Lecture-4

1 - Transfer Function

Definition

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

the transfer function $\mathbf{H}(\omega)$ of a circuit is the frequency-dependent ratio of a phasor output $\mathbf{Y}(\omega)$ to a phasor input $\mathbf{X}(\omega)$

since the input and output can be either voltage or current at any place in the circuit, there are four possible transfer functions

$$H(\omega) = \text{Voltage Gain} = \frac{V_O(\omega)}{V_i(\omega)}$$
$$H(\omega) = \text{Current Gain} = \frac{I_O(\omega)}{I_i(\omega)}$$
$$H(\omega) = \text{Transfer Impedance} = \frac{V_O(\omega)}{I_i(\omega)}$$
$$H(\omega) = \text{Transfer Admittance} = \frac{I_O(\omega)}{V_i(\omega)}$$

Zeros and Poles

the transfer function $\mathbf{H}(\omega)$ can also be expressed in terms of its numerator polynomial $\mathbf{N}(\omega)$ and denominator polynomial $\mathbf{D}(\omega)$

$$\mathbf{H}(\omega) = \frac{\mathbf{N}(\omega)}{\mathbf{D}(\omega)}$$

- **zeros**: the *roots* of the numerator polynomial $\mathbf{N}(\omega)$
- **poles**: the *roots* of the denominator polynomial $\mathbf{D}(\omega)$

2 - Decibel Scale and Bode Plots

Decibel Scale

$$\begin{aligned} G_{dB} &= 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2/R_2}{V_1^2/R_1} \\ &= 10\log_{10}\left(\frac{V_2}{V_1}\right)^2 - 10\log_{10}\frac{R_1}{R_2} \\ &= 20\log_{10}\frac{V_1}{V_2} - 10\log_{10}\frac{R_1}{R_2} \end{aligned}$$

for the case when $R_1 = R_2$, we can get the basic equation

$$G_{dB} = 20\log_{10}\frac{V_1}{V_2}$$

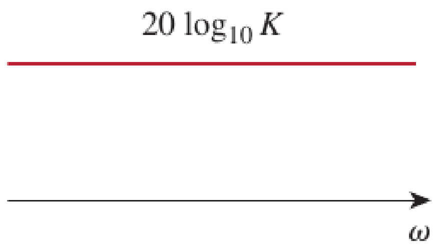
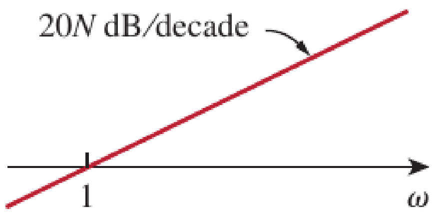
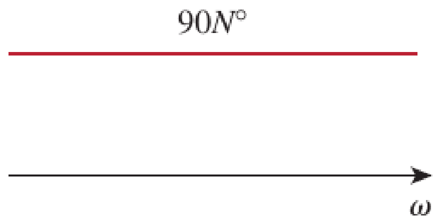
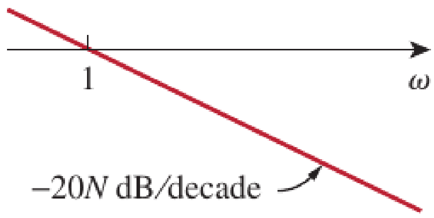
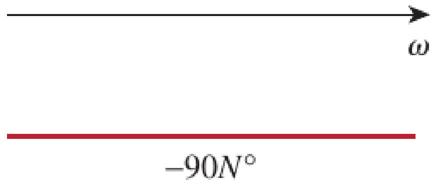
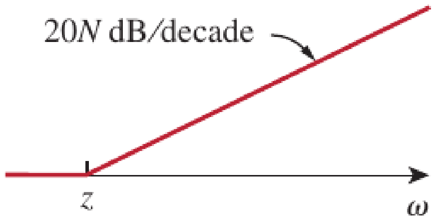
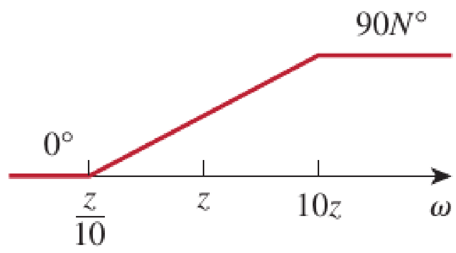
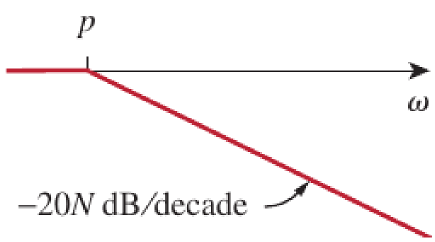
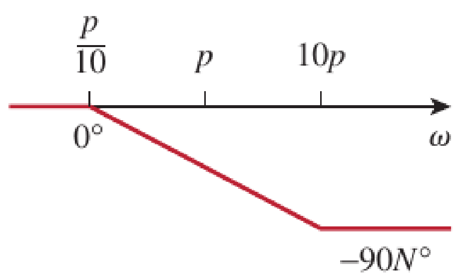
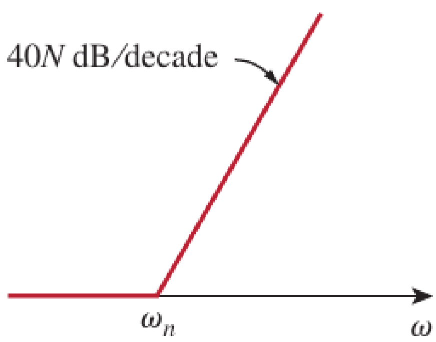
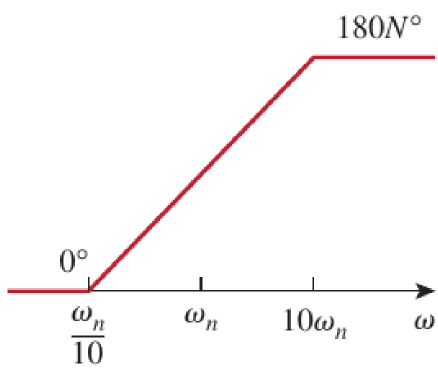
Bode Plots

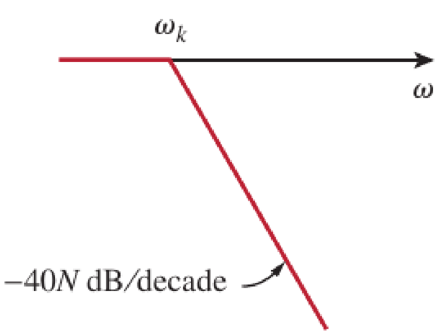
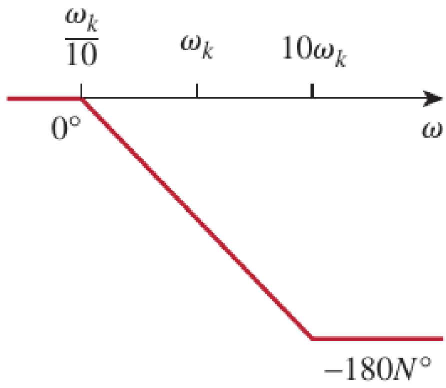
one transfer function could be written in terms of factors that have real and imaginary parts

$$\mathbf{H}(j\omega) = \frac{K(j\omega)^{\pm1}(1 + j\omega/z_1)[1 + j2\zeta\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

which combines several types of factors:

- gain K
- zero $(j\omega)$ or pole $(j\omega)^{-1}$ at the origin
- simple zero $(1 + j\omega/z_1)$ or pole $1/(1 + j\omega/p_1)$
- quadratic zero $[1 + j2\zeta_2\omega/\omega_k + (j\omega/\omega_k)^2]$ or pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$

Factor	Magnitude	Phase
K		
$(j\omega)^N$		
$\frac{1}{(j\omega)^N}$		
$\left(1+\frac{j\omega}{z}\right)^N$		
$\frac{1}{(1+j\omega/p)^N}$		
$\left[1+\frac{2j\omega\zeta}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]^N$		

Factor	Magnitude	Phase
$\frac{1}{[1+2j\omega\zeta/\omega_k+(j\omega/\omega_k)^2]^N}$		

3 - Power Considerations

Instantaneous Power

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

where we can see that the first part is constant while the second part is periodic

Average Power

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Maximum Average Power Transfer

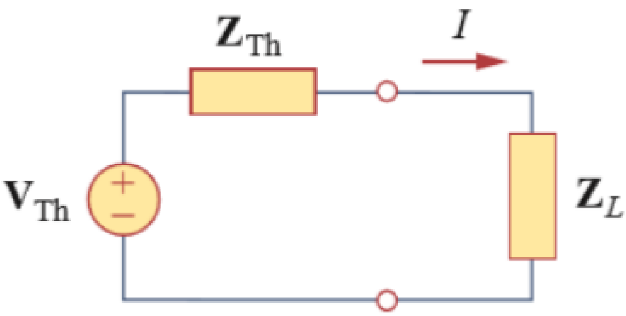


Fig. Circuit with a load and its Thevenin equivalent.

$$P = \frac{1}{2}|I|^2 R_L = \frac{|V_{Th}|^2 R_L/2}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

How to adjust the load parameters R_L and X_L so that **P is maximum**

$$\begin{cases} X_L = -X_{Th} \\ R_L = \sqrt{R_{Th}^2 + (X_{Th}+X_L)^2} \end{cases}$$

Therefore, $Z_L = R_{Th} - jX_{Th} = Z_{Th}^*$

4 - Series & Parallel Resonance

Series Resonance

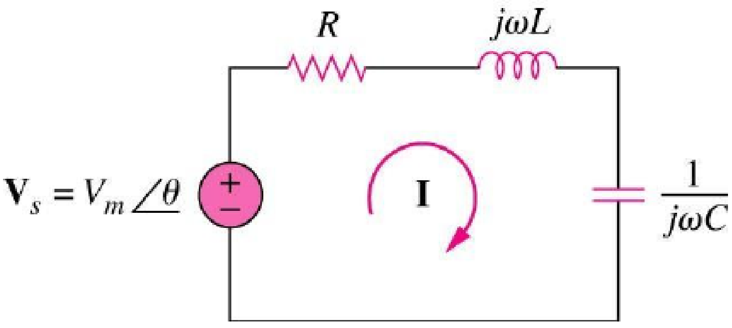


Fig. The series resonant circuit.

$$Z = R + j(\omega L - \frac{1}{\omega C})$$

Resonance is a condition in an RLC circuit in which the **capacitive and inductive reactances are equal in magnitude** thereby resulting in **a purely resistive impedance**

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Bandwidth

Half-power frequencies ω_1 and ω_2 are frequencies at which the dissipated power is **half the maximum value**

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

where we can find

$$\omega_0 = \sqrt{\omega_1\omega_2} \qquad B = \omega_2 - \omega_1$$

Quality Factor

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

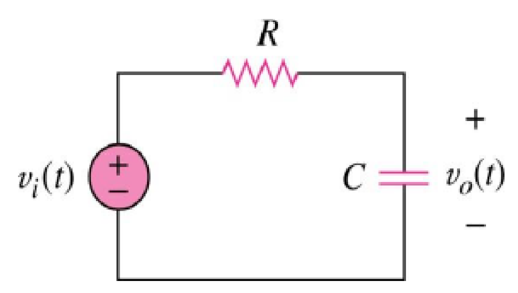
and the relationship between the B, Q and ω_0

$$B = \frac{R}{L} = \frac{\omega_0}{Q} = \omega_0^2 CR$$

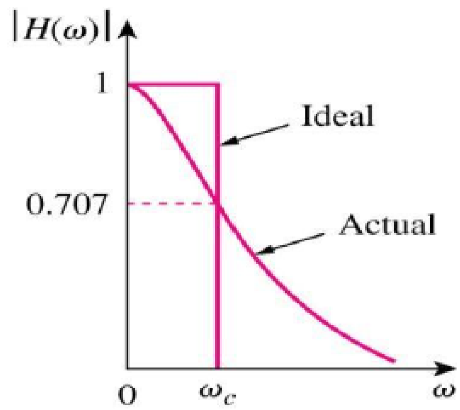
5 - Passive & Active Filters

Passive Filters

1 - LPF



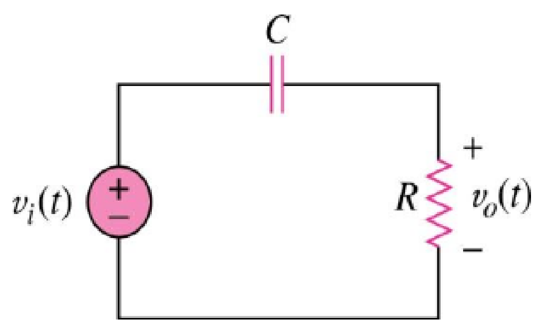
Circuit



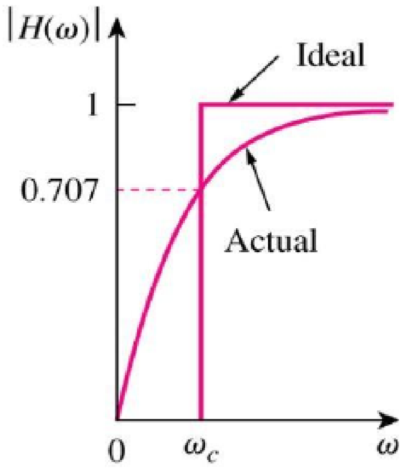
Frequency Response

$$\omega_C = \frac{1}{RC}$$

2 - HPF



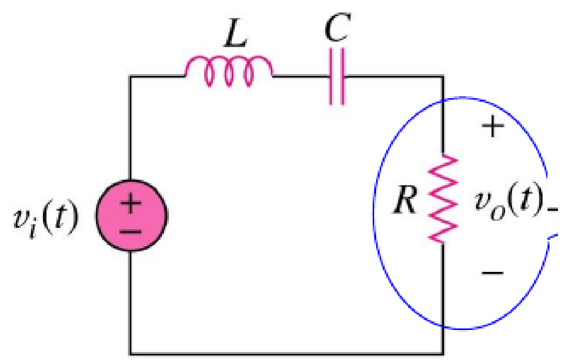
Circuit



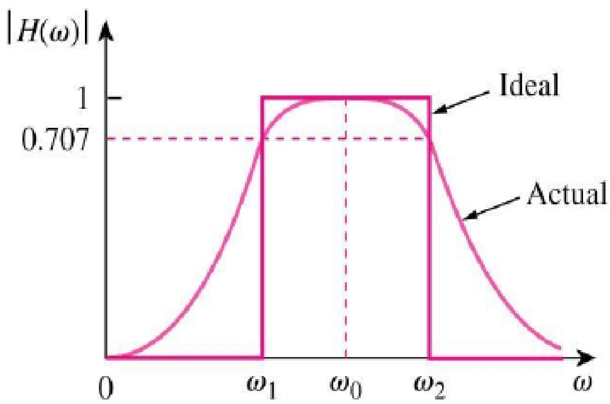
Frequency Response

$$\omega_C = \frac{1}{RC}$$

3 - BPF



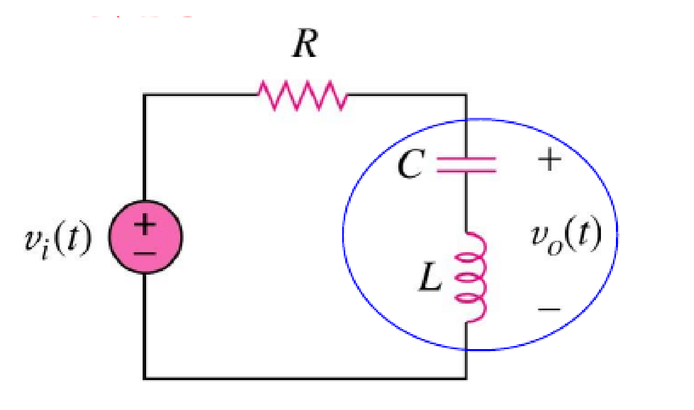
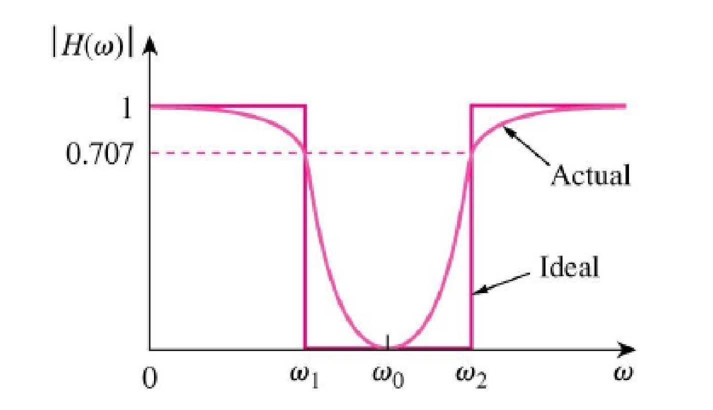
Circuit



Frequency Response

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

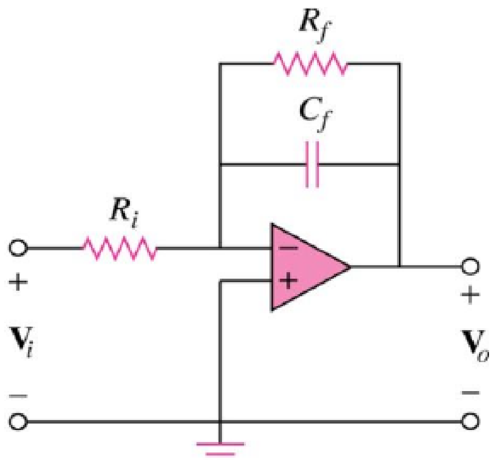
4 - BSF

	
Circuit	Frequency Response

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

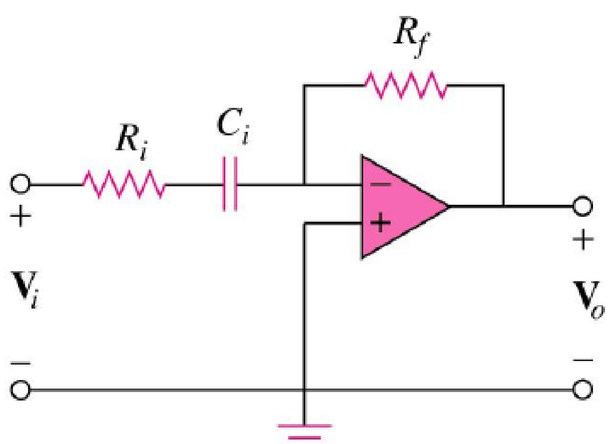
Active Filters

1 - First-order Low-pass Filter



$$\omega_C = \frac{1}{R_f C_f}$$

2 - First-order High-pass Filter

> 
>
>
> Fig. first-order high pass filter

$$\omega_C = \frac{1}{R_i C_i}$$