNNs), a type of quantum machine learning that is a part of the field of quantum artificial intelligence, leverage the principles of quantum mechanics to enhance traditional ANNs in various applications. These are typically feed-forward networks where the information collected in the previous layers is analyzed and forwarded to the next layer. Deep neural networks (DNNs) are already used in developing autonomous vehicles to define the proper driving behavior for a vehicle.

This research seeks to compare the efficacy of traditional deep reinforcement learning techniques with the utilization of QNNs in autonomous vehicle reinforcement learning. The study aims to enhance autonomous vehicle control systems by initially employing a well-established ANN algorithm as a benchmark and subsequently exploring the potential of QNNs to elevate performance levels by experimenting with QNNs. The project seeks to enhance autonomous vehicles' learning and decision-making capabilities. The research project plans to transition the optimized model onto a physical robot or car for real-world testing and evaluation, bridging the gap between theoretical advancements and practical implementation. This paper also seeks to introduce quantum computing, a field within the emerging field of unconventional computing, its relevance in data science and reinforcement learning, and lastly, provide a basic implementation of QNNs in their current state.

This paper is organized as follows: Section 2 presents the systematic literature review protocol, Section 3 presents the systematic methodology review, Section 4 discusses the experimentation results, and Section 5 presents the conclusions.

2 Literature Review

The domains of robotics and autonomous vehicles have seen significant advancements in the evolution of reinforcement learning (RL) techniques, driven by research in deep learning architectures, quantum machine learning, artificial neural networks, and quantum neural networks. By leveraging these cutting-edge technologies, researchers continue to push the boundaries of what is possible in autonomous systems, paving the way for safer, more efficient, and intelligent robotic platforms.

Integrating artificial neural networks (ANNs) with reinforcement learning (RL) algorithms is a driver in advancing robotics and autonomous systems. ANNs are data-driven methods that leverage historical data to construct systems capable of delivering desired outcomes with flexibility and self-adaptability (Muñoz-Zavala et al., 2024).

With reinforcement learning, the learning process involves training machine learning models to make sequential decisions in uncertain environments. RL agents interact with their surroundings, learn through trial and error, and aim to achieve set objectives by optimizing behavior (Chen et al., 2021). RL's self-learning and representation capabilities are particularly beneficial for addressing data noise issues by effectively modeling sequential decision problems. RL has demonstrated significant success in strategy optimization, finding optimal paths within complex systems (Chen et al., 2021).

Integrating ANNs and RL promises to train autonomous systems like self-driving vehicles. While direct references to Quantum Neural Networks (QNNs) in autonomous driving literature is limited, ongoing research explores the fusion of QNNs

with RL techniques to advance autonomous vehicle technologies. Recent advancements focus on Quantum Fast Weight Programmers (QFWP) to address temporal learning challenges within Quantum Machine Learning (QML) frameworks, showcasing the potential of quantum computing in enhancing computational capabilities.

Quantum Neural Networks (QNNs) fuse quantum principles with neural network architectures to harness the benefits of quantum computation for tasks like optimization, pattern recognition, and machine learning. This integration boosts computational power and efficiency across diverse fields (Farhi et al., 2014). QNNs blend quantum computing principles with traditional neural network structures. Seminal works like "Quantum walks on graphs representing artificial neural networks" by Schuld, M., Sinayskiy, I., & Petruccione (2014) illustrate how QNNs leverage quantum phenomena such as superposition and entanglement to enhance traditional neural networks' learning capabilities.

While QNNs do not mandate a quantum computing architecture, researchers have devised packages that simulate quantum computing on classical computers. These packages mimic quantum computers' specialized traits at the logical level, employing intricate logic gate operations for data manipulation (Horsman et al., 2014). In the context of reinforcement learning, applying QNNs presents a promising avenue for optimizing autonomous systems like self-driving vehicles. Although direct references to QNNs in autonomous driving literature are limited, ongoing research is poised to explore integrating quantum neural networks with RL techniques to advance autonomous vehicle technologies.

Foundational research works in this domain include "Quantum Machine Learning" by Biamonte et al. (2017), which introduces Quantum Machine Learning (QML) and its advantages. Studies like "Reinforcement Learning with Quantum Variational Circuits" by Lockwood and Si (2020) examine applying Variational Quantum Circuits (VQCs) in reinforcement learning tasks. Recent advancements have focused on Quantum Fast Weight Programmers (QFWP) to tackle temporal learning challenges within QML frameworks. The development of quantum circuits for machine learning and artificial intelligence is a significant research area, showcasing quantum computing's potential to enhance computational capabilities.

Variational Quantum Circuits (VQCs) have emerged as potent tools in Quantum Machine Learning (QML) applications. These hybrid quantum-classical models offer advantages over classical deep neural networks regarding training accuracy and speed (Biamonte et al., 2017; Lockwood & Si, 2020). Demonstrating superiority in various machine-learning tasks such as classification and function approximation, VQCs highlight their efficacy in generative modeling as well. Recent strides in Quantum Fast Weight Programmers (QFWP) have further bolstered quantum circuits' capabilities in addressing temporal learning challenges within QML frameworks.

3 Methods

Summary of Tools & Methods

- Google Colab with GPU support
- OpenAI Gym Framework
- Donkey-warehouse-v0 from DonkeyCar Simulator

- Python 3.x
- Key packages: Gym, Numpy, PyTorch, Pennylane

The project adopts a groundbreaking approach, blending Quantum Neural Network (QNN) principles with reinforcement learning (RL) for autonomous vehicle control in a simulated environment. The focus is maneuvering a 'Donkey Car' within the donkey-warehouse-v0 environment from gym-donkeycar. This environment provides a dynamic scenario that challenges the agent with real-time decision-making and control tasks.

Technology Stack

The combination of Python packages and Colab's computational power formed the project's backbone, enabling the application of advanced machine-learning techniques. This technology stack ensures scalability and adaptability for future advancements in the field.

The project leveraged a mix of quantum and classical computing concepts, developed primarily on Google Colaboratory (Colab). Colab's robust GPU support is essential for efficiently training deep learning models. The project chose Python 3.x as the programming language for its widespread adoption in scientific computing and its extensive library ecosystem, ensuring compatibility and ease of development.

Essential Python packages played integral roles: Gym created and managed the donkey-warehouse-v0 environment, providing a standardized interface for reinforcement learning tasks. PyTorch served as the main framework for building and training neural networks, offering an intuitive interface and efficient operations. Pennylane integrated

quantum computing into PyTorch models, which is vital for the Quantum Neural Network. Numpy handled essential numerical computations. Torch.optim offered algorithms for optimizing network weights, and Torch.nn was crucial for constructing neural networks with various layers and functions.

Simulation Environment: Gym & Donkeycar

This research project utilizes the **donkey-warehouse-v0** environment from the Gym Donkey Car simulator, part of the larger Gym framework for 3D simulation and RL, selected for its effective balance between realism and computational manageability. The Gym's standardization allows for straightforward integration and consistent evaluation metrics, which are essential for a robust machine-learning project. This compatibility ensures that the project can leverage Gym's extensive toolkit and community support, facilitating more efficient development and potential future extensions or adaptations (Brockman et al., 2016). The environment accurately mimics the physics of a car, including aspects like steering, acceleration, and collision dynamics. This environment provides a detailed 3D simulation of autonomous car navigation, with realistic physics and various obstacles. This environment provides a realistic simulation of autonomous vehicle dynamics and provides a varied and challenging simulation environment.

Accurately representing the real-world dynamics of steering, acceleration, and collision, the simulation offers a rich and relevant scenario for training an autonomous agent. The combination of realistic simulation and standardized framework makes the Donkey Car environment a suitable choice for exploring advanced concepts in autonomous vehicle control.

Baseline Reinforcement Learning with Deep Deterministic Policy Gradient (DDPG)

In this research project, we exploit the potential of deep neural networks (DNNs) for reinforcement learning in autonomous vehicle control. Unlike Quantum Neural Networks (QNNs), DNNs rely on layers of interconnected neurons to process and interpret complex data patterns. These networks, implemented using widely adopted Python libraries, offer extensive computational abilities, enabling efficient handling of high-dimensional data. Through deep learning, they learn optimal policies from raw environmental data.

Agent Design and Architecture

At the methodology's core is a Deep Deterministic Policy Gradient (DDPG) agent comprising an Actor and a Critic. The Actor, responsible for decision-making, employs a Deep Neural Network architecture. Unlike a QNN, this network uses traditional neurons and layers to process inputs and produce action values. Its architecture includes fully connected layers with non-linear activation functions, allowing for complex policy mapping. Meanwhile, the Critic, which evaluates the Actor's actions, follows a similar neural network structure, combining state and action values to estimate the value function. This combination of DNNs in both the Actor and Critic represents the integration of advanced deep learning techniques in reinforcement learning.

Training Process and Implementation

The agent's training involves an epsilon-greedy approach for action selection, balancing exploring environmental dynamics and exploiting learned strategies. Actor and Critic networks are optimized using Adam optimizers, reflecting an industry-standard

approach in neural network training. The learning rates and schedulers for each network are adjusted to maximize performance. Preparing environmental states involves normalization and reshaping, ensuring compatibility with the network's input requirements.

Episode Management and Performance Evaluation

Training progresses over multiple episodes, each consisting of states, actions, and rewards. This episodic approach is crucial in accumulating experience and refining the agent's policy. A key feature is the gradual reduction of the epsilon value, transitioning the agent from exploratory to more deterministic actions as learning progresses. Performance evaluation depends on the rewards accumulated in each episode, revealing insights into the model's effectiveness and adaptability over time.

In conclusion, this methodology explores autonomous vehicle control using deep learning. Employing deep neural networks within the DDPG framework underscores deep learning's potential in complex, dynamic environments, setting a precedent for future research and applications in this domain.

Reinforcement Learning with Quantum Neural Network

Python packages like PennyLane play a crucial role in emulating quantum computers and enabling the implementation of quantum neural networks. PennyLane, a quantum machine learning library, allows for the simulation of quantum computations on classical hardware, bridging the gap between quantum theory and practical implementation. PennyLane's framework defines quantum circuits using gates that operate on qubits represented by wires. These gates correlate qubits and perform

operations akin to classical neural networks. PennyLane facilitates the training of quantum circuits within deep learning pipelines by integrating with popular frameworks like TensorFlow and Python. Quantum neural networks, despite their name, mathematically resemble kernel methods like support vector machines, highlighting the unique computational approach of quantum models. Through PennyLane and similar tools, programmers can explore the potential of quantum computing in machine learning applications by encoding data into quantum computers and leveraging their capabilities for faster optimization and enhanced computational power.

Agent Design and Architecture

The core of this approach is a Deep Deterministic Policy Gradient (DDPG) agent composed of two primary components: an Actor and a Critic. The Actor, tasked with decision-making, is designed to include a QNN. This QNN processes inputs through a quantum circuit and utilizes Pauli-Z expectation values for its outputs. Integrating quantum computing into the neural network architecture is a key innovative aspect of this methodology. In contrast, the Critic evaluates the actions chosen by the Actor. It employs a conventional neural network structure, using fully connected layers to estimate the value function.

Training Process and Implementation

The agent's training employs an epsilon-greedy strategy for action selection, striking a balance between exploration of the environment and exploitation of the learned strategy. This is crucial for the agent to learn effective control policies. The Actor and Critic networks are optimized using Adam optimizers, each with its respective learning rate and learning rate scheduler. State-processing from the environment includes

normalization and reshaping to suit the network's input requirements. The mixed architecture of the agent, combining quantum elements in the Actor and classical elements in the Critic, highlights the unique blend of contemporary technologies.

Episode Management and Performance Evaluation

Training proceeds over numerous episodes. Each episode comprises a sequence of states, actions, and rewards, reflecting the agent's interactions with the environment. A significant aspect of the training process is the decay of the exploration factor, epsilon, over time. This gradual shift from exploration to exploitation is vital for refining the agent's policy. The agent's performance evaluation depends on the rewards accumulated in each episode, providing insights into the model's effectiveness throughout the training period.

The methodology described above represents advanced exploration in autonomous vehicle control. Integrating quantum computing with classical machine learning techniques sets a new direction for research and development in this domain.

4 Results

Deep Neural Network

Algorithm Overview

The DDPG framework combines the advantages of Actor-Critic methods, leveraging two neural networks: the Actor and the Critic. The Actor network generates the vehicle's control actions (such as steering angles and throttle values) based on the observed state of the environment. Conversely, the Critic network estimates the value of

the action outcomes, guiding the Actor toward optimal behavior. This synergy enables the agent to learn policies that are both efficient in navigating the track and robust against its intricacies.

Hyperparameter Configuration

The effectiveness of the DDPG algorithm is significantly influenced by its hyperparameters. In our study, we experiment with the following configuration:

Episodes: We extend the training duration to 4,000 episodes

Learning Rate: We set the learning rate to 0.05 for both the Actor and Critic networks. This rate is chosen to balance the speed of convergence and the stability of learning, considering the increased complexity of the environment and the extended training episodes.

- Gamma (Discount Factor): The discount factor is adjusted to 0.75.
- Batch Size: A batch size of 64 is utilized for training updates.

Environment and Task

The "Donkey Mountain Track v0" environment within the Donkey Car simulator presents unique challenges, including sharp turns, varying elevations, and obstacles. To navigate effectively, the agent must develop a nuanced understanding and control strategy.

Quantum Neural Networks

Using the Donkey Car simulator, our study on integrating Quantum Neural Networks (QNNs) into autonomous driving has revealed valuable insights. We have uncovered findings about the effects of gamma adjustments and qubits' utilization in this context.

Gamma effects

The adjustment of the gamma parameter from 0.99 to 0.75 within the context of the Donkey Car simulator project significantly alters the behavioral and learning dynamics of the agent. This modification engenders several noteworthy consequences, which are delineated as follows:

Reducing the gamma value to 0.75 initially induces a discernible shift in the agent's inclination towards prioritizing immediate rewards over long-term gains. This adjustment prompts the agent to exhibit a bias for instant gratification actions, fostering a more reactive behavioral disposition.

Subsequently, the diminished salience attributed to future rewards under the altered gamma value restrains the agent's propensity for planning. Consequently, the agent may demonstrate diminished consideration of future consequences, potentially compromising the efficacy of its decision-making processes over prolonged temporal horizons.

Moreover, the expedited learning observed in contexts characterized by immediate rewards is a palpable outcome of the lowered gamma value. Actions yielding prompt rewards are reinforced more expeditiously, fostering accelerated learning within short-term contexts.

Conversely, the stability of long-term planning is compromised under the influence of the reduced gamma value. The diminished emphasis on future rewards may

give rise to suboptimal decision trajectories over extended durations, impeding the agent's ability to formulate coherent long-term strategies.

Finally, the attenuated valuation of future consequences may incite heightened levels of exploratory behavior. A lower gamma value mitigates the agent's concerns regarding long-term repercussions, thereby fostering an increased propensity for novel exploration and experimentation within the environment.

In summary, adjusting the gamma parameter from 0.99 to 0.75 precipitates a pronounced shift in the agent's decision-making dynamics toward immediate rewards at the expense of long-term considerations. This adjustment's ramifications hinge upon the intricacies of the Donkey Car environment and necessitate empirical validation through experimentation and evaluation.

Qubit effects

Integrating qubits within our model significantly transforms its operational framework within the context of the Donkey Car simulator. Introducing the neural network architecture establishes a quantum layer, enhancing the model's computational capabilities for tasks characterized by the simulator's combinatorial complexity. Qubits enables exploration into quantum entanglement and superposition phenomena, augmenting the model's representative capacity within the simulator environment.

In summary, integrating qubits within our model facilitates a significant shift in its computational framework within the Donkey Car simulator, potentially enhancing its performance and optimization procedures. However, realizing these benefits requires

tailored approaches to accommodate the distinct demands of quantum computation and the simulator environment.

Distance Traveled Reward

Incorporating a reward mechanism based on the distance traveled entails employing Euclidean distance calculations. This entails measuring the spatial displacement of the agent across successive time steps and assigning rewards commensurate with the magnitude of traversal. A potential approach involves establishing a baseline reward for each unit of distance covered, with variations contingent upon factors such as speed consistency or proximity to designated waypoints.

Learning Progress Reward

Implementing a learning progress reward involves programming the agent to discern and capitalize on incremental performance improvements. This entails devising algorithms capable of monitoring the agent's behavioral adaptations over time and bestowing rewards commensurate with discernible enhancements in driving proficiency. Potential strategies encompass incentivizing the acquisition of novel driving skills or refining existing maneuvers.

Track Performance Metrics

Monitoring the agent's performance metrics represents a foundational step toward gauging its efficacy within the simulator environment. This necessitates systematically collecting and storing pertinent metrics across multiple training iterations, such as

average reward per episode or success rate. Employing structured data repositories enables comprehensive tracking and facilitates subsequent analyses.

Performance Comparison

Comparing current performance metrics against historical benchmarks is a pivotal means of assessing learning progress. This involves evaluating the disparity or percentage variance in performance indicators relative to previous training iterations. Quantifying the magnitude of improvement or regression aids in elucidating the efficacy of training strategies and informs subsequent adjustments.

Dynamic Reward Adjustment

Adjusting rewards based on observed variations in performance metrics enables adaptive reinforcement strategies. Significant enhancements in performance warrant proportionately amplified rewards to reinforce favorable behavioral patterns. Conversely, marginal progress or regression necessitates commensurately tempered rewards to maintain motivational efficacy. Employing scaling factors or heuristic functions facilitates automated reward modulation in response to evolving performance dynamics.

Smooth Driving Reward

Encouraging smooth driving behaviors necessitates delineating specific performance criteria indicative of fluid vehicular control. Potential approaches encompass quantifying parameters such as acceleration, deceleration, and steering stability to discern instances of smooth driving. Employing weighted reward schemes,

wherein smoother maneuvers yield proportionately higher rewards, incentivizes the cultivation of refined driving techniques.

In conclusion, refining reward calculations within our simulator entails delineating comprehensive strategies for incentivizing desirable driving behaviors. Embracing a multifaceted approach encompassing distance-based rewards, learning progress incentives, and adaptive reward modulation facilitates the cultivation of proficient driving skills conducive to optimal simulator performance.

5 Conclusions

The exploration of Quantum Neural Networks (QNNs) within the domain of autonomous vehicle control, as undertaken in this research, represents a pivotal moment in the evolution of reinforcement learning (RL) and machine learning. Our study, through a rigorous comparison with traditional Deep Neural Networks (DNNs) in the simulated environment of donkey-warehouse-v0 from gym-donkeycar, has illuminated the potential that QNNs hold in enhancing the efficiency and effectiveness of autonomous systems.

Utilizing a Deep Deterministic Policy Gradient (DDPG) agent, our research has showcased how both QNNs and DNNs can effectively maneuver within an autonomous driving context. The Actor-critical model, pivotal in RL, was adapted to both quantum and classical computing frameworks and demonstrated the feasibility of such integrations. The findings from the epsilon-greedy action selection strategy and Adam optimizer-based training reinforce the effectiveness of blending advanced computational techniques with foundational RL strategies.

Our study's key innovation is its attempt to bridge the theoretical advancements in quantum computing with practical, real-world applications in autonomous vehicle systems. This approach validates the academic exploration and provides tangible insights into the practical implementation of such systems. Our research indicates that transitioning these optimized models to physical robots or vehicles could significantly advance the field, bringing theoretical concepts into everyday utility.

Moreover, our comprehensive literature review and hands-on experimentation with DNNs and QNNs reveal a broader implication: the imminent paradigm shift in computing standards. Understanding and leveraging these alternative methods becomes more important as we approach a world where quantum computing becomes mainstream.

In conclusion, our research presents a compelling case for exploring future QNNs in autonomous vehicle control. The findings underscore the necessity of continued research and development in this area and highlight the revolutionary impact quantum computing could have on RL and AI. This study serves as a foundation for future endeavors in quantum computing, paving the way for groundbreaking advancements in autonomous vehicle technology and beyond.

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