Lecture-1

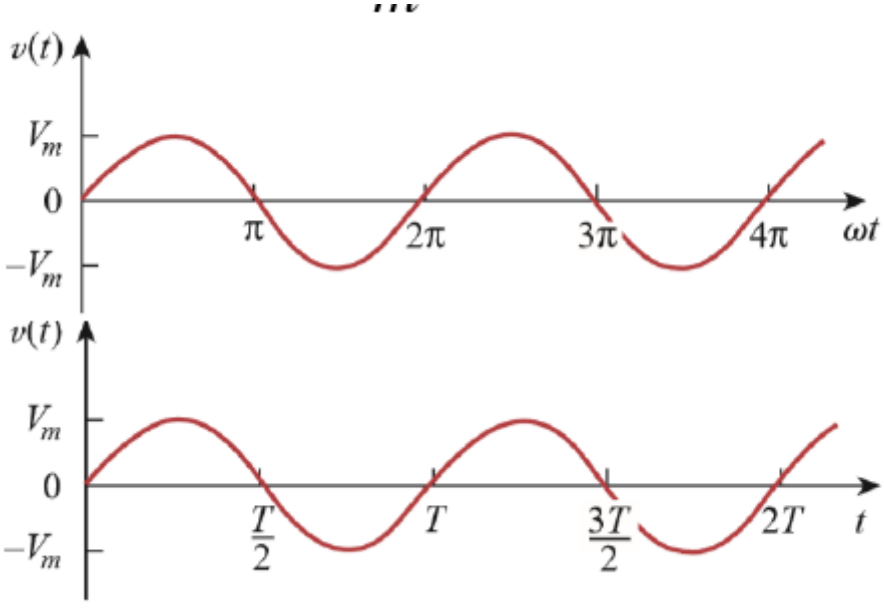
1-Intro

A **sinusoid** is a signal that has the form of the sine or cosine function

2-Sinusoids

Definition

$$v(t) = V_m \sin \omega t$$



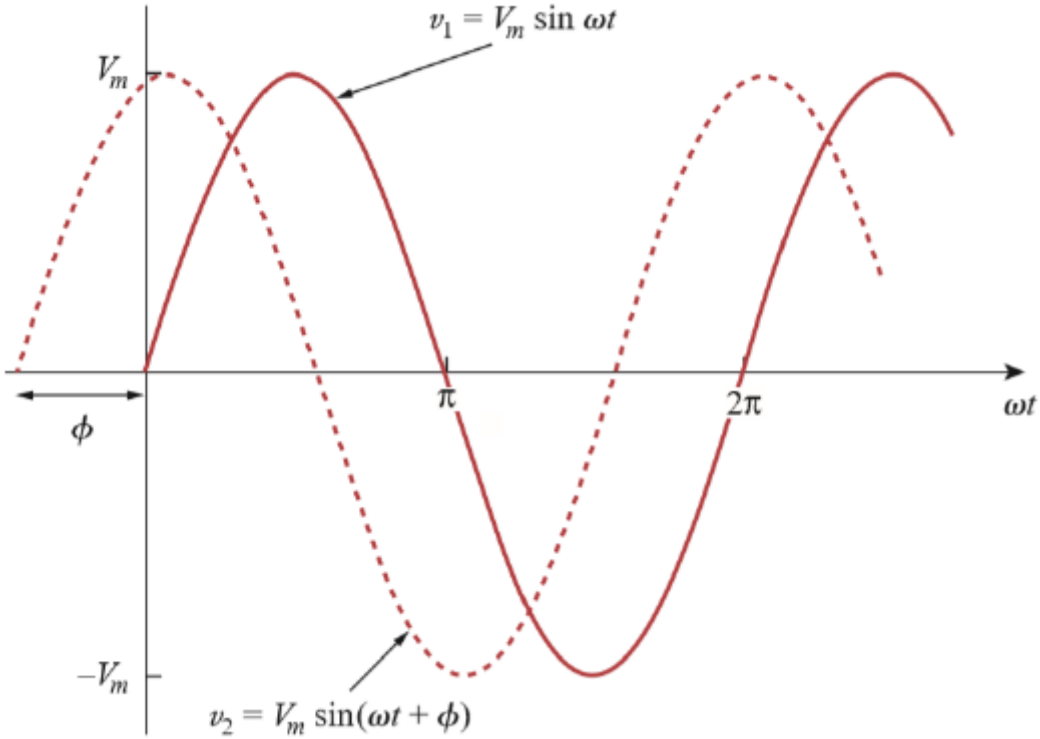
- $\omega = \frac{2\pi}{T}$

- $T = \frac{2\pi}{\omega}$
- $f = \frac{1}{T}$
- $\omega = 2\pi f$

sinusoids are periodic, which implies $v(t+T) = v(t)$

Phase

$$v_1(t) = V_m \sin \omega t v_2(t) = V_m \sin(\omega t + \phi)$$



where ϕ is the **phase**

Combining Sinusoids

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \cos(\omega t - \theta)$$

where $\theta = \arctan \frac{B}{A}$

3-Phasors

Definition

A **phasor** is a complex number that represents the amplitude and phase of a sinusoid

$$z = x + jy = r\angle\phi = re^{j\phi} = r(\cos\phi + j\sin\phi)$$

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

Calculation

$$\frac{\mathrm{d}v}{\mathrm{d}t} \longleftrightarrow j\omega V$$

$$\int v\mathrm{d}t \longleftrightarrow \frac{V}{j\omega}$$

Phasors and Circuit Elements

Inductor

if the current in the inductor is $i(t) = I_m \cos(\omega t + \phi)$ then the voltage across it will be

$$\begin{aligned} v = L\frac{\mathrm{d}i}{\mathrm{d}t} &= -\omega L I_m \sin(\omega t + \phi) \\ &= \omega L I_m \cos(\omega t + \phi + 90^\circ) \end{aligned}$$

transforming as a phasor and take $I_m e^{j\phi}$ as I

$$V = \omega L I_m e^{j(\phi + 90^\circ)} = j\omega L I$$

Conductor

similarly since the current on the conductor is $C\frac{\mathrm{d}v}{\mathrm{d}t}$

$$V = \frac{I}{j\omega C}$$