CH-1

1 - Continuous Time Signal and System

Useful Signals

Unit Step Function

$$u(t-t_0)=\begin{cases}0 & t<t_0\\ 1 & t>t_0\end{cases}$$

Unit Impulse Function

$$\int_{-\infty}^{\infty}\delta(t)\mathrm{d}t=1$$

- sampling property:
  - $$\begin{cases}\delta(t)=x(0)\delta(t)\\ \int x(t)\delta(t)=x(0)\end{cases}$$
  - $$\begin{cases}x(t)\delta(t-t_0)=x(0)\delta(t-t_0)\\ \int x(t)\delta(t-t_0)=x(t_0)\end{cases}$$

- differentiation and integration
  - $$\begin{cases} u(t) = \int_{-\infty}^{t} \delta(\tau)\mathrm{d}\tau \\ \delta(t) = \frac{\mathrm{d}u(t)}{\mathrm{d}t} \end{cases}$$

2 - Fourier Transformation

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\mathrm{d}t \qquad f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{jn\omega t}\mathrm{d}\omega$$

Useful Fourier Transform Pair

Aperiodic Signal	Fourier Signal	Signal	Fourier Transform	Fourier series coefficients
$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$	a_k
$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$a_1 = 1\ a_k = 0$
$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$a_1 = a_{-1} = 1/2\ a_k = 0$
$x^*(t)$	$X^*(-j\omega)$	$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$a_1 = -a_{-1} = 1/2j\ a_k = 0$
$x(-t)$	$X(-j\omega)$	$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1\ a_k = 0$
$x(at)$	$\frac{1}{\|a\|}X(\frac{j\omega}{a})$	$x(t) = \begin{cases} 1 & \|t\| < T_1 \\ 0 & \|t\| > T_1 \end{cases}$	$\frac{2\sin\omega T_1}{\omega}$	
$x(t) * y(t)$	$X(j\omega)Y(j\omega)$	$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1 & \|\omega\| < W \\ 0 & \|\omega\| > W \end{cases}$	
$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$	$\delta(t)$	1	

Aperiodic Signal	Fourier Signal	Signal	Fourier Transform	Fourier series coefficients
$\frac{\mathrm{d}}{\mathrm{d}t}x(t)$	$j\omega X(j\omega)$	$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$	
$\int_{-\infty}^{t}x(t)\mathrm{d}t$	$\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$	$\delta(t-t_0)$	$e^{j\omega t_0}$	
$tx(t)$	$j\frac{\mathrm{d}}{\mathrm{d}\omega}X(j\omega)$	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	
$X(jt)$	$2\pi x(-\omega)$	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	

Energy Theorem

$$W=\int_{-\infty}^{\infty}|x(t)|^2\mathrm{d}t=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|X(j\omega)\right|^2\mathrm{d}\omega$$

3 - Discrete Linear Displacement Invariant System

Species

- FIR: finite impulse response system
- IIR: infinite impulse response system

Linear Property

$$x_1[n]\rightarrow y_1[n]\qquad x_2[n]\rightarrow y_2[n]$$

$$ax_1[n]+bx_2[n]\longrightarrow ay_1[n]+by_2[n]$$

Displacement Invariant Property

$$x[n]\rightarrow y[n]$$

$$x[n-n_0]\rightarrow y[n-n_0]$$

Convolution

$$y[n]=x[n]h[n]$$

$$=\sum_{k=-\infty}^{\infty}x[n]h[n-k]$$

4 - Sampling

Normalized Frequency

the signal with an analog frequency of F is sampled at a rate of F_s

then the relationship between the **normalized frequency** f and the **analog frequency** F is

$$f = \frac{F}{F_s}$$

Sampling Theorem

- If maximum frequency of the signal is known to be F_{max}, the **minimum sampling rate** would be

$$\text{Nyquist Rate} = 2 \times F_{max}$$

- If the sampling rate is know to be F_s, the maximum frequency in the signal must not exceed

$$\text{Nyquist Rate} = \frac{1}{2} F_S$$