Lecture-2

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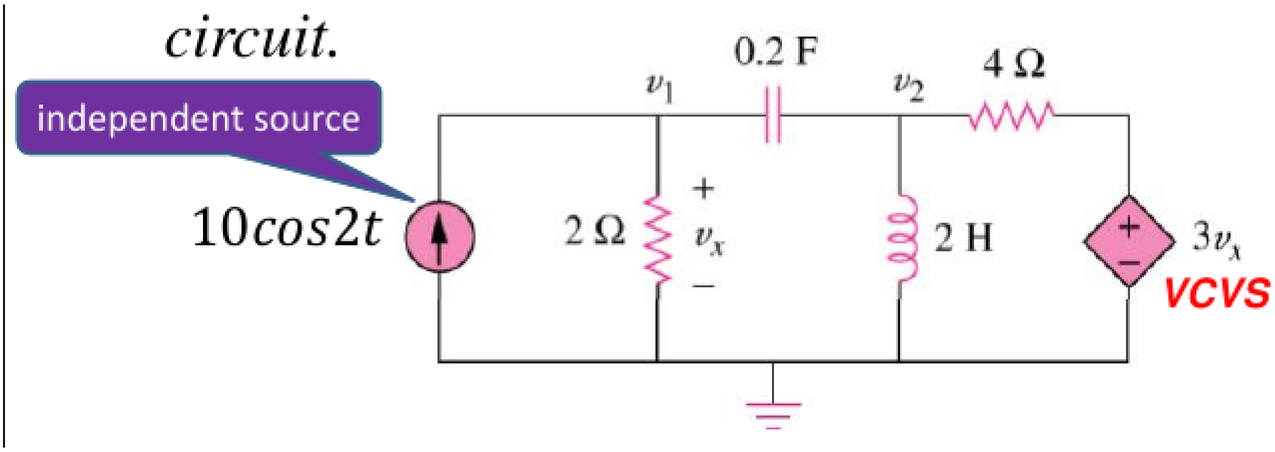
Node Analysis

- define node voltage
- define node current ($I_{in} = I_{out}$)

EX 10-1

Using nodal analysis, find v_1 and v_2 in the following circuit



$$\begin{cases} 10 = \frac{V_1 - V_2}{-2.5j} + \frac{V_1}{2} \\ \frac{V_2}{4j} = \frac{V_1 - V_2}{-2.5j} + \frac{3V_1 - V_2}{4} \end{cases} \implies \begin{cases} (-2 + 2.5j)V_1 + 2V_2 = 50j \\ (-4 + 7.5j)V_1 + (1.5 - 2.5j)V_2 = 0 \end{cases}$$

solve the equations by using Carmer's Rule

Consider a system of *n* linear equations for *n* unknowns, represented in matrix multiplication form as follows:

$$Ax = b$$

where the $n \times n$ matrix A has a nonzero determinant, and the vector $x = (x_1, \ldots, x_n)^\mathsf{T}$ is the column vector of the variables. Then the theorem states that in this case the system has a unique solution, whose individual values for the unknowns are given by:

$$x_i = \frac{\det(A_i)}{\det(A)} \qquad i = 1, \ldots, n$$

where A_i is the matrix formed by replacing the i-th column of A by the column vector b.

$$\det A = \begin{vmatrix} -2+2.5j & 2 \\ -4+7.5j & 1.5-2.5j \end{vmatrix} = 12.87\angle{-29.05^\circ}$$

$$\det A_1 = \begin{vmatrix} 50j & 2 \\ 0 & 1.5-2.5j \end{vmatrix} = 145.77\angle 30.96^\circ$$

$$\det A_2 = \begin{vmatrix} -2+2.5j & 50j \\ -4+7.5j & 0 \end{vmatrix} = 425\angle 28.07^\circ$$

$$V_1 = \frac{\det A_1}{\det A} = 11.33\angle 60.01^\circ V$$

$$V_2 = \frac{\det A_2}{\det A} = 33.02\angle 57.12^\circ V$$

$$v_1 = 11.33\cos(2t + \angle 60.01^\circ)\ V$$

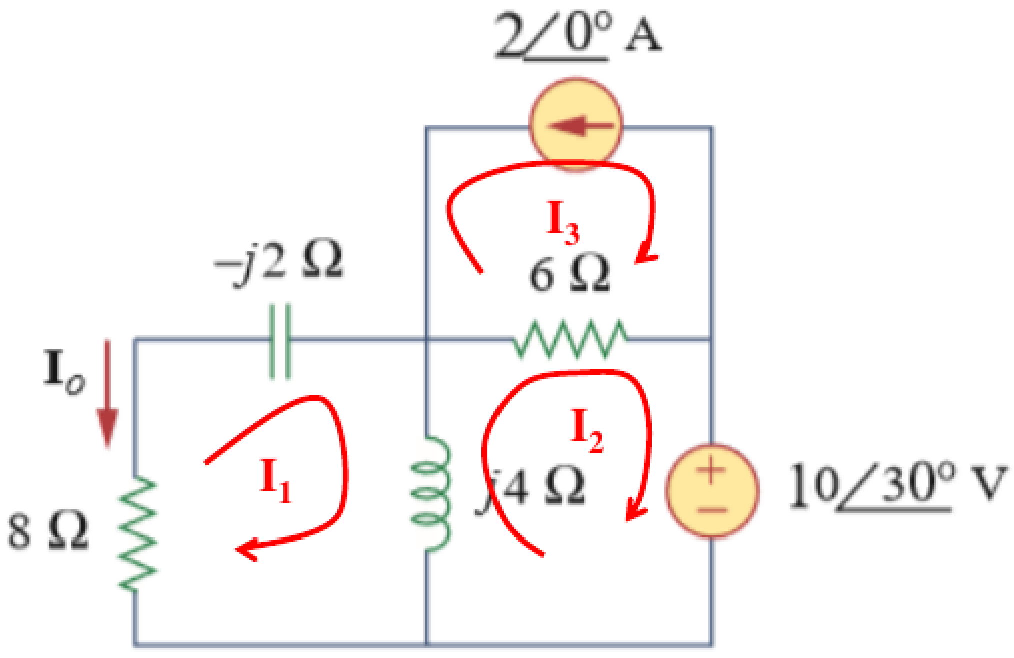
$$v_2 = 33.02\cos(2t + \angle 57.12^\circ)\ V$$

Mesh Analysis

mesh analysis also works well for phasors

EX 10-2

Finding I_0 using mesh analysis



$$\begin{cases} -2j \times I_1 + 4j \times (I_1 - I_2) + 8 \times I_1 = 0 \\ 4j \times (I_2 - I_1) + 6(I_2 + I_3) = -10\angle 30^\circ \implies I_1 = 1.194\angle -114.55^\circ \\ I_3 = 2 \end{cases}$$

$$I_0 = -I_1 = 1.194\angle 65.45^\circ$$

Superposition Theorem

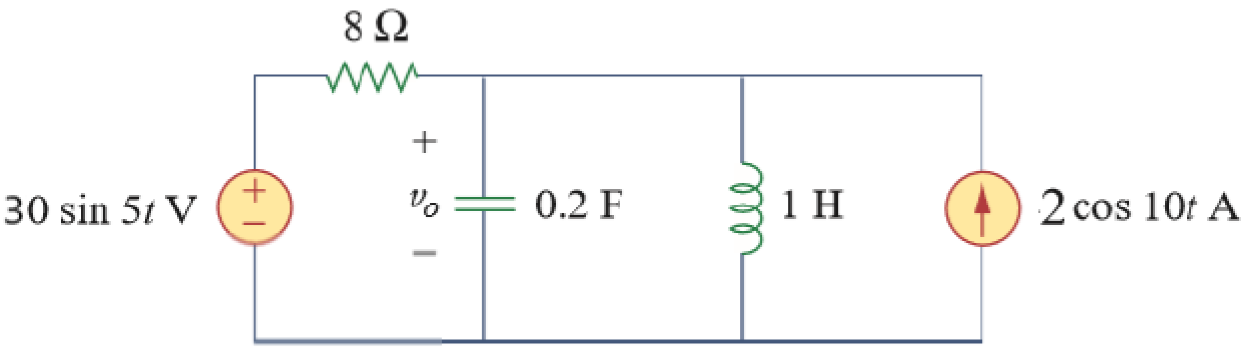
since as circuits are **linear**, the superposition applies to ac circuits the same way it applies to dc circuits

this theorem becomes important if the circuit has sources operating at different frequencies

the total response must be obtained by adding the individual responses in the time domain

EX10-3

Using superposition theorem to calculate v_0



Considering the circuit without voltage source

$$V_1 = 2 \times \frac{1}{\frac{1}{8} + \frac{1}{-0.5j} + \frac{1}{10j}} = 1.05\angle -86.24^\circ$$
$$v_1 = 1.05\cos(10t - 86.24^\circ)$$

Considering the circuit without current source

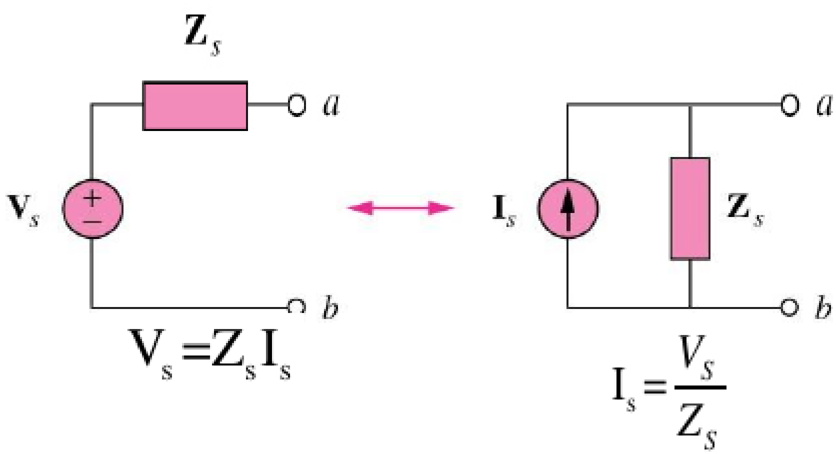
$$V_2 = 30 \times \frac{-1.25j}{8 - 1.25j} = 4.63\angle -81.12^\circ$$
$$v_2 = 4.63\sin(5t - 81.12^\circ)$$

Therefore the voltage is $1.05\cos(10t - 86.24^\circ) + 4.63\sin(5t - 81.12^\circ)$

Source Transformation

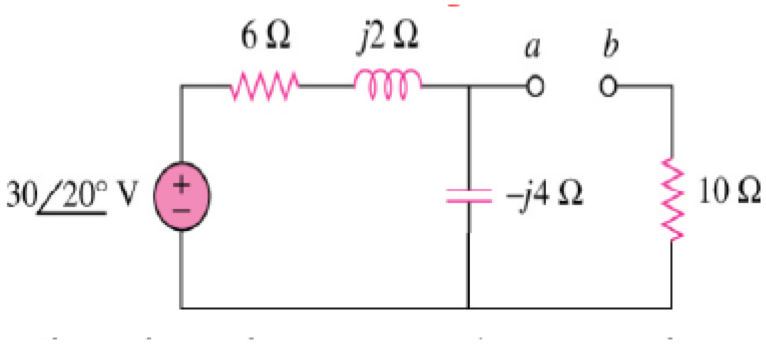
Thevenin and Norton Equivalent Circuits

- A linear circuit is replaced by a voltage source in series in series with an impedance
- A linear circuit is replaced by a current source in parallel with an impedance



EX10-4

Find the Thevenin equivalent at terminals a-b in the following circuit



$$Z_{th} = 10 + (-4j)||(6+2j)$$

$$= 12.81\angle -14.47$$

$$V_{ab} = \frac{-4j}{6-2j} \times 30\angle 20^{\circ} = 18.97\angle -51.57^{\circ}$$